Acoustic Materials and Metamaterials
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Lecture – 02
Sound Wave Propagation in Fluids - I

Welcome to the second lecture of our course and this lecture we are going to study about Sound Wave Propagation in Fluids.

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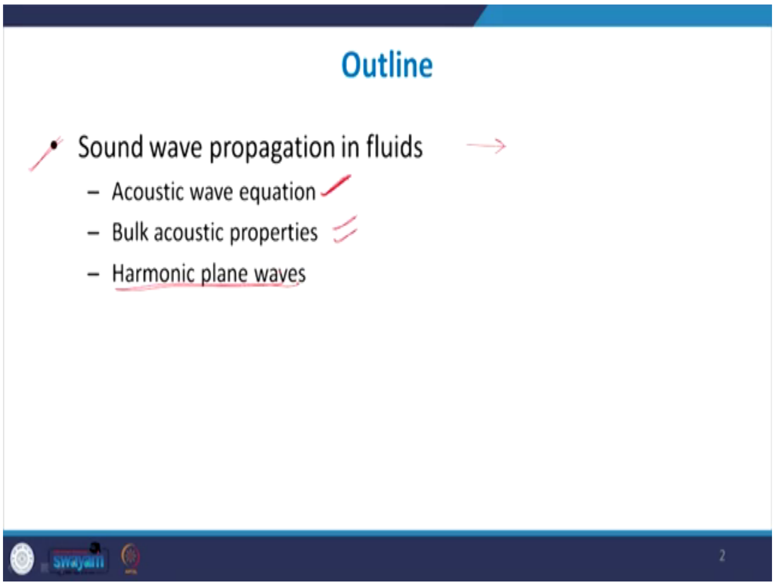

So, the outline for this course is as follows. So, we will study about what happens when the sound wave propagates through a fluid medium and then we will study something called as the Acoustic wave equation. So, this wave equation is a wave equation that can be used for any fluid medium and any kind of sound wave propagation, so a general equation will be derived. Then will study what are the various Bulk acoustic properties and then we will study one special type of wave which is called as Harmonic plane waves.

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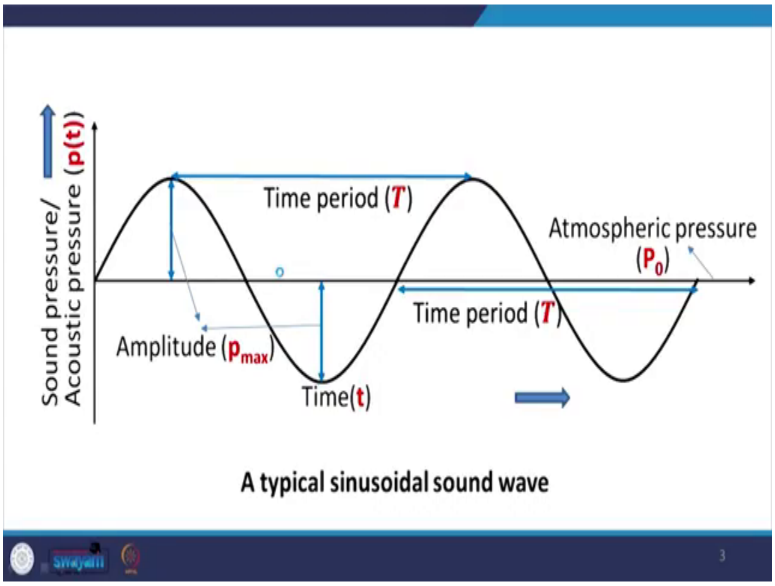

So, let us begin our class. So, as we discussed in the last chapter in the last class was that, any wave can be represented as a disturbance over time and a disturbance over space. So, it can be represented as a sinusoidal disturbance over space and time provided it is a harmonic periodic sound wave.

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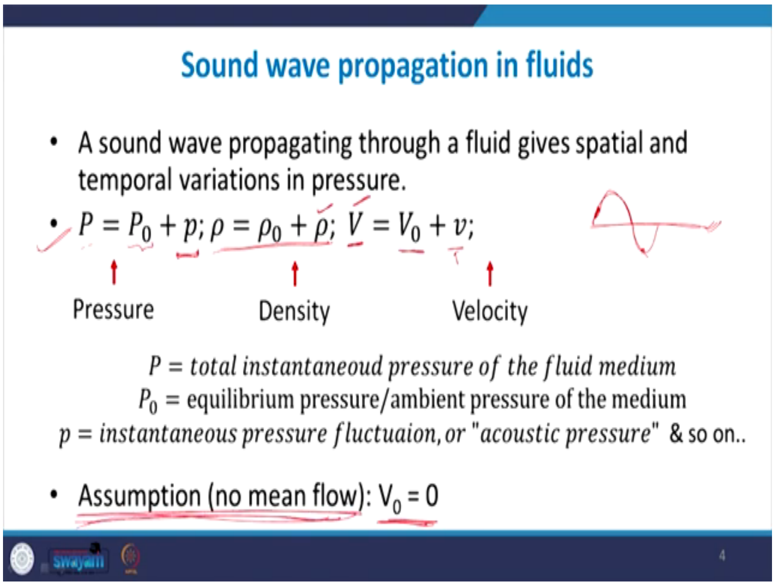

So, when the sound wave is generated in a fluid medium, it will cause both disturbance over space as well as the disturbance over time. So, it will be a function of both space and time. So, let us see, let us define some quantities here. So, for a fluid medium; the total pressure at any time or the instantaneous pressure will be whatever is the mean pressure of the fluid plus the acoustic pressure. Because acoustic pressure as we told was the mean pressure plus whatever is the fluctuation. So, the total pressure becomes whatever is the mean pressure plus whatever fluctuation is created. So, the total pressure can be calculated like this.

Similarly because the pressure is changing, the density of the medium will change. So, here we are resuming that we have a medium where we are not adding any additional mass. So, the mass of the medium is fixed, no additional particles are being added to the medium. So, when a zone of high and low pressure is created; so the density will also change and the velocity of the particles will also change, because this disturbance propagates by the oscillation of particles. So, when the particles are oscillating, the velocity is also going to change.

So, similar equation can be derived; the total density then becomes whatever is in mean density of the medium plus the density due to the acoustic pressure. Similarly we have V; so this becomes whatever is the density or the total velocity of the particle becomes the mean particle velocity plus the acoustic velocity. And now we are assuming that there is no mean flow. So, the mean velocity is 0. So, we take this assumption when we come to derive an equation for sound wave propagation.

So, as we said in the earlier class I had said that, the fluctuations are very small. So, they can be of the order of 10 to the power minus 10 to 10 to the power minus 4 times the actual pressure of the medium, and even such very minute fluctuations are perceived as sound. So, because this acoustic process always involves very small compression and very small expansion; therefore, when the when such expansion and compression is taking place, then the temperature gradient that is created is not sufficient enough. So, it is almost negligible. So, for such very small compressions and expansions of the fluid particle, we neglect the temperature gradient. And we neglect the thermal conductivity.

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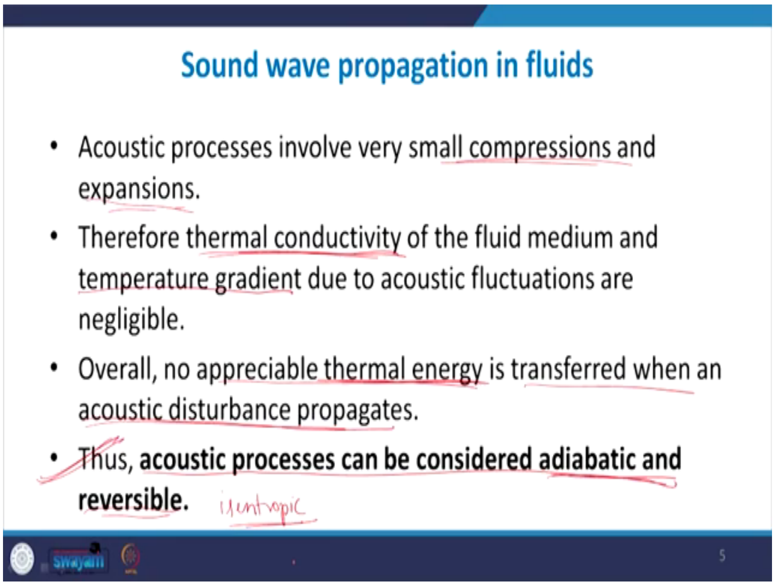

So, overall there is no change in the thermal energy, when a acoustic disturbance propagates.

Because of these very small values of fluctuations that are happening; there is no sufficient gradient generated and that is why there is no heat transfer during such process. So, for all the study of a acoustic, we assumed that the acoustic process they are either; they are considered as adiabatic and reversible or we can name it as isentropic. So, isentropic is the name for a process which is both adiabatic and reversible in nature. So, this is what is assumed in physics that because the process involves such small fluctuations, the process can be considered as adiabatic and reversible.

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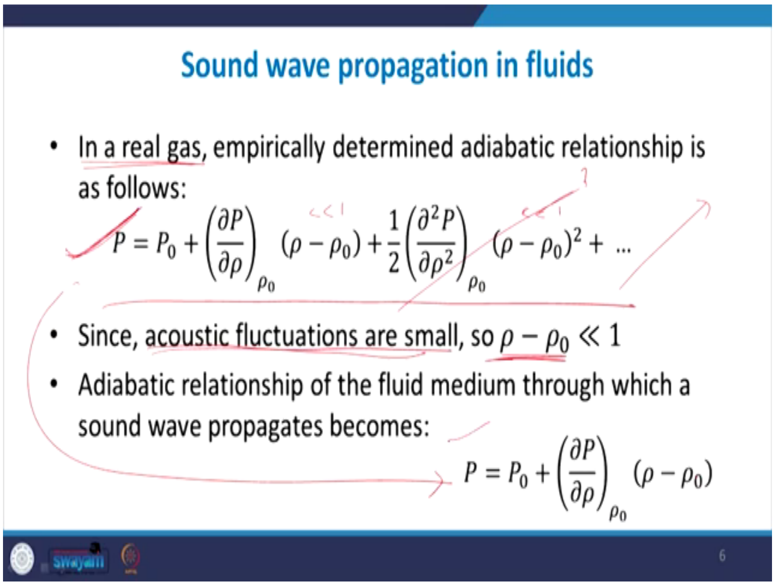

Empirically from the field of thermodynamics a relationship has been derived. So, if we have a real gas, then the relationship between the pressure and the density variation is given by this equation. So, this is the derivation of this is out of course, because it is from the field of thermodynamics. So, this is a typical relationship between this is called as the adiabatic relationship of a real gas. So, we will directly take this equation into our derivation. Now we know that the acoustic fluctuation they are very small. So, this rho minus rho naught is going to be very small. So, this quantity is extremely small and this is a Taylor series expansion.

So, the higher order terms they can be neglected; all the higher order terms they can be neglected, we only retain the first order terms. So, this equation can be approximated to P is equal to P naught plus del P by del rho and rho naught into rho minus rho naught and we are neglecting all the higher order terms. So, this is the equation we have got. Now let us talk about another parameter of the fluid medium; it is a very important parameter which we call as adiabatic bulk modulus.

Now you already know about what is a modulus in the field of solid mechanics. So, when we have a solid medium; the modulus of elasticity, then we have bulk modulus, modulus of rigidity and so on. So, the usually measure what is the resistance to deformation. So, bulk modulus measures what is the resistance to compression or expansions. So, it is a resistance to a change in the volume or change in density.

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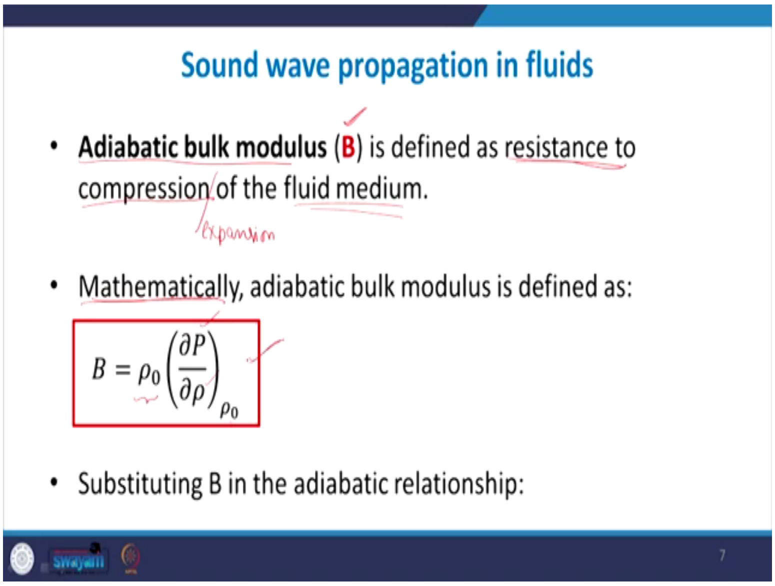

So, in the same way for the fluid medium, this adiabatic bulk modulus which is represented by the symbol capital B is the resistance to compression or expansion as the case may be for the force of this fluid medium. And mathematically this particular quantity is defined as B is simply whatever is the mean density of the medium multiplied by the rate of change of pressure with respect to the density. So, rho naught del P by del rho, the value of this at the mean density. So, this is what, this is how adiabatic bulk modulus is defined.

So, we had this particular relationship, this was the adiabatic relationship. So, if we replace this value now with the definition of bulk modulus this entire thing. So, this has been replaced. So, what do you get here? So, bulk modulus is this. So, this quantity becomes B by rho naught, this quantity becomes B by rho naught.

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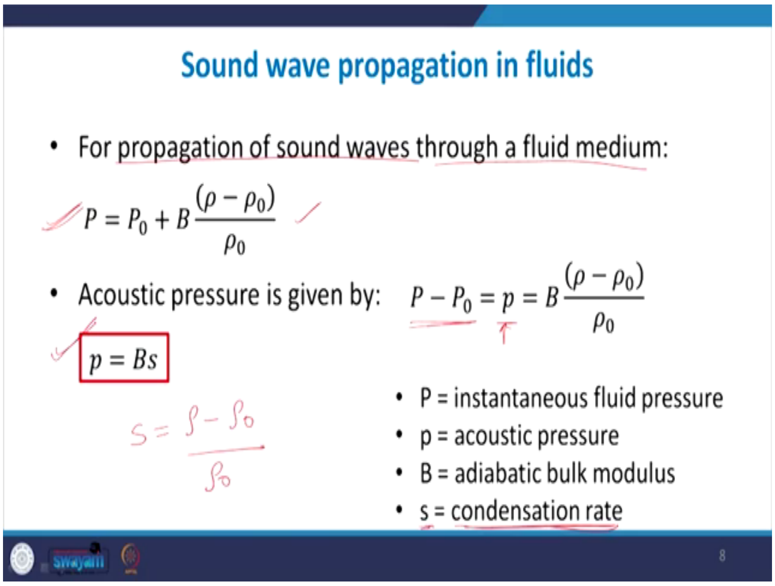

So, we replace this quantity here. So, now, the equation we get. So, this is the adiabatic relationship for the propagation of sound waves through a fluid medium; this is the relationship we are getting. And now we know that acoustic pressure is what; it is the difference between the total pressure and the mean pressure. So, this is the acoustic pressure. So, this becomes B times rho minus rho naught by rho naught.

And now we are going to for the sake of simplicity of derivation we have defined a new term which is called as the condensation rate. So, it is a rate at which the fluid particle undergoes condensation which is change in the density. So, it is the rate of change in density which is rho minus rho naught by rho naught. So, here s is rho minus rho naught by rho naught. So, we have defined a new variable. So, we can replace this as acoustic pressure becomes B into the rate of condensation. So, this is the first equation will use it later and I will go through it, so we will recall this.

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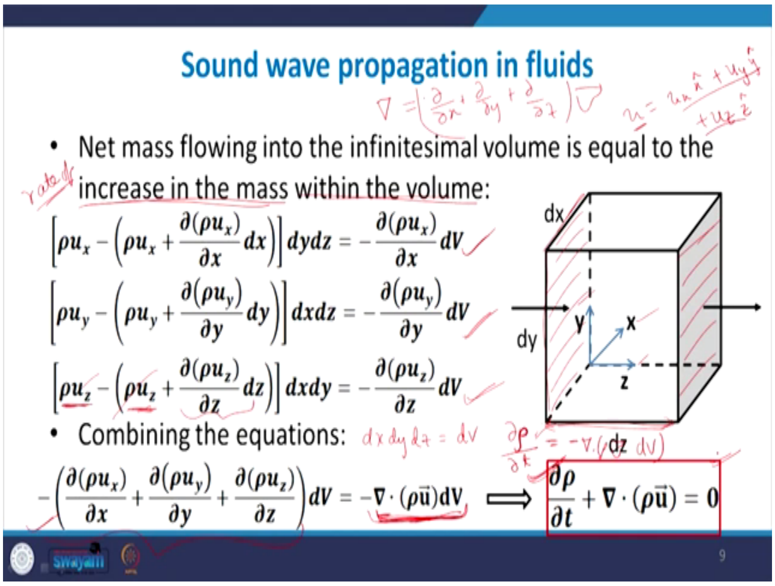

Now, let us take a small volume element within the fluid medium. So, now, we will see it is the equation of conservation of mass. Now as I said earlier that, we are not adding any additional mass to the fluid medium only some disturbance is propagating.

So, in that case the rate of change; in that case suppose the particles they come and go, so they are oscillating and some particles they enter in this elemental area and some particles go outside the elemental area. Then the net increment in the mass is what were deriving; so the increase in the mass within this volume element. Let us say increase in the mass per unit time. So, it is the rate of increase in the mass.

So, what is the rate of increase in the mass is what we are deriving. So, the rate at which the mass increases in this particular small element of a fluid is whatever mass is entering minus the mass going out. So, the rate of increase in mass will be the rate at which the mass enters that element minus the rate at which the mass goes out of the element. So, for every, then we have decomposed it. So, for the different directions x, y and z we are seeing what is the rate of increase in the mass along the x direction, y direction and z direction.

So, let us take for example, one particular direction, z direction. So, here you see that the rate at which the mass is entering this space; this is the phase if you are taking the direction. So, the mass is entering from this phase and it is going out from this phase. So, the rate at which the mass is entering will be the density times the velocity at this point. So, rho u x this is the, because we are I am describing here about x directions. So, rho u z is the rate at which the mass is entering this space and then it is going out; then this particular. So, let us see, then it can simply be given as whatever is this rate plus whatever is the change that occurs over the distance of d z. So, this variable we simply differentiates over d z multiplied by the distance over d z.

So, what is a rate at which it is changing over z direction multiplied by the distance that it covers. So, this becomes and d x by d y becomes the area. So, we have rho into velocity into the area through which it is entering and the area in this case is d x multiplied by d y. So, this becomes a total increase in the mass along the z direction. So, total. So, if you solve this, this quantity cancels out, you left only with this term. So, this is what you get d x, d y and d z becomes the net volume of this element. So, we have replaced it like this. And we get similar equations for y and x axis.

So, if you combine these equations together, this is what you are getting; del by del x of rho u x, del by del y of rho u y, del by del z of rho u z d V is equal to minus lambda times. So, this is what we are getting. Now we know about the Laplacian operator nabla. So, nabla is the Laplacian operator which is defined as del by del x plus del by del y plus del by del z; and it operates over some vector quantity.

So, here this can be written in the form of this Laplacian operator as it is the negative times the nabla of rho into u; where u is, it has a x component along the x axis, it has a y component along the y axis, and it has a z component along the z axis. So, it is simply nabla times rho of u. So, you get, the x component gets multiplied with, the x component is differentiated with respect to del by del x; the y component with del by del y; and z component with del by del z

So, the overall equation that you get here, this is the quantity that you are getting. So, this is the net increase in the mass; and the net increase in the mass per unit time can also be written as del rho by del t. So, del p by del t is equal to this. So, the quantity that we have derived is the increase of mass per unit time. So, which means that, the increase of mass per unit time is also the increase in density per this is the rate in; the rate at which the mass is increasing per unit time can be written as del rho by del t and this will be simply minus nabla times of the quantity that we have derived, this quantity.

So, you put it in the left side and this is the equation you get del rho by del t plus nabla times of rho u is equal to 0. So, this equation we are going to carry forward and we are going to use.

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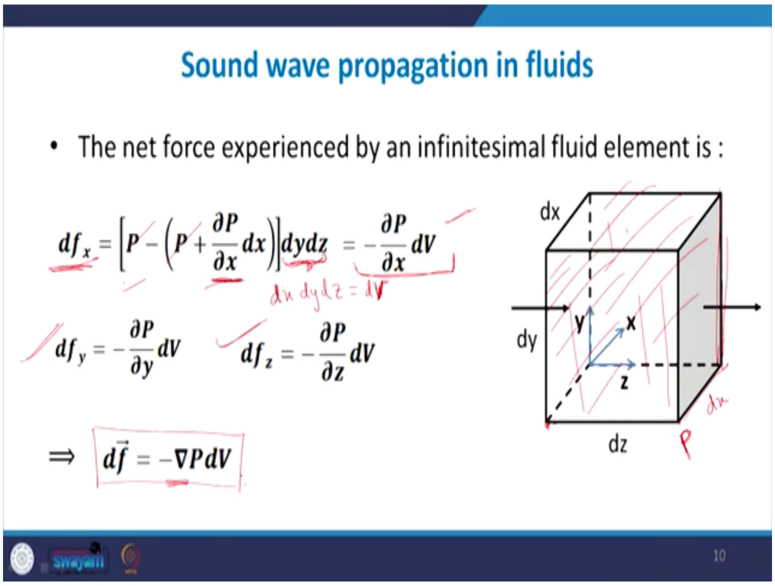

Now, the net force experience let us again consider the same fluid element. So, now the force can be given as simply the pressure multiplied by area, right. So, if you consider the elemental force of this speed elements. So, it will be simply whatever is the change in pressure multiplied by the area.

So, here let us consider the x axis here I have considered; so the change in pressure over this phase and the change in the pressure. So, we are taking this phase and this phase; so these two different phases. So, let us see the pressure at this phase is P which was a total pressure of the medium fluid particle and then as it goes through a distance of d x; if this is the pressure gradient along x axis. So, this is the rate at which the pressure is changing along x axis; then this multiplied by the distance will give you what is the pressure at this particular point, the change in pressure. So, the pressure at first phase minus the pressure at second phase times the area of the phase; so pressure, the change in pressure multiplied by the area is given us whatever is this differential force.

So, again this quantity cancels out and d x, d y and d z becomes d V. So, what we get is, we get this particular equation. And similarly it can be done for y axis and z axis. So, the total force again becomes del by del x you have minus del by del x minus del by del y and minus del by del z. So, again we introduced the Laplacian operator. So, it is this Laplacian operator applied over P d V. So, this is one equation we are getting.

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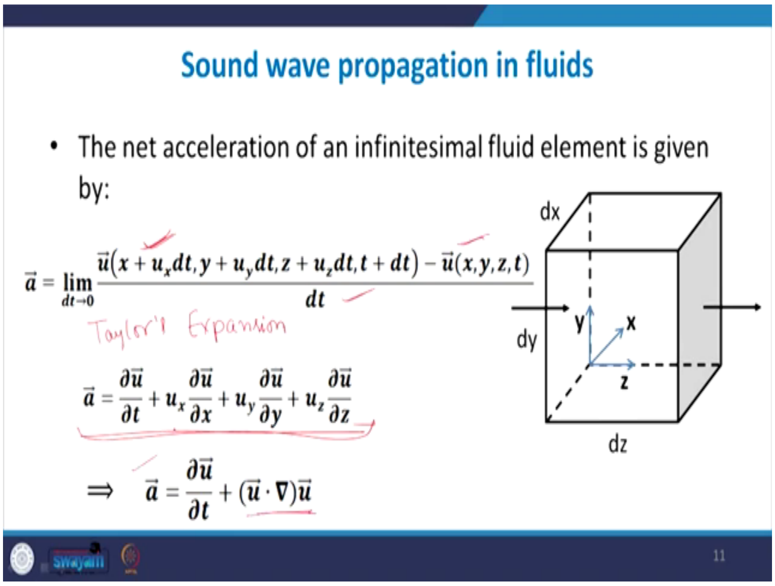

Now, the acceleration of this particle can be whatever is the change in the velocity of the particle; so the change in the velocity, so velocity final velocity minus initial velocity, so it is d u by d t.

So, the expansion of this is you can use again the Taylor series expansion, the Taylors expansion. And this derivation again is quotes on mathematics; so it is out of course. So, when you expand it to a left with this quantity. So, it is simply del u by del t times u nabla u.

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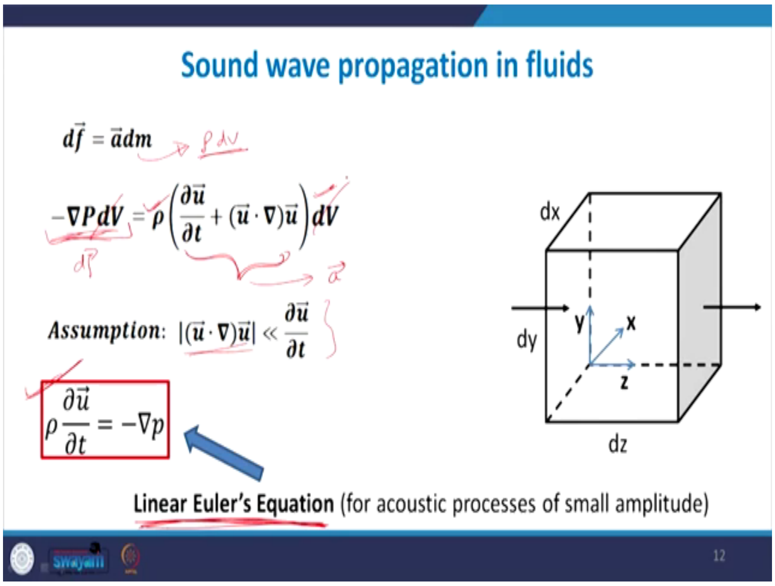

And now force is simply acceleration into mass by Newton's second law. So, we already had the equation for force here; this was the equation for force and this was the equation for the acceleration. So, this force is becomes acceleration times the mass. So, this is the mass; mass is rho d V, this becomes rho times d V density multiplied by volume.

So, we have a place d m by rho d V and we already have the expression for the d f and the expression for net acceleration. So, this becomes the expression for the net acceleration, this becomes the expression for d f and this quantity is d m. And assuming this double gradient is very small compared to a single derivative. So, the overall equation that we get this is called as a Linear Euler's equation; this is rho times del u by del t is equals to minus nabla time of p, the d V the d V cancels out here.

So, as you can see few portions of this derivation is what you study in the basic course of mathematics. So, this derivation is just for the sake of understanding, it is not a part of the assignment, it is not the part of your course. So, it is not going to be a part of your assignment or papers; but it is more for the sake of understanding. So, now, we have got these two equations.

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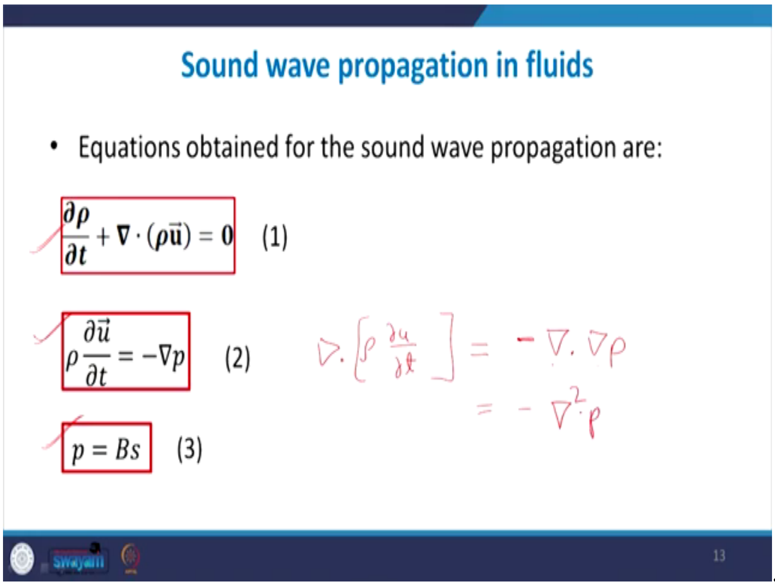

So, the three equations we got; we got this equation, we got this equation and we got the first this one equation. So, let us use these three equations and find the acoustic wave equation. So, let us solve this first one; del rho by del t nabla dot rho u.

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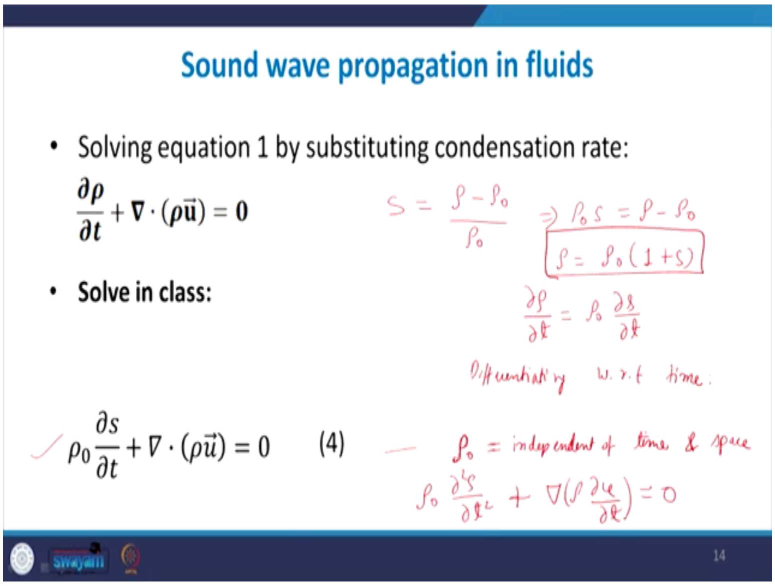

Now, I am going to write this in terms of del s. So, we know that the condensation rate is rho minus rho naught by rho naught; so which means what, rho naught s is equal to rho minus rho naught. So, rho then becomes rho naught plus rho naught s; so it is rho naught into 1 plus s, this becomes a relationship. So, del rho by del t can be represented as rho naught times del s by del t. So, we have used this particular relationship and we replace this del rho by del t by this quantity, and this is the equation we get. So, now, let us derivate this, let us differentiate this equation with respect to time; when you differentiate this equation with respect to time what you will get is, differentiating with respect to time the same equation becomes, this one becomes rho naught.

Now here the mean, this rho naught is assumed to be independent of time and space. So, what we are assuming is that, we have a homogeneous medium. So, all this equation we are deriving with the assumption that the medium is homogeneous throughout. So, the density, the bulk modulus it remains the same throughout irrespectable of what point of space you are considering. So, irrespective of, it is independent of time and space. So, since rho naught is now independent of time or space. So, in that case if you differentiate it you get is; rho naught del square s by del t square plus nabla times of rho into del u by del t is equal to 0. So, this is the equation you are getting.

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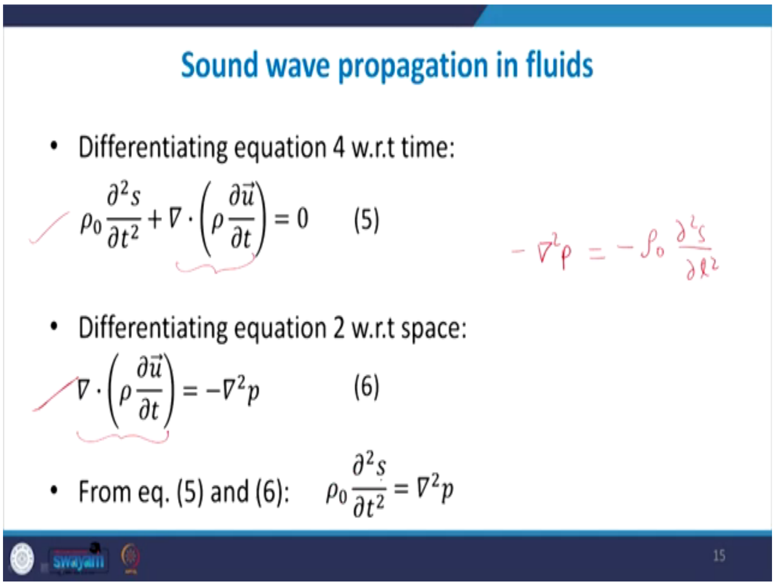

Now you differentiate this equation. So, this was the equation rho del u by del t minus nabla times of p; if you differentiate this equation with respect to space, so you are applying nabla dot operator both on the left hand side and the right hand side. And nabla dot nabla is represented as the 3 D Laplacian operator which is minus nabla square. So, this is the equation you get; nabla dot rho del u by del t is equal to minus nabla square p that is a equation you get, when you differentiate with respect to space.

Now this quantity can be equated. So, this quantity is equal to. So, which means that minus nabla square p is same as this quantity becomes minus rho naught times del square x by del t square. So, we are equating the value of this quantity.

So, from the two equations finally, we get nabla square p is equal to rho naught square, nabla square p is equal to rho naught times del square x del t square.

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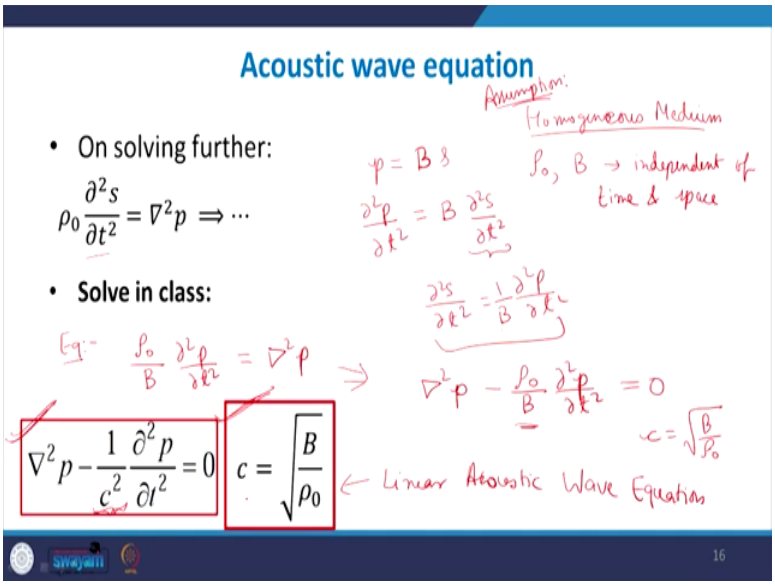

Now, we solve further. So, what we do here is that, we use the first relationship; we had a acoustic pressure as the bulk modulus multiplied by s, which is the condensation rate. Now we know that the medium is homogenous; homogenous medium, this is our assumption, is the homogenous medium. So, both rho naught and the bulk modulus they are independent of time and space.

So, both are independent of time and space. So, this B is a constant. So, when you derivative it with respect to time; so what you get is, del square p by del t square will then become this constant B times del square s by del t square. So, we can replace this quantity as del square p sorry, del square s by del t square becomes del square p by del t square into 1 by B. So, if you put now this particular thing into the equation. So, the equation that you get is rho naught by B. So, we have replace this quantity with this value. So, it is rho naught by B times of del square p by del t square which is going to be nabla square times p.

So, we can overall write something like this. So, what we do is that, nabla square p minus rho naught by B is equal to del square p by del t square is equal to 0, this is the equation we get. So, now, if we take c as under root of B by rho naught. So, this quantity becomes 1 by c square. So, the overall equation we are getting is nabla square p minus 1 by c square times del square p by del t square is equal to 0. This is a very important wave equation; this is called as the linear acoustic wave equation.

So, it is a very important equation and here the purpose of putting this in terms of this is because; when you solve such differential equations, then you can easily get the solution for this would be a sinusoidal function of this particular quality. And that is why we have represented this particular equation by replacing it and replacing it in this particular form. If you replace it in this particular form, then the solution you get will be a sinusoidal function with respect to c.

So, here c is the thermodynamics speed of sound.

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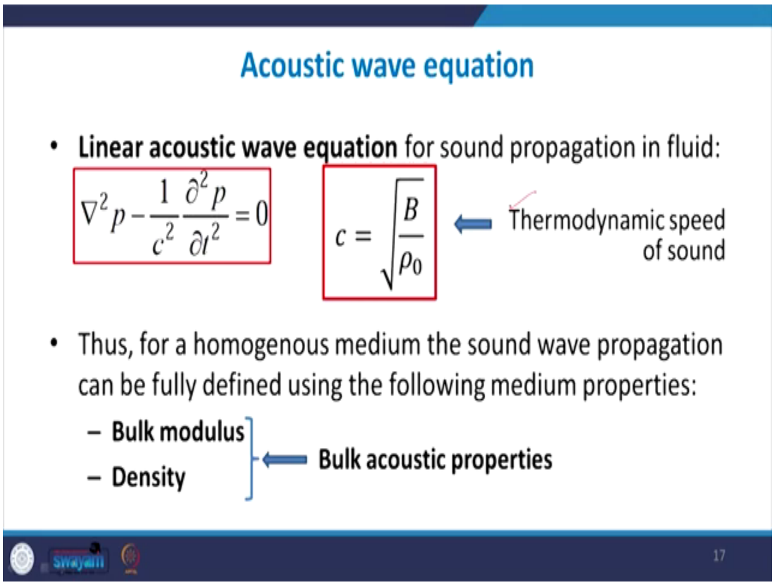

So, as you can see overall c is thermodynamic speed of sound, which is actually this was the originally equation, this is the equation. So, as you see the behaviour of the acoustic pressure it depends upon rho naught and B. So, for a homogeneous medium if you can, if you fully know this bulk modulus and density; then you can fully define how the sound wave will propagate through this medium and that is why they are called as the bulk acoustic properties.

So, why are we called the bulk acoustic properties? Because here we are assuming that throughout our domain of study, the properties of the medium remain constant. So, both B and rho naught remains constant. So, when we know these two quantities, we can define how the wave is going to propagate in a medium. So, these are the bulk acoustic properties.

Now let us, now we have derived a general wave equation; the only assumption was that there was no mean flow in the medium and the medium was homogeneous throughout.

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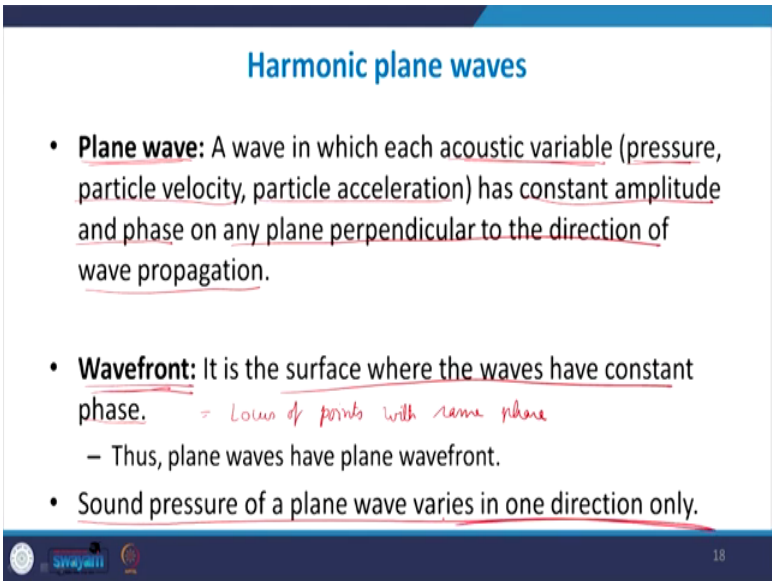

Now, we will consider a specific case and that will be of harmonic plane wave. So, what do you mean by a harmonic plane wave?

It is a wave in which each acoustic variable; whether it would be the acoustic pressure, the acoustic particle velocity or acceleration. So, any variable of study, it has constant amplitude and phase on any plane that is perpendicular to the direction of wave propagation. So, that is the definition of a plane wave. So, the direction in which it is propagating, you can take any particular plane that is normal to the direction of wave propagation; then throughout that plane the wave will have a constant amplitude and a constant phase.

Now here we are introducing a new term which is called as the wavefront. The wavefront is simply the surface where the waves have constant phase or it can also be called as the locus of points with same phase.

So, in a particular waveform all the locus of a point having the same phase will constitute a wavefront. So, I will show you a few figures and animations to make these definitions more clear. So, you have to remember that in a plane the wave that it varies; here the sound wave it varies only in one direction. So, there is only a single direction of wave propagation and perpendicular to that we have the wavefront where the amplitude and the phase both are constant.

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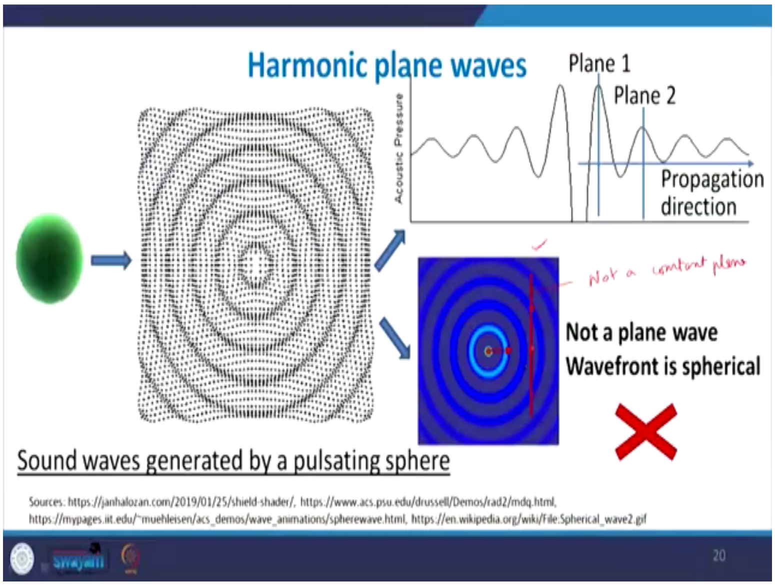

So, just to tell you a few examples; let us say we have a pulsating sphere. So, we have a ball that is pulsating. So, when it is pulsating, then this is the kind of motion that is creating throughout in 3 dimensions. So, is this a harmonic plane wave, if you freeze it overtime? So, let us say we have cut a plane here; this red line is the plane here red line. So, this is. So, at any point of time the direction of wave propagation is radially outwards. So, we cut a plane normal to the direction. So, you see that here at this point there is a zone of low pressure; at this point there is a zone of high pressure and low pressure.

So, at different points the wave is undergoing, the wave is under different phases. So, this is not a constant phase plane. So, because here the plane is not constant; here the plane the wave front here if you draw the plane normal to the direction of wave propagation, you see that at different points we have different phases. Sometimes the phase, at some points the wave is undergoing a undergoing a low pressure; at some point it is undergoing a high pressure and the phase difference becomes here pi by 2. So, this is not a spherical wave.

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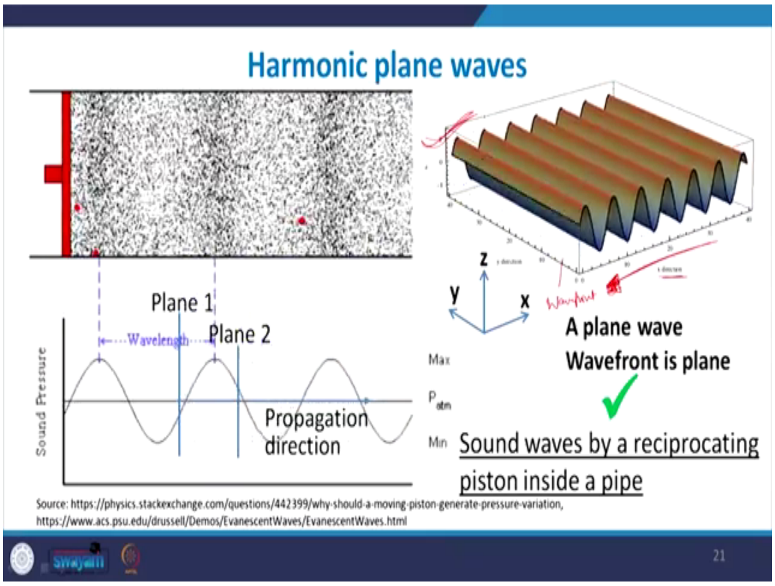

This is a harmonic wave. So, what we see here is that, the previous example where it was being generated inside a long tube and a reciprocating piston. So, this is the kind of waveform we are getting; if this is the kind of this is a 3 D view of the waveform and it needs a little bit of visualization. So, here as you can see this is the x direction. So, the wave is propagating in this direction, it is going towards in this direction. So, if you cut any plane this becomes the wave wavefront.

So, if you cut any plane normal to this whether you cut it here or there. So, let us freeze it and see; if you cut a plane here or a plane here anywhere, so at every point within this plane. So, all the particles here they are under the same phase. So, either here if you cut a plane here all of them are undergoing, they are at the highest level they are at the peak. So, highest pressure, if you cut a plane here all the points together will be undergoing a minima; all the points together will undergo a maxima and then if you cut a plane here all the points there at the same phase and the amplitude is also constant.

So, this is a typical waveform of a harmonic plane wave this.

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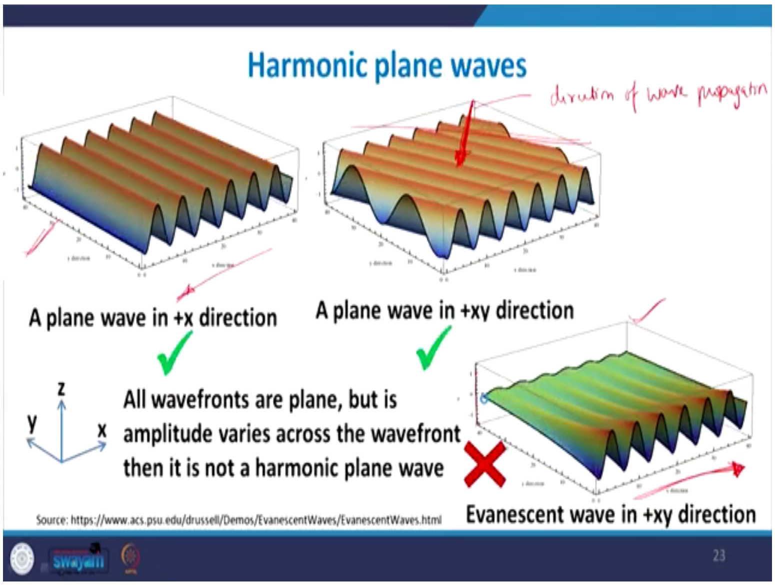

Similarly, we have different types of harmonic plane waves; we have a harmonic plane wave which is going towards a positive x, then we have a harmonic plane wave that is going. So, here also although in this case the direction is in this way; in this case the direction of wave propagation is somewhere here, this is the direction of wave propagation. So, as you can see; if you cut any plane in this direction, so it is like x y direction, then you will see that all the waves together undergo minima, they together they undergo maxima. So, this is also a plane wave and the amplitude is fixed.

Now let us see a differentiation here; here also you have a wave and if you cut any plane. So, this is propagating along this direction as you can see; this is the direction in which the wave is moving. If you cut any plane normal to this, then the phase is going to be the same; they are together undergoing minima and maxima, but the amplitude is same. So, here in this phase if you see, the amplitude of this and the amplitude of this is different. So, the amplitude changes over changes in the plane and the phase remains the same.

So, although here the wave front is; so here we have the same phase of this becomes the wave front, but still the amplitude is not constant. So, it is not a harmonic wave. So, we will continue with this discussion in our next lecture and will study more about harmonic plane waves and the equation for sound propagation of harmonic plane wave.

Thank you for listening.