Mechanical Measurement Systems
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Lecture - 23
Higher Order System

I welcome you all in this course on Mechanical Measurement Systems. Today we will discuss the Higher Order Systems. So, topics to be covered today are Higher Order Systems and if example on Higher Order System; we have amply discussed about the analysis of whether it is periodic signal or transient signal analysis of first and second order systems.

So, we today we will discuss Higher Order System. And for these two types of systems, we have done transient analysis and we have done periodic input analysis. Now for higher order system, if you want to do some transient analysis it definitely, equations will become more and more complicated.

Because if you go for the Higher Order System, you suppose you go for the third order system, so in third order system, the n is going to be 3. So, d raised to power 3 it will start with the d raised to power 3 and system, the differential equations will continue to become more and more complicated and the solution also at the same time, the solution of these equations will also become a little more complicated.

So, for this purpose, you can use the number of software's are available and MATLAB is one of the software where you can have solutions for such type of equations. Regarding the frequency response, frequency response is still we can do with the calculator and a pen.

For frequency response for Higher Order Systems, we can draw a logarithmic plot for c frequency response and it is known as Bode plot, Bode plot. And this plot, we can use for the frequency response of Higher Order Systems.

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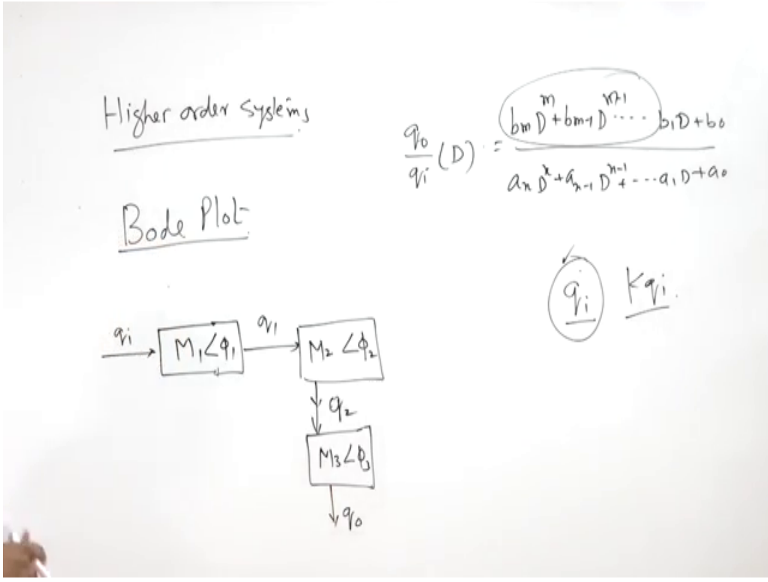

So, first method is, you just now, now the advantage of this method advantage of this method is you simply keep on responses of different components of the system because a system is a measuring system an actual measuring system, it is a complicated system. So, it may have number of components and what you can do, you can keep on adding the response of different components right and then can do further analysis.

So, a response, a frequency response of Higher Order System, suppose it has 3 components, amplitude of first component is M 1, this is and this is transfer function and angle phi 1 that is legging angle or phase angle and input is q i.

So, first component of measuring system is this one and it gives the output as q 1 right and then it goes to another system which has amplitude ratio M 2 and phase again phi 2. These again suppose goes to another system where amplitude is M 3 and phase is phi 3.

And now this is q 1 and we can say this is q 2 and now this is q o and we get the final output. For transient analysis, we have already done q o upon q i is equal to b m if you remember, D to power m plus b m minus 1, D 2 power n minus m minus 1, b 1 D plus b o.

And then a a to power n D to power n plus a n minus 1 D n minus 1 a 1 D plus a o. This is a not a ratio; it is transfer function. So, q upon q i, now if you go for the higher order.

Then suppose we go for third order, then this n is going to be 3, where we go for the fourth order then n is going to be 4 right. Input as I said earlier that we take input only q i right and this is and we normally neglect the higher order systems because this first order or the only K q i is sufficient K q i is sufficient to for representing most of the mechanical systems for input.

Now here for say transient response of higher order system definitely we will have to use some software because solutions will become more and more complicated, you have seen for the second order system itself.

There are number of equations which were quite complicated and lengthy; however, in the case of frequency response, this lag angle we can always add. Suppose second system, first system develops a lag angle of phi 1, another system follows phi 2, not this is not phi 2 from phi 1 it is phi 2 and the third system again you develops a lag phi 3.

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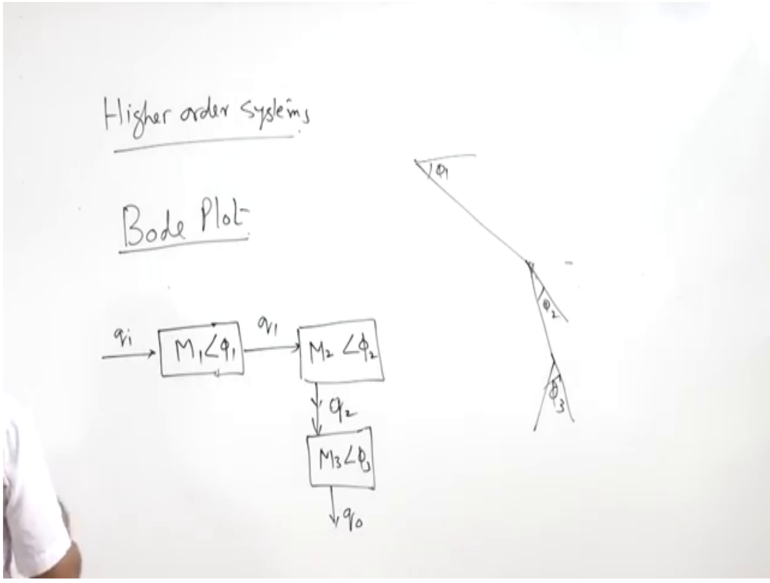

So, total lag is going to be phi 1 plus phi 2 plus phi 3 right. Now second issue is about the amplitude, how we will take the amplitude into account alright.

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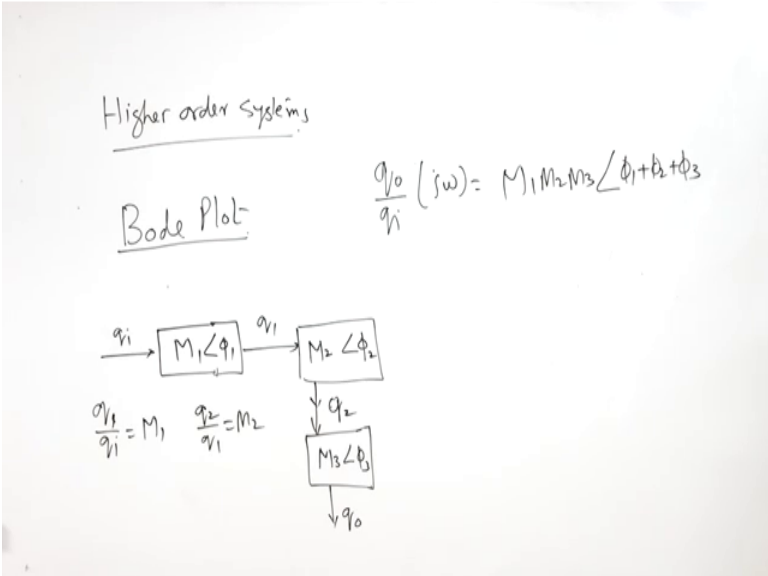

In order to find the amplitude, let us say q o upon q i i omega. So, if you look at the amplitude that q 1 by q i is equal to M 1 and then q 2 upon q 1 is equal to M 2 and q o by q 2 again M 3.

So, if you take product of these three, that M 1, M 2, M 3, we will be getting q o q o upon q i right. And then once we have q upon q i then phase angle will be phi 1 plus phi 2 plus phi 3.

So, this is the response of higher order system, frequency response of higher order system. Now we will do the analysis of higher order system using bode plot.

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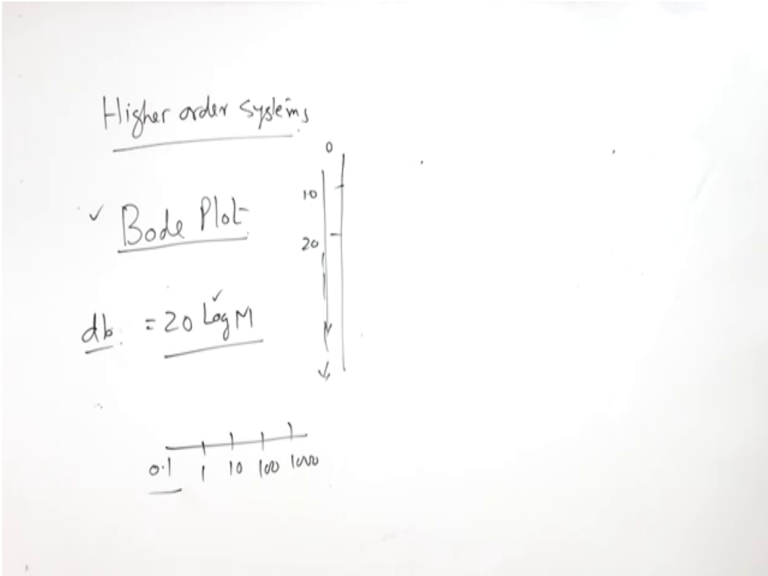

So, before we going before going for the bode plot, we will try to find the decibel value of the magnitude M and decibel value is 20 log M; this log is base 10.

It is not a natural log. So, 20 log M is the decibel value db, decibel value because the frequency when we take if you, when you want to have frequency response of higher order system frequency can be of any order. It can start from 10 hertz to 1 million hertz.

So, for frequency response, x axis we always take log scale in terms of log. So, every subsequent number is the multiple of 10, say this is 1, this is 10 and this is 100 then 1000 then, 10 to power 4, 10 to power 5 if and this side it is 0.1.

Log scale starts from 0.1. It does not start from 0. Now y axis, y axis all the angles will be mine negative because it is always phase lag. So, phase lag will start from 0 to certain value, it can go minus 1, 180 makes 1 minus 180 right.

It goes, it can go further go, but normally up to second order system, it goes up to minus 180; lag can be of any order. So, let us not v confine it to minus 80 so, but it starts from 0, 0, minus 10 minus 20 and so on.

Now let us talk, let us start with the because, a system can be broken in number of first order and second order systems. Any higher order system can be broken into a number of first order and second order systems. That is why the analysis of first order and second order system is very important to understand.

So, let us start with a with a first order system. Now I want to plot a bode plot for first order system. So, amplitude ratio for first order system is A o by A I is equal to K by under root 1 plus omega lambda whole square.

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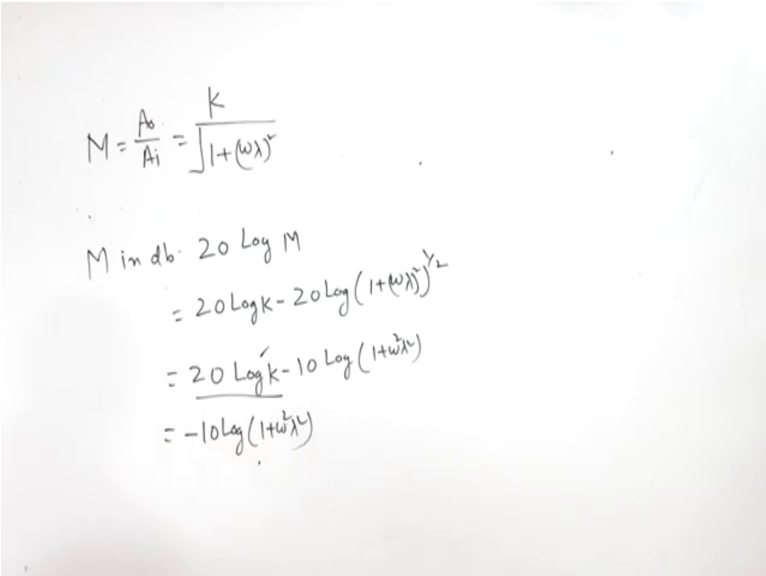

This is amplitude ratio and it is often less than 1. Another is M in decibel. Now if I want to have M in decibel, then I should have 20 log M and that is equal to 20 log K minus 20 log 1 plus omega lambda square raise to power 1 by 2 is equal to 20 log K minus 10 log 1 plus omega square lambda square. This is the value of decibel where M is in decibel. Now suppose K is 1.

Many of the cases, you must have observed we have taken K is equal to 1. If K is equal to 1, then it is going to be minus 10 log 1 plus omega square lambda square. Now if you want to draw a plot for this, plot for this for amplitude.

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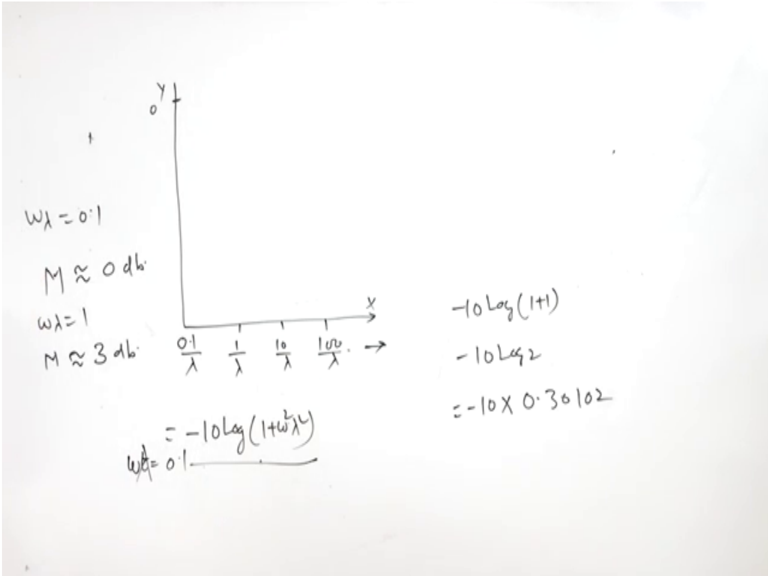

This is x axis and this is y axis and x axis has to be in logarithms. It has to be log axis right. So, we will take 0.1 by lambda sorry 1 lambda, 10 times 1 by lambda, 10 by lambda, 100 by lambda and so on.

This is I have taken because omega t is equal to 0.1, so omega lambda. So, omega lambda is 0.1. So, 0.1 well lambda will give the omega frequency. So, that we are getting on this axis, y axis we will calculate this.

Suppose omega lambda when omega lambda is 0.1; in that case it is going to be log of 1.01 and approximately this M decibel value of M is going to be approximately 0 decibel. So, we will start from 0, 0 is somewhere here. Now we will take omega lambda as 1.

The moment we take omega lambda as 1, it becomes minus 10 log 1 plus 1 and log which is going to be minus 10 log 2 and log 2 is minus 10 into 0.30102 or it is going to be approximately.

When omega lambda is equal to 1, then M is a decibel value is going to be approximately 3 decibels. Now we are increasing the value of omega lambda.

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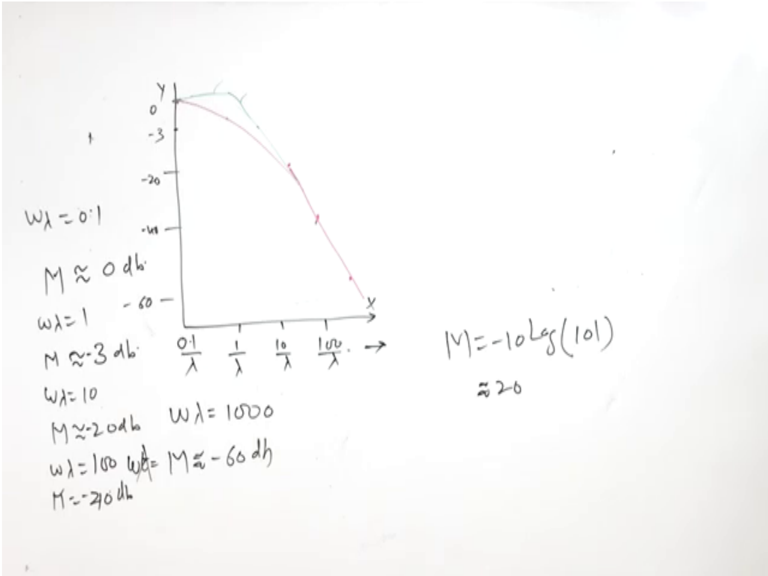

Now, we go for omega lambda is equal to 10. When omega lambda is 10, then M is going to be minus 10 log 101; then is square plus 101. So, when we take.

Log 101, It is 2 multiplied by 10 is equal to 20 approximately that is also approximately 20. So, it is when omega lambda is equal to 10, then M is going to be approximately 20 db.

Now we take omega lambda 1000; so, 1000 square plus 1. So, log of 1000 squares plus 1 and multiplied by 10 right and then we will get for 1000. We are going to get the value; 100 we have done. We are not done 100; we start with the 100.

Right, so for M omega lambda is equal to 100 then M is equal to 40 db and this is minus they are all minus right and when it is omega lambda is 1000.

Then M is going to be minus 60 db and this is also approximately very close to minus 60 db. So, now after certain values say omega 1 it is 3; it is minus 3. So, minus 3 will be somewhere here right let us say minus 20 minus 40 minus 60 like this.

So, for 100 it is minus 40. So, for 100 it is minus 40 for 100 it is minus 40 for 10 it is minus 20 for 60 that is not 100 it is thousand for thousand it is going to be minus 410sixty.

So, there is a linear relationship now we have got the linear relationship a difference of 20 decibels 4 point one it is approximately 0 for this it is approximately 3. So, the system is behaving like this or if you want to have asymptotes these are the asymptotes after this.

10 omega lambda is equal to 10 if the variation is linear with the with frequency right. So, this is for the amplitude now for frequency plot again for the frequency plot I will rub it off for the frequency plot phasing will now we have already done the amplitude frequency same not frequency board is a phase angle broad.

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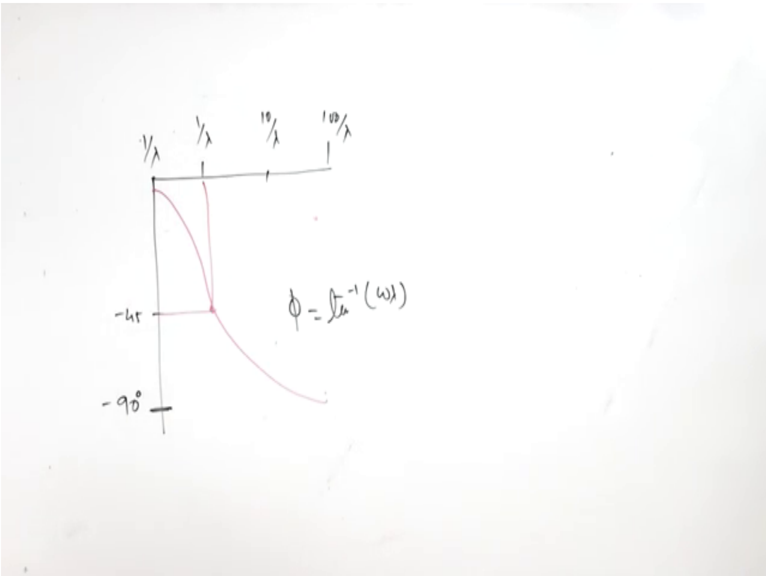

So, for the phase angle, it is 1 by lambda, this is again 10 by lambda and this is 100 by lambda. It is on n x axis and this is a linear scale and this is of course, starting from 0.1 by lambda.

Vertical scale is phase lag and simply because the phi is tan inverse omega lambda. So, just by putting different values of omega lambda, we can get the different values. I will take certain salient values for example, minus 45; minus 45 will be there when tan phi is equal to 1. So, that is 1 point right. And like this we can find easily find other points and the response of the system is going to be like this. So, this is about the phase angles and the here in it is in first order system.

It is 90 degree minus 90 degree. So, the behaviour of the system is going to be like this. Now after the first order system, we will pick up the second order system. So, bode plot for second order system.

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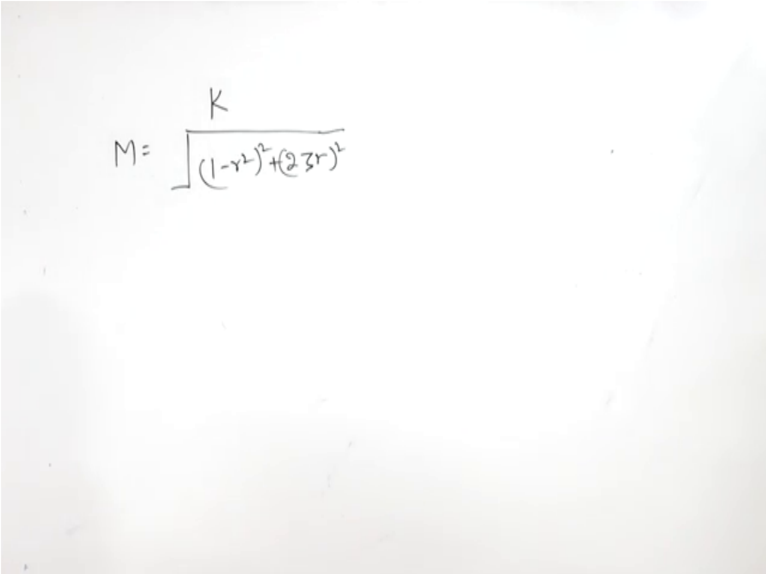

Now, in second order system as you know that for frequency response amplitude ratio M is equal to K over under root 1 minus r square whole square plus 2 zeta r whole square. Now, we will start with the second order system.

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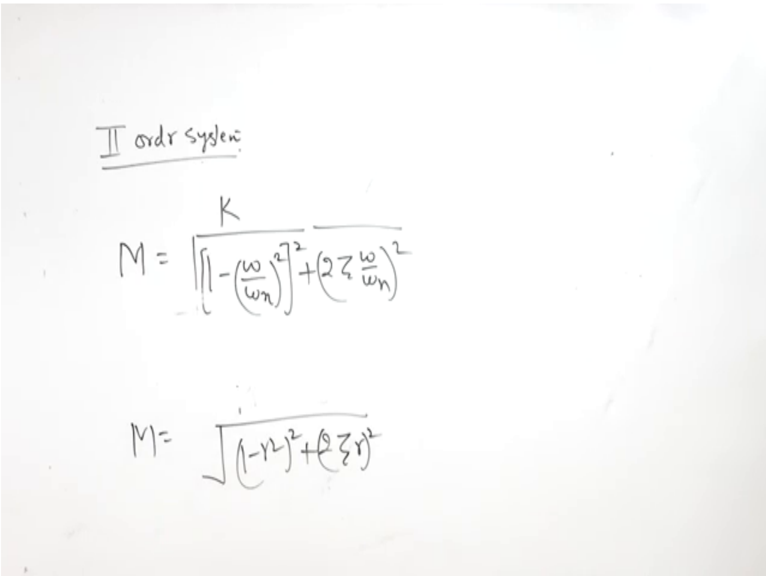

Bode plot for second order system.

For second order frequency response, the magnitude ratio is equal to K by 1 minus omega by omega n whole square plus 2 zeta omega by omega n square and this is under root. And for the sake of convenience in the previous lecture as I stated that omega by omega n can always be replaced by f r. So, M is equal to K by under root 1 minus r square whole square plus 2 zeta r whole square.

Now, here in this case also, we can take the log of M; The log of M in order to find decibel.

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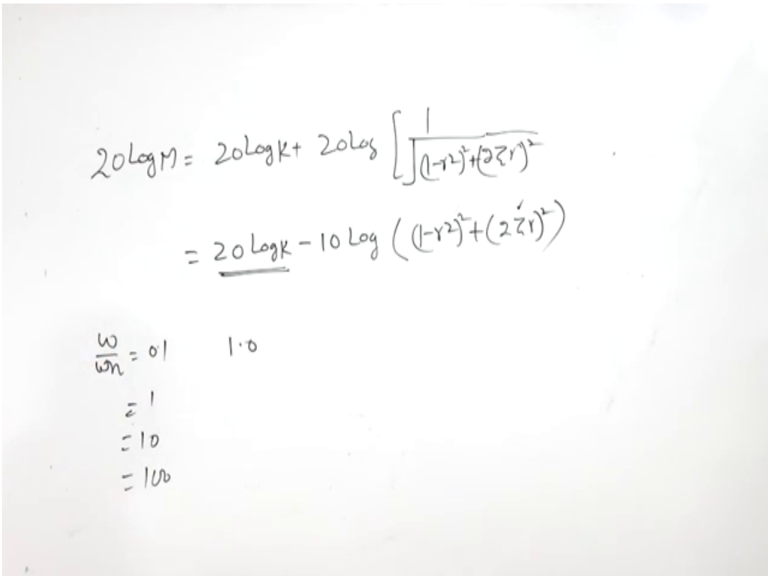

So, 20 log M oh, this is K. So, it is going to be 20 log K minus 20 log 1 by sorry plus 20 log 1 by under root 1 minus r square whole square plus 2 zeta r multiplied by whole square sorry, whole square of this.

If you further simplify this, we are going to get 20 log K minus half. So, it is going to be 10, log of 1 minus r square whole square plus 2 zeta r whole square.

Right, now here in this case, if K is equal to 1, this is going to be 0 right. Now only this term will be remaining; this is only this term will be remaining now again here we will start with omega by omega n is omega n is equal to 0.1 right and then we can go for 1, then is equal to 10. And then is equal to 100 like this and if we keep on putting these values in this equation, we will find that the there is a change in a in the value of decibel value of M; that is one thing, second thing is here damping ratio is also important because there is a.

One factor damping ratio; so, first of all, we will have to assume the damping ratio constant. Let us start with a higher damping ratio that is critically dam 1.0.

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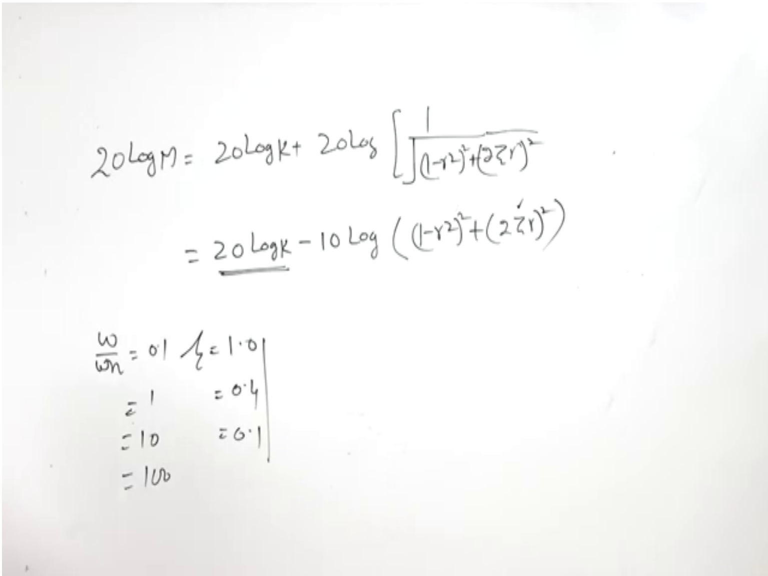

So, for this, for just to have an idea we can take this damping ratio 1 and 0.4 or 0.1; we can draw 3 curves. So, we will have fairly good idea about the trend.

So, if we draw the trend using these information’s, the final graph is going to be like this.

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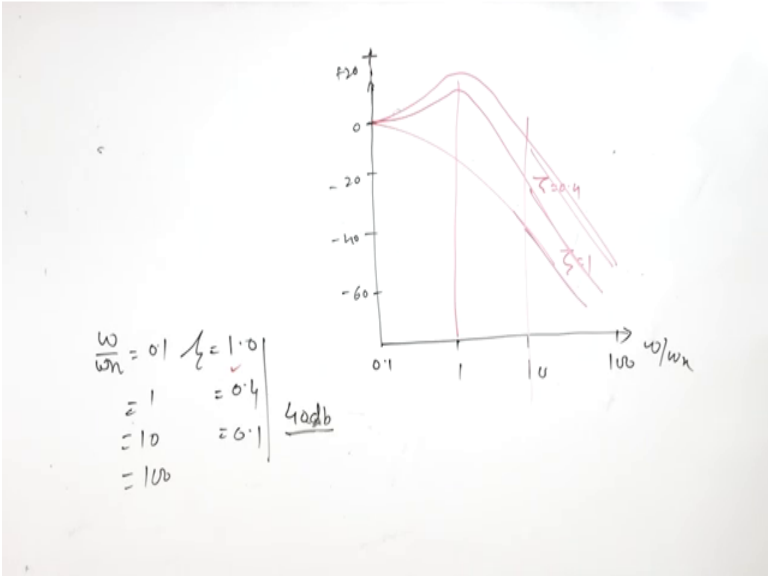

On the x axis this, we can take omega lambda or say omega by omega n; omega by omega n here it is 0.1, this is 1, this is 10, this is 100, this is omega by omega n.

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Omega by omega n; On the y axis, we will start from 0 right and minus 20 minus 40 and minus 60 right; here also it becomes the curve becomes a straight line and the difference is 40 decibel; for every decade in first order system it was 20 decibels.

Here in second order system it is 40 decibels and characteristics curve for zeta is equal to 1; for zeta is equal to 1 this is omega by omega n 1. So, for zeta is equal to 1 it is going to be like this, this is zeta is equal to 1.

Now, we will change the equation, will change the value of zeta; we will go to the zeta is equal to 0.4 the curve is going to be something like. This is now on the y axis we can further shift it to plus 20 right. So, this is for zeta is equal to 0.4 and if you further reduce the damping ratio.

We will get the curve like this, but after this passing this stage, when it passes this omega by omega n by is equal to a 10; after that if there is a linear variation of 40 decibels in subsequent multiple of 10 of frequency rise.

And regarding the phase curve also, now, this is about the an amplitude for phase angles we will be simply added. So, when the phase angle is added and we have value of phi this is omega by omega n omega by omega n.

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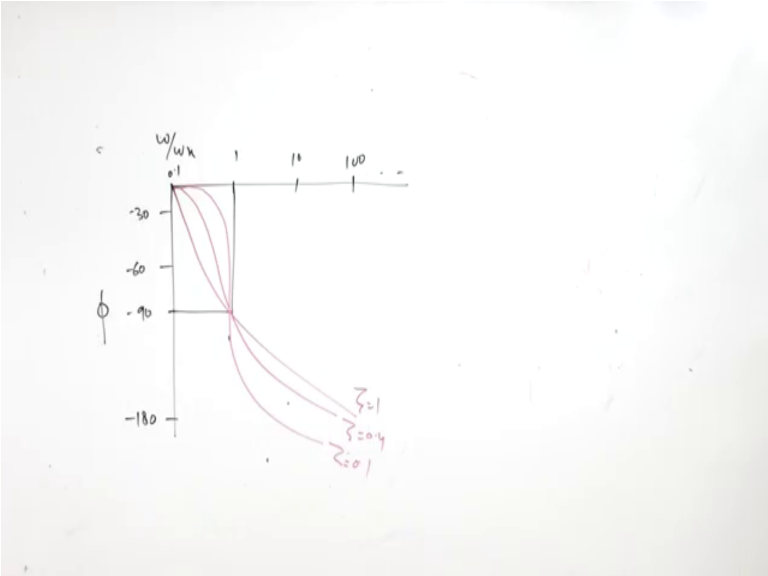

Starting from 0.1, 1 10, 100 and so on.

And on the y axis, there is a phase lag or phase lag is minus or phase lag is or. So, phase angle minus 30, minus 60, minus 90, it will go up to minus 180 right. For us minus ninety is critical.

So, minus 90 and 1; with all the characteristics curve for the phase, let us start for 1 it will. This is zeta is equal to 1 right. For zeta is equal to 0.4, it is going to be like this, zeta is equal to 0.4 and if you further reduce a zeta this graph is going to be like this, zeta is equal to 0.1. So, these are the characteristic curves.

They are only representatives so that you can have idea how the phase lag is varying and how the magnitude of the system is varying and it is very interesting to note that after 10 decibels, in both the system whether it is a first order system or the second order system, the variation becomes linear.

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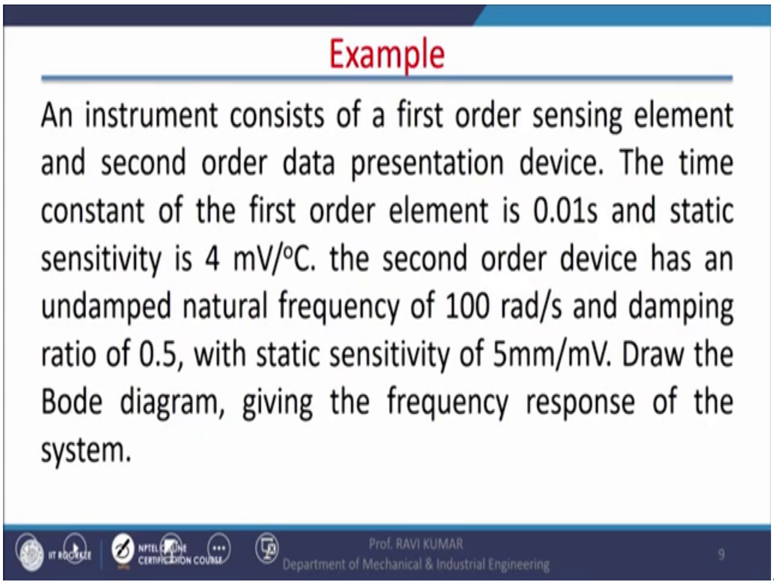

After that we can take one numerical; it states that an instrument consists of a first order sensing element and second order presentation device. So, a system is a combination of first order system and a second order system. The time constant of first order element is 0.01 seconds and the static sensitivity is 4 millivolts per degree centigrade.

So, first order system as time constant of 0.01 seconds.

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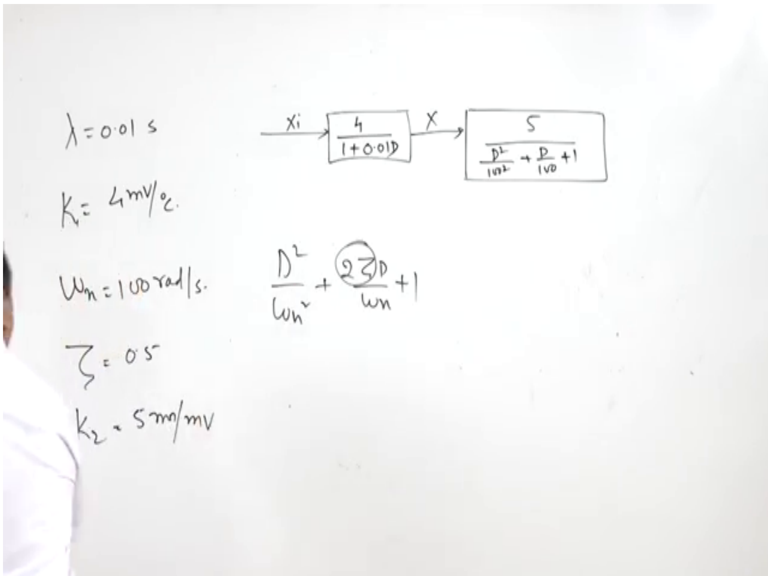

So, lambda is equal to 0.01 seconds and static sensitivity is equal to 4 millivolts per degree centigrade. So, input is in degrees centigrade, output in millivolts. Now; it has display unit also that is a second order device which is which has undamped natural frequency of 100 radians per second; undamped natural frequency of 100 radians per second. This is for the second instrument.

And they are coupled and the damping ratio 0.5, 0.5, this is damping ratio for a second instrument with the static sensitivity of 5 mm K 2, this is K 1 is equal to 5 mm per millivolt.

So, this instrument is taking input from here. So, here the output is 4 in millivolts these millivolts are going to this instrument and we are getting output in displacement that is 5 mm per millivolts. So, both the instruments are coupled. Now for the frequency response of the both the instruments; for example, first instrument this is let us draw a block diagram.

So, if you wish to draw a block diagram, then this is x i and first order instrument transfer function is a static sensitivity divided by 1 plus what is omega; omega is not over the lambda is 0.01.

This is transfer function of forces; this is how it has to be represented then we are getting output x, some output x from this instrument and this output is going to another instrument which has transfer function 5 d square by 100 square plus d by 100 because two zeta will be the one

Two zeta omega n omega by 2 zeta is 2 zeta. zeta is 0.5. So, it is 1 n plus 1 right because this equation. If you remember it is this 1 is D square by omega n square plus 2 zeta D by omega n plus 1. So, 2 zeta is because zeta is 0.5. So, we have taken 1 already considered that.

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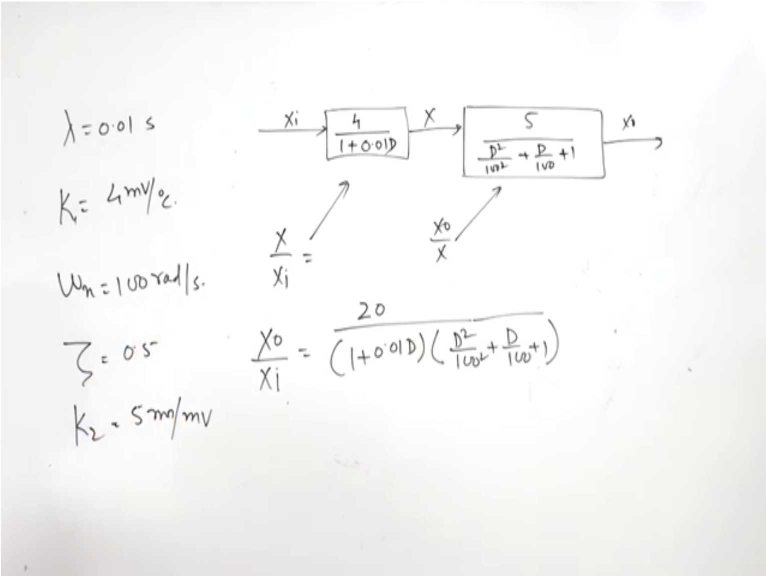

And this is transfer function for second instrument and output of this instrument is X o right. So, X upon X I is equal to this and X o upon X will come from here.

Now, we want to have output by input. So, output by input it is X o by X i, it is going to be equal to multiple of this 20 divided by 1 plus 0.01 D multiplied by D square by 100 square plus D by 100 plus 1.

Now, simply we will take the log of this equation and when we take log of this equation and we have to replace D by i omega because we are going for the frequency response right.

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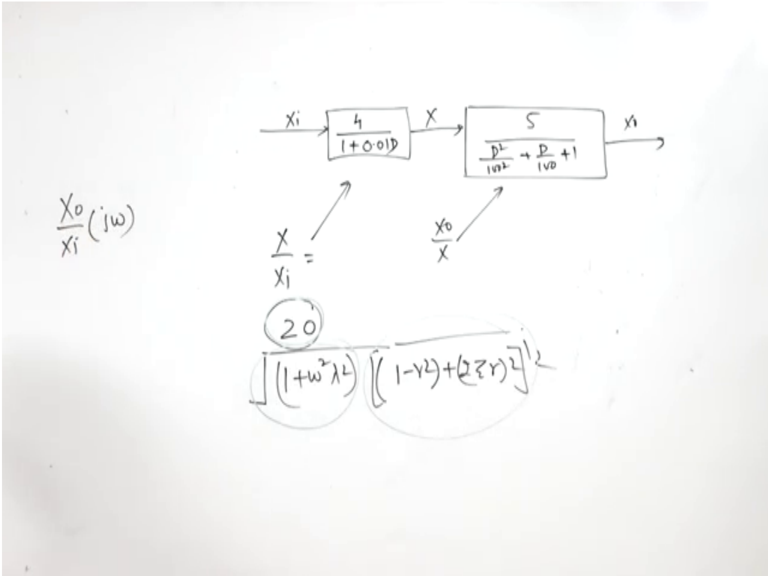

So, once we are going for the frequency response, then we will have to take X upon X i, frequency response of the system. So, M is equal to M 1 M 2.

Right and 20 log M is equal to 20 log M 1 plus 20 log M 2. So, net output is 20 divided by 1 plus omega square lambda square and then 1 minus r square plus 2 zeta r whole square raise to power 1 by 2 right.

Now, this 20 is M 1, 20 is M M 1, 4 into 5 this is M 2; 1 by this is M 2 and 1 by this 3, M 3. So, simply we will take log log of. So, the for those purpose of bode plot.

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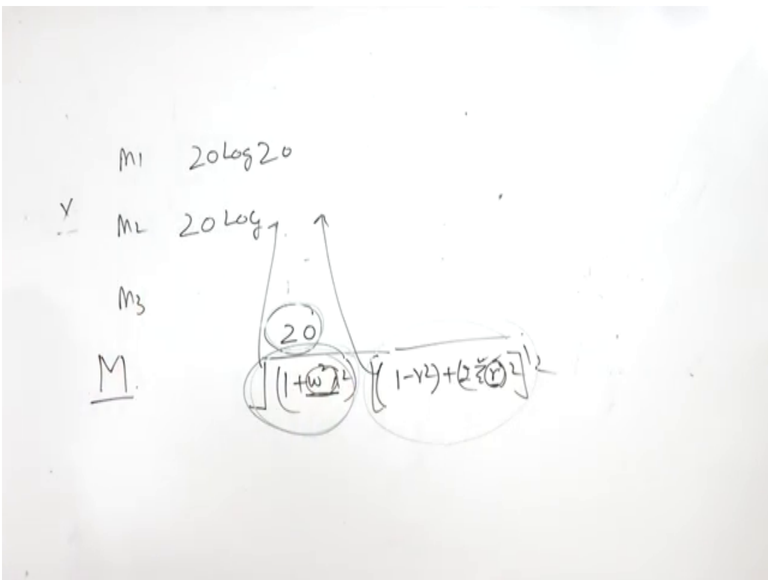

We will write 20 log M 1, it is 20 20 log of this and then 20 log for this; this is. So, they are going to be M 1, M 2, M 3. All the values are with us and using these values.

We can find the value of final magnitude M right and. So, with the help of this we can draw the diagram and regarding frequency with the help of this information we can draw the bode plot for different values of omega lambda and different values of r. So, now, the M will be product of.

So, it will be sum off this log of 20 log 20, 20 log this plus, 20 log this and we have the value of omega lambda and zeta and r for sorry r zeta and for so, for different values of r and omega lambda, we can draw the bode plot and if you try to for the both part, it is going to be like this, it will appear to be like this.

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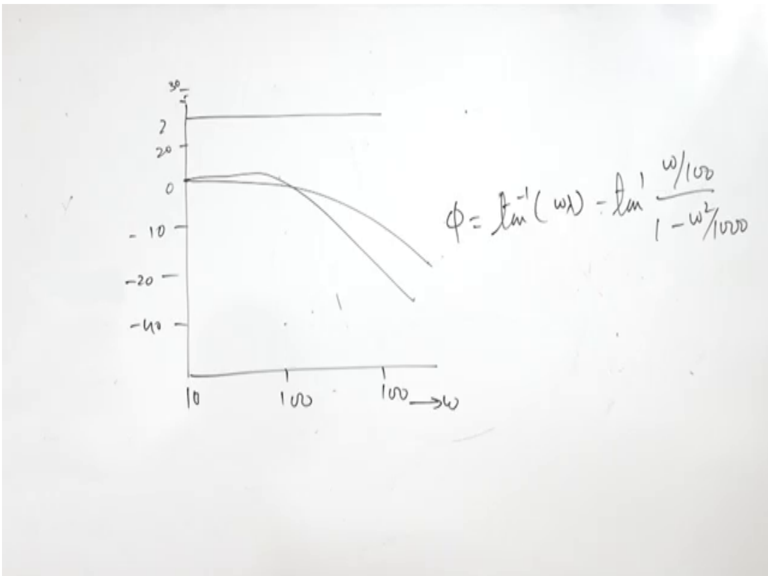

For that is it start from here 10, this is 100 and this is 1000 right and this is the value of omega.

For M 1 this is 20 log 20, this is provided is constant. So, 20 log 20 it will something a little more than 20, this is 30 and this is 20 let us say, this is 20 log 20 will come somewhere here.

And then first order system the first order system will start from 0, it will like it will be like this we have already done that and the second order system will be like this.

So, for any frequency we can take the sum of M 1, M 2 and M 3 and that is how we can find the final value of M and here it is 0 the minus 10, minus 20, minus 40 and so on.

Now, regarding the phase lag phase lag is going to be here is equal to tan inverse minus omega lambda plus this is phase lag. So, if you are taking phase lag, then it will not be minus omega lambda, is going to plus omega lambda minus tan inverse omega by 100 divided by 1 minus omega by 1000.

So, this is going to be the final phase lag for different values of lambda is given, for different values of omega right. This is all for today. From the next class or in the next class, we will start with the compensation in first order and second order systems.