Acoustic Materials and Metamaterials

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Lecture - 24
Introduction to Acoustic Metamaterials - 1

Welcome to lecture 24 in the series of Acoustic Materials and Metamaterials I am Doctor Sneha Singh an Assistant Professor in the Department of Mechanical and Industrial Engineering at IIT Roorkee. So, today till now in this course we have studied about some concepts on acoustics and how the sound propagation takes place, how does it interact at boundaries, about reflection, transmission, dissipation and then we studied one by one about some conventional materials.

Today will be our first lecture on introduction to acoustic metamaterials. So, in this lecture, I will go through first what why do we need. So, I will try to address the question why do we need some new kind of metamaterials for noise control.

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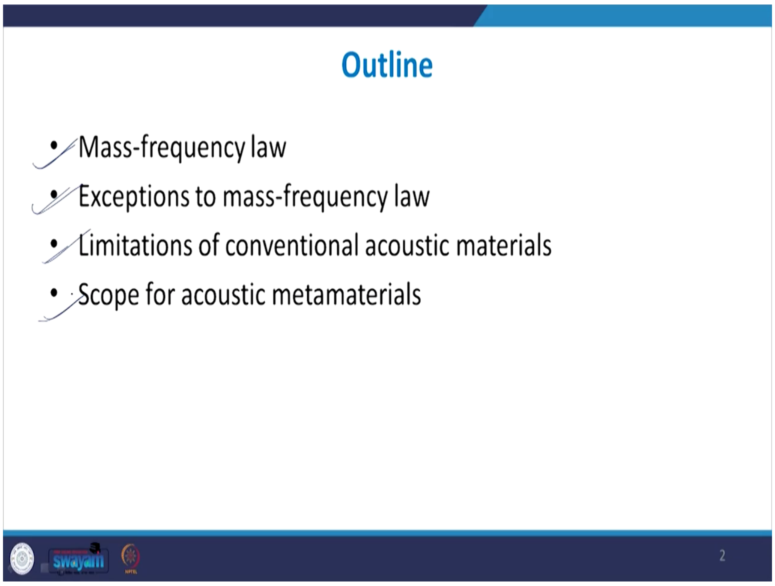

So, for that I will discussed with you a special law called as the mass frequency law, then some exceptions to the mass frequency law, the limitations of conventional acoustic materials. And, then finally, we based on the limitations will define what is this scope for creating some new materials, called as the matematerials.

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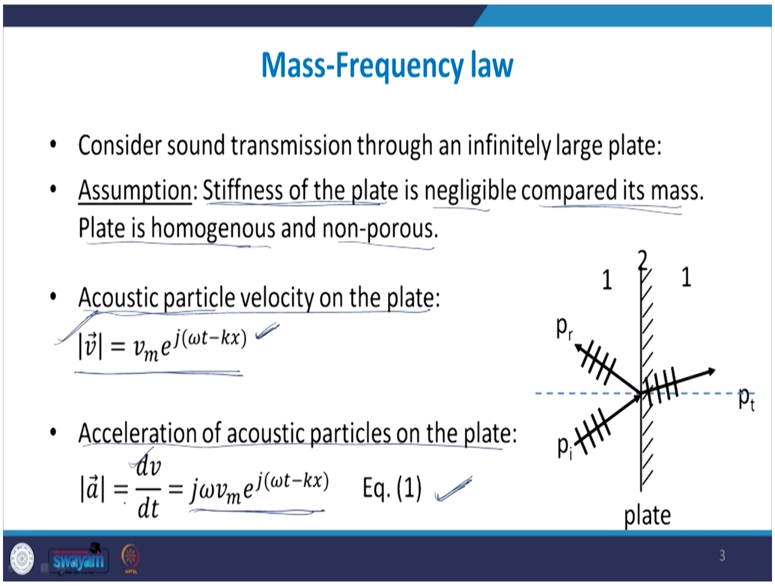

So, mass frequency law, this is a law that most of the tradition materials they have to obey. So, let us say that through what does it mean through. So, how is how it is derived? So, let us say we have a infinitely large plate and that in the sound hits this plate. So, we have to find out what is the impedance of this plate.

So, the first assumption we begin with is that, the stiffness of the plate is negligible compared to it is mass and for most of the real life material. So, if we are using specially if we have using some barrier material or a material for enclosure, then such kind of hard materials, they obey this law that is in that case the stiffness of the material is usually smaller compare to the mass and that the plate is homogeneous it is non porous.

So, this is a few assumptions that we take so, in this case the particle velocity on the plate. So, let us say some sound wave incident on the plate and we are studying how it gets transmitted.

So, the particle velocity on the plate can be given a simple harmonic form. So, as I have a emphasized again and again all the acoustic processes they are adiabatic in nature. And, at the same time small fluctuations they adiabatic, and because here if you are studying some harmonic plane wave front, then the solutions are also harmonic, so, we usually study harmonic solutions.

So, small acoustic fluctuation they are studied as harmonic solutions. Although, they could be random noise which may not be harmonic in nature, but they can always be represented as a sum of a number of harmonic solutions based on the Fourier series. So, we take a harmonic solution assumption not the assumption, but this is the this is what happens for every acoustic wave. So, here v is taken as some amplitude v m this is the velocity amplitude into e to the power j omega t minus k x. So, because here we are studying about a harmonic plane wave front, the same concepts can also be applied to spherical wave front or any other wave front.

So, because a harmonic plane wave is incident, so, the velocity profile is also similar to a harmonic plane wave. So, this is the expression. Then, the acceleration of the acoustic particles on the plate so, this a can be given as dv by dt. So, if we differentiate this expression what you get is this becomes j omega, v m e to the power j omega t minus k x. So, when you differentiate only this term comes out which is the constant multiplied by the time variable. So, this becomes the expression for the acceleration of the acoustic particles in on the plate.

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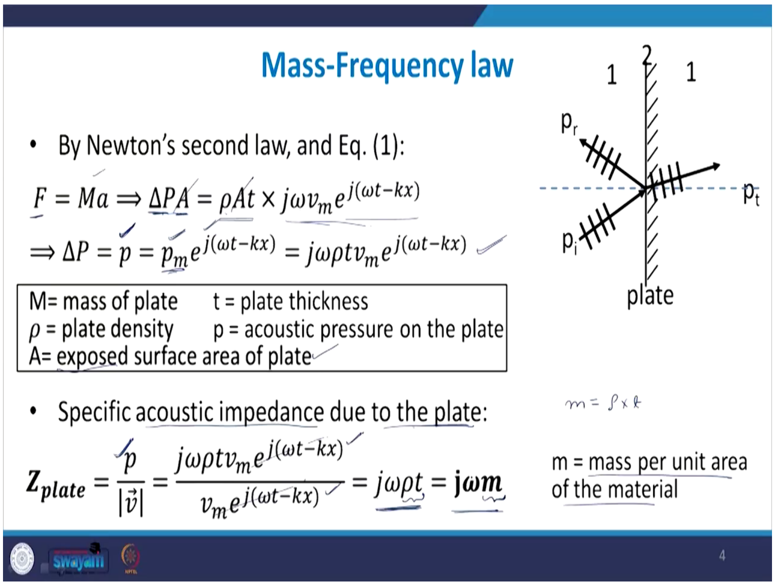

Now, by Newton's second law force is equal to mass into acceleration. So, if we take a simplified version of Newton's law and apply to this plate. Then, this force can be given as the net pressure acting on the plate into the acceleration of the plate and the net pressure acting on the plate. So, let say we have a thin plate and there is certain pressure p 1 uniformly acting on the plate and the pressure p 2 acting on the other end, then the resultant pressure will be p 1 minus p 2 that will be the resultant pressure acting on the plate.

So, we have presented this F as del p into A where A is the exposed surface of the surface area of the plate and this mass into acceleration. So, we use this previous equation 1, where we derived the equation for acoustic particle acceleration. So, this equation is used for acceleration and the mass. So, F is represented as this A is represented as this and this mass then becomes this expression.

So, here mass is the density of the material multiplied by it is volume. So, the density is rho and the volume is the exposed area of the material multiplied by what is the thickness of the material. So, now, if this area cancels out from both ends so, this cancels out. So, what we have left with is del p is equal to p m. So, here del p.

Now, we know that, in this case the acoustic pressure is actually the pressure difference, it is the fluctuation from the mean value. So, here we are representing del p as the acoustic pressure or the net fluctuation or difference in the pressure that is created in the plate. So, the pressure difference is actually created because of the acoustic waves flowing through the plate and that is why del p is equal to the acoustic pressure.

So, this del p again will be a; will be of the form of this equation. So, here you have this p can be represented as the acoustic pressure will become the pressure amplitude multiplied by e to the power j omega t minus k x. So, this becomes the expression for p and this is same as, if you take this right hand side this becomes j omega rho v m e to the power j omega t minus k x. So, this is the equation we have reduced to.

So, the specific acoustic impedance or simply the acoustic impedance of this plate is then given by the plate sorry the acoustic pressure acting on the plate divided by the particle velocity. So, we write the expressions for the pressure and the velocity here. So, what we get is this entire expression cancels out and the and the net value that we are getting is j omega rho into t.

So, this comes out to be the simplified form of acoustic impedance of a plate considering it is homogeneous in nature and the stiffness is not as big as the mass. So, we get j omega rho t. So, rho into t can also be written by the variables. So, this variable is same as this variable. So, here M which is equal to the density multiplied by the thickness is actually mass per unit area of the material. So, this is the ultimate equation we get or the expression we get for the Z or the acoustic impedance of the plate that is j omega into it is mass density per unit area. So, the expression for Z plate came out to be j omega m, where M was the mass density per unit area.

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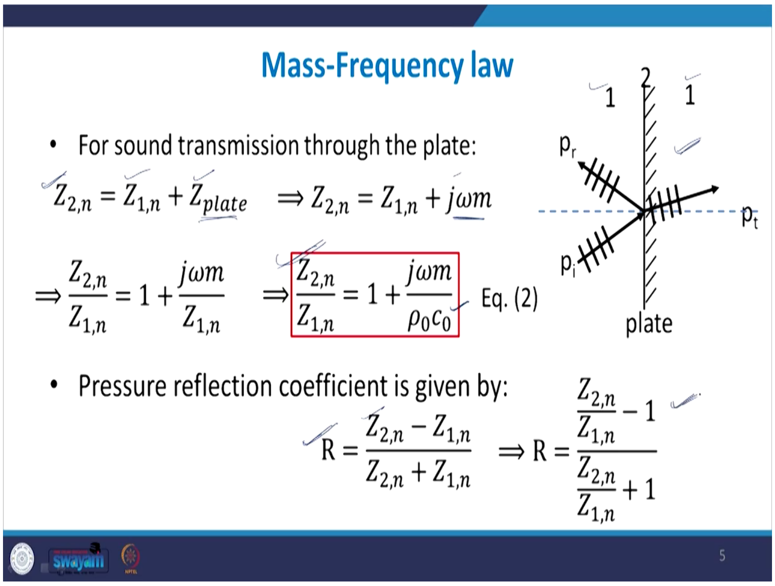

So, if the sound transmission takes place through this plate. So, in that case the total impedance due to this particular this boundary will be the impedance due to the mass of the plate or simply the plate vibration plus the impedance of this corresponding fluid medium. So, Z 2 n can be written as Z 1 n plus Z of plate, because there is a medium 1 on both ends. So, Z plate we have found as j omega m, we replace this expression here.

So, Z 2 by Z 1 then becomes 1 plus j omega m by Z 1, which if Z 1 and this Z 1 n, we have replaced with the expression here rho naught c naught. So, the specific acoustic impedance of any fluid medium is the product of it is density and the speed of sound in that medium. So, we are using that thing. So, this is rho naught c naught here. So, this finally, with get this value for this expression. Now, in our lecture on the sound propagation through medium boundaries, if you go through that we have derived the expression for the reflection coefficient and absorption coefficient, in terms of the impedance of the 2 media.

So, reflection coefficient was given by Z 2 n minus Z 1 n upon Z 2 n plus Z 1 n, which is equal to Z 2 n. So, if we divide the numerator and denominator by the Z 1 n this becomes the final expression.

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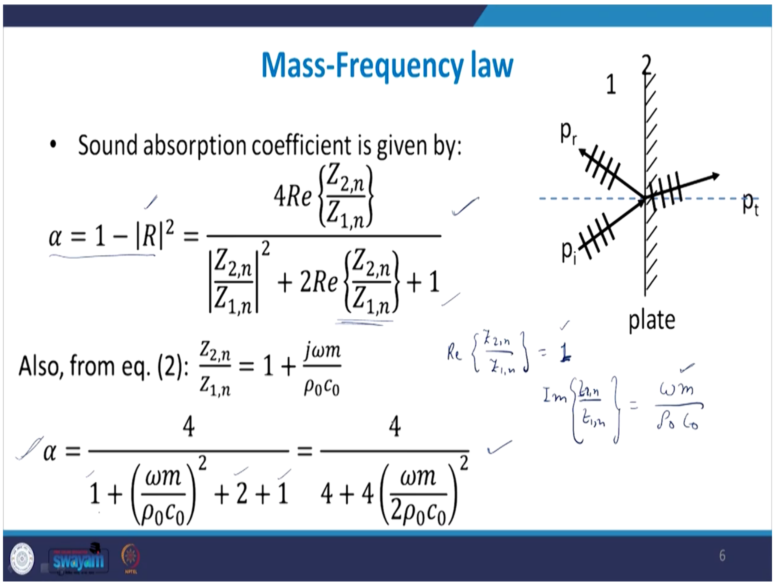

So, this is our expression for R. And, alpha is given by 1 minus mod of R square. So, you can try this as an exercise at your home just input this value here as mod of R square. So, this expression if it is input here, then this is what you will end up with this is 4 times the real part of Z 2 by Z 1 divided by mod of Z 2 by Z 1 whole square plus 2 re Z 2 by Z 1 plus 1. Now, here Z 2 by Z 1 we have already found as 1 plus j omega m by rho naught c naught.

So, using this particular value so, the real part of this particular ratio of complex number becomes 1. So, here 1 is the real part and the imaginary part is omega, the imaginary part of this Z 2 n by Z 1 n comes out to be omega m by rho naught c naught.

So, using this these 2 this particular expression here 4 times the real part of Z 2 by Z 1 is what 4 times of 1, 1 is the real part of this. And, mod of this expression will be this mod of any complex quantity is the mod square of this is simply the real part whole square plus the imaginary part whole square. So, it becomes 1 plus omega m by rho naught c naught whole square plus 2 times of 1, the real part being 1 here plus 1. So, using this expression this is what we end up with for alpha.

So, again if you this expression comes out to be 4 divided by 4 plus and we have taken this 4 out. So, it becomes omega m by twice rho naught c naught whole square. So, this is the expression for absorption coefficient. Now, if you take out this common factor 4 from both numerator and denominator.

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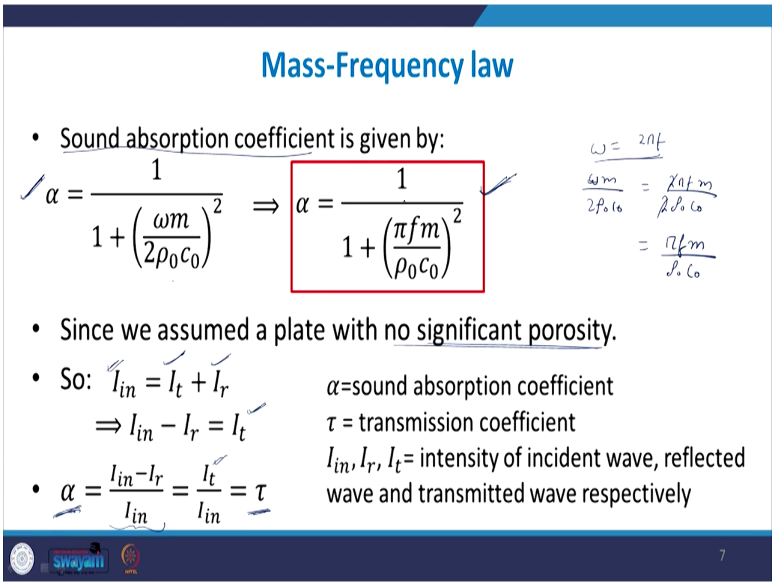

So, ultimately this is the expression of alpha or the sound absorption coefficient and omega is 2 pi f. So, omega m by twice rho naught c naught will be 2 pi f m by rho naught c naught, which will be pi f m by rho naught c naught. So, using this omega equals to 2 pi f, we can reduce this expressions in terms of the frequency. So, what we get from this exercise is we get a simplified version of sound absorption coefficient in terms of the frequency and the mass density.

Now, here we have assumed that the plate was non porous in nature, it was homogeneous, it was a solid plate. So, there was no significant porosity in the beginning we have made this assumption. So, in that case because there is no significant porosity it is just blocking the sound, because of it is mass property or inertia.

So, in that case there is no heat dissipation inside the pores. So, we neglect heat dissipation. So, all the incident wave becomes in transmitted intensity plus the reflected intensity. So, I in minus I r becomes I t, here no heat dissipation because of no porosity. So, alpha which is given as which is defined as the difference between intensity of the incident wave minus the reflected wave divided by intensity of the incident wave will now become I t by I in.

So, in case of no other means of heat dissipation whatever is being absorbed is been actually transmitted to the other end. So, that is the thing that is happening. So, in that case alpha will be same as the transmission coefficient tau. So, the expression that we got for alpha can be used for the transmission coefficient also.

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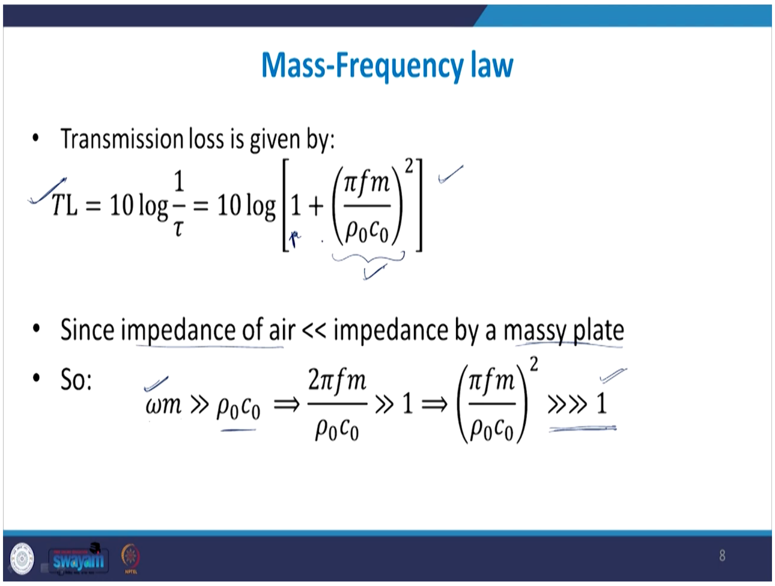

So, the transmission loss for this particular plate then becomes 10 log of 1 by tau which is going to be 1 by this particular expression here. So, it becomes 10 log of 1 plus pi f m by rho naught c naught whole square. Now, the impedance of the so, here, the medium fluid medium is a air medium or even if it is not an air medium, it is any other fluid medium then the impedance of this fluid medium will in general be much smaller than the impedance of a massive plate.

So, we have assumed the thick solid massive plate. So, the impedance due to this massive plate will; obviously, be much larger it will our more resistance compare to the over just a uniform fluid medium.

So, therefore, and what is the impedance, what is the magnitude of the impedance of the plate it is impedance of the plate was j omega 1, I am sorry j omega m. So, the magnitude of the impedance of the plate is omega m, the magnitude of the impedance of air is rho naught c naught. So, omega m is much greater than rho naught c naught. So, which means this 2 pi f m by rho naught c naught will be much greater than 1.

So, overall this whole square will be much greater in order than the quantity 1. So, we can neglect this particular expression here and we can only use this one here to reduce it or further simplify this transmission loss.

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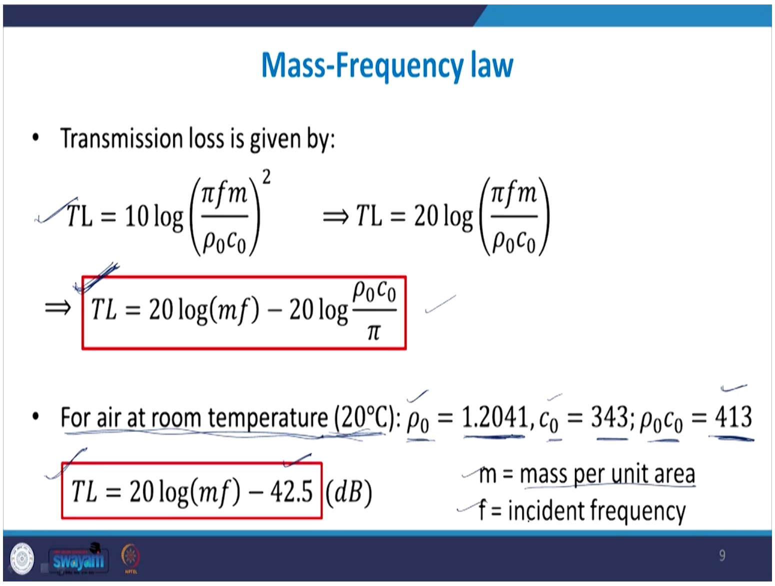

So, this is what we get, by the property of law this becomes 20 log pi f m by rho naught c naught, you separate the 2 numbers. So, you get 20 log. So, this is the final expression of transmission loss it is 20 log m into f minus 20 log of rho naught c naught by pi, if, we are considering air at the room temperature.

So, if you know what is the fluid medium? Then, you can just input the value of that fluid medium and the most common medium, in general is an air at the room temperature. So, for that case rho naught is given by this c naught is given by this. So, there are tables of air density and air speed of sound you can, it is available online or in books. So, you can easily find them and you can find out value of rho naught and c naught. So, this becomes the value of rho naught c naught for air at room temperature.

So, when you input this value here. So, minus 20 log of 413 by pi will then come out to be approximately 42.5. So, for air at room temperature the transmission loss can be simplified to 20 log m into f minus 42.5 decibels and for general medium this will be the expression the first expression. So, as you see this is mass per unit area and this is incident frequency.

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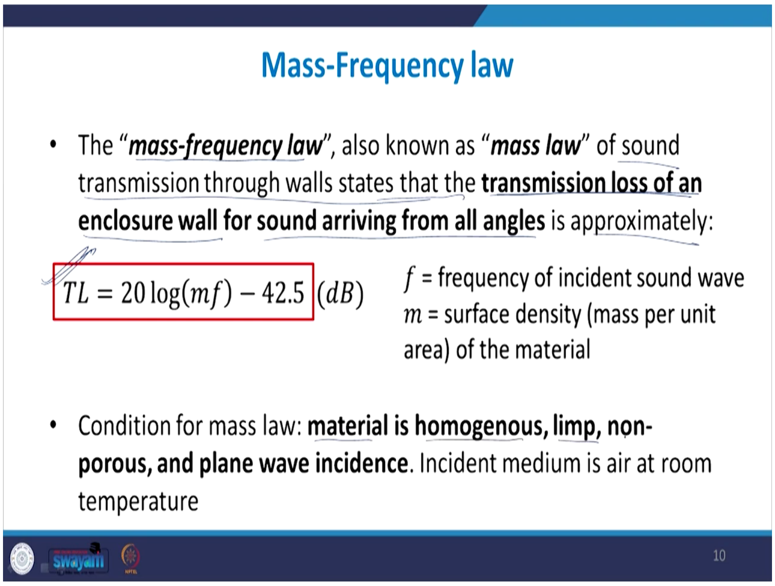

So, now based on this whole derivations of absorption coefficient and transmission loss the mass frequency law can be stated as it is the mass frequency law for sound transmission through the walls, any walls that are usually acting as the barrier or enclosure.

So, the sound Trans so, the transmission loss for such enclosure wall for sound arriving from all angles is approximately given by this expression. And, the conditions here is at the material should be homogeneous limp non porous and plane wave incidence.

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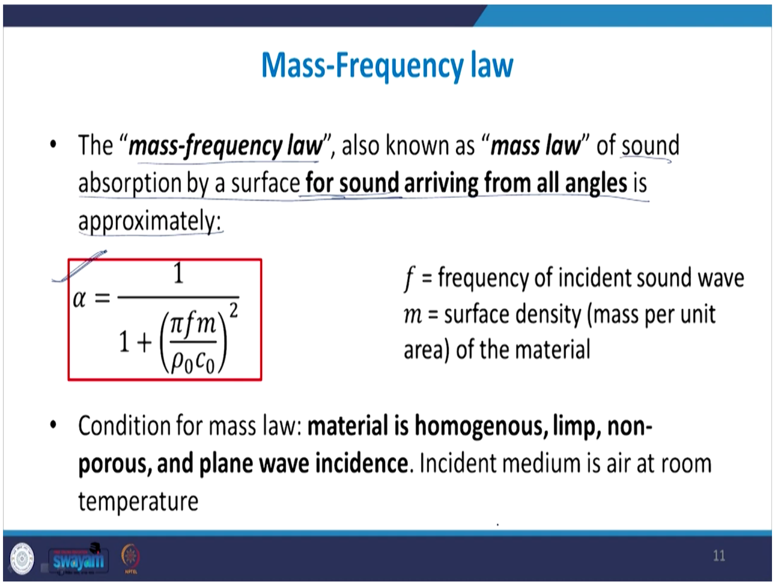

It can also be stated using the absorption coefficient expressions. So, the same mass law can also be stated as the total sound absorption by a surface of by a surface for sound arriving from all the angles is approximately given by this particular expression 1 by 1 plus pi f m rho naught c naught whole square.

So, the two expressions we got alpha and T L, they are used to in that the mass frequency law. So, from the mass frequency law what we get is that approximately this alpha is if you neglect this value because it is small compared to this expression.

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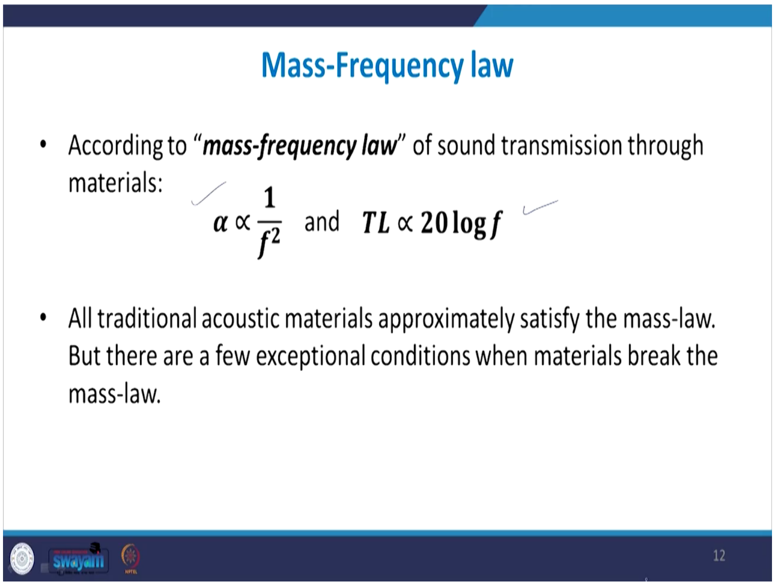

So, alpha is 1 by inversely proportional to 1 upon f square here and transmission loss is proportional to 20 log of f. So, the conclusion is that at low frequencies both alpha value is going to be extremely low and the transmission value is also going to be low. So, at low frequencies the transmission loss is less at high frequencies the transmission loss is more.

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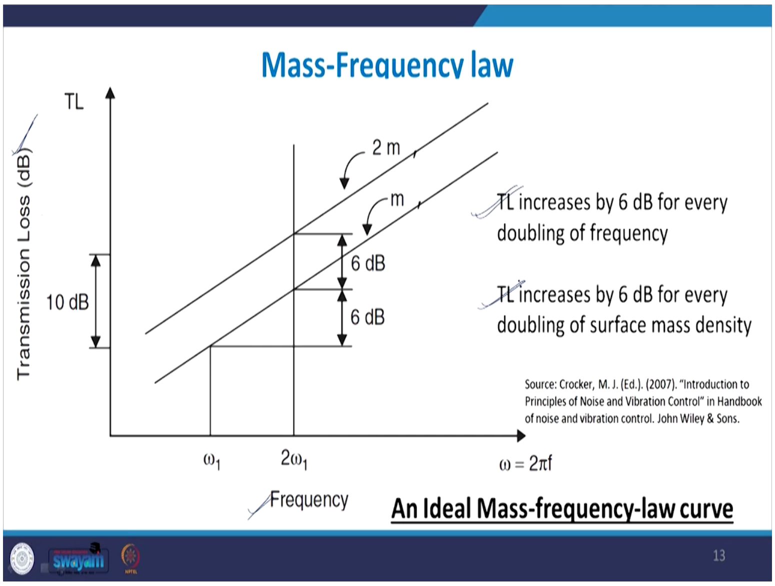

And, that is why most of the materials they perform better at high frequencies. So, this gives you a table. So, this is the variation of transmission loss with frequency and all the traditional and non-porous materials they follow this law. So, which means that at low frequency is their performance is always going to be poor and it will increase with the frequency.

And, suppose you double the frequency what will the effect on the transmission loss it will be 20 log of 2 f becomes double. So, 20 log of 2 is 6 decibels. So, every doubling of frequency increases TL by 6 decibels, similarly every doubling of surface mass density will increase the TL by 6 decibels.

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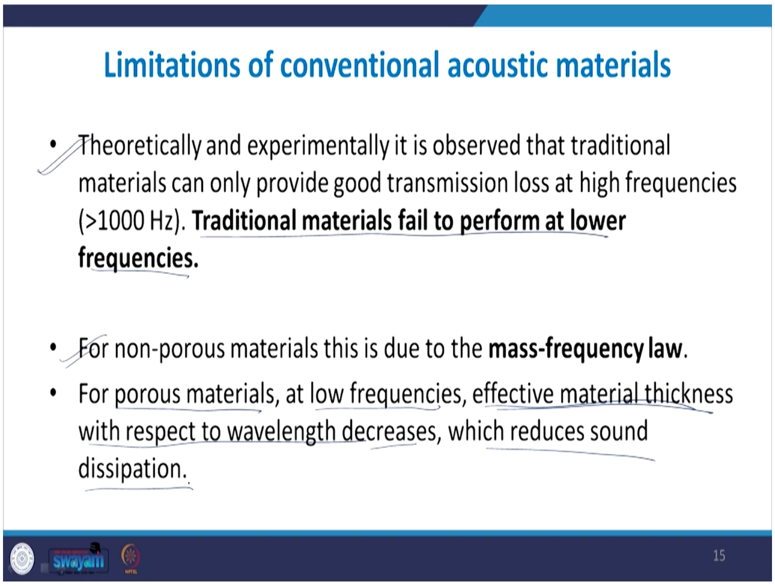

So, this is the traditional mass frequency law, but there are certain exceptions to this law and what are the exceptions? So, usually when a material it is transmitting sound, then typically there are 2 types of transmission that takes place, it is one is the non-resonant transmission and the other one is the resonant transmission.

So, when whenever the material is in a normal condition there is no resonance. Then, it will follow this typical mass frequency law and it is transmission loss will be heavily dependent on the frequency of the wave, but if at certain frequencies the material achieves the phenomenon of resonance, then in that case the material will vibrates.

So, what so, what is resonance? When the incident frequency becomes equal to the natural frequency of the material. So, when both the frequencies become the same, then the particular material offers minimum resistance or minimum impedance to sound flow, and it starts vibrating heavily, and sound, and maximum transmission takes place. So, only at certain resonant frequency is this law is broken otherwise this law is followed. So, this is what has been observed.

So, usually for non-porous materials so, the limitation can be summed up as for non-porous materials, they fail to perform at lower frequencies. And, why is this? Because of the mass frequency law and for porous materials at low frequencies, the effective material thickness with respect to the wavelength decreases. So, at low frequency means very high, very large wavelength. So, compared to the wavelength the material thickness is very very less and therefore, less loss. So, both porous and non-porous materials they perform bad at low frequencies typically below 1000 Hertz.

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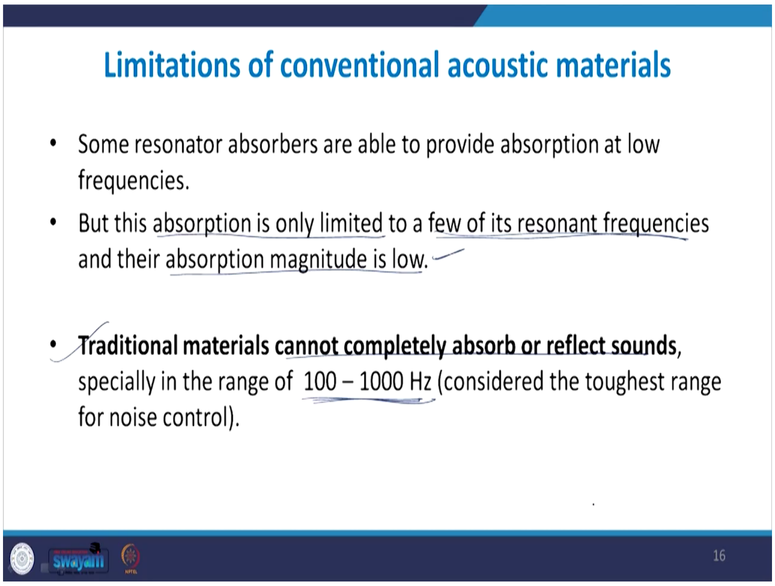

And, even if so, we studied some of these materials and then we studied about some resonators. So, among absorbers there were porous absorbers and then there were resonator absorbers. And, the resonator absorbers included the Helmholtz resonator, the panel resonator and the micro perforated panel.

So, there what we observed was that all the porous material it does not perform, it performs very poorly at low frequencies, but these particular resonators the Helmholtz the panel or the micro perforated panel it. They can give you a sharp absorption at low frequency, but even then that absorption is only limited to a few of it is resonant frequency it is not a broadband. So, overall performance is not good only at a limited number of frequencies, they have some sharp peaks and the absorption magnitude in that case is low.

So, overall what you can say is that the traditional materials they cannot completely absorb or reflect sounds, in the low frequency range typically 100 to 1000 Hertz, which is considered as the most critical range for noise control.

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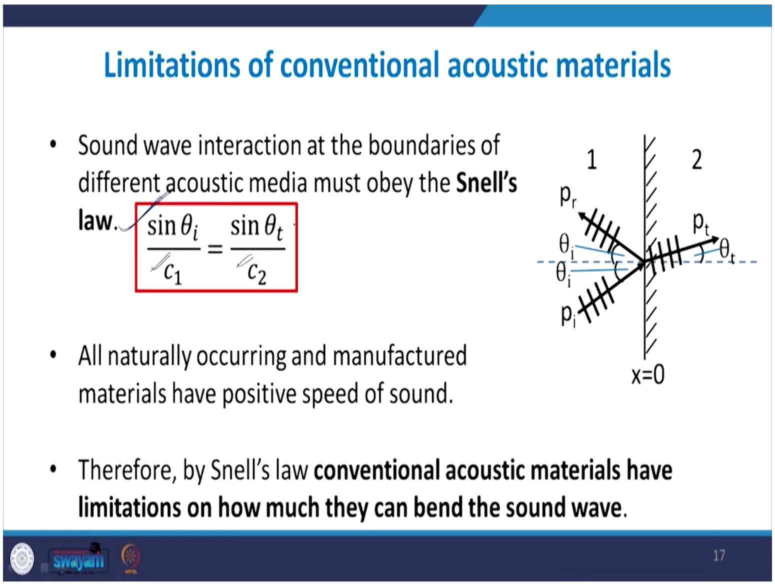

And, due to this limitation a new type of material is desired. The second form of limitation of the conventional acoustic material is that, whenever it interacts with the boundary it has to obey the Snell's law.

So, by the Snell's law, this is the Snell's law here. And, and the speed of sound is positive for both medium. And, theta i can vary only between 0 to 90 degree by definition, because if it is more than ninety degree which means then the rays going into the other medium. So, when theta varies only between 0 to 90 degree and both are positive. So, theta t is has a very limited value it can only lie within this region. So, they are not able to bend the sound waves properly.

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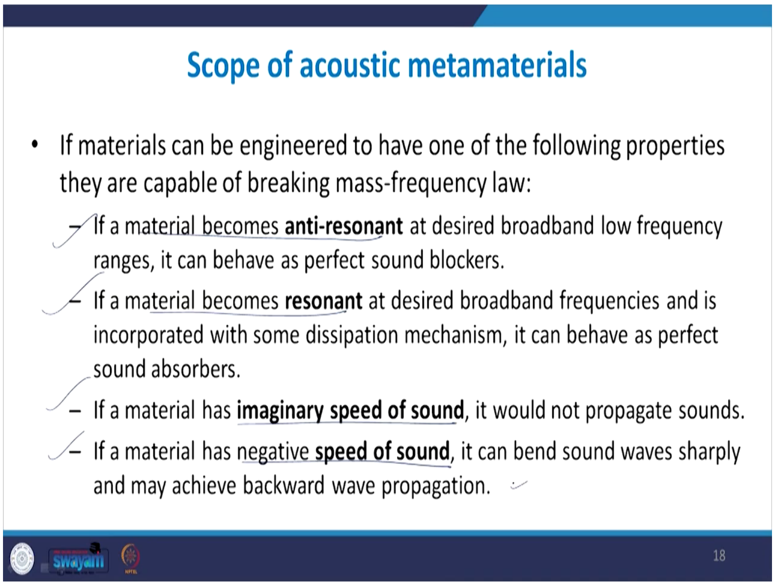

So, these are the two limitations poor performance at low frequency and unable to bend the sound waves sharply. And, that is why certain materials are desired, that are the acoustic metamaterials. And, these metamaterial they try to break or eliminate these conventional limitations. And, this can be done either, if the material becomes anti resonant or it becomes resonant at certain low frequencies. So, when it becomes anti resonant which means that at that frequency no matter how much excitation you give there will be no sound propagation.

So, the material will be a blocker. If, it becomes resonant at certain desired frequencies which means that now lot of transmission will take place. So, the it will become like a perfect absorber or if the material can have imaginary speed of sound or negative speed of sound. So, these are certain new concepts introduced.

So, what acoustic metamaterial tries to do is it tries to obey one of these principles to eliminate the limitation of the conventional material. So, I will explain this last point to you here. So, we studied about the Snell's law.

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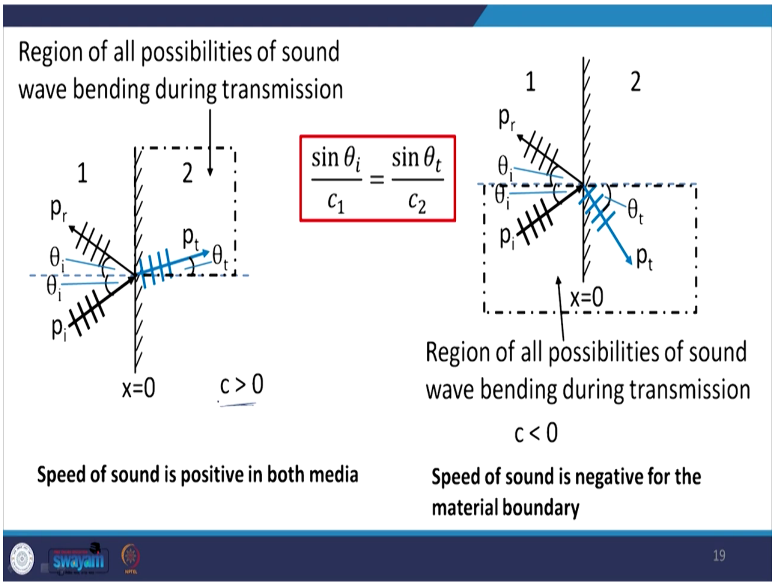

So, suppose when the speed of sound in both the mediums is. So, when the speed of sound in both the medium is positive. So, as I told you this theta t will have a limitation it can only lie between here, but if suppose the second medium has a negative speed of sound. In that case this is positive this is positive this is positive, but this one becomes negative. So, see theta t could be overall negative and it could be anywhere between this domain.

So, you observe. So, negative theta t means, it is having a very sharp turning sometimes even reverse turning. So, this is these are the various means through which these are the various scopes for acoustic metamaterial. So, we can have new materials which have either a negative speed of sound or which become locally resonant at some broadband low frequencies. So, in the next class we will introduce to you formally what is acoustic metamaterial.

Thank you.