Mechanical Measurement Systems
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Lecture - 40
Problem solving- 2

Hello I welcome you all in this course on mechanical measurement systems, today we will solve a few new miracles related with the mechanical measurement system. Now starting with the numerical based on the first order system, the statement of the numerical is a signal described by the relation theta is equal to 3 sin 2 t.

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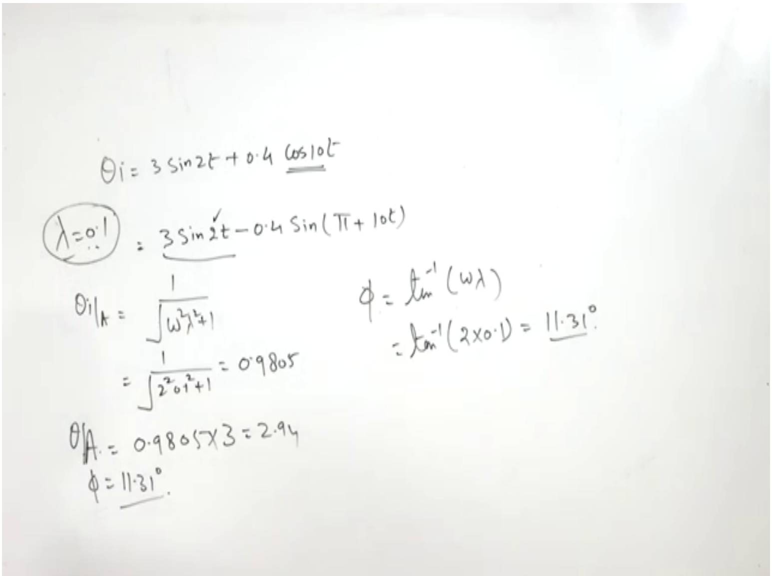

This is theta I is equal to 3 sin 2 t plus 0.4 cos tan t this is a input signal. It requires to be measured by using a first order system, having a time constant of 0.1 second. So, lambda is 0.1 develop expression for the corresponding output. So, first order system is being used for the measurement and we have to write equation for the response of the instrument. First of all here when we do this first of all first order analysis of this type of input cyclic input we add only sine equations. So, cos has to be converted into the sin 3 sin 2 t plus 0.4 or we can write minus 0.4 sin pi plus 10 t right lambda is equal to 0.1.

Now, let us take theta 1 A. Now for this component the amplitude ratio is 1 by under root omega square lambda square plus 1 right and here in this case the omega is 2 omega is 2 in this case omega is 2 lambda is 0.1 right. So, amplitude ratio is going to be under root 2 square 0.1 square plus 1 and that is going to be equal to 0.9805. It means the response of the system for this input was for particularly for this input the amplitude is going to be 0.9805 into amplitude here is 3 it is going to be 2.94 ok.

And regarding the phase lag phi, phi is tan inverse omega lambda this is phase lag or phase angle is tan inverse minus omega lambda either way we can write. So, phase lag is tan inverse 2 into 0.1 it is going to be 11.31 degree. So, here. So, a 1 amplitude of A and this is phase angle is 11.31 degree.

Similarly, we can do for this, this is part A now for part B amplitude ratio for part B it is going to be again 1 by 1 plus omega lambda whole square under root omega lambda whole square under root.

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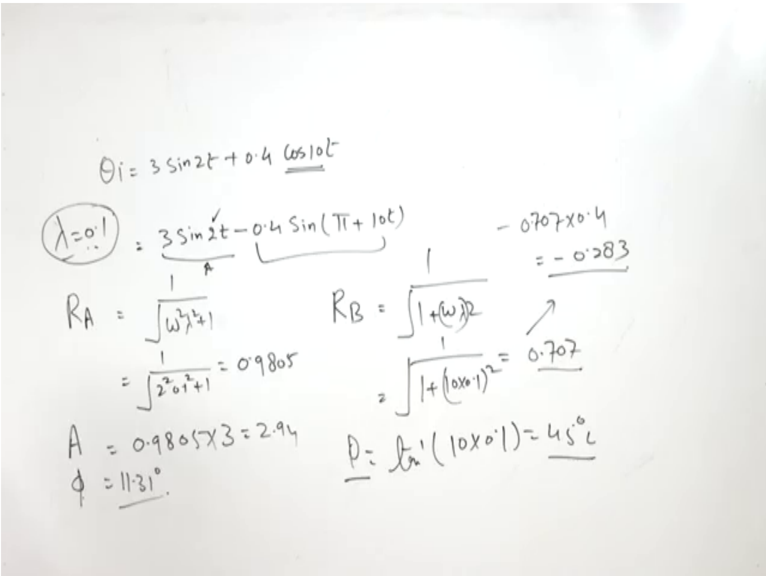

Now here omega is that and lambda is 0.1. So, it is going to be under root 1 plus 10 into 0.1whole square and that comes to be 0.707.

So, here amplitude is attenuated by its 0.707 and here it is 0.98 right because frequency is high. So, when the frequency is high that amplitude ratio will be low phi phase angle is equal to tan inverse omega lambda what is omega 10 into 0.1 is equal to 45 degree and this because it is amplitude is an end attenuated by a 707. So, 0.707 multiplied by 0.4 will give the amplitude and that is going to be equal to this is minus 4. So, this is minus.

So, minus 0.283. Now we have amplitudes and phase angle for or phase lag of responses in both the cases.

Now, here we can conclude that here the amplitude ratio amplitude ratio of A let us say R A is this much amplitude of ratio A is this much and this will give the amplitude of A dash or a this is phase lag of A, this is part A, for part B this is amplitude ratio R B right and amplitude in the part B is this and phase angle in part B is this.

Now, we have 4 values for amplitude this is for amplitude phase and this one is for amplitude and this is for phase angle right and we can find we can write the finalized equation as theta o that is output. Now this is input signal now the output signal is going to be 2.94 sin 2 t minus 11.31 this is the phase let minus 0.283 sin 10 t plus pi minus 45 degree right.

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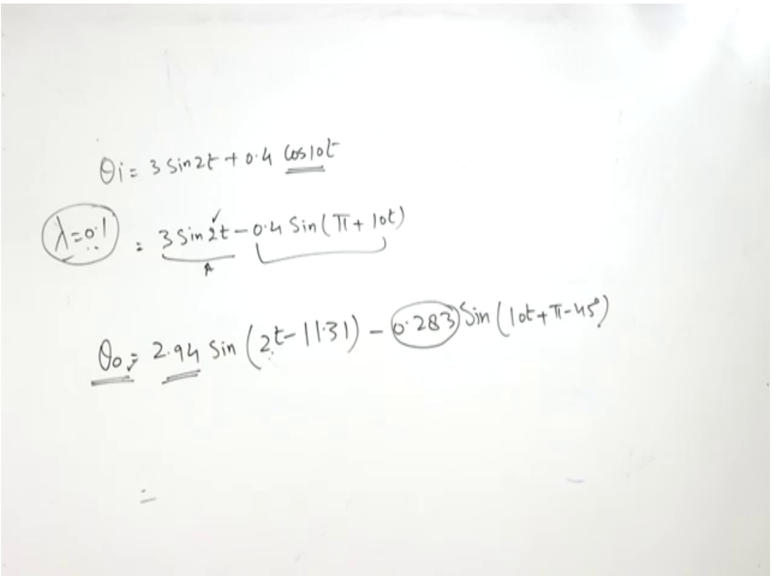

And now this is the final response; now here we can see when the omega is high the amplitude ratio of output and input is low when the omega is low when omega is only 2 the ratio has gone up to 0.98. Now this is an observation in this case. So, lambda is equal to 0.1 in this case will give large measurement errors right larger measurement error.

So, now we will take another example, this is for the second order system. During the act of force measurement by a pressure transducers of diaphragm type.

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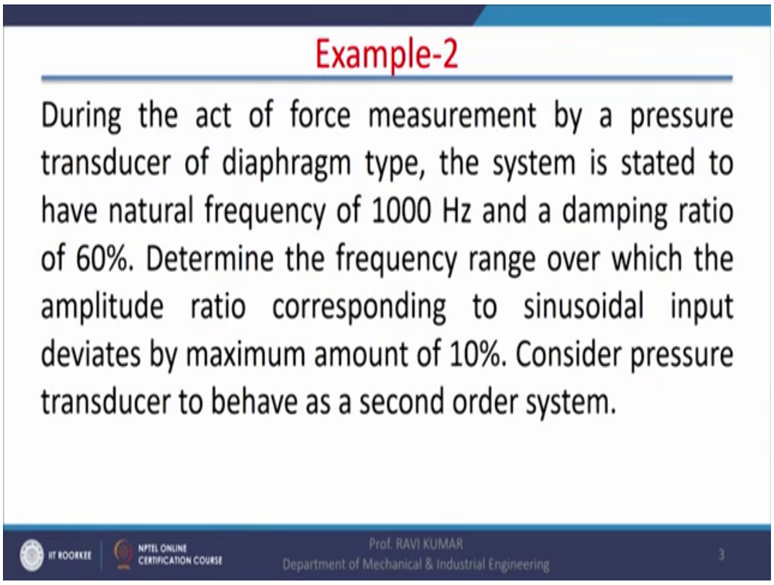

So, during the act of force measurement by a pressure transducer of diaphragm type the system is stated to have a natural frequency of 1000 hertz. So, natural frequency of the system is 1000 hertz and damping ratio 60 percent. So, damping ratio 60 percent means zeta is equal to 0.6.

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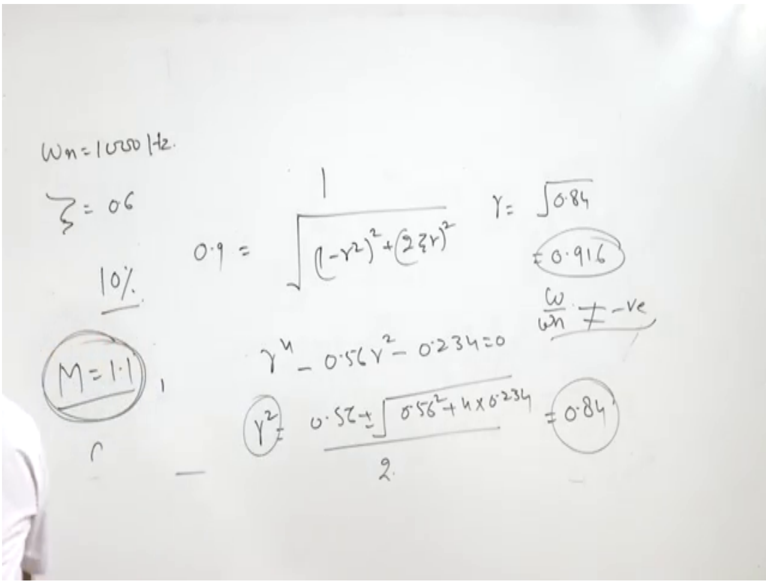

Determine the frequency range over, which the amplitude ratio corresponding to sinusoidal input deviates by maximum amount of 10 percent. So, division is 10 percent in amplitude, for sin response for the sinus order input.

So, consider transducers to behave with to consider pressure transducer to behave as second order system right. So, amplitude ratio in a second order system the amplitude of a second order system can be expressed as 1 by this is the amplitude ratio, this m is amplitude ratio is 1 minus r square whole square plus 2 zeta r whole square, where r is equal to omega by omega n.

Now, if we consider amplitude ratio as 1 by 1.1 right here because it is a if we have to go for the this amplitude ratio 1.1. So, when it is 1.1 then we can put here M is equal to 1.1 and try to find the value of r because eta is already known it is 0.6 right. If you simplify this equation this equation becomes r raised to power 4 minus 0.56 r square plus 0.174 is equal to 0.

Now, here if we take the solution of this equation, it has or both imaginary roots right both imaginary roots. So, it implies that that this is not a possible solution means we cannot go for M is equal to 1.1. Here also it is stated that the damping ratio is 0.6. So, for damping ratio 0.6 output will never be 1.1, because we are getting roots here all there are imaginary roots right. So, the if there is a there is a response of the system. So, rest of m is not exceeding 1 it means. So, we should go for the 0.9 because deviation can be on both sides plus 10 percent to minus 10 percent.

So, here m we can put 0.9, once we are putting m is equal to 0.9 then we are getting another equation then this equation is about r raised to power 4 minus 0.56 r square minus 0.234 is equal to 0.

Now, let us take solution of this equation, this is r square is equal to 0.56 plus minus under root 0.56 square plus 4 into 0.234 divided by 2, and this will give the r value r square value as 0.84. So, we have replaced 1.1 by 0.9 and the equation is modified like this r square we are getting 0.84. So, r is equal to under root 0.84 and it is going to be 0.916. We will not take negative because w by w omega n omega can never be negative right.

So, this is r is 0.1916.

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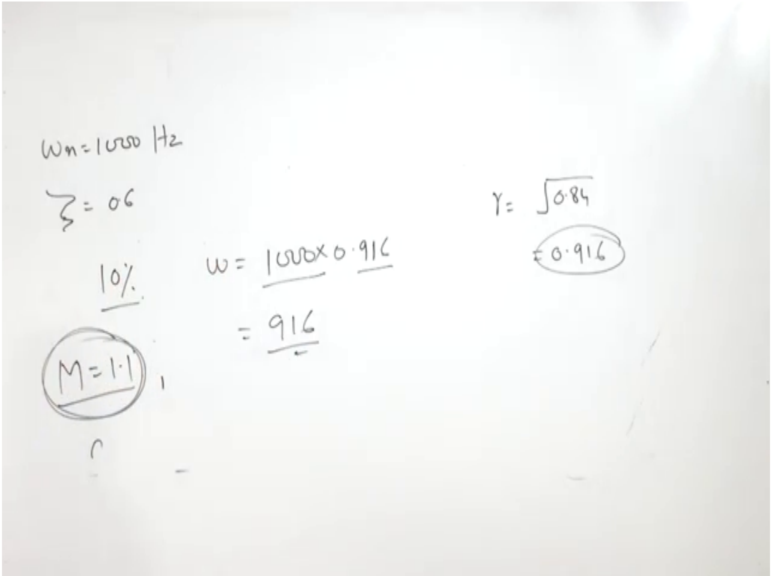

So, 0.916 means if you multiply this by 1000, we will getting the omega s 1000 multiplied by 0.916 it is going to be 916 this is not hertz cycles per second. So, it is 916, 916, 916 it is 916 cycle per second. So, this omega n is 1000 hertz. So, [FL]. So, omega is 1000 hertz. So, this 9.916 is multiplied by hertz and we are getting 916.

So, this is the answer for this numerical. Now another numerical will take up about the estimation of error, this is the power measurement is to be conducted by measuring voltage and current across the resistor r.

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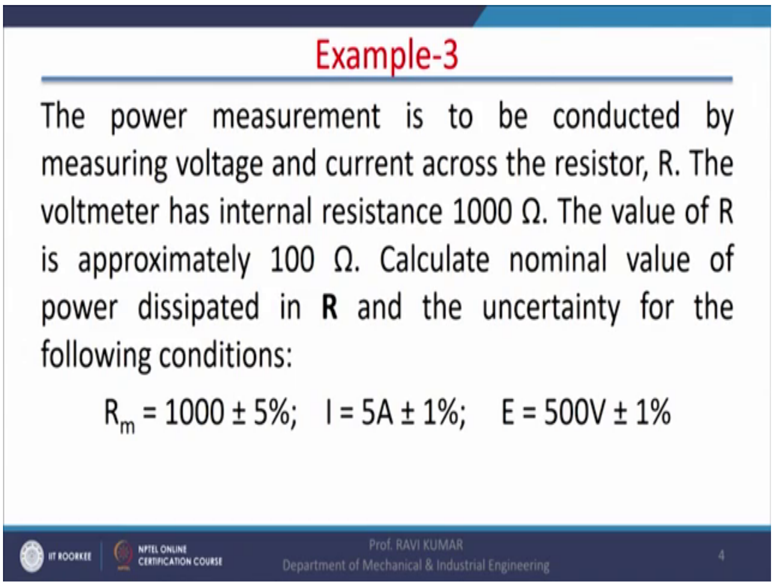

So, there is a resistor R, voltage and current has to be measured across the resistor R. So, there is a emitter right R parallel to R there is a voltmeter and voltmeter will have certain resistance right.

the voltmeter has internal resistance 1000. So, this R m is 1000 ohms ok. Calculate the nominal value of the power dissipated in R and uncertainty for the following condition. So, how much power is dissipated in R?

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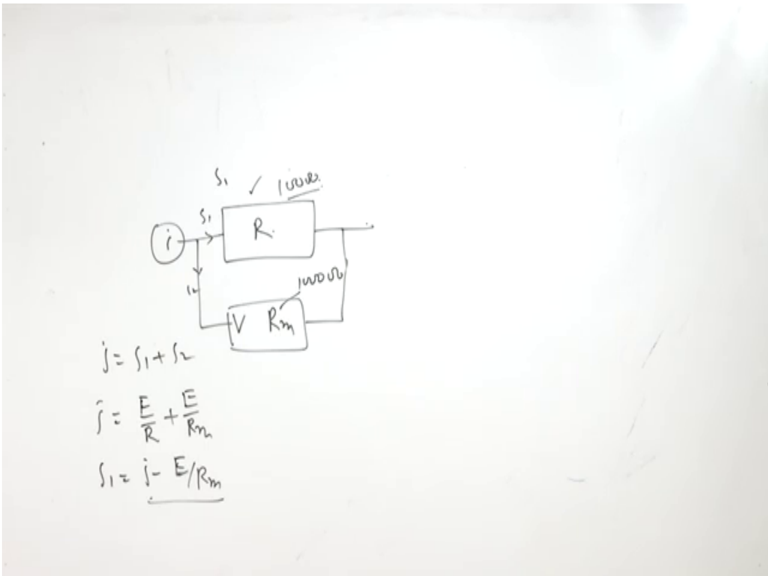

So, i is equal to let us say that, the current is splitted in 2 parts i is equal to i 1 plus i 2 this is total current it is i 1 and this is I 2 and potential difference between these 2 is E. So, i is equal to E by R plus E by R n because they are connected in parallel. So, E is will remain common or we can say that current here i 1 or E by R i 1 is equal to i minus E by R m right because R we do not know r we do not know, but we have measured the I 1 right.

So, E is the value of E is with us, value of i is with us and value of R m is with us. So, we have made 2 measurements, value of R is approximately it is approximately 100 ohm it is approximately not correct one right calculate the nominal value of power dissipated in the R what is that nominal power dissipated in the R. So, for dissipation of the power and R m is when R m is 1000 plus minus 5 percent I will right here R m is 1000 plus minus 5 percent ohms current is 5 ampere plus minus 1 percent and E is 500 volts plus minus 1 percent ok.

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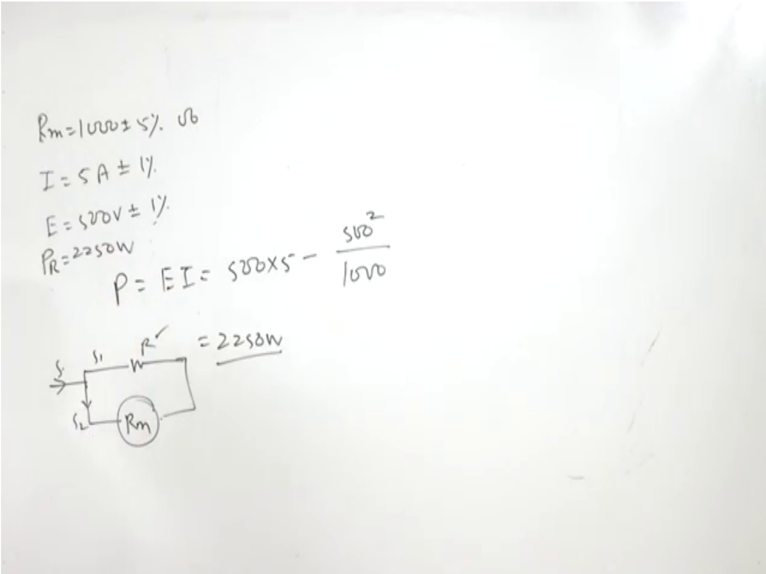

So, P is equal to simply E I. So, E is 500 into 5, 500 into 5, but this I is shared between because this I is splitted it is going to R m and 1 I is going to the resistance all I is not entering here it is I 1 and I 2. So, power dissipated will be by I 1 only. So, P is going to be this minus 500 square by 1000 E square upon r this power dissipated by this will be subtracted.

So, final power is going to be 2 2 5 0 watts that is a power dissipated by R. So, P R is 2250 watts. Now we have to find uncertainty in uncertainty in the measurement.

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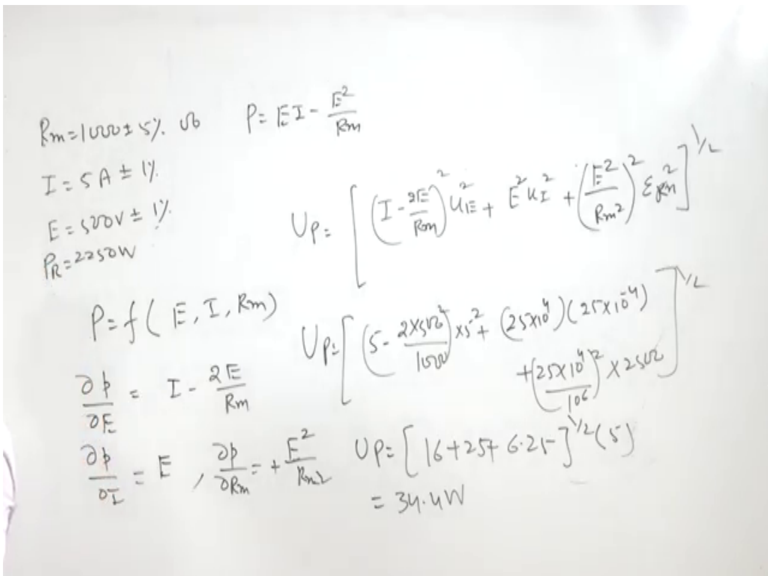

So, power is a function of here the power is a function of E I and R m because we calculated P is equal to E I minus E square upon R m. So, it is a function of E I and R m right. Now we will take del p by del E now the equation remains same E Ip is equal to E I minus E square upon R m right now del p by del E is going to be equal to I minus 2 E by R m. Now del p by del I is going to be E and del p by del R m this is going to be this is 0 minus E square by R m square and this will become plus.

Now, uncertainty in the measurement of power Up. Uncertainty in the measurement of power is going to be I minus 2 E by R m whole square, uncertainty in the measurement of E square plus E square q I square right and we will use plus this is E square is del p by del a uncertainty in the measurement of I, then del p by del R m square. So, E square by R m square whole square E square r square whole square and then error in R m whole square raise to power 1 watt.

Now, we can put because respective values are known to us. So, here we can put the values uncertainty in the measurement of power is 5 minus 2 E, 2 times E is 500 divided by R m 1000 into 5 square because ue 1 percent. So, 5 100 volts and 1 percent of 5 volts is 5 ok. It is E square is 25 into 10 to power 4 into 25 into 10 to power minus 4 plus 25 into 10 to power 4 divided by 10 to power 6 whole square into 2500 raise to power 1 by 2.

If we simplify this then we will be getting U p is equal to 16 plus 25 plus 6.25 raise to power 1 by 2 multiplied by 5 and final uncertainty is 34.4 watts.

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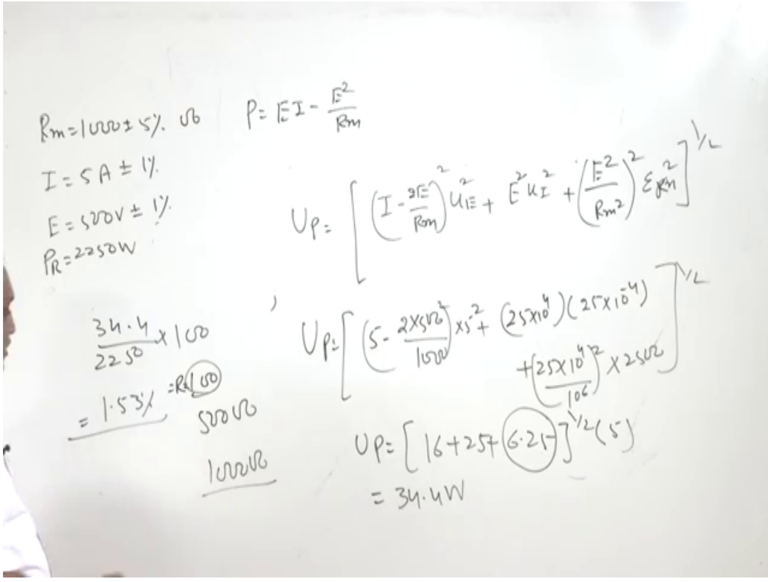

If you take percentage wise now percentage wise uncertainty is 34.4 divided by 2250 into 100 and that is going to be 1.53 percent.

And uncertainty in the measurement is lowest in or bearing of the error in the measurement resistance of the voltmeter is minimum. So, if we are now here we are measuring 100, R is equal approximately R is equal to 100 100 ohms approximately and voltmeter resistance is 1000 volts 1000 ohms.

If we were measuring R of 500 ohms and voltmeter resistance is 1000 ohms then this error would have increased because substantial part of the current would have been going to the voltmeter. Contrary to this you are measuring 5 volts 5 ohms and the voltmeter resistance was 1000 volts, this would have further reduced ok. So, this is about this example of error analysis.

Now, one more example will take about the design of venturimeter.

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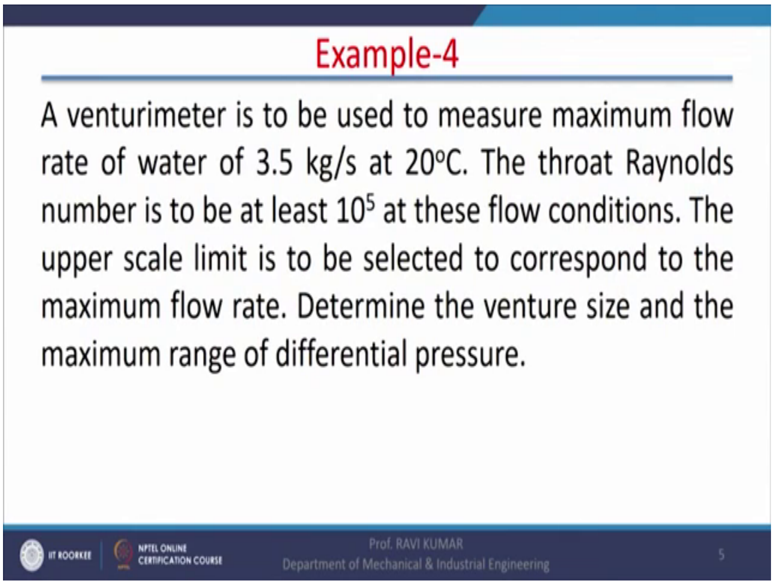

Venturimeters are used for the flow measurement for metering the flow through a pipe. In this example a venturimeter is to be used to measure maximum flow rate of water of 3.5 kg per second that is a maximum flow rate 3.5 kg per second.

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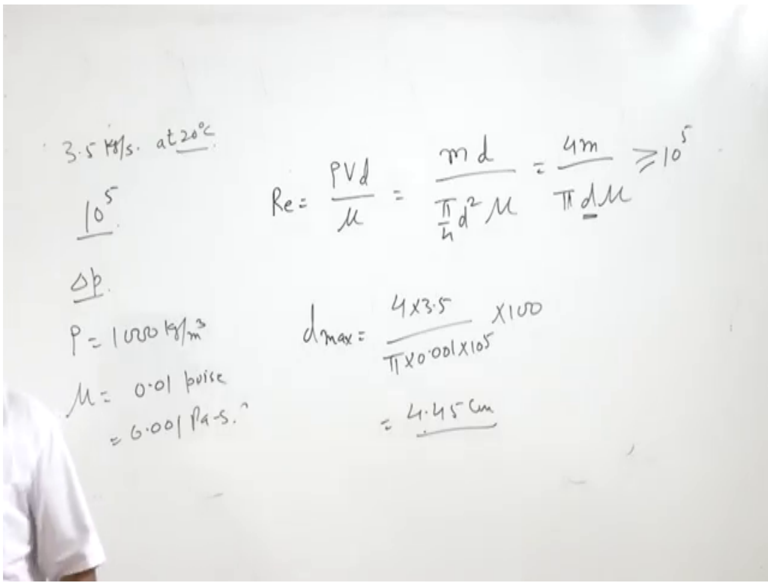

That is a mass flow rate of water at 20 degree centigrade. So, temperature is also given at 20 degree centigrade right.

The throat Reynolds number is to be at least 10 to power 5 minimum Reynolds number at throat has to be 10 2 power 5, it can be 10 to power 6, 10 to power 7, to power 10 to power 8, but minimum Reynolds number has to be maintained 10 to power 5, the upper scale limit is to be selected to corresponding to the maximum flow rate, determine the venturi size and the maximum range of differential pressure. So, venturi size and the maximum range for differential pressure.

So, it is a little design problem, is not totally numerical it is a little design problem because here some informations are to be required. For example, density of water density of water let us assume 1000 kg per meter cube density of the water. Viscosity of the water is 0.01 poise or 0.001 pascal seconds and Reynolds number here. Reynolds number here the Reynolds number are going to be rho V d b by mu is the viscosity v is the velocity of water in the pipe, d is the diameter of the pipe, and rho is the density of the pipe.

And we know that V d sorry V A and rho is mass flow rate by continuity equation and A is pi by 4 d square. So, Reynolds number can be written as mass flow rate multiplied by d divided by pi by 4 d square mu. So, m by a will give the rho v. So, rho v is replaced by m. So, this will give 4 m divided by pi d mu and that has to be greater than or equal to 10 raise to power 5.

Now, here mass flow rate is given 3.5 kg per second right mu is all also with us right and what is required d? D we can find from here. So, d in order to find d that is d maximum is equal to 4 into 3.5 divided by pi into 0.001 into 10 to power 5 into 100 is equal to 4.45 centimeters we have converted meter was normally the pi diameter is the order of a few centimeters. So, we have converted meter into the centimeter it is 4.45 centimeters.

If you reduce the diameter the Reynolds number will increase, because not go by this formula mass flow rate. Because here if you reduce the diameter it appears the Reynolds number will reduce, but when you are reducing the diameter, velocity will increase if you go by this formula mass flow rate is constant. So, if you reduce the diameter Reynolds number will increase.

So, in any case we take the diameter is smaller than this, the Reynolds number will be more than 10 to power 5, but this is not a standard size in the market 4.445, it is difficult to get such type venturi. So, pipe into throat section if we go for the standard venturi it is could be going to be 5 centimeter in 2.5 centimeter.

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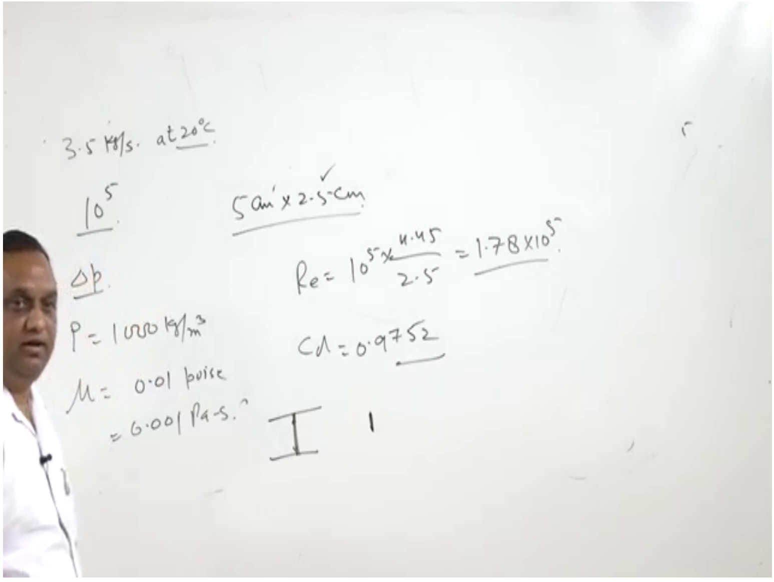

So 5 centimeter into 2.5 centimeters and if you go for the throat diameter 2.5 centimeter definitely the reynolds number will be higher than 10 to power 5 right.

Now, with this selection of 2.5, the Reynolds number will be modified as 10 to power 5 into 4.45 divided by 2.5, 1.78 into 10 to power 5. This condition is satisfied for this venturi coefficient of discharge is going to be 0.9752 for this reynolds number 57, this is 9752. Now we have coefficient of discharge a 1 area here which is which is the diameter of the pipe is 5 centimeter, area 2 of diameter of the pipe 2.5 centimeter, coefficient of discharge is already with us we can make use of this equation for the actual mass flow rate through the venturimeter that is rho cd A 1 A 2 divided by under root A 1 square minus A 2 square under root 2 g P 1 minus P 2 divided by rho g right.

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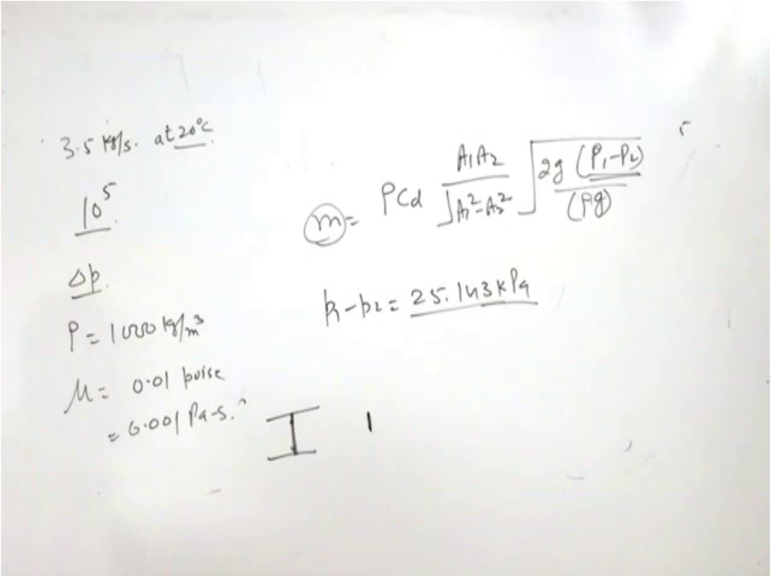

G and g will be cancelled ok. So, this is the mass flowrate actual mass flowrate from this venturi meter, A 1 A 2 we can easily calculate because the diameter of 1 pipe is 5 centimeter another pipe diameter is 2.5 centimeters, the rest of the values are known, but we do not know the value of P 1 minus P 2. So, this is only unknown in the entire equation and if you put the values with respective values, then you will be getting P 1 minus P 2 as 25.143 kilopascal.

So, whatever value you get from this equation divided by 1000, then you will be getting the pressure drop as 25.143 kilopascal right. So, this is the actual design of the venturi for the flow measurement, pipe diameter 5 centimeter venturi throat diameter is 2.5 centimeter and pressure drop is going to be 2.5143 kilopascal for the flow measurement of 3.5 kg per second. I hope you have enjoyed this course and in the following week you will have exam of the course, best of luck for examination.

Thank you.