Mechanical Measurement Systems
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Lecture - 22
Second Order System - Impulse and Frequency Responses

So today, we will be covering second order system impulse response second order system frequency response.

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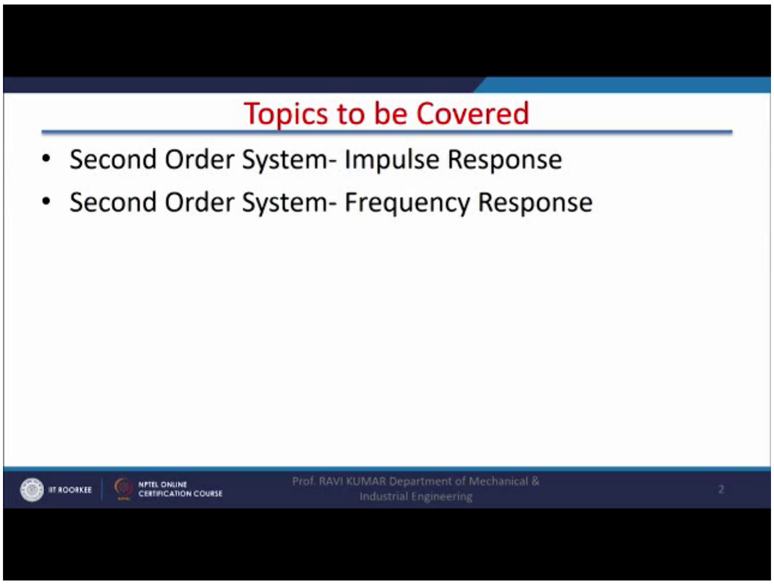

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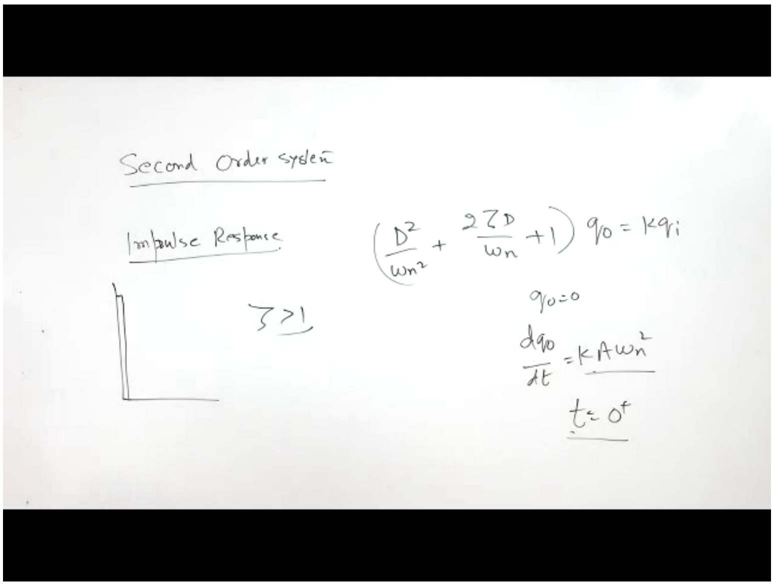

So, first of all we will start with impulse response of response of second order system.

Now, as we read in the first order system, we considered that there is input for a infinitesimally short time and after that the system response or system works on unloaded conditions, there is no imposed input on the system and then we analyze the response of the system.

Now, here in this case also, we will start with the governing equation for second order system that is D square by omega n square plus 2 zeta D by omega n plus 1 qo is equal to K q I.

The governing equation is same for all the cases only boundary condition will change and boundary condition is q o is equal to 0 and b qo by dt is equal to K A omega n square because a term has come a term has come due to area of that triangle which we have taken into account while doing the first order analysis.

So, the area and that is representing the energy input that is coming into the picture and this is at p is equal to just after t t is equal to 0 plus and this is in this case with the these boundary conditions, if we analyze the system for the cases, let us say over damped when zeta is equal to greater than 1 when the when there is a over damped system, then the output or the response of the system is going to be qo by qo by ka omega n and I will write on the board it is going to be when the system is over damped.

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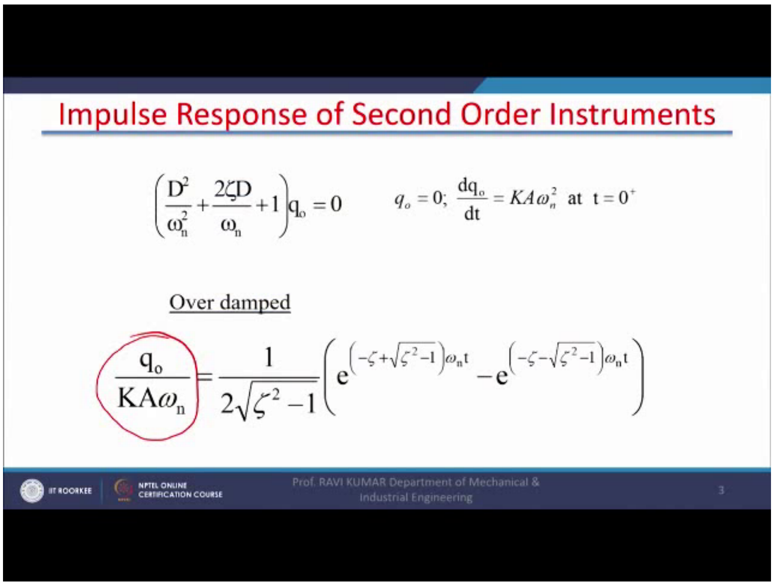

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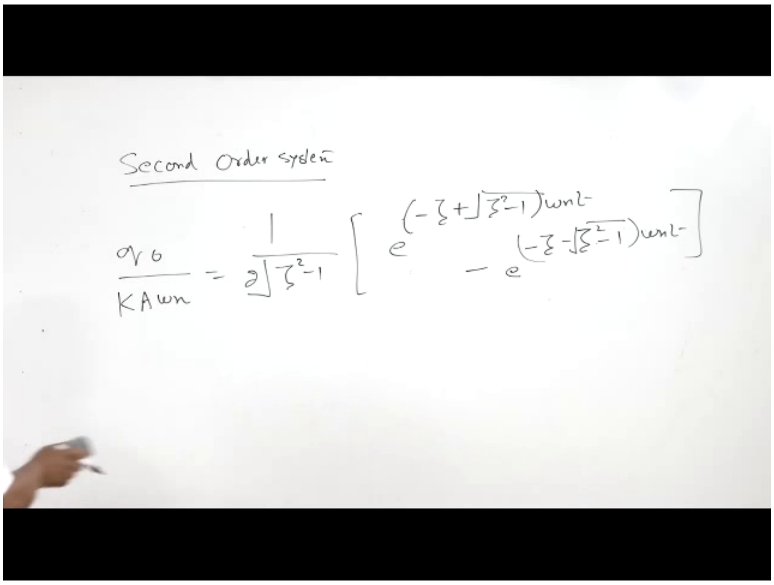

Qo by K A omega n is going to be equal to 2 times under root zeta square minus 1 plus we will take the roots e raise to power minus zeta plus zeta square minus 1 omega nt minus e raise to power minus zeta minus zeta square minus one omega nt.

So, this is the response where the system is over damped. Now, in this type of system, if we look at the transfer function the transfer function is the block representation of This system is or the second order system is this.

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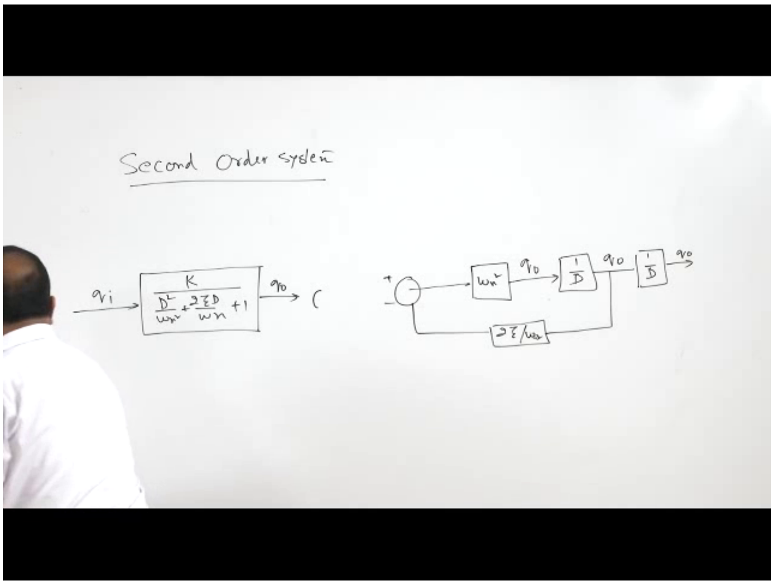

This is qi and this is your transfer function D square by omega n square plus 2 zeta D by omega n plus one and then we get qo.

So, this is how the input output relationship in such type of systems is depicted. Now if we break this block diagram if you break this block diagram, then we will get this. This is omega n square, right and omega n before omega n square and after omega n square, there is a differentiation, if you remember, if we differentiate the step input it becomes impulse right and then this qo is this is qo, then qo is again differentiated.

Now, we will connect this here minus plus and first differentiation 2 zeta by omega n, sorry, omega n it is connected here after first differentiation and then before that before that.

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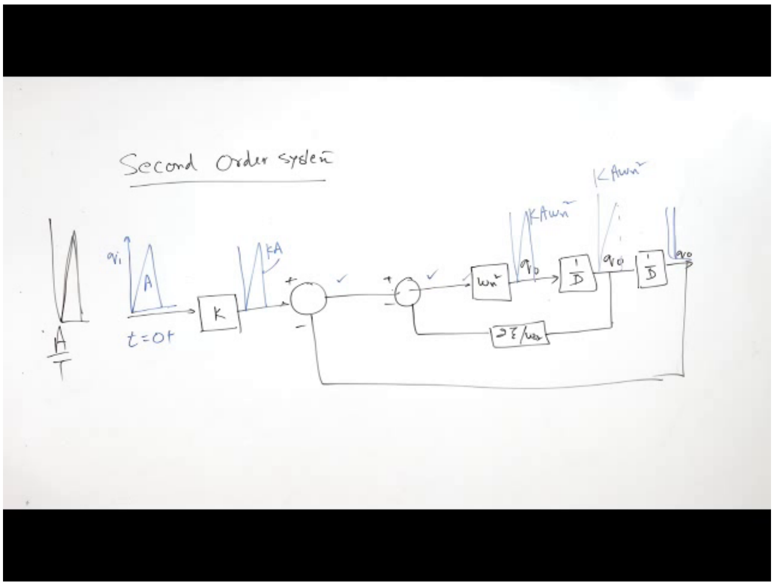

It is again this is this is plus and this is minus and this signal is coming here and then it is K and then it is qi.

Now, we have broken this system in the parts after first differentiation we are taking this omega n by omega 2 then after second differentiation of the signal, then omega n square has been differentiated 2 times ome; this one has been differentiated 1 times and this is how we can represent the signal in block diagrams.

Now, the input is as in the case of impulse input. Input we always defined by a by t input is defined by a by t or there is a pulse can have any shape of the pulse, it does not matter. So, there is a pulse. So, input is a pulse at t is equal to 0 plus.

So, input is a pulse of area a right at t is equal to 0 plus. Now after this transfer function the pulse becomes K, this is area, this is a is area right and this continues here and this continues here as well and then after this omega n square this block, this pulse become K A omega n square just we have broken transfer function in the number of parts.

Now, this is going to get differentiated here when the pulse is differentiated. Now, this is the case when the pulse is differentiated and when the pulse is differentiated it is going to be like this and this is K A omega n square and from here, we will get the qo out and this, after differentiation it is going to be multiplied by this right and after that it is further is differentiated then it becomes a vertical line and this is how the entire block diagram is completed right.

Now, if the system is critically damped instead of over damped system is critically damped when the system is critically damped, then we can write the equation as q o sorry qo divided by K A omega n is equal to omega n t e to power minus omega nt

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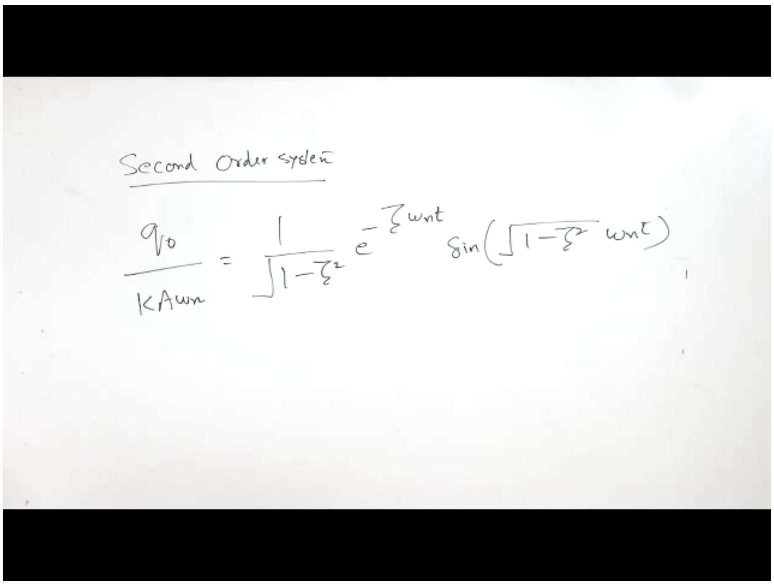

This is the case when damping ratio is one and similarly with the available boundary conditions we can drive the equation for under damped system also for under damped system this equation is going to get modified as 1 by root 1 minus zeta square e raise to power minus zeta omega nt sign under root 1 minus zeta square omega nt.

So, depending upon the value of damping coefficient we can use any of this, we will use the particular equation which is applicable for the value of damping coefficient. Now after this, we will start with the frequency response of second order system right now in frequency response of second order system frequency response.

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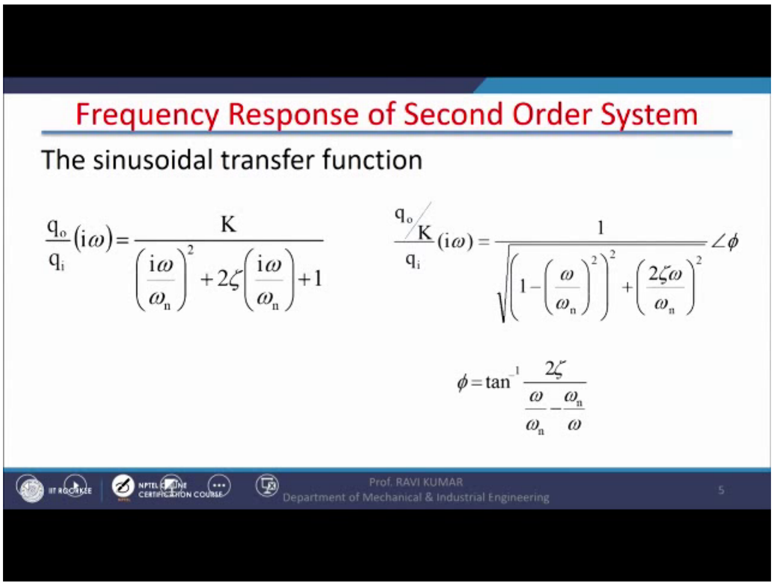

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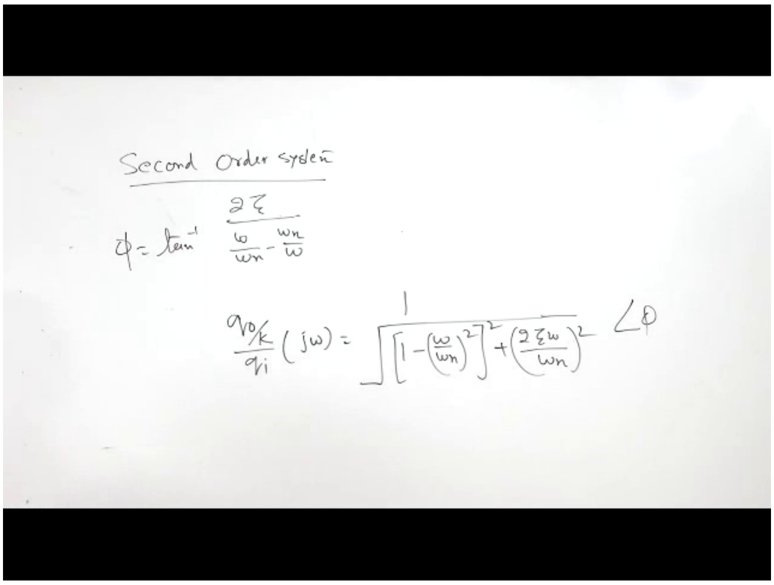

For first order system if you remember we have taken the amplitude ratio, let us say m is equal to K by under root 1 plus lambda omega whole square this was for the first order system.

Now, here the governing equation is qo by q i i omega is equal to K by i omega by omega n whole square plus 2 zeta omega by omega n plus 1. This is the transfer function for frequency input or sinusoidal input or a frequency input or the sinusoidal transfer function, this shall be used to finding the solution for we need only 2 information in the frequency response when there is a sinusoidal input what is the amplitude ratio and what is the phase lag here the phase lag was or phase angle was tan inverse omega lambda minus omega omega lambda or phase lag was tan inverse omega lambda.

There is never phase begin the response right because the system cannot respond before the input is given to the system. So, there is always ideal cases phase lag is 0, but the replication of input input to the system is more important than the phase lag, suppose, we reduce the phase lag and the signal is distorted and we are not giving the actual output, what is given to the input the system is not giving actual output which is being give given to the input then the purpose of measurement is lost right because this phase lag can be compensated, but distortion in the signal cannot be compensated.

So, the instrument a well designed instrument replicate it can be mirror image of the input to the instrument the output of the instrument or measurement by the instrument can be mirror image of the input, but it can have certain amount of phase lag and that can be taken care of by certain compensations right.

Now, here the amplitude ratio in a second order system the amplitude ratio is.

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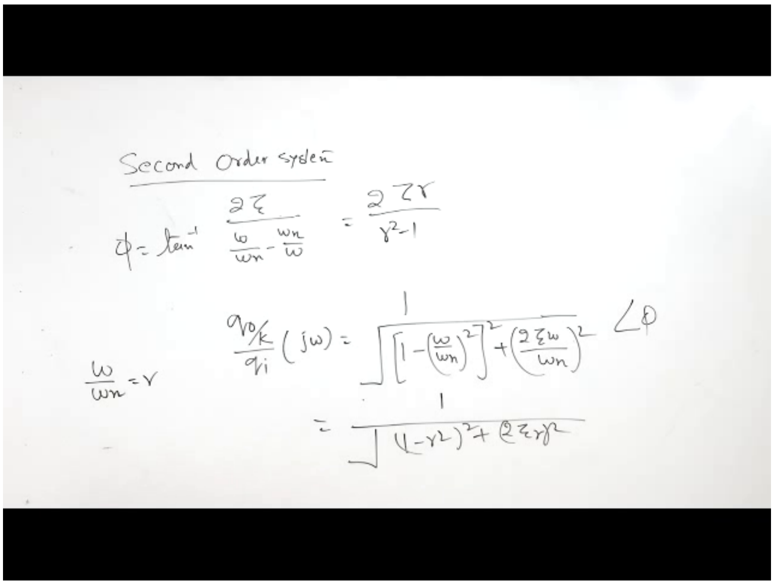

Qo by K divided by q i i omega is equal to 1 by under root 1 minus omega by omega n whole square of this plus 2 zeta omega n 2 zeta omega by omega n whole square or and this output may be lagging by a certain angle phi and phi is here phi is I am writing here the phi is tan inverse 2 zeta omega by omega n minus omega n by omega.

So, now we have both the expression for expression for both the values this is amplitude ratio and this is phase angle phase lag or we can if you replace omega by n for the sake of convenience if we replace by omega by omega n by r then this is going to be one by under root one minus r square whole square plus 2 zeta r whole square just for the sake of remembering the thing said here also we can write 2 zeta this is omega by omega n omega r square minus one and right you multiply numerator and denominator by r then you will be getting 2 zeta r by r squared minus.

So, this is easy to remember otherwise both the expressions are same now response of the system these are the equations and always the damping ratio plays a mirror major role in deciding the ratio of output and input amplitude and deciding the phase lag in the measurement.

Now, let us see the characteristic curves for output and input qo by K upon qi.

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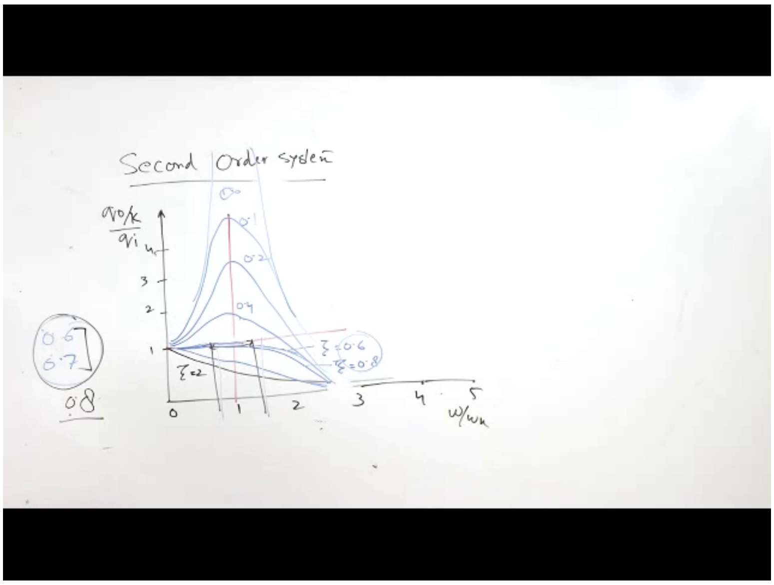

Right and this side we have taken omega by omega n and we have started from 0, 1, 2, 3, we can go up to 5 4, 5 omega by omega n.

So on abscissa we have taken omega by omega n and ordinate we have taken output ratios. Now this is one this is important this is one and vertical line we can have markation as this is also one 2 three four and. So, on, but this also this is the characteristic point. So, here also we will try one horizontal line.

Now, zeta is equal to 2. Now zeta is equal to 2 output and input ratio because the damped case because damping is there. So, the variation is with omega by omega n is going to be like this is zeta is equal to 2.

Next is critically damped zeta is equal to 1, now critical damp will also go something like this, but it will soon; it will come here right then it will come under damped system 0.6, 0.8, 0.4. So, up to 0.6, later this is 0.8, this is points. So, no 0.6 will not cross, it will just be just a little over little over here and then it will come down, it will come down somewhere here.

Now, I am not able to draw the exact figure approximately they are going to be like this we will convert that one point we will convert that one point this is for zeta is equal to 0.6 and this is for zeta is equal to 0.8, now damping ratio will further reduce if the damping ratio is further reduced, then we get there is a hump 0.4 we further reduce the damping ratio then there is a 0.2, then 0.1 like this when there is no damping when there is no damping this will go to the infinity, there is a case of resonance will take place if the damping is 0 and at the same time phase angle now I will draw another figure for.

Now, here we can see that between 0.6 to 0.8. In fact, actually between 0.6 to 0.7, we get this almost flat somewhere here more frequencies are available that is why I stated earlier also while choosing an instrumenting instrument, we should look for the damping ratio between 0.6 to 0.7, we can go up to 0.8 also, but not more than 0.8. So, best range is 0.6 to 0.7, we are overshoot will not be there significant overshoot will not be there and we get more frequencies in this range either the curve is almost flat in this range or for the range of frequency for which the curve is flat is higher for this zeta when the zeta is lying between these 2.

Now, we will draw the characteristics curve for phase lag as well for rec frequency response now for phase angle.

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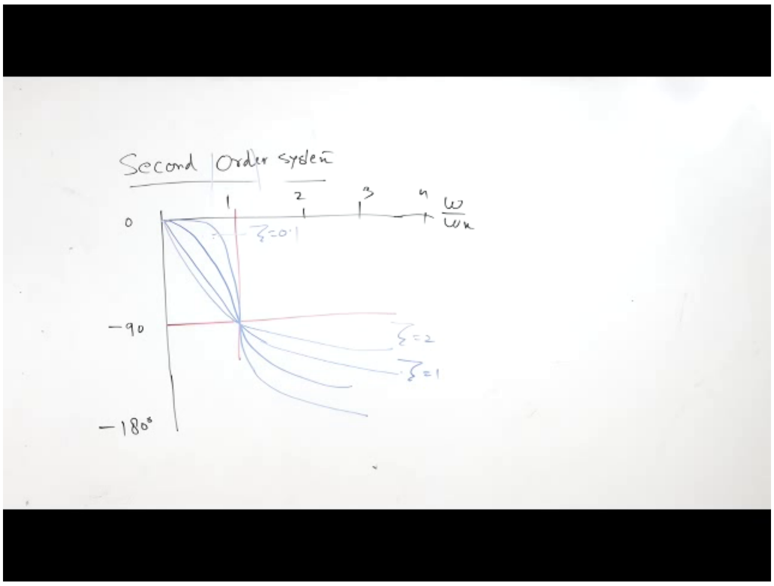

Now, phase angle is always negative as I say that the response cannot lead the input. So, it is always negative to 0 and it will go up to minus 180 degree, but in phase angle also 90 minus 90 is important, I will draw a one line at minus ninety degree and 1 line will draw at 1 here and then it can be 1, 2 and 3, 4 and so on right.

Now, first curve will draw for zeta is equal to 2 zeta is equal to 2. So, zeta is equal to 2 curve will come like this, this is zeta is equal to 2 right, now we will go move towards red reducing the damping ratio right. So, zeta is equal to 1 zeta is equal to one will something like this slope will start increasing.

Now, this is going to be for zeta is equal to 1, this is zeta is equal to 1, then we will go for under damped underdamped system when the under damped system is there then it will take starting in this type of curvature right. So, both sides it will be it will be having the if you reduce the damping ratio the both sides it will be nature of the curve will be sharper there will be sharp terms in the curve this is zeta is equal to or zeta is equal to 0 point one right.

So, here in this is response characteristic curves by referring this characteristic curve we can have we can get certain idea about the normal response or the general response of second order visiting system for frequency input.

Now, if we take some critical points if we take certain critical points for measuring instrument.

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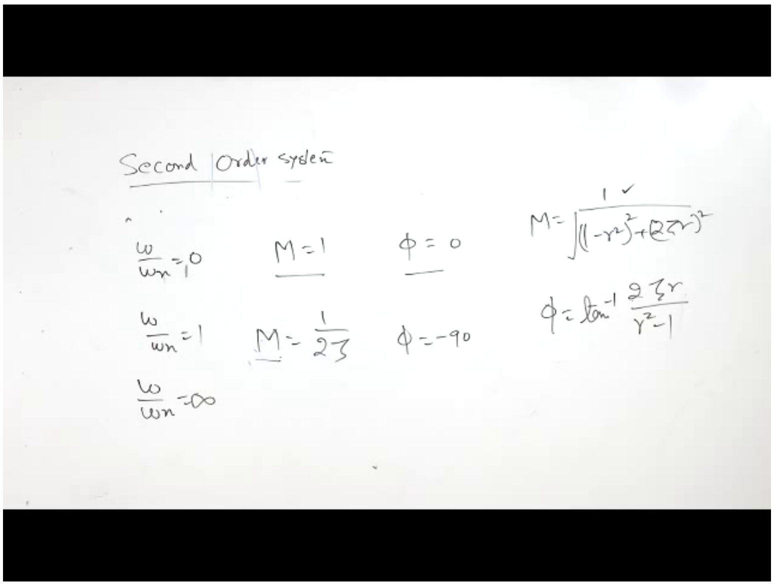

Then suppose omega by omega n is equal to 0 when omega by omega n is equal to 0, M is going to be 1 and phi is going to be 0 because M, we have calculated M is equal to 1 by 1 minus under root 1 minus r square whole square plus 2 zeta r whole square right and when omega by omega n is equal to 0.

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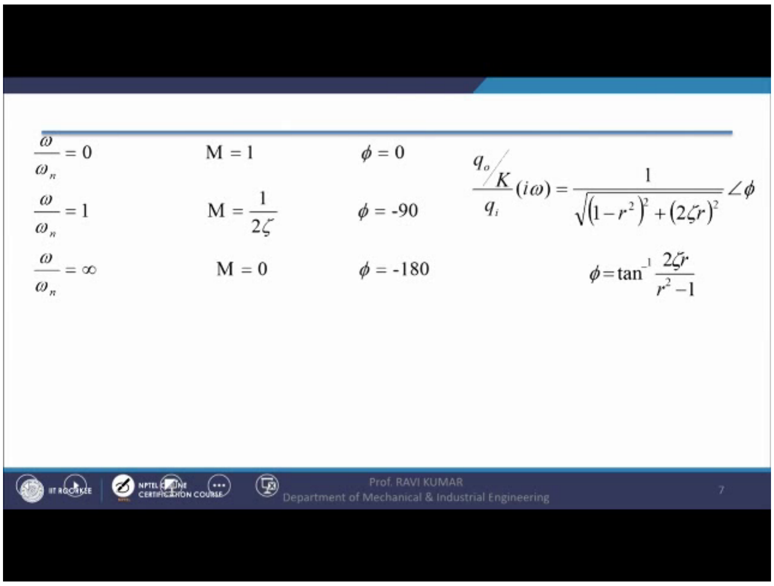

Then, this expression is going to be 1. So, we will get M is equal to 1, here phi is equal to 0 because phi is equal to tan inverse 2 zeta r by r square minus one. So, this is going to be 0.

Now, omega by omega n is equal to 1 when omega by omega n is equal to 1 M is going to be 1 by 2 zeta, if this no damping is there, then it is going to be infinity that is perfect resonance because sorry, this is omega by omega n because it means the imposed frequency is equal to the natural frequency when imposed frequency is equal to natural frequency of the system M is going to be 1 by 2 zeta, if there is no damping or damping is say zeta is 0.01 in that case and we will have any exorbitantly high value then phi is equal to in this case is going to be minus 90, this is lag I mean 90 degree or phi is going to be minus 90 means it be it will be lagging by 90 degree.

Now, if omega by omega n is equal to infinity impose frequency is very high 1 lakh cycles per second natural frequency is thousand cycles per second and impose frequency is some millions cycles per second or million radians per second.

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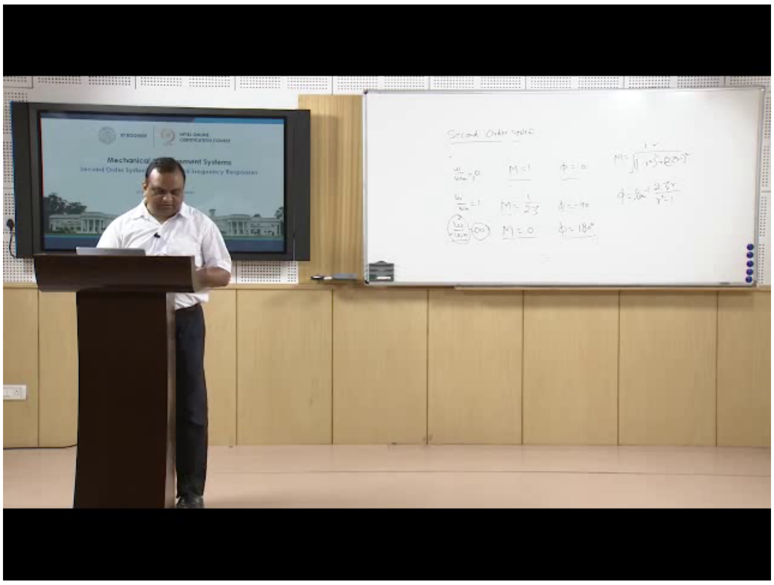

This is imposed frequency divided by natural frequency. So, imposed frequency is much much higher than the natural frequency right. So, in that case it is going to be infinite when omega by omega n is infinite. So, definitely m is going to be 0 when imposed frequency if physically if you try to understand this phenomena if I impose the physical frequency on this table and the frequency is say if 1 million cycles per second the table has got its own inertia.

So, take if you cannot feel any movement in the table right see is the case here when omega by omega n is an infinite the ratio is going to be 0 and the phi is going to be minus one eighty degree, these are the values at certain salient points, and there is a case when we call resonant frequency the resonance frequency is the frequency when dM by d omega by omega n is equal to 0 that is known as frequency of resonance right.

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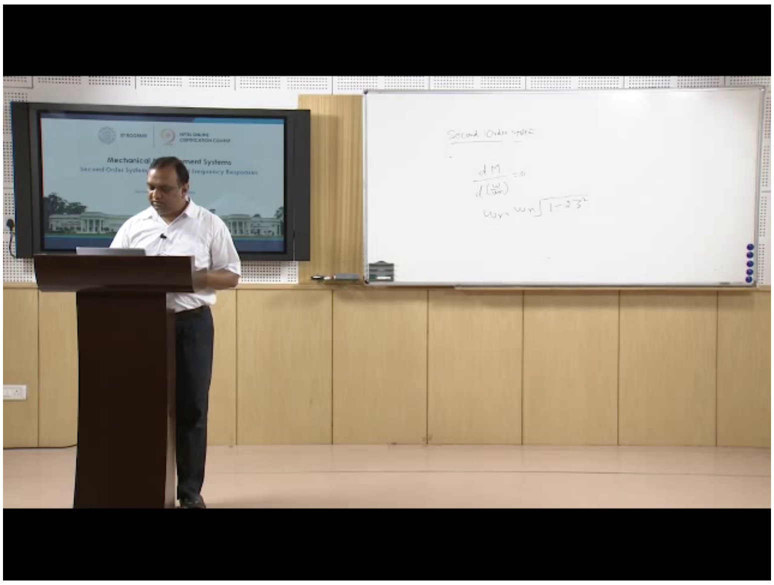

So, resonant frequency here is the resonant frequency wr is equal to omega n under root 1 minus 2 zeta square. So, once we have the damping ratio, we can find the rest the resonant frequency of the system, there is one more phenomena that is known as a dead time element dead time.

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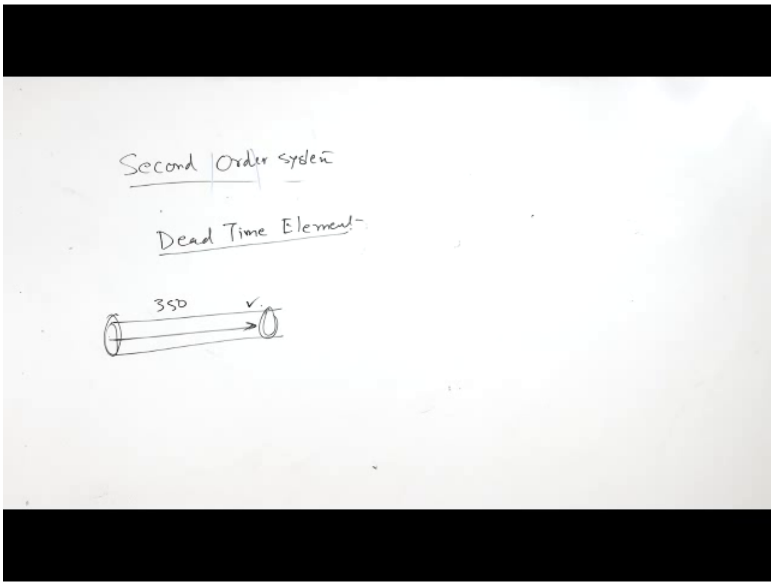

Now in dead time element the system it is a system which is defined as the system in which the output is exactly as the same input output is exactly same as the input, but after certain time lag right for example, there is a pneumatic system, suppose, there is a there is a tube filled with air there is a pneumatic system and tube is pressurized here the air pressure is exerted here at the length of the tube let us say is 350 or 360 meters.

So, when the pressure pulse will move suppose it moves with the velocity of sound right. So, after one second you will be hearing a simply the same sound there is a time lag, but we are getting the exactly same input as I said earlier the nature of the nature of the signal should not be distorted. So, nature of the signal is not distorted, but we are getting a signal after a pause.

So, this is known as dead time element this is all for today from the next lecture we will start with the higher order systems.

Thank you.