Mechanical Measurement Systems
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Lecture – 24
Compensation

Hello! I welcome you all in this course on Mechanical Measurement Systems. Today, we will discuss about the compensation in measuring systems, compensation in the process of improving the dynamic characteristics of measuring system.

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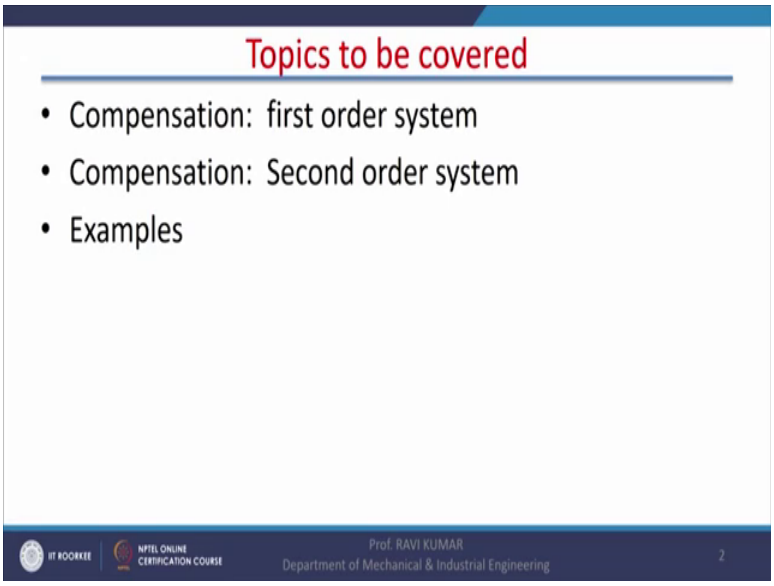

Topics to be covered in today's lecture are, compensation of a first order system, compensation with the second order system and I will take one example if the time is left.

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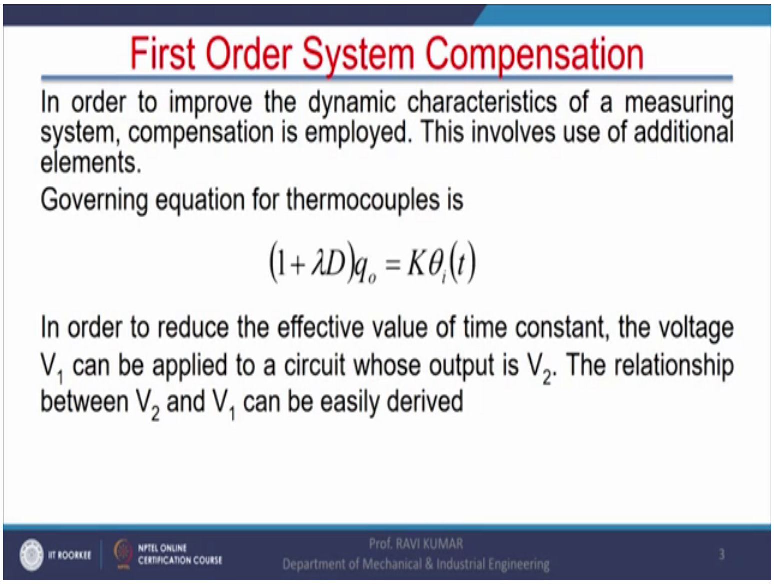

So, in a first order system, as we know the response of the first order system can be written as.

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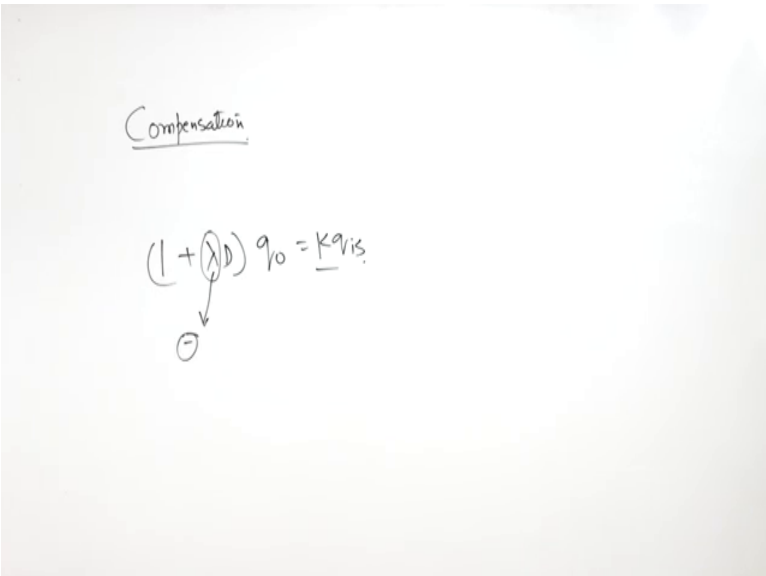

This is the governing equation for the first order system and in this equation, if I want to improve the dynamic response of the first order system, this lambda has to be changed. In fact, lambda has to be reduced; time constant has to be reduced in order to improve the response of the system right.

So, in order to do that, suppose we are getting output form certain input on certain measurement.

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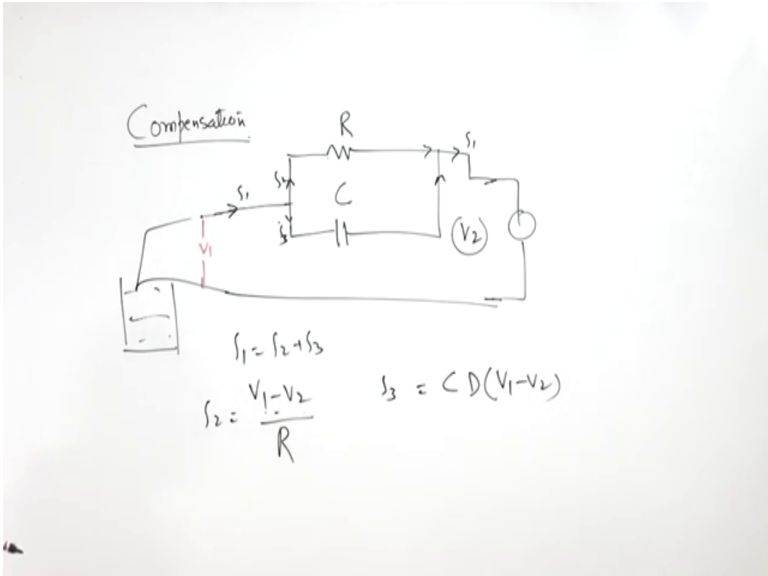

There is a suppose some fluid and the temperature is the fluid is being measured with the help of a thermocouple, thermocouple is giving output let us say output is, so output of this thermocouple is let us say V 1 right.

Now what we will do; we will impose another voltage V 2 in this output and this V 2 is here and there is a display right. And in between, we will provide an RC circuit. So, there is a resistance and there is a capacitance in the circuit and this is also how for connected to this one.

Now; the current which was earlier going directly to the display as V 1, now it is going as a V 2 to the display and this V 1 has been manipulated to be 2 by using this RC circuit. The output of this output of current is of this is suppose is I 1, then is it will be splitting here in I 2 and I 3 and then it will be going to again, it will combined here and then again it will become I 1 and then it will go to the display.

Now, resistance R is R is the resistance of the resistor is and C is the capacitance of the capacitor. So, I 1 is equal to I 2 plus I 3 right. Now I 2 now for finding out the I or I 2, it is going to be V 2 minus V 1 divided by R. This is the value of I 2.

Now regarding the value I 1, I 3 I 3 is going through this. So, for I 3 it is going to be C D again current is flowing. So, it is not V 2 by minus 1, it is V 1 minus v 2, V 1 minus v 2, the current is flowing in this direction, so, potential difference, so V 1 minus V 2. So, C D again, V 1 minus V 2; this is I 3.

Now, if we combine these two, then we will get V 2 and V 2 is equal to I R and; V 2 is going to be in this case, now V 2 will also have certain resistance R. So, this system will also have certain resistance R. So, this we can take R o to differentiate between these two R and R o.

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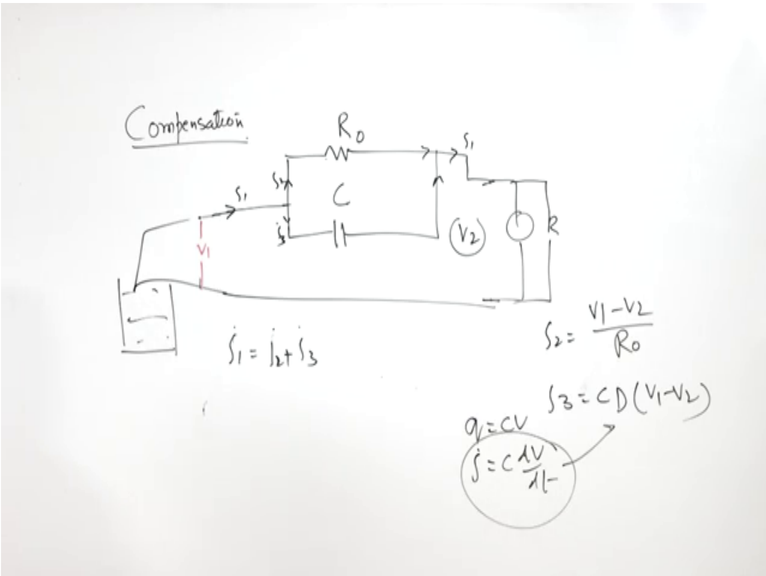

So, here, again I will write I 1, I 2 is going to be equal to V 1 minus V 2 upon R o and I 3 is going to be equal to C D V 1 minus V 2 because resistance of display unit also has to be taken into account right.

Now here, I is equal to, I 1 is equal to I 1, I 2 plus I 3. Now this D is differential of this. For a capacitor, for a capacitor q is equal to C V. So, I is equal to C dV by dt. From here we have got this. So, it is derivative of V 1 minus V 2.

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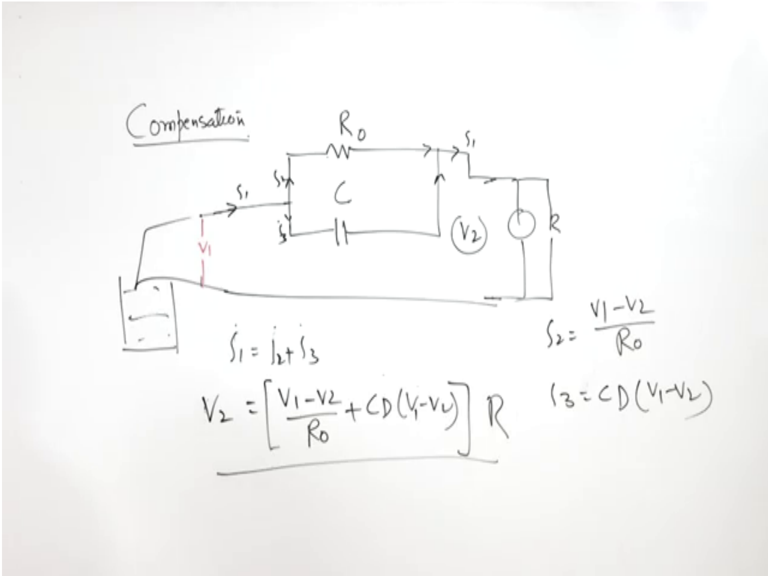

So, I 1 is equal to I 2 plus I 3 and this is going to be equal to V 1 minus V 2 upon R o plus C D V 1 minus V 2. And this V 1 is equal to this current multiplied by R, V is equal to IR this is not this is not V 1, V 2; V 2.

. So, what we have done here I am just repeating. This is current I 1 and it is splitted to I 2 and I 3. We have calculated the value of I 2 from this, resistance R o and that is V 1 minus V 2 upon R o. We have calculated the value of I 3 as C D V 1 minus V 2.

Once we have the value of I 2 and I 3, we are just added them and we got the window of I 1 and voltage V 2 is nothing but resistance multiplied by I 1 and this is the total resistance multiplied, this is the resistance and this is the current this is the final equation we have got.

Now using this equation, we can find the value of V 2 by V 1 using this equation, we can get the value of V 1 by V 2 by V 1 sorry V 2 by V 1 as alpha 1 plus lambda o D divided by 1 plus alpha lambda o D.

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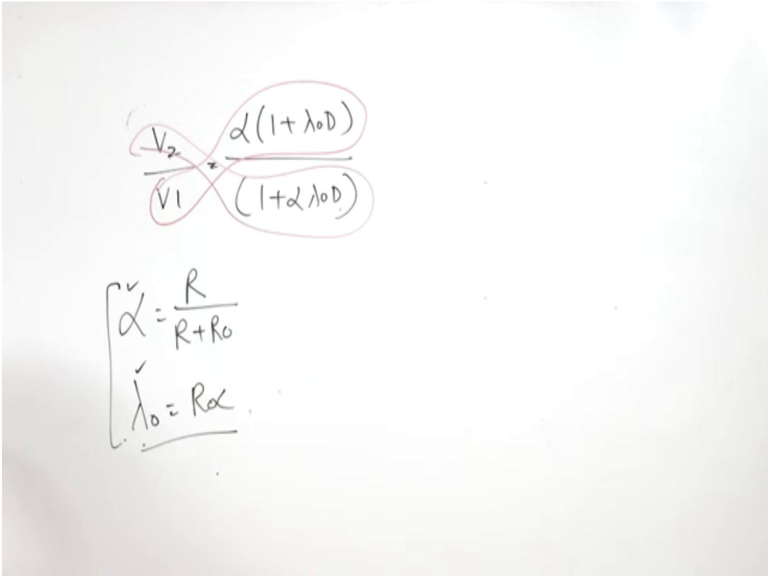

Now, alpha what is alpha? Now alpha here is, alpha here is R upon R plus R o, R is the resistance of display unit and R o is the resistance added we added through RC circuit and lambda o is equal to R o C. Now, if we have these two values, now we have these values because R and R o are known to us then we can find the value of lambda and here alpha is required and lambda o is required, both values are with us.

So, and this V 1 alpha lambda o D is equal to V 2 multiplied by this.

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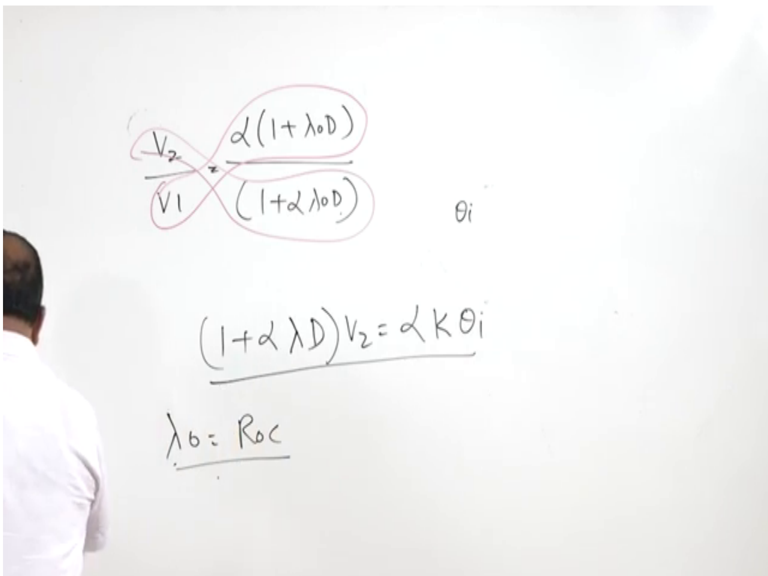

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Or we can write or we can write 1 plus alpha lambda D V 2 is equal to alpha K i oh q i or q i is let us say this temperature difference to theta i, it is temperature difference. So, it is theta i. So, this is the input value right and this is the response of the system lambda we choose, the value of because the lambda naught is R o C. So, lambda we should choose in such a manner that it is equal to the lambda o or if it is equal to R o C. So, we will put here lambda o. So, that we have to choose like that right and then we will get this response of the system.

Now, for this response of the system, here you can see, the time constant has been modified to alpha lambda o and alpha is less than 1. So, the response of the system has improved, but on the other side sensitivity of the system has been sacrificed here. So, we have improved the performance of the system. So, time constant has reduced, but it is the same time a static sensitivity of the instrument has also reduced.

So, it is always a conservation law which the applicable everywhere; if here giving somewhere, if you are giving somewhere, we are giving in the form of quick response of the instrument and we are sacrificing the sensitivity of the instrument. Now same thing can be done for second order system.

Now, second order system, if you want to improve the dynamic performance of second order system, then damping has to be reduced. The damping is the main issue in the second order instrument. Even if the system is under damped in this case is suppose the damping ratio is 0.05 or 0.1, a lot of oscillations are going to be there. So, that has to be compensated or it has to be improved or the dynamic performance of instrument has to be improved for a second order system.

Now, for the second order system, in the output form the primary sensing element or from the thermocouple, tip of the thermocouple, it will be the form of some millivolts and this output will go to now I will make a simpler circuit; this is R o, this is resistance we are adding and this is R, resistance of display unit and in series of that we take L and C.

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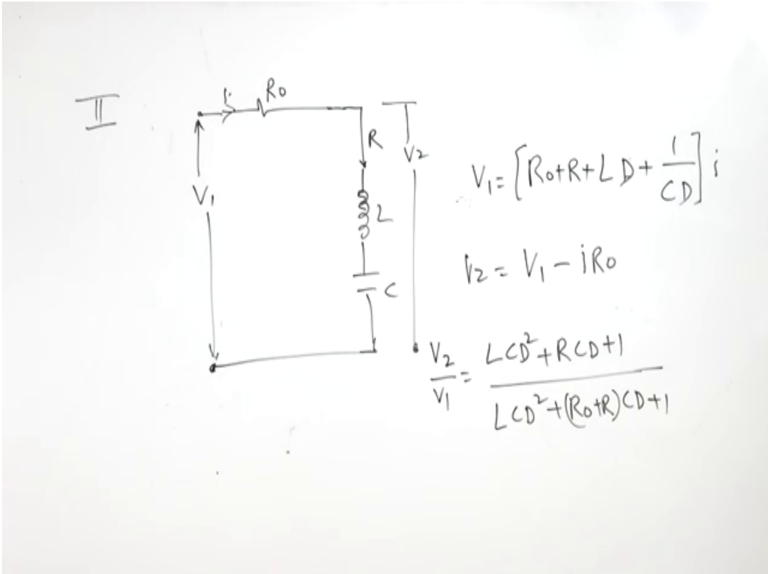

So, we make it in the RLC circuit and this is V 1 across this, we are getting V 1 and for V 2 across this, we are getting V 2. So, in the existing circuit, R o L and C are added.

Now, due to addition of this, the response of the system will change, the response of the system will change again. We will calculate the value of V 1. V 1 is there is a current right.

So, V 1 is R o plus R plus L C plus 1 by C D, L D sorry multiplied by L D and multiplied by i. So, this is the V 1 and for V 2 is equal to V 1 minus i R o or just we can remove this from here, then we will get the value of V 2.

So, now, we have two values V 1 and V 2. Now we can take the ratio of these V 2 by V 1. If we take ratio of these two V 2 by V 1, we are going to get LCD squared plus RCD plus 1 divided by LCD square plus R o R CD 1; if we take the ratio, the ratio is going to be like this.

Now, we have to further compare this, I will rub this off.

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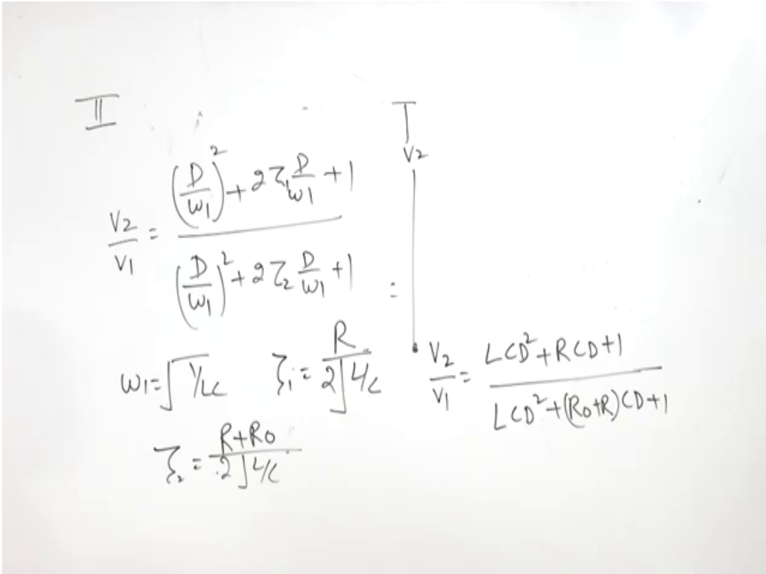

V 2 by V 1 is equal to D by omega 1 whole squared plus 2 2 zeta D by omega 1 plus 1. This zeta 1 and D by omega 1 square plus 2 zeta 2 D by omega 1 plus 1; only difference is this damping ratio.

Now, if you compare these two equations, we will get omega 1 as 1 by under root LC from here. And if we take the value of zeta 1 is equal to 2 by sorry 2 by not is 2 it is R, it is R by 2, R by 2 under root L by C. That is the value of the zeta 1 and for zeta 2, for zeta 2 is going to be R plus R o by 2 under root L by C right and now, if we write the response of the system.

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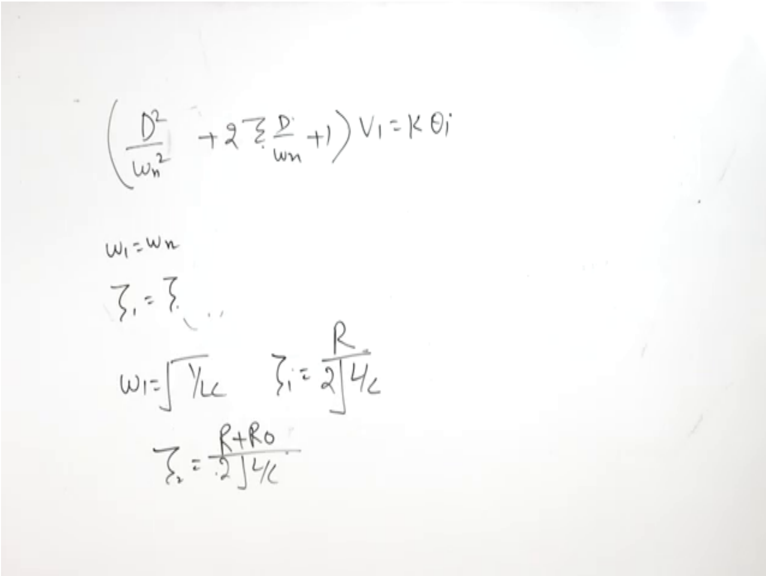

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That is initial response of system D by omega 1 whole square plus 2 zeta D by omega n or D square by or D square D square by omega n square plus 2 zeta D by omega n plus 1 into q o that is V 1 is equal to K, some theta i. This is the initial response of the second order system.

Now, here the omega 1 is equal to omega n. If we take omega one is equal to omega n, that is the one boundary condition and this is zeta 1 is equal to zeta right and then we can replace this equation for second order by this equation.

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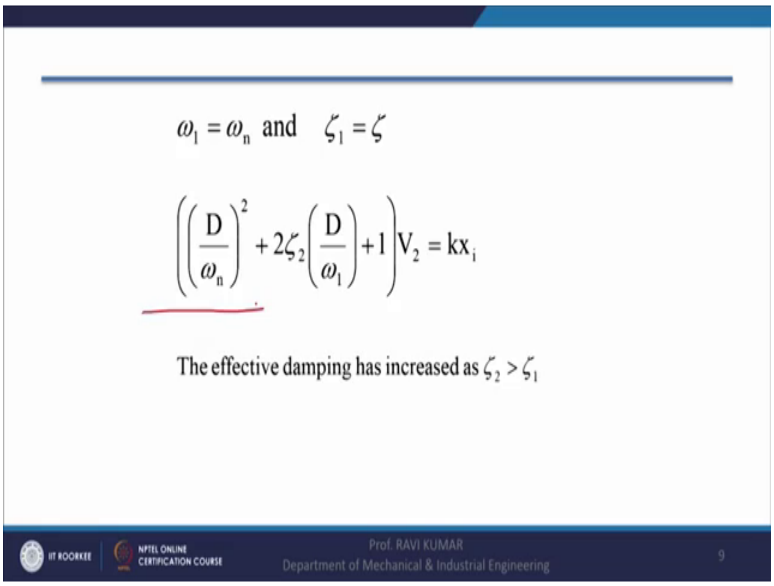

And here we can see that the effective damping, the zeta 1 is equal to zeta, omega 1 is equal to omega n and this equation can further be modified as this one and here the effective damping has increased from zeta 2 to zeta 1 to zeta 2 right.

So, here in this case, what does happen just by putting a LRC circuit, we have increased the damping of the system and these techniques this technique has been used or is being used in many of the electrical appliances. We are simply just by putting RC circuit LRC circuit, this we are using in measurement also; just by putting this RC circuit and RC circuit by putting RC circuit in the first order system, we have improved the response of the system because in the first order system, the main drawback or the main restriction in the first law first order system is the response of the system.

So, in order to make the response quicker, we have reduced the time constant. Now here in this case in the second order system, we have improved the or increase the damping ratio thus the system was under damped and the zeta venue was very low. So, in this case, we have just introduced the RLC circuit in the in the in the output and this RLC circuit has increase or increase the damping of the system. So, damping ratio has increased right and system has become more stable as far as the dynamic response is concerned.

Now, I will take one example of thermocouple because it is of interest for the mechanical instruments. A thermocouple with a time constant of 0.3 seconds and static sensitivity of 0.04 millivolts per degree centigrade, so it has time constant a thermocouple has time constant of 0.3 seconds and static sensitivity and let us say K 1 is equal to 0.04 millivolts per degree centigrade. This is for thermocouple.

It is suddenly immersed in bath of hot oil which is 800 degree centigrade. So, t 2 is 100 degree centigrade. The initial temperature of thermocouple measuring and reference junction was 25 degree.

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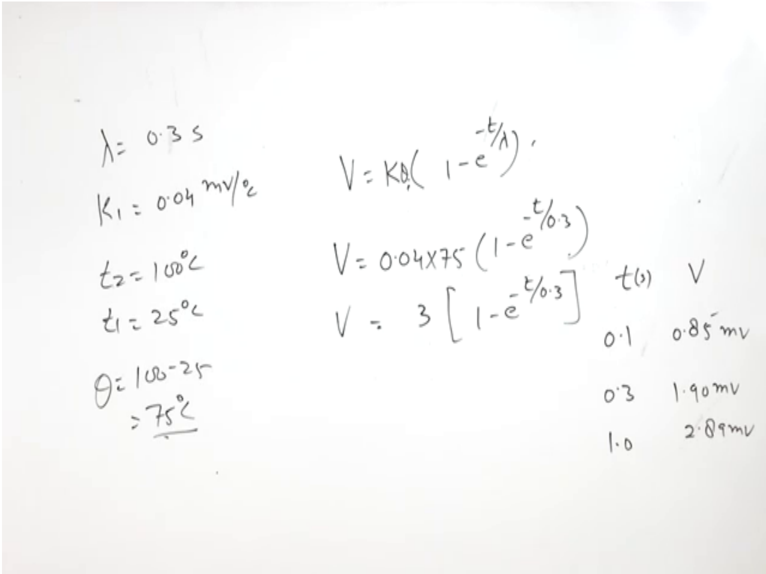

So, reference temperature is 25 degree centigrade. So, theta is equal to 100 minus 25 is equal to 75 degree centigrade right.

So, there is a input of 75 degree centigrade, what is the output at t is equal to 0.1, 0.3 and 1.

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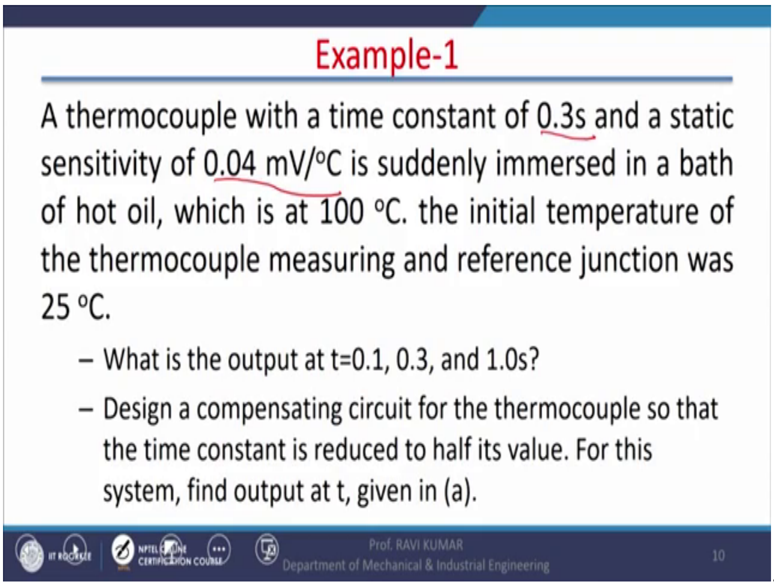

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Then it is being asked, design the composition circuit for thermocouples. So, that a time constraint is reduced to half it is value right.

So, we will start with the response of thermocouple. So, response of the thermocouple V is equal to K multiplied by e, 1 minus e raise to power minus t by lambda multiplied by K theta e, K theta e this is input, e raise to power minus t by lambda. So, V is equal to k is 0.04, theta is 75 1 minus e to power minus t by 0.3.

We further simplify this, this is going to be 150 300, 3, 1 minus e raise to power minus t by 0.3. So, this is going to be response of thermocouple. So, just we will keep on putting the value of t and we will be getting different values of output.

We will note it down somewhere. Let us noted down here when t and output t is 0.1, output is 0.85, t is 0.3 seconds, output is 1.90 millivolts, this is millivolts and t is equal to 1.0 seconds, then output is 2.8 time millivolts.

So, these values we have calculated just simply putting the value of t here. When we are putting t is equal to 0.1, then it is shift when t is equal to 0.1, then it is 0.85; it is correct. So, similarly we can find when the values of V by putting different values of t right.

And then compensation circuit comes into the picture. Now we will compensate this with this is the first order system. So, RC circuit will be applied. So, if you are compensating this with RC circuit and time constraint time constraint has to be reduced by 50 percent.

So, it means alpha is equal to 0.5 and lambda o is equal to R o C. This equation it is well read the derivations, we did it we did this equation and this is equal to lambda is original lambda this is 0.3. So, R o C has to be 0.3 because here lambda is 0.3, initial value of lambda. So, that is equal to R o C.

An alpha is equal to 0.5. So, alpha is equal to 0.5 is equal to R divided by R plus R o. The two equations and in these two equations they are 3 unknowns, in these 2 equations there are 3 unknowns; I mean one is R o C.

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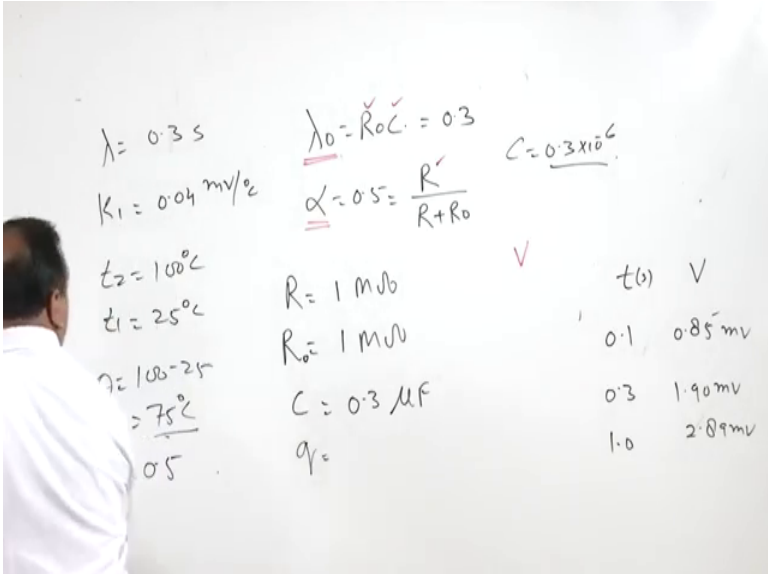

And third one is R, all 3 are unknown because we know only time constant of thermocouple and this K 1 and this is a step input which is given and we want to improve the response of thermocouple by 50 percent or time constraint has to reduce by 50 percent alpha, we have taken 0.5.

Now, we have three unknown, two equations. So, only when therefore, this is trial and error method. We will keep on assuming one value and then we will get two simultaneous equations for two values and then trial and error method, we will do that and suppose R is equal to I will I will just coming to the final value mega ohm, then R o is going to be how much 1 mega ohm, 0.5.

When R o is 1 mega ohm, then C is going to be how much; 0.3 farad micro farad. This is 1, this is 1, 1 divided by 1 plus 1, 0.5. R o is 1 mega ohm and this is 0.3; so, so C is equal to 0.3 into 10 to the power minus 6; so, c is 0.3 micro farad.

Student: R o (Refer Time: 25:16).

So, I am repeating here. First of all we have assume the certain values of R because there are two simultaneous equations and three unknowns. We have assumed the value of R is 1 mega ohm R o is also 1 mega ohm because the ratio of R and R plus R o is 0.5 V and C, the value of C will get here; it is 0.3 micro farad.

So, we have all 3 values. Now for all these 3 values again, we will calculate the value of voltage with the revised values, revised value of lambda and this is alpha. So, lambda will be L multiplied by alpha and static sensitivity will also be multiplied by alpha. So, static sensitivity will be reduced. So, here it will be 0.02 right.

But the same time the lambda will be changed to 0.15 and the response of the system will be q is equal to alpha K 1. So, alpha, so first of all I will write the governing equation then final equation.

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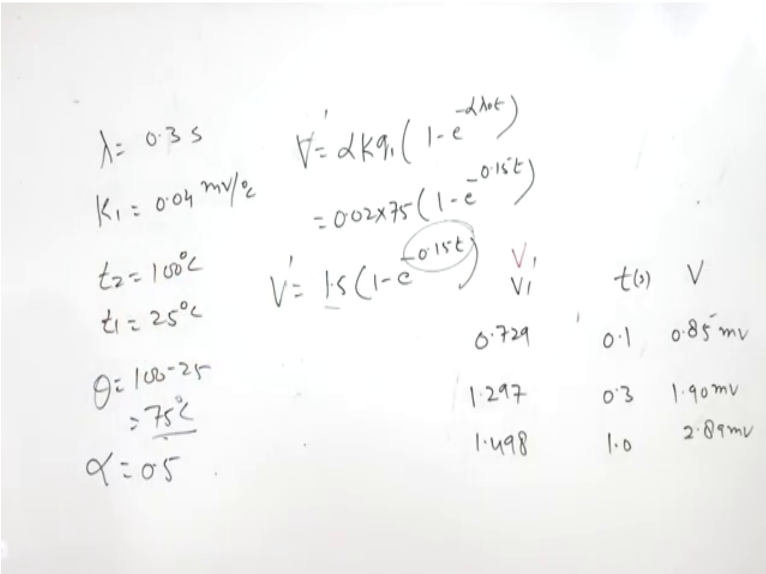

So, q o is V, V is equal to alpha K q i 1 minus e to square minus alpha lambda o t, alpha is here 0.5 and K is 0.04. So, 0 0.02 multiplied by theta it is 75 1 minus e raise to power minus alpha lambda o, then it is 0.15 t or you can see here that the final version of the voltage is unusually it was see, so 1.5 1 minus e raise to power minus 0.15 t.

Now, here again, we will be putting the different values of t when we are V dash, let us say this is V dash. So, we will be getting certain values of V dash for the same values of t and once you calculate these values, it is going to be 0.729 this is for V 1 0 point sorry 1.297 and 1.498.

So, output is affected, but at the same time system, this can be further calculated right, but the same time response of the system has improved response of the system has improved because the time constraint has been changed from 0.3 to 0.15.

So, we have found this in this numerical output was required. So, output we have calculated for compensated circle and; obviously, the output for compensated circle is significantly lower than the output of uncompensated circle because the sensitivity of the instrument is compromised in order to improve the a dynamic performance of the system. That is all for today.

Thank you very much.