

**Acoustic Materials and Metamaterials**  
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**Lecture - 18**  
**Helmholtz Resonators**


Welcome to lecture 18 of our series on Acoustic Materials and Metamaterials. So, in this week we began our discussion about acoustic materials and then we studied about porous sound absorbers and panel absorbers. So, panel absorbers were basically designed to reduce the limitations of the porous absorbers because porous absorbers they are inefficient in low frequencies, but they provide a broadband high frequency absorption.

So, with the same principle as panel resonators, now we I will discuss with you in this particular lecture on Helmholtz Resonator and we will study about the working principle of this resonator. So, let us begin our discussion.

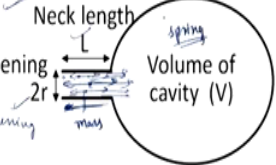
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### Helmholtz resonator

- **Helmholtz resonator** consists of an acoustical cavity contained by rigid walls and connected to the exterior by a small opening called the neck. It is a special type of air-spring oscillator.
- Conditions:  $V \ll \lambda^3$   
 *$d \ll \lambda$  [spherical cavity] = radius of opening*  
 *$l, b, h \ll \lambda$  [cuboidal cavity]*



Hermann L.F. von Helmholtz



A Helmholtz Resonator

So, Helmholtz resonator is like a acoustical cavity that is enclosed by rigid walls. So, you have a big volume of cavity it can be the circle spherical in shape, it can be rectangular in shape or any other shape desired and then that cavity is then exposed to the outside environment by a small opening or an orifice called neck. So, if you have a look at the figure here. So, this is the circular cavity.

So, here you see that this is the volume of the cavity, it is enclosed within this volume and then you have a opening, this is called as the neck and this is the neck length  $L$ , and this is the diameter of this opening the radius being,  $r$  being the radius of the opening and the diameter becomes twice the radius of this opening. So, any such kind of cavity that can be made will be called as a Helmholtz resonator.

And this particular resonator is a special type of air spring oscillator and from now on we will study about Helmholtz resonator, then we will study about pan perforated panel absorbers and then we will continue our discussion into micro perforated panel absorbers. So, the remaining discussion we do in the field of traditional acoustic materials. All of this will employ the same principle as a Helmholtz resonator. So, this is an important topic. And all of them will be air spring oscillators, and the meaning of that will be clear as I explain about it.

So, the first condition before we begin to study such absorber is that the dimension of the absorber has to be smaller than the target wavelength or the wavelength which we want to target. So, if it is a spherical cavity then the volume is directly proportional to  $d^3$ . So, the diameter of the spherical cavity has to be smaller than the wavelength. Similarly, if it is a rectangular cavity then the length breadth and the height of the rectangular cavity has to be smaller than the wavelength.

So, if it is a spherical cavity, then the diameter has to be smaller than the wavelength and if it is a cuboidal sort of cavity then its individual dimensions, they all have to be much smaller than the wavelength. Now, as you know that the wavelength is inversely proportional to frequency. So, if we construct a small cavity then in that case the frequencies  $\frac{1}{\lambda}$ , so it will be able to cut or be effective for a frequency for the incident sound whose wavelength is larger than the dimensions of the cavity.

So, all the larger wavelengths which means if you convert it into frequency domain which means that all the frequencies which are so, which are smaller than what it is set out for. So, usually it is used for low frequency absorption. And the scientist who proposed this resonator was Hermann Von Helmholtz. So, he was he was the one who came up with this concept of Helmholtz resonator. So, let us see how this works.

Now, here I will again go back to this figure to explain. So, I had explained to you what is acoustic coupling and how acoustic coupling helps in a very high absorption. So, with the panel resonators we studied that let us say when the when the frequency of the sound wave matches with the natural frequency of the panel in that case the couple acoustically. So,

coupling always takes place when both the frequencies are same. The incident or the driving frequency and the natural frequency of the system that is being driven.

So, when the driving frequency becomes equal to the frequency of the driven system acoustic coupling takes place, and similarly for panel resonators whenever the room modes matched with the panel resonators natural frequency they coupled and then it was as if the incident sound energy is being used to drive the panel and in that way a lot of sound energy was used up in doing work against the panel. The same way this also works.

So, the way it works is that whenever the incident sound frequency becomes equals to the fundamental frequency of this Helmholtz resonator, then some sort of acoustic coupling takes place and whatever is the incident sound energy then will be used to directly drive the air molecules back and forth through this Helmholtz resonator.

So, there will be, they will, so the incident sound energy will be lost in doing all the work against the air molecules to drive the air molecules through the neck of the Helmholtz resonator. So, here effectively this is the kind of oscillations that take place in the particles here. So, they do back and forth motion. So, the incident sound frequency it causes large amount of such back and forth motions throughout the neck, so the cavity is going inside the, so the air volume inside the neck is going into the cavity and then coming back again into the cavity.

So, this is the sort of sound, this is the sort of oscillations that are taking place and at the resonant frequency they are maximum the coupling takes place these oscillations are done. So, the incident sound energy is driving these air molecules and hence a heavy absorption will take place lots of energy will be lost.

So, that is what I have explained to you, that whatever is the incident sound waves they will cause air molecules in the neck of the cavity. So, here it is the air molecules inside the neck of the cavity which becomes the mass of the system. In the panel case it was the mass of the panel.

So, here air molecules they are the ones that are in the neck that will start to oscillate back and forth when the acoustic coupling takes place, so they constitute the mass and the incident sound energy will be lost in driving this mass. And what is the restoring force here?

So, as you see when the air molecules they go inside the cavity they when they are oscillating towards the cavity then some mass addition will take place, so a little bit of, so the inside volume will be compressed or the density will slightly increase for the cavity inside. And when the air molecules they go outside then the density of the cavity will decrease slightly or you can say the inside volume is undergoing an expansion the inside cavity.

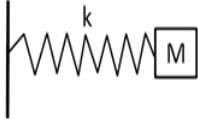
So, the compression of the gas molecules and then the expansion of the gas molecules takes place periodically as the mass inside the neck keeps oscillating back and forth. So, it is and because they are resistant to this compression and expansion, the particular volume inside there we have the air inside the cavity, so due to the bulk modulus of the air it is resistant to this compression and expansion. So, this acts as the restoring force or this acts as the spring element.

So, this one becomes this spring element and this is the mass element, so this is the volume in the mass of the air inside the neck is actually being driven or a oscillating back and forth. And it is and the opposing force or the restoring force is provided due to the resistance to compression and expansion of the air molecules inside the cavity.

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### Working principle of Helmholtz resonator

- Incident sound waves causes air molecules in the neck to vibrate back and forth, while air inside the cavity provides the restoring spring force.
- Helmholtz resonator behave like an acoustical mass-spring system, where
  - air mass in neck = mass
  - air bulk modulus in the cavity = spring
- This oscillator has its unique fundamental frequency.



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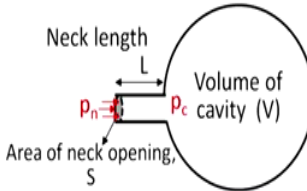
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So, this becomes the air spring oscillators. So, it behaves as a oscillator and acoustically couples at its fundamental frequencies. So, let us find out what are what is the fundamental frequency of this Helmholtz resonator.

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### Natural frequency of a Helmholtz resonator

- In steady state  $P$  and  $V$  inside the cavity remains spatially uniform and varies only with time.
- Acoustic pressure inside the neck and inside the cavity is denoted as:  $p_n(x,t)$  and  $p_c(t)$
- As air oscillates in and out in along the neck, the acoustic pressure causes air in the cavity to compress and expand.



The diagram illustrates a Helmholtz resonator. It consists of a neck of length  $L$  and cross-sectional area  $S$ , which is connected to a cavity of volume  $V$ . The acoustic pressure inside the neck is denoted as  $p_n$  and the acoustic pressure inside the cavity is denoted as  $p_c$ . The neck is shown as a horizontal tube with arrows indicating the direction of air flow. The cavity is represented by a circle to the right of the neck.

So, in this steady state we assume that the pressure and the volume inside the cavity it is constant throughout, so we are assuming here that the cavity is big enough, compared to the neck the cavity is big enough. So, whatever addition will take place and whatever removal of air molecules will take place we will still be small. So, because the cavity is big enough compared to the neck volume.

So, in that case in the steady state we assume a homogeneous system inside the cavity. So, the pressure and the volume they are independent of the space inside the cavity they are only dependent on time. So, this is the assumption that we make that they are spatially uniform or the cavity is a homogeneous medium.

Now, let us indicate the acoustic pressure inside the neck and the acoustic pressure inside the cavity by this expression. So, we will derive the expression both for what is the acoustic

pressure in the neck of the cavity and the pressure in the cavity itself. So, now as air starts oscillating in and out, then the acoustic pressure it will cause the cavity to compress and expand as explained earlier.

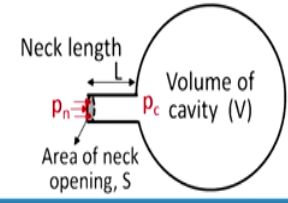
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### Natural frequency of a Helmholtz resonator

- Assuming this compression/expansion in cavity as adiabatic:

$$p_c(t) = B \frac{\rho - \rho_0}{\rho_0} \quad \text{and} \quad c^2 = \frac{B}{\rho_0}$$

$$p_c(t) = c^2(\rho - \rho_0) = c^2 \rho_0 c$$

$$p_c(t) = c^2 \rho_c \quad \text{Eq (1)}$$


The diagram shows a Helmholtz resonator consisting of a neck and a cavity. The neck has a length  $L$  and an area of neck opening  $S$ . The pressure in the neck is  $p_n$  and the pressure in the cavity is  $p_c$ . The volume of the cavity is  $V$ .

So, because it is an acoustic process and we have small oscillations because the incident source itself was an acoustic wave, so within the acoustic process we studied in the lecture two that they follow a adiabatic process. All of these acoustic compressions, expansions or density fluctuations all of them they are adiabatic in nature. And if you go to lecture 2, we had derived this equation that the pressure is equal to this particular expression. So, this was derived in lecture 2. If you can refer to that lecture 2, when we were discussing about sound wave propagation. So, this is the adiabatic relationship.



Here  $\rho$  is the overall density and  $\rho_0$  is the mean density at equilibrium position. So, and the thermodynamic speed of the sound was found to be  $c^2 = B/\rho_0$  or bulk modulus by the mean density. So, if we replace this expression  $B$  by  $\rho_0 c^2$  with  $c^2$  then what we get is, so if  $B$  by  $\rho_0$  is replaced with this in this particular expression is replaced here we get the pressure inside the cavity as  $c^2(\rho - \rho_0)$  and because the pressure fluctuations are very, small similarly the density fluctuations are also very small for the acoustic processes.

So,  $\rho_0$  is almost approximate to the  $\rho$  or the overall density is approximately same as  $\rho_0$  because acoustic fluctuations are very very small fractions of the actual value. So, we replace this  $\rho - \rho_0$  by  $\rho_0$ , so this is what we get. The pressure inside the cavity from this adiabatic relationship comes out to be  $c^2 \rho_0$ . So, this is one equation we have got.

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### Natural frequency of a Helmholtz resonator

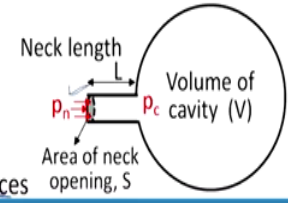
- As mass flows in, density in cavity increases and vice versa:

$$\frac{d\rho_c}{dt} = \frac{\dot{m}}{V} \Rightarrow \rho_c = \frac{1}{V} \int_0^t \dot{m} dt \quad \dot{m} = \text{mass flow rate entering the cavity}$$

- From Eq (1):  $p_c(t) = \frac{c_0^2}{V} \int_0^t \dot{m} dt$  Eq (2)
- Applying Newton's second law for air movement in the neck:

$$\rho_0 \frac{dv}{dt} = -\frac{\partial p_n}{\partial x} - F \quad \text{Eq (3)}$$

$F = \text{viscous and resistive forces}$



The diagram shows a cylindrical neck of length  $L$  and cross-sectional area  $S$  connected to a spherical cavity of volume  $V$ . The pressure in the neck is labeled  $p_n$  and the pressure in the cavity is labeled  $p_c$ . Arrows indicate the direction of air flow from the neck into the cavity.

Now, we know that whatever mass flows into the cavity will increase its density. So, the net increment in the density is the mass flow flowing per unit volume. So, the net increment in the density of the cavity. So, when the air inside the neck is oscillating towards the cavity there will be an increment in density, when air oscillates back then there will be a decrement in the density and in both case it is given by whatever if the mass of the rate of mass entering the cavity divided by the volume. So, this becomes the case.

So, you can replace this like this. So, this implies that the overall, so if we integrate this equation with respect to time then the total density or the density can be found as 1 by volume and the integral of  $m$  dot with respect to time. So, with this equation integrating with respect to time this is the expression we are getting for the density of the density of air inside the cavity. And from the last equation we saw that  $p_c$  is equal to  $c^2$  into  $\rho_c$ . So, if

we put this  $\rho$  naught value now, so  $p_c t$  will be  $c$  square times the expression for  $\rho$  naught. So, this becomes the equation for the fresh acoustic pressure inside the cavity.

So, we have obtained an expression for the acoustic pressure inside the cavity. Now, we will obtain an expression for the acoustic pressure in the neck. So, for that first of all we apply Newton's second law to the air movement that takes place in the neck. So, by Newton's second law it is that the total force acting towards the direction of motion will be equal to mass into acceleration.

So, using that now the pressure gradient that is generated throughout the neck length will actually be opposite to the direction of the velocity because the air will always move from a zone of high pressure to low pressure. So, it is the negative pressure gradient which is the force which generates this air flow. And similarly as the air flows through this particular neck, you will see that this is a solid neck and then we have a fluid air that is flowing across it, so there will be viscous losses or resistive losses.

The viscosity, due to viscosity these opposing forces will they oppose the motion of air over the neck. So, we take the summation of all the forces that are acting and because both of these are opposing forces or in the opposite direction of the motion, so we take a minus sign. So, it is the negative pressure gradient along the neck minus the resistive forces due to viscosity and any other resistive forces.

So, any resistance faced by the air molecules while flowing through the neck are represented by this expression. So, this total force then is equal to the density. So, we are doing this per unit volume. So, this is  $\rho$  naught into  $\frac{dV}{dt}$  or simply the density into the acceleration. So, this is with Newton's second law. So, that is the expression we have got.

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### Natural frequency of a Helmholtz resonator

$$\rho_0 \frac{dv}{dt} = -\frac{\partial p_n}{\partial x} - F \quad \text{Eq (3)}$$

- Based on nature of viscous and resistive forces:  $F = \frac{Rv}{L}$
- Assuming harmonic oscillations in the neck:
 
$$p_n(t) = p_{n,max} e^{j\omega t}; v(t) = v_{max} e^{j\omega t}$$

$$\frac{\partial p_n}{\partial t} = j\omega p_n; \frac{\partial v}{\partial t} = j\omega v$$
- From eq. 3:  $j\omega \rho_0 v + \frac{Rv}{L} = -\frac{\partial p_n}{\partial x}$

$F \propto v, F \propto \frac{1}{L}$

Neck length  $L$   
 Volume of cavity  $(V)$   
 $p_n$   $p_c$   
 Area of neck opening,  $S$

Now, all the viscous and the resistive forces that act whenever a fluid medium flows through a solid. So, just like we had the concept of friction, so where we had two solid bodies moving against each other and friction was acting viscosity acts when a fluid is flowing across the boundary of a solid surface.

And the nature of this forces that if you study the theory of viscosity, then you will find that through all the experiments and to analytically also the nature of this resistive force is that it is directly proportional to the velocity with which the fluid particle is flowing over it. So, the velocity of the layer of fluid and it is inversely proportional to the length through which it flows.

So, in that case I mean from the theory of viscosity, we know that  $F$  will be directly proportional to  $V$  and  $F$  is inversely proportional to the length over which in this flow takes

place. So, this is the nature of the resistive force which acts. So, if we introduce a constant of proportionality and combine these two equations, so it is some constant into  $V$  by  $L$ . So, this will be the general form of the force equation. So, we use this in this particular equation here and also let us now because this is an acoustic process, so we are deriving expressions for the acoustic, we are deriving the expression of acoustic pressure and the acoustic particle velocity.

So, there are two things with acoustic processes. First of all the fluctuations corresponding to acoustic processes they are very very small compared to their actual mean values. The second thing is because of this very very small; very very small fluctuations the process is adiabatic in nature. And the third one is that in such small cases in the small fluctuations a common solution can be harmonic, so we usually take a harmonic solution and even if there is a random sound wave it can by Fourier series by Fourier's theorem it can be represented as a combination of sinusoidal waves. So, we start with a harmonic solution and we derive for a harmonic solution.

So, we are taking a harmonic solution for this acoustic pressure. Then  $p_n t$  will become some amplitude into  $e$  to the power  $j \omega t$ ,  $v t$  will again be some amplitude into  $e$  to the power  $j \omega t$ , so it is sinusoidally varying with respect to time. So, in that case now we have a because it is a acoustic process, so we have assumed a harmonic solution or a seems like a simple harmonic motion sort of. So, in that case if we assume this form of solution. Then  $\frac{dp_n}{dt}$  will become  $j \omega$ , in it will be  $j \omega$  times  $p_n \max$  into  $e$  to the power  $j \omega t$ , right. If you if you differentiate this with respect to time. So, this simply becomes  $j \omega$  into whatever is this value.

Similarly, if you differentiate this particular equation with respect to time again this becomes  $j \omega$  into  $e$  to the power. So, this becomes  $j \omega$  into  $v \max$  into  $e$  to the power  $j \omega t$ , so which is simply  $j \omega$  into whatever is the expression for  $v$ . So, in this form if this is the form of solution, then the differentials with respect to time or simply  $j \omega$  times the original function.

So, now that we have these values we will put all these values in this equation 3 and we put when these values in the equation 3, then  $\rho_0 \frac{dv}{dt}$  will be what?  $\rho_0$

naught into  $j\omega v$ . So,  $\rho$  naught into  $j\omega v$  and this force is we take it on the other hand side, so we get  $Rv$  by  $L$  is equal to minus  $\frac{dp_n}{dx}$ . So, this is what we get.

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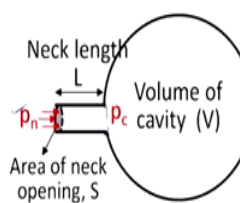
### Natural frequency of a Helmholtz resonator

- Integrating the previous equation w.r.t. 'x' along the neck 0 to L:
 
$$[p_n]_0^L = p_n - p_c = \frac{j\omega\rho_0 vL + Rv}{j\omega}$$

$$p_n = p_c + \frac{j\omega\rho_0 vL + Rv}{j\omega}$$
- Differentiating above equation w.r.t. 't':
 
$$j\omega p_n = \frac{dp_c}{dt} - \omega^2\rho_0 vL + j\omega Rv$$
- From Eq. (2):  $\frac{dp_c}{dt} = \frac{c_0^2 \dot{m}}{V}$ 

$$j\omega p_n = \frac{c_0^2 \dot{m}}{V} - \omega^2\rho_0 vL + j\omega Rv$$

$j^2 = -1$       $j = \sqrt{-1}$



Neck length L  
Volume of cavity (V)  
Area of neck opening, S  
Pressure at neck  $p_n$   
Pressure at cavity  $p_c$

So, integrating this equation with respect to  $x$  now. So, this was the equation we have got. Now, we integrate this equation with respect to  $x$ , then this if we integrate this with respect to  $x$  we get the overall pressure from one end to the other end of the neck.

So, let us say from 0 to L. So, this is the domain of integration. So,  $p_n$  from 0 to L which is equal to let us say the equilibrium value at the point when the sound is incident is  $p_n$  and the value at this point we will be same as the value due to the continuity of pressure this becomes the value of the pressure at the cavity. So, this is  $p_n$  and at this end due to continuity of pressure the value has to become  $p_c$ .

So,  $p_n$  minus  $p_c$  upon integrating this along, then length of the neck is what? It is going to be  $j\omega\rho naught v$  and this expression is also integrated from 0 to L and this expression is also integrated from 0 to L. So, this is the ultimate form we are getting. So, that is the form we get.

Now, we have obtained this expression. So,  $p_n$  becomes  $p_c$  plus  $j\omega\rho naught v L$  plus  $R$  times of  $v$ . Now, again differentiating this equation with respect to time what we get is now we had assumed a harmonic solution here. So,  $\frac{dp_n}{dt}$  will be  $j\omega$  of  $p_n$ . So, when we differentiate it with respect to time this expression becomes  $j\omega p_n$ , this becomes  $\frac{dp_c}{dt}$  we are differentiating with respect to time and this becomes if you differentiate this with respect to time then  $v$  is again  $j\omega$  times 3.

So, it will be  $j\omega$  it will be  $j^2\omega^2\rho naught v L$  and  $j^2\omega^2$  becomes minus  $\omega^2$ ,  $j^2$  is minus 1 where  $j$  is obviously, root of minus 1. So, this becomes this value and this becomes  $j\omega R v$ . So, this is what we get. And in the very beginning we had derived the expression for  $p_c$  at the acoustic pressure, the homogenous acoustic pressure in the cavity and this was the expression for this,  $c naught^2$  by  $v$  integral with respect to time  $m \dot{d}t$ .

So, now if we differentiate it with respect to time, this is this is a constant with respect to time, this is a constant with respect to time. So, only this expression is differentiated. So, integral again differentiated will be ending up with this integral sign will go off. So, we will get this expression from equation 2,  $c naught^2 m \dot{d}t$  by  $v$ . So, again this expression becomes  $c naught^2 m \dot{d}t$  by  $v$  plus this, minus this, plus this. So, this is the final form of equation we are getting.

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### Natural frequency of a Helmholtz resonator

- Obtained equation:  $j\omega p_n = \frac{c_0^2 \dot{m}}{V} - \omega^2 \rho_0 v L + j\omega R v$
- $p_n = -j \frac{c_0^2 \dot{m}}{\omega V} + j\omega \rho_0 v L + R v$
- $p_n = R v - j \rho_0 c_0 \left[ \frac{c_0 S v}{\omega V} - \frac{\omega L}{c_0} \right]$  (From eq (2):  $\dot{m} = \rho_0 S v$ )
- Acoustic impedance of this oscillator:
- $Z = \frac{p_n}{v} = R - j \rho_0 c_0 \left[ \frac{c_0 S}{\omega V} - \frac{\omega L}{c_0} \right]$

$\frac{1}{j} = -\frac{j^2}{j} = -j$   
 $\frac{1}{j} = -j$

Now, to get  $p_n$  we divide every, we divide throughout by  $j\omega$ . When we divide throughout by  $j\omega$ , then we get  $p_n$  as you divide by  $j\omega$ . So,  $1$  upon  $j$  is equal to minus  $j$  square by  $j$  which is equal to minus of  $j$ . So,  $1$  upon  $j$  is minus of  $j$ . So, we use that value, so we get minus of  $j c_0^2 \dot{m}$  by  $\omega v$  and then again divided by  $j\omega$  what we get is plus of  $j\omega \rho_0 v L$  this is the equation that we are using, this is the property of the imaginary root the imaginary unit quantity and then this divided by  $j\omega$  becomes  $Rv$ . So, we divide throughout by  $j\omega$  and this is the equation we get.

Now, let us separate the real part and the complex part. So, when we separate the real part and the complex part. So, this is the real part and the complex part together, we can write we take this quantity as common  $\rho_0 c_0$ , then what we get is now  $\dot{m}$  the



total rate of flow of mass can be written as what is the density, mean density into the surface area through which it is entering.

So, what is the rate at which the mass is flowing into the cavity will be the density multiplied by the surface area through which the mass is flowing, multiplied by the velocity at which the mass is flowing. So, it is  $\rho$  naught into the surface area of the surface area of the neck into the velocity. So, this is the rate at which the mass is flowing. So, we put this equation for mass here and we take this constant. So, if you do this is what you end up with, that is the expression you end up with.

So, now, this is the expression you have got for the net pressure in the neck and we know that it is the air molecules in the neck that are undergoing this harmonic motion and it is the air molecules through. So, this is the sound wave is propagating via the oscillations of these air molecules, then the acoustic impedance of this particular oscillator will be whatever is the pressure divided by the velocity. So, this becomes if you divide this expression by  $v$ , this is what we are left with. So, this is the net acoustic impedance of this cavity, all of this particular oscillator or similar or simply you can say what is the net acoustic impedance of the neck. So, this is the expression for the acoustic impedance.

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### Natural frequency of a Helmholtz resonator

- $Z = \frac{p_n}{v} = R - j\rho_0 c_0 \left[ \frac{c_0 S}{\omega V} - \frac{\omega L}{c_0} \right]$
- At resonance Z is minimum:  $\rho_0 c_0 \left[ \frac{c_0 S}{\omega V} - \frac{\omega L}{c_0} \right] = 0$

$$\omega_r^2 = \frac{c_0^2 S}{VL} \Rightarrow \omega_r = c_0 \sqrt{\frac{S}{VL}} \Rightarrow f_r = \frac{c_0}{2\pi} \sqrt{\frac{S}{VL}}$$

$\omega_r = 2\pi f_r$

$$f_r = \frac{c_0}{2\pi} \sqrt{\frac{S}{V(L + 1.7r)}}$$

End correction factor

Neck is like an open-open pipe

Neck length L  
Volume of cavity (V)  
Area of neck opening, S

So, at resonance is the condition when suddenly the resistance offered by the system becomes minimum and because the system offers no resistance or very minimum resistance to the flow of sound waves, so the sound waves are very large amplitude then flow through the system. So, by definition that is resonance the condition of resonance that is when the system offers minimum resistance to the flow of sound waves.

So, if this is the resistance expression when will this be minimum? If  $r$  is a fixed quantity and this is the only frequency dependent quantity here then this expression has to be 0 for resonance. So, putting this as 0 what you get is here the  $\omega$  at resonance. So,  $\omega^2$  will be  $\frac{c_0^2 S}{VL}$ . So, if this expression is put as 0, then  $\omega_r^2$  will be this expression, so  $\omega_r$  will become  $c_0 \sqrt{\frac{S}{VL}}$

by  $V L$ . So, the frequency  $\omega r$  is twice  $\pi$  of  $f r$ . So,  $f r$  will become  $c$  naught by  $2 \pi$  under root of  $S$  by  $L$ . So, that is the expression we are getting.

We have replaced it with a new value now. So, it is near  $c$  naught is the speed of sound in the air or the speed of sound in the medium of the Helmholtz resonator,  $S$  is the surface area of the neck, and  $V$  is the volume of the cavity enclosed and  $L$  is the length of the neck. But here we have replaced this length of the neck with a new expression  $L$  plus  $1.7$  times of  $r$ . So, what we have done is we have added additional factor of  $1.7 r$ , this is the end correction factor. So, I will briefly explain it to you that why do we add an end correction factor.

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### End correction factor

- Natural frequency of a pipe is dependent on the length between two medium boundaries.
- Ideally, medium boundary should occur at the closed end and the open end of the pipe.

For open-closed pipe:  $f_n = \frac{(2n + 1)c}{4L}$   $n = 0, 1, 2, 3, \dots$

For open-open pipe:  $f_n = \frac{nc}{2L}$   $n = 1, 2, 3, \dots$

$L =$  distance between medium boundaries = pipe length

$z = 0$  Medium boundary  
 $z = L$   
 Open-closed pipe  
 Ideal first mode  
 $z \rightarrow \infty$   
 Medium boundary

12

So, let us say what is this neck. This neck is like, the neck is like open-open pipe. So, it is like a pipe with both ends open, both ends are open, now no end is closed. So, that is a neck. So, in case of pipes, so when we were doing the lecture on standing waves and resonance, so we

derived the equation for the natural frequency of a closed-closed long tube and it was found to be  $n \lambda = 2L$ . So, how did we find out the resonance of that tube?

What we did was that we impose the condition that at every end the impedance suddenly reaches infinity or the suddenly the acoustic particle velocity becomes 0 at the end and  $p$  if  $v$  become 0 which means  $p$  by  $v$  is that is  $Z$  which tends to infinity. So, we had assumed that condition that at the rigid end, suddenly impedance is the maximum a hard surface will have maximum impedance, it will not allow any further propagation of sound particles and therefore, that condition we imposed we put  $v$  equals to 0 and we derived an expression for resonance frequency.

Similarly, when a tube had one end closed and one end open, we had derived another equation. So, all of this was done in the lectures on standing waves and resonance and the following lecture on numericals. And there also, when we derived the expression for natural frequency of this particular long tube or pipe. What we assumed was that one end had maximum, one end had almost infinite impedance and the other end suddenly the impedance is 0 because it offers no resistance. Here also this it is air, here also it is air and therefore, in that case  $p$  was said to be 0. So, we saw we said  $v$  as 0 for the rigid backing and  $v$ , so  $v$  equals to 0 for the rigid backing and  $p$  equals to 0 for the open end.

So, using these conditions we had derived the resonance frequency for both the tubes. But, so the assumption was that, so here this length  $L$  was actually the distance between the two boundaries which corresponded to the distance or the length of the tube. So, it is actually the distance between the two medium boundaries which is equal to the length of the tube.

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### End correction factor

- Longitudinal acoustic waves formed in the pipe start scattering at the pipe end, which lead zone of medium expansion where density ( $\rho$ ) changes, impedance ( $\rho c$ ) changes.
- But impedance change happens not exactly at the opening but at a small distance ahead of it, due to slow change of medium density.
- Thus, actual medium boundary is at a distance ahead of opening.

The diagram illustrates an open-closed pipe. Inside the pipe, blue arrows represent particle oscillations. At the top, a dashed line indicates the 'Actual medium boundary' at a distance ahead of the pipe's opening. A red arrow points to the 'Scattering' zone at the opening. Labels include  $\rho c$  for impedance at the opening and  $\rho' c$  for impedance at the actual boundary.

However, this is not the actual case it is just an approximate case. What happens in actual situation is that, suppose we have one end closed and one end open tube and let us say the particles are oscillating. So, this is the propagation of sound waves that is taking place to the tube. Just at the edge we will have scattering and diffraction.

So, the particles they will start to scatter around and while propagating. So, when the scattering takes place let us say  $\rho c$  is this particular value which is the impedance of this medium and as the particles start to scatter and say expand, there will be a zone of low density created just above the tube and after a certain length the difference between  $\rho c$  and  $\rho' c$  will be significant enough to be considered as a boundary.

So, this change in density is very slow. So, instead of that typical change in the medium happening here this is what we assumed in ideal situation and we took

this length as the length of the pipe. But in reality the actual, the actual change in the medium takes place a little bit ahead of the pipe. So, this is where the second boundary takes place.

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### End correction factor

- Thus, a velocity antinode is formed at a distance away from the pipe opening.
- To calculate correct natural frequency of the pipe, some correction factor is added to the pipe length. This is called **end correction**.

For open-closed pipe:  $f_n = \frac{(2n + 1)c}{4L'} ; L' = \underline{L + e}$

For open-open pipe:  $f_n = \frac{nc}{2L'} ; L' = \underline{L + 2e}$

$L'$  = corrected pipe length;  $L$  = actual pipe length;  $e$  = end correction factor

So, while we derive the equation any modes that have formed. So, we had, let us say this is the first velocity mode, this is the velocity mode or the shape of the velocity function. So, in the rigid end it is 0 and then suddenly towards the open end it should become maximum, but it actually becomes maximum at a certain length above the actual pipes open end. So, it becomes maximum a little bit ahead of the opening where the actual medium boundary is.

Therefore, the total length in that case should be this for the formula. So, if we have this resonance frequency this length should not be the length of the pipe, but it should be the corrected length which is the actual length of the pipe plus some end correction. So, this is for a closed open tube. For an open-open tube this will be the end correction because similar thing

there will be an e here and there also there will be an e, the medium boundary will be little bit separated from both the openings. So, that is the; so that is the rationale or the reason for using this end correction.

So, suppose some problem is given to you and the end correction value is not known to you, you can simply take the actual length of the neck as an approximate solution, but in reality you will need to have the end correction factor added. This table shows to you what are the various end correction factors.

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End correction values	
$\delta$	Notes
0.85	Single hole in a baffle
$0.8(1 - 1.4\sqrt{P})$	For $P < 0.16$
$0.8(1 - 1.47\sqrt{P} + 0.47\sqrt{P^3})$	Includes $P = 1$
$0.85(1 - 1.25\sqrt{P})$	Square apertures; for $P < 0.16$
$-\ln[\sin(\pi P/2)]/\pi$	Slotted plate; in Eq. (20) $v = 0$ and $r =$ width of slots
Here, <b>P = porosity</b>	

Source: Crocker, M. J. (Ed.) (2007). "Use of sound absorbing materials" in Handbook of noise and vibration control. John Wiley & Sons.

End correction:  $e = \delta r$

For open-closed pipe:  $L' = L + \delta r$

For open-open pipe:  $L' = L + 2\delta r$

$L' = L + 1.7r$

So, if you have a single hole in a baffle this is our case. 0.85 is the end correction, so here this 0.85 gives the value of delta and the e is simply delta times of r. So, for a open-open pipe this is L plus twice of delta r. So, in case of neck what will it that be? L plus 2 into 0.85 which is 1.7 times of r, so we have taken that value. So, this is the fun resonant frequency that was

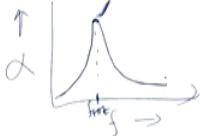
found for the Helmholtz resonator and then the working principle is as I had explained to you it is same as the panel absorbance.

So, whenever the incident sound frequency matches with the fundamental frequency of this Helmholtz resonator, then all the incident energy is then used to drive the air molecules through the neck of the oscillator. So, the air molecules inside the neck they keep oscillating back and forth at large amplitudes in this resonance condition and all the energy is absorbed in doing work against it.

(Refer Slide Time 34:48)

**Working principle of Helmholtz resonator**

- Thus, sound absorption by Helmholtz resonator peaks at its natural frequencies.
- The resonance acoustic pressure peaks are sharp, hence absorption characteristics are also sharp and narrow around the resonator's natural frequencies.



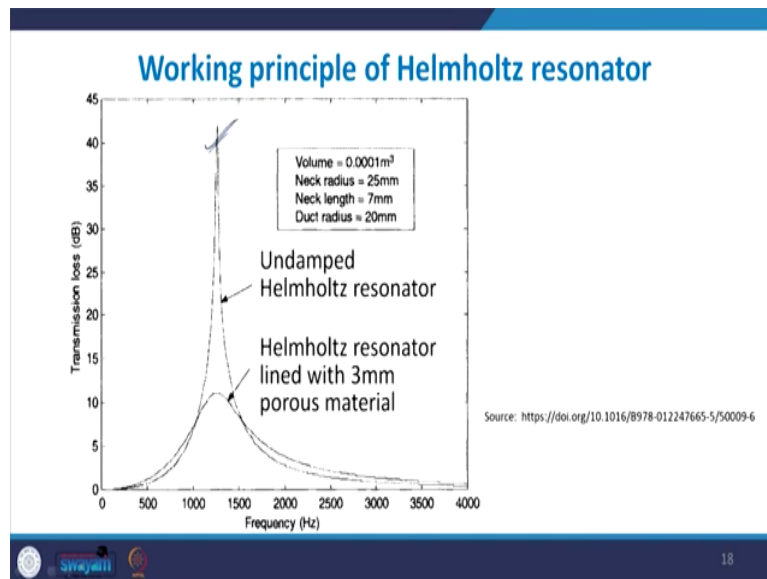
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So, that is why these Helmholtz resonator's, they are very selective, they have peak at the resonance resonators natural frequencies. So, if we draw the alpha or the absorption of this Helmholtz resonator, then wherever the incident sound frequency matches with the actual frequency of the resonator.



See if this is the natural frequency of the resonator, it is only there that suddenly the acoustic coupling takes place and the energy that is incident will then be used to drive the molecules of the resonator. So, suddenly there will be a jump in, there is a jump in the absorption a lot of energy will be lost and in all the other frequencies it will be very very low. So, this is a kind of a typical absorption characteristic of a Helmholtz resonator.

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So, this figure shows to your typical absorption characteristic. So, a common practical of the example of a Helmholtz resonator is that suppose you have an empty bottle with a fine neck and you blow over the empty bottle you always hear a whistle kind of a sound. So, the air that you are blowing like that it here, it is a white noise, it has sounds you know it has noise in all the frequency, so it does not sound like a whistle. But a whistle is typically a single tone noise.

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### Working principle of Helmholtz resonator

- Practical example of working of a Helmholtz resonator:
  - *Pure tone is heard when you blow on the top of a bottle with narrow neck opening.*



Source: <http://science4kid.blogspot.com/2009/07/shooting-backwards.html>

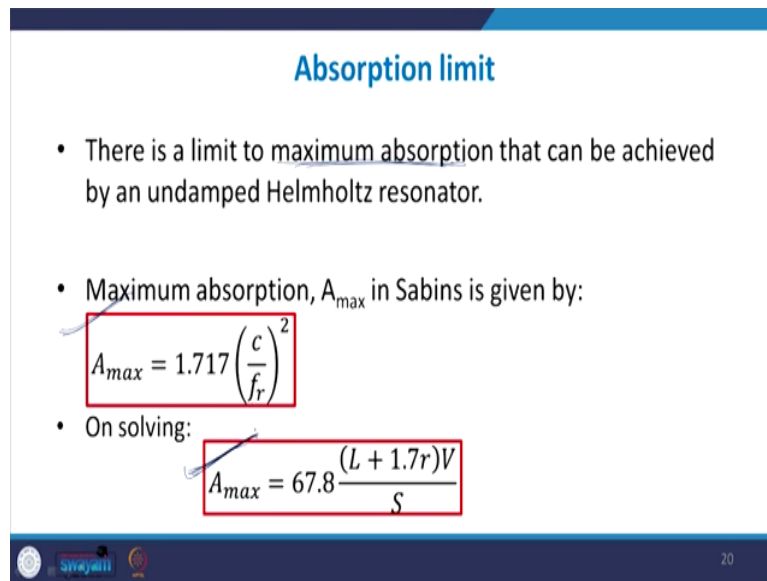
19

So, whenever you blow the air into the bottle you hear a tonal noise. Why? Because this becomes a Helmholtz resonator. So, when you are blowing the air what you are providing is a broadband excitation, but it is only at the natural frequency of the bottle that suddenly there will be large oscillations and the sound will propagate and you will hear a tonal noise. So, this is this observation of daily life is based on the principle of Helmholtz resonator, ok.

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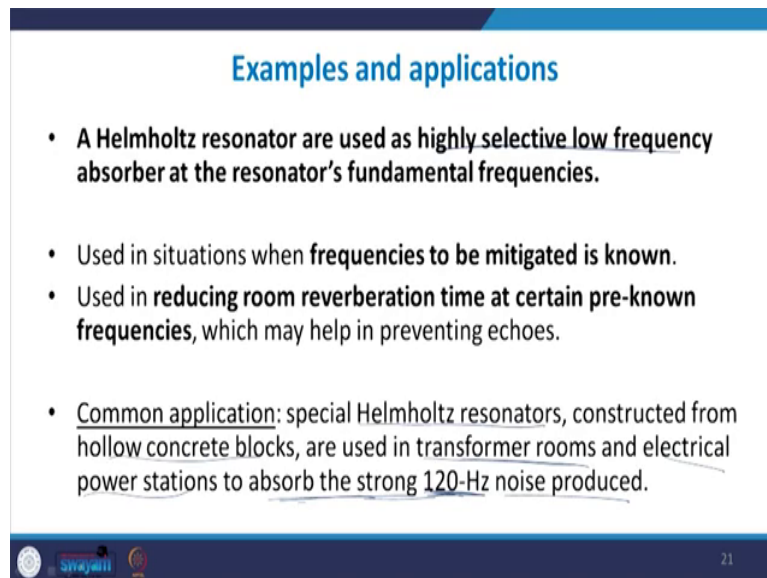
### Absorption limit

- There is a limit to maximum absorption that can be achieved by an undamped Helmholtz resonator.
- Maximum absorption,  $A_{max}$  in Sabins is given by:  
$$A_{max} = 1.717 \left( \frac{c}{f_r} \right)^2$$
- On solving:  
$$A_{max} = 67.8 \frac{(L + 1.7r)V}{S}$$



So, lastly the absorption due to the Helmholtz resonator. There is always a maximum and this is the value for the maximum absorption which can be written as this, if you solve for  $f_r$ . So, this is the value for the maximum absorption of a Helmholtz resonator. So, here we end the discussion before ending the discussion I will just give you some examples.

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**Examples and applications**

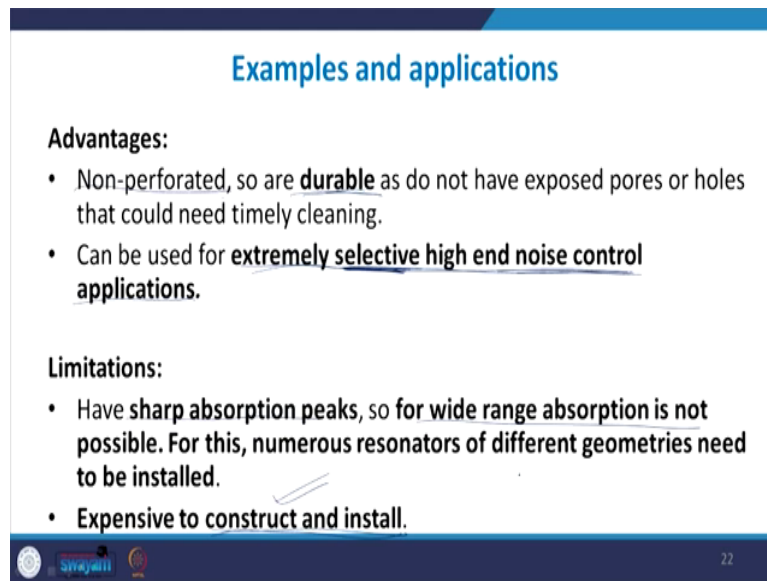
- A Helmholtz resonator are used as highly selective low frequency absorber at the resonator's fundamental frequencies.
- Used in situations when frequencies to be mitigated is known.
- Used in reducing room reverberation time at certain pre-known frequencies, which may help in preventing echoes.
- Common application: special Helmholtz resonators, constructed from hollow concrete blocks, are used in transformer rooms and electrical power stations to absorb the strong 120-Hz noise produced.

swayam 21

So, Helmholtz resonator is commonly used whenever the frequency that has to be mitigated is known to you because we know that it can offer very high, it can offer a good absorption, but only at a very narrow selective frequency range. So, it can only be used in certain applications where you already know that, this is the frequency where the noise lies and this is the frequency we have to mitigate.

For example, the Helmholtz resonators they can be constructed inside transformer rooms or electric power stations because we know that they generate a tonal noise at 120 Hertz. So, just to control that noise it can be used. So, it can be used only for highly selective purposes where the frequencies are already known to us which frequency needs to be reduced.

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**Examples and applications**

**Advantages:**

- Non-perforated, so are  **durable**  as do not have exposed pores or holes that could need timely cleaning.
- Can be used for  **extremely selective high end noise control applications** .

**Limitations:**

- Have  **sharp absorption peaks** , so  **for wide range absorption is not possible** . For this, numerous resonators of different geometries need to be installed.
- **Expensive to construct and install** .

swayam 22

So, the advantage with respect to porous absorber is that it is an all perforated system. So, it does not need any cleaning and it is durable and it is highly selective. So, if the frequency that is to be controlled is already known to us, then it will work perfectly fine. But this advantage also becomes as limitation because since the it can only this, it can it is only selective and works only at particular frequencies it cannot be used for a wide range purpose and we know we need to know the frequencies beforehand.

So, if we do not know the characteristic of the noise we have to mitigate then Helmholtz resonators cannot be used. And they are expensive to construct and install, they cannot be such kind of cavity cannot be installed in every form every every machinery component or every system. So, these are some of the advantages and limitations. So, with this I would like to end the discussion on Helmholtz Resonator.

Thank you.