Acoustic Materials and Metamaterials

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Lecture – 25
Introduction to Acoustic Metamaterials-2

Welcome to lecture 25 in the series of Acoustic Materials and Metamaterials. So, this is the last lecture of week 5 and the 2nd lecture on Introduction to Acoustic Metamaterials. So, in the last class, we studied about the limitations of conventional materials. So, the main limitations can be summed up as that the conventional materials they follow, they perform really, they do not perform well at low frequencies. Typically, below 1000 Hertz and this could be due to for the non porous materials, it is due to the mass frequency low which says that the lower the frequency poorer is the transmission loss and absorption.

Similarly, for the acoustic porous materials, the low frequency performance is low because at low frequencies the wavelength is very high. So, the effective depth of the material becomes extremely small and the losses are very less. So, in both the cases, the conventional acoustic materials, they perform bad at low frequencies, below 1000 Hertz and even end. So, we do not get a good broad band low frequency noise control and at the same time, if in certain frequencies they perform well, but the absorption magnitude is not high enough.

So, what we need is materials that can give us a wide range of high absorption or wide range of high transmission loss in the low frequency region. The second limitation was that these conventional materials, they are they follow the Snell's law whenever they are interacting with any boundary. Therefore, according to that law, they are unable to bend the sound waves of beyond a certain extent. So, they cannot achieve a sharp bending of the sound waves and this gives us a scope to design some new materials which can overcome these limitations.

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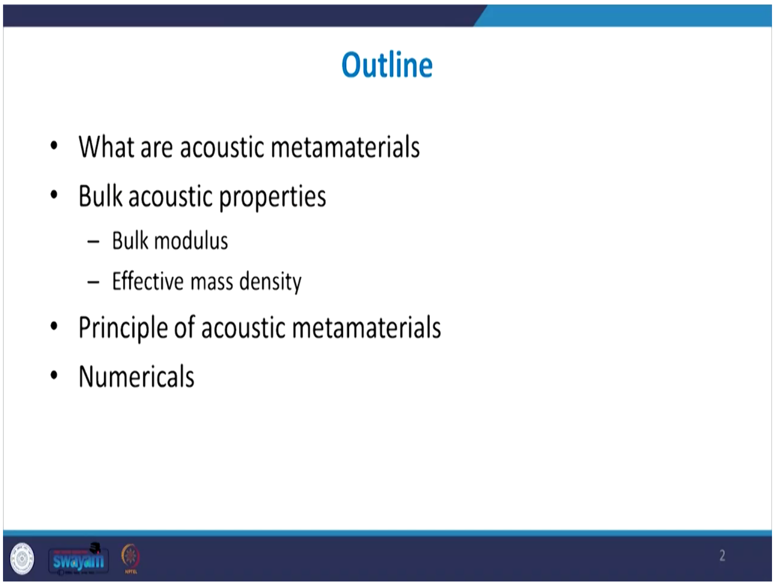

So, today, we will discuss about what are acoustic materials and what are the different bulk acoustic properties and the principle of acoustic materials followed by which will do some numericals to gain a better understanding of whatever we studied in this week.

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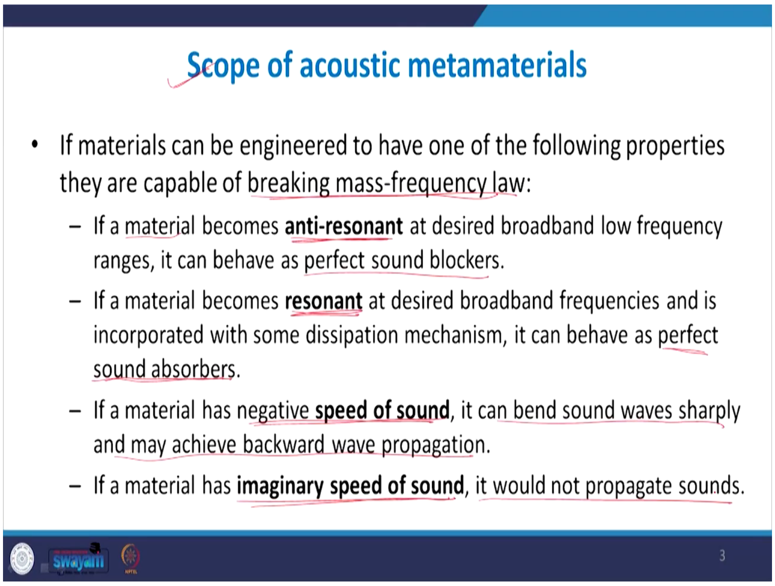

So, again, I would like to summarize that the based on the learnings of the last class, the acoustic where the scope of the acoustic materials, so here if the material if the materials can be engineered so that they can break the mass frequency low. So, if you look here if they can be engineered in some way and this can either be achieved by if a material can become anti resonant at some desired frequencies. So, when it becomes anti resonant which means it can act as perfect sound blocker or a material becomes resonant at some desired broadband frequencies and in that case, it can behave as perfect sound absorbers.

And, in the material or if the material has a negative speed of sound, then it can bend the sound waves sharply and may achieve backward wave propagation also or if a material has imaginary speed of sound. So, if the speed of sound is imaginary, then the material then that means, that the sound waves are not able to propagate through the material. So, it is acting as a perfect blocking material. So, we will investigate more and discuss more about this concept of negative speed of sound and imaginary the speed of sound.

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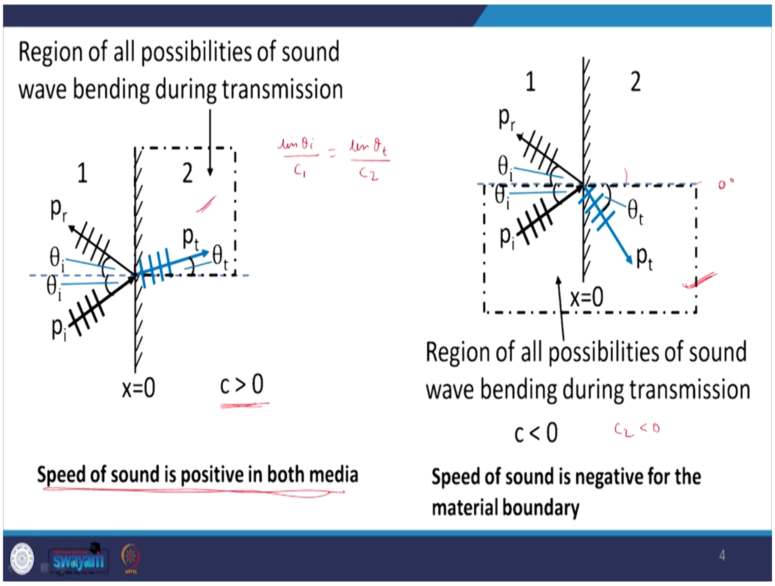

So, as already said in the last class also, so if this is the conventional interaction at the boundaries when c is positive; so, by Snell's law if c is positive and Snell's law tells that sin of theta i by theta i is equal to sin of theta t by theta t and this can be given as and this angle ratio can also be represented in this form sin of theta sorry. So, I have written the wrong equation here. So, it is a sin of theta i by c 1 and sin of theta t by c 2. So, this is the Snell's law.

So, c 1 is positive and c 2 this of the second medium is also positive, then theta i will only have certain values between 0 to 90 degrees. So, in that case, we can only achieve the bending like this; only in this region all the possible transmission will take place. So, the bending can only take place along this region. So, this is the region of all possibilities of transmission. But if this c 2 becomes negative here, so here in this case c 2 is negative. So, the second medium speed of the sound is becoming negative. So, what happens is that now this sin theta t can have a negative value which means that this region; so, here we are from here this is the point where 0 degrees starts and we start measuring theta t.

So, from here to here sin is all positive and it is only in this region, where the sin theta t is going to be negative. So, if c 2 is negative which means sin theta t is negative. So, this is the region where all of all the possibilities of transmission. So, what it means effectively is that when the sound wave is propagating directly and if that medium which it is incident on the speed of the sound becomes negative in that medium. So, what happens sin theta t becomes negative which means that theta t is going to lie somewhere over here.

So, what you see is that you see a either a very sharp bending or even a reverse propagation. So, it can bend sharply or so, it can be a very sharp bending here or even a reverse propagation back into the incident medium. So, extremely sharp bending already and natural wave manipulation can be observed when you make the speed of sound in a medium negative.

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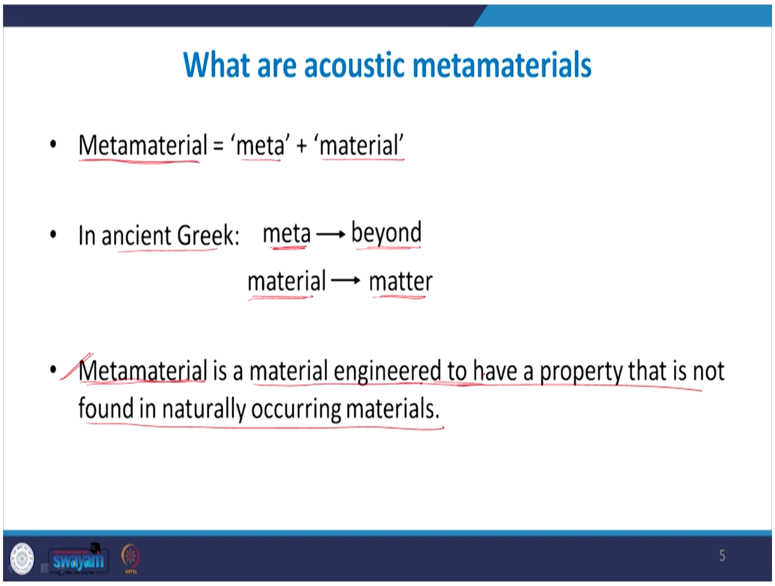

So, with this concept metamaterial came into existence and metamaterial if you go into the etymology of the word, if you see here, metamaterial comes from meta and material. And in ancient Greek meta meant beyond and material meant matter. So, it is beyond conventional material or beyond conventional matter. So, that is the meaning of this word metamaterial. And metamaterials, they are defined they usually came up in the field of optics and electromagnetic waves and from electromagnetic waves a similar concept came for acoustic waves.

So, in general the metamaterial is a material that is engineered to have a property that is not found in naturally occurring materials. So, something it has some properties which goes beyond what can be found in conventional materials.

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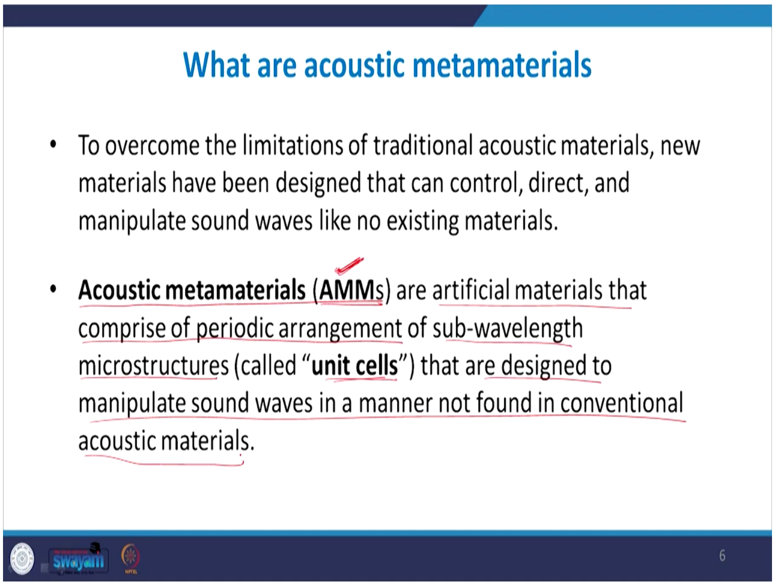

So, for what is the; so, we studied that what are the limitations and what is the scope for a new kind of material. So, to overcome the limitations of conventional materials, a new form of material has now been designed and these new form of materials, they can manipulate, control and transmit. So, overall, they can control direct and manipulate the sound waves like no other conventional material and they are called as acoustic metamaterials.

So, here I have defined acoustic metamaterials and I will be frequently using the short form AMM throughout the rest of the course; because throughout the rest of the course now we will be discussing about such acoustic metamaterials. So, AMMs is the this is the short form that I will be using. So, they are defined as artificial materials that comprise of periodic arrangement of sub wavelength, micro structures or unit cells that are designed to manipulate sound waves in a manner not found in conventional acoustic materials.

So, overall a metamaterial can also be thought of like a composite. So, metamaterial what it has is? It has the small micro structures or the units cell. So, these this is the prime building block. So, we have a conventional unit cell and this and the repetition or periodic arrangement of this unit cell forms the entire metamaterial and together that material can then manipulate the sound waves like no other conventional material. And how do they manipulate the sound wave; how is this extraordinary sound wave manipulation achieved?

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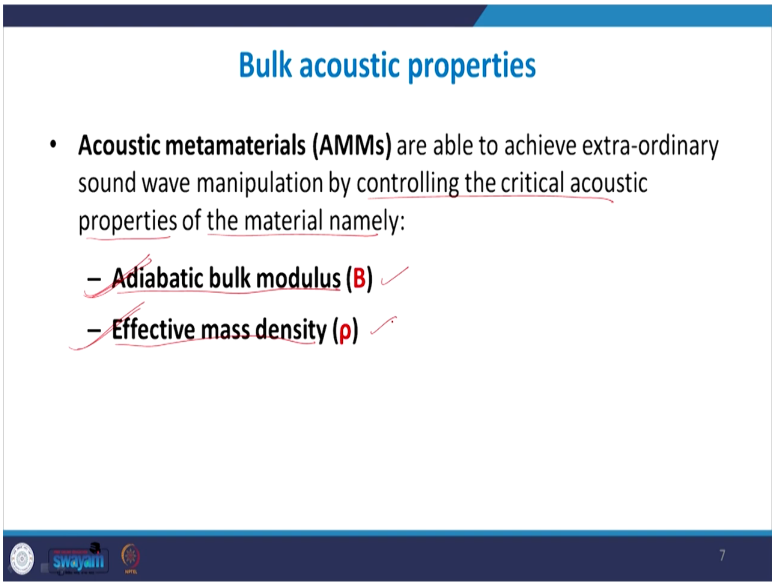

So, the it is achieved by controlling two critical acoustic properties of a material and these two critical properties are adiabatic bulk modulus here and the Effective mass density rho. So, B and rho, these are the two critical bulk acoustic properties that these metamaterials control.

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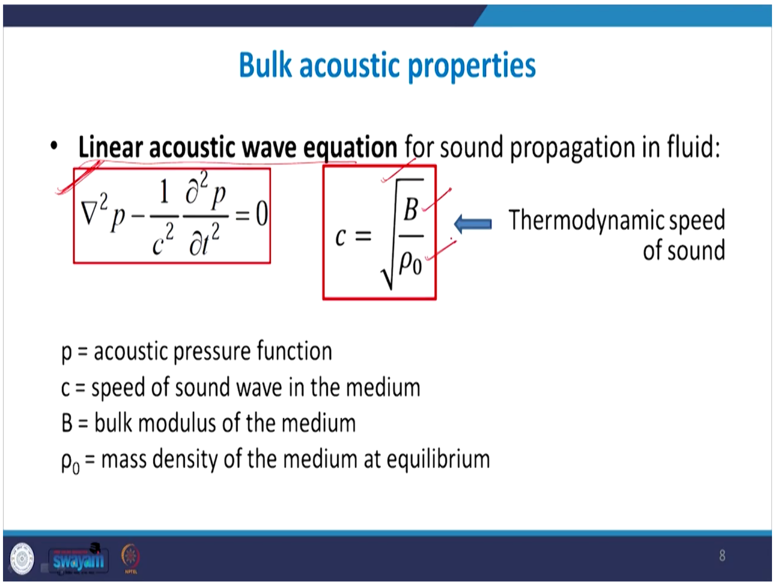

So, why these two properties? Now, if you have a look at the linear acoustic wave equation that we have studied right at the beginning of this course in our lecture 2 and 3. So, what we will see is that any sound wave propagation in a homogeneous medium can be defined by a general equation, irrespective of what medium you are using, irrespective of what wave front you are talking about or the nature of the wave; but any propagating sound wave can be represented as this equation here.

So, this is the overall equation of sound wave propagation through a fluid, homogeneous fluid medium. So, here you see is that the entire nature of acoustic pressure is dependent on this variable c which is under root B by rho naught. This is also called as the thermodynamic speed of sound. So, the entire nature of the wave, then is dependent upon these two properties that is bulk modulus and the density of the material.

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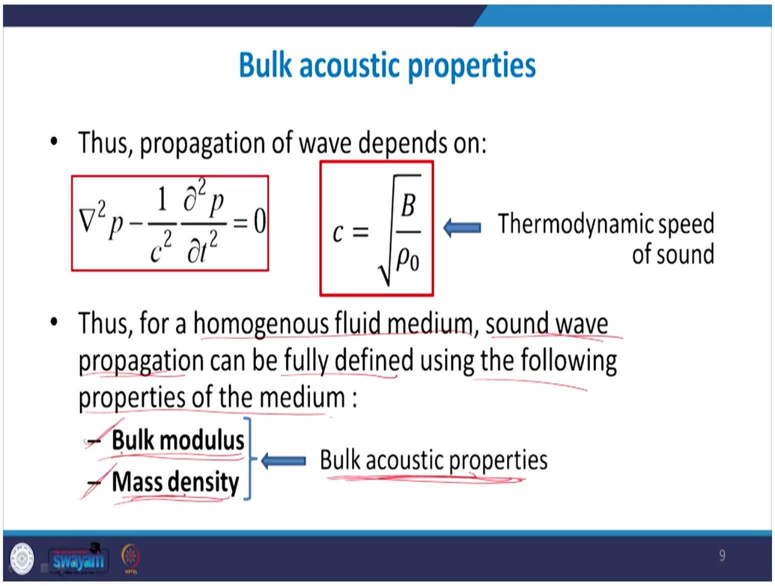

So, that is why they are also called as the bulk acoustic properties because in a homogeneous fluid medium, the entire wave propagation equation and its nature can be fully defined through these two properties itself. So, on a macro scopic level, how does a material behave can we define using these two properties.

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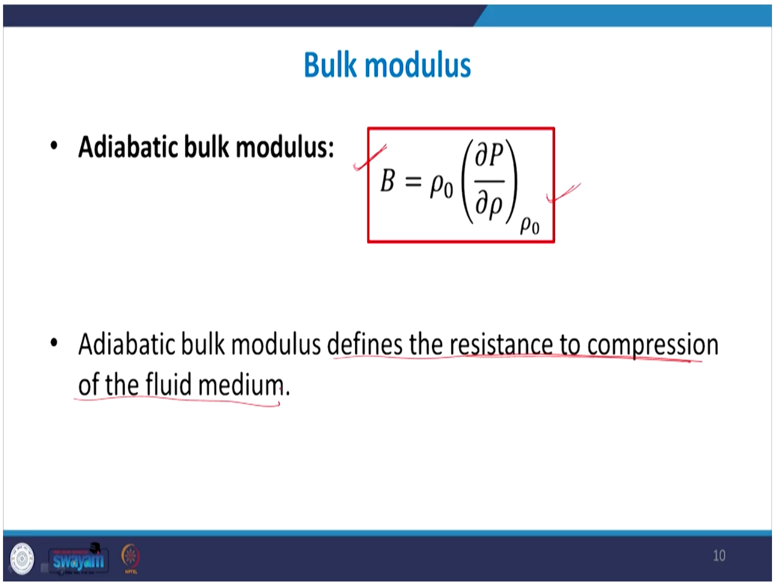

So, that so, let us go through one by one through what is a bulk modulus and mass density. So, will be a quick revision of what we have studied. So, bulk modulus, as I have already told you previously that the acoustic processes, they are antibiotic in nature because they involve very small fluctuations with respect to mean values. So, in such antibiotic process the bulk modulus, where we have a special kind of bulk modulus defined for the material, for the medium and that is called as the antibiotic bulk modulus.

So, the adiabatic bulk modulus is given as B is equal to rho naught dell P by dell rho naught which is the mean density times the rate of change of pressure of the medium with respect to density at the mean density. So, overall, what it measures is: what is the resistance to compression of the fluid medium. So, how resistance? It is to being compressed or expanded because acoustic waves are effectively these are the periodic expansions and compressions of the fluid medium propagating longitudinally which we finally, hear is sounds and the adiabatic bulk modules measures what is this resistant to compression or expansion of the media.

And then the second bulk acoustic property is the mass density. So, I will now onwards refer to mass density as effective mass density. So, how is our effective mass density defined? You already know that a mass density of any material or any body for in classical mechanics, mass density is given by what is the mass contained per unit volume. But, for dynamic systems and for other such for dynamic system specially, a new kind of noise mass density can be defined or a effective mass density. So, there are two ways to define mass density.

You have mass equal mass densities mass per unit volume. But if you use the Newton's Second Law, then mass will be force is equal to mass times acceleration; F equals to m a.

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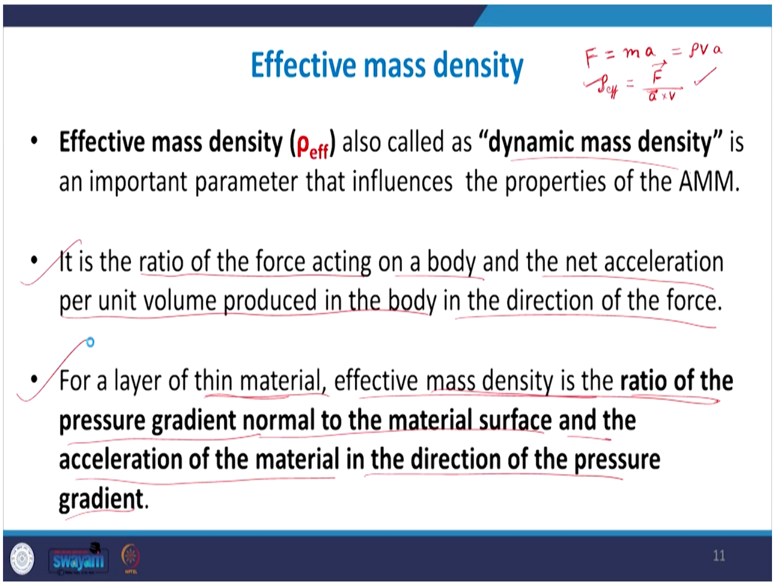

So, using this Newton’s Second Law, we can come up with a new derivation of mass density. This m can be written is the density multiplied by the volume times the acceleration. So, the new mass density which is the effective mass density will become whatever is: what is the net force acting on a body divided by its acceleration per unit divided by the acceleration of a unit volume of that element; so, this could be a new definition of mass density.

So, this is there are two definitions, I have given you here and I will go one by one through the derivations of them. So, first is that a effective mass density is the ratio of the force acting on a body and the net acceleration than that force produces per unit volume in the body, in the direction of force; so, effectively this one. So, the net force acting on the body and because, this is a scalar quantity so a in the direction of a force into the volume.

Similarly, if you have a thin material then this mass density becomes the ratio of the pressure gradient normal to the material surface and the acceleration of the material in the direction of the pressure gradient. So, let us see why? So, if you go through these. So, these are the two definitions of effective mass densities.

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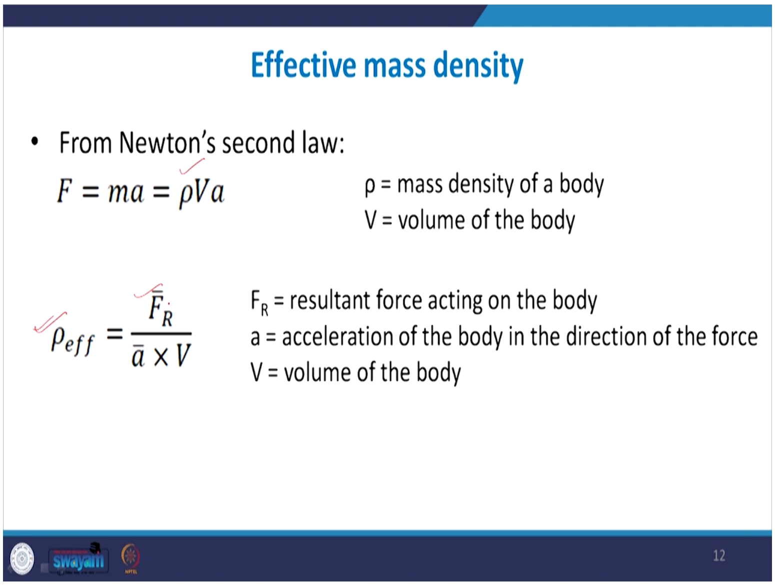

Let us go through them one by one. So, from Newton's second law as I explained to you F is equal to m a. So, it can be written as rho V into a and rho effective, then becomes whatever is the net resulting force acting divided by the acceleration in the direction of x of the force multiplied by the volume of the body. So, you have got this equation.

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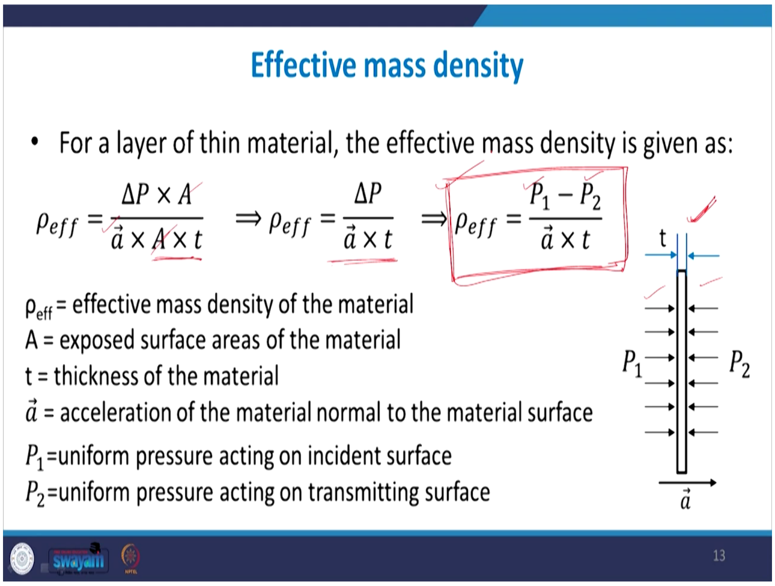

So, now what we can do is that now because were defining the mass density for a particular material acoustic material. So, let us define it in the context of a layer of thin material. So, if you take a layer of thin material. So, if you see in this figure here. So, we have a layer of thin material of thickness t some uniform pressure is acting on this side and some uniform pressure is acting on the material on the other side. So, we know that rho effective is F R by a into V. So, from this F R can be in the net force can be written as the pressure gradient that is acting across this material.

So, this can be written as rho. This F R can be written this pressure multiplied by the area of the material on which it is acting. So, dell P multiplied by area or the net change in the pressure multiplied by the area will give you the total pressure force that is acting along the pressure gradient. So, this will be dell P multiplied by area divided by a and the volume of the material will then become whatever is the area of the material multiplied by the thickness. So, this becomes the volume.

So, rho effective can be written as this cancels out. As this expression which for a thin layer of material, we can write if uniform pressure is acting on both the ends then this dell P is simply P 1 minus P 2. So, this is the equation for rho effective for a thin layer of material. So, rho effective is this. So, it is the difference between the uniform pressure acting on the incident surface and the uniform pressure acting on the transmitting surface divided by the acceleration that is produced in the direction normal to the material surface multiplied by the thickness of the material.

So, this is how we have defined mass density, effective mass density and the antibiotic bulk modulus.

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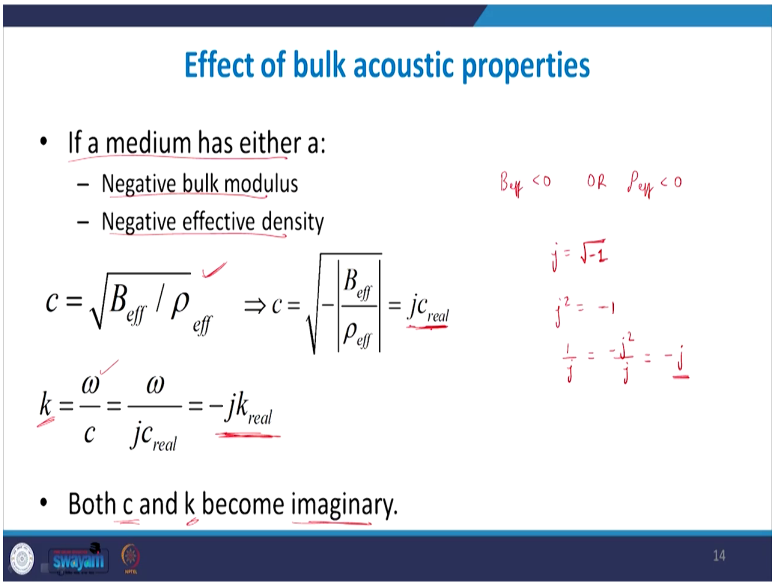

Now, let us see what is the effect of these properties and how they can be used to manipulate sound waves or control sound waves. So, let us study what happens. Suppose, if a medium has either a negative bulk modulus or a negative effective density. So, either B effective becomes negative. So, what we are studying here it is that here B effective becomes negative or rho effective becomes negative; so, any one of these properties when it becomes negative, then within the under root sign because this is the speed of sound.

So, within this under root sign, we will get some negative quantity. Because initially both were positive, now if you make one of them negative, so it will be some positive value of B by rho. So, the absolute value of B by rho minus; so, we will get a negative value overall. So, you know that j is equal to under root of minus 1. So, under root of minus 1 can be taken out. So, this becomes j times some real c c real. So, now, we get this c as purely imaginary. So, what will be k for that medium? It will be omega by c. So, it will be omega by j times of c real.

So, j is equal to minus 1; j square is equal to minus 1. j is equal to root of minus 1 sorry. So, j square is equal to minus 1. So, 1 by j will be equal to minus of j square by j which will be minus of j. So, 1 by j becomes minus of j. So, what we see here is that both the c and the k of the medium which is both the speed of sound and wave propagation vector, they become purely imaginary. So, even the you can interpret from this imaginary value that both are becoming imaginary means that probably the wave propagation does not take place because the propagation vector itself is becoming imaginary and they are given by these two quantities.

So, let us see what happens if you put this value into a proper acoustic wave equation.

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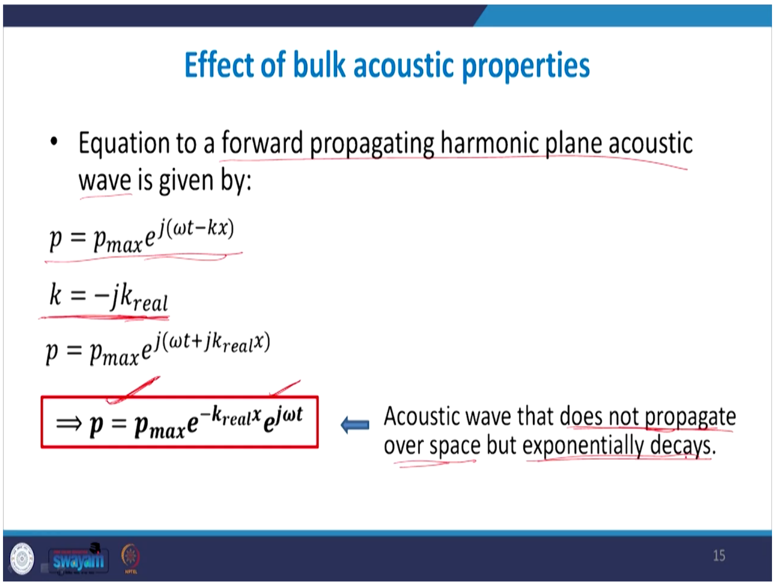

So, if you take for example, a harmonic plane wave front. So, if we have a forward propagating harmonic plane wave. So, this is the equation for this harmonic plane wave; some amplitude into e to the power j omega t minus k x. Now, if we input this value now because c has become negative. So, B or rho, one of them becomes negative. So, this is what we get. So, we put this value here. So, if you put this value over here. So, k is minus j times of some real value. So, minus of k is plus j times that real value. So, you put this minus k as plus j times some k real into x.

Now, if you take these equations, you separate the two equations. So, this is e to the power omega t and this is e to the power j square times of k real x and j square is minus 1. So, overall this is the equation you are getting p is equal to p max e to the minus k real times of x into e to the power j omega t. So, when you put any imaginary proportional when you put an imaginary propagation vector, then the equation of the wave that comes out is that it shows that it is it does not propagate over space.

It is exponentially decaying; it is an exponentially decaying function. So, there is no propagation or sinusoidal variation over space. So, what you can see is that the propagation does not take place.

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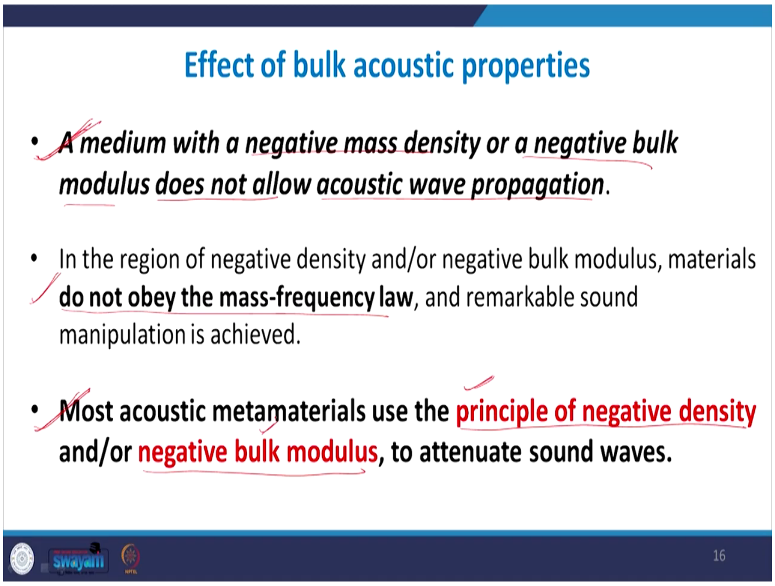

So, do summarize if a medium has a negative mass density or it has a negative bulk modulus, then it does not allow the acoustic wave propagation and this is the quality which is now exploited for by the acoustic metamaterials. So, whenever this thing happens that either a negative mass density or negative bulk modulus, so in that region, the materials they do not obey the mass frequency law.

Because suddenly irrespective of what frequency it is, the propagation will be 0. So, it does not obey the traditional mass-frequency law and some remarkable observations can be seen. So, most acoustic metamaterials, they are based on this principle that is either the principle of making the density as negative or the principle of making the bulk modulus is negative to attenuate sound waves.

So, in our next lectures and the subsequent lectures, we will go into the detail of how a negative mosque density can be achieved and what does it mean for noise control and how a negative bulk modulus can be achieved and what does that mean for noise control. So, this is the two principles; the principle of negative mass density and the principle of negative bulk modulus.

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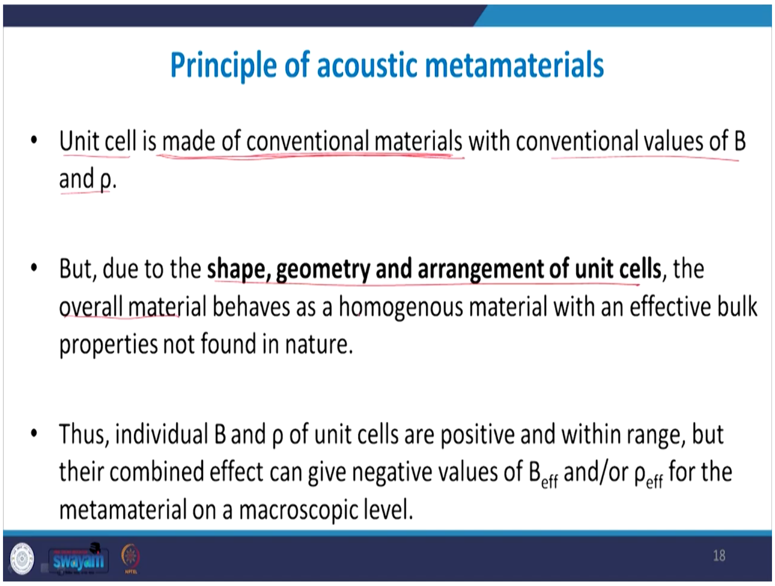

So, once again as I told you that they are acoustic metamaterials, it comprises of some unit cells which are arranged periodically and they achieve the manipulation by making the bulk modulus and the mass density as negative. So, what happens here is that for this such a acoustic metamaterials, the unit cell is made up of some conventional materials. Because, we do not have naturally occurring materials where, we can either have a negative mass density or a negative bulk modulus.

So, in general how it is achieved? We combine the conventional materials together into a unit cell. So, the unit cell will be composed of conventional materials and these materials will have conventional values of B and rho. But the way they are arranged together and the way the overall arrangement is or the shape or the geometry is that the overall material now behaves as a homogeneous material with the effective bulk properties that are not found in nature.

So, B and rho of unit cells will be positive within the normal range, but the combined effect can have negative values of B effective or rho effective; this making it a metamaterial on a macroscopic level.

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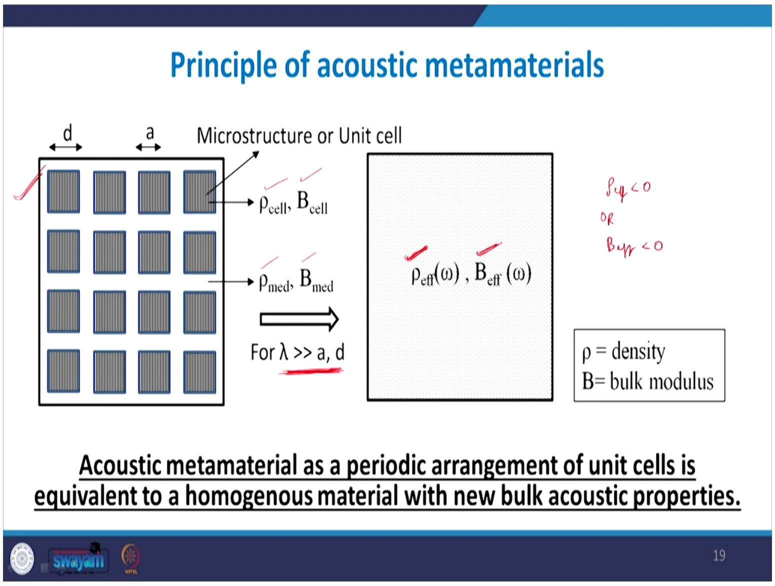

So, this is just a pictorial description of what I have told you. So, this is like a metamaterial it has such certain unit cells, which are arranged in some which are arranged in a certain way, certain geometry. So, their individual rho and B are all positive and the medium rho and B is also positive.

But at the sub wavelength dimensions or at whenever the lambda becomes much greater than the individual dimensions of the unit cell, in that particular range suddenly what can be observed is that the rho effective or B effective in that case, they behave as if a material has a new kind of rho effective or the new kind of B effective which can be negative. So, here rho effective can be negative or B effective can be negative.

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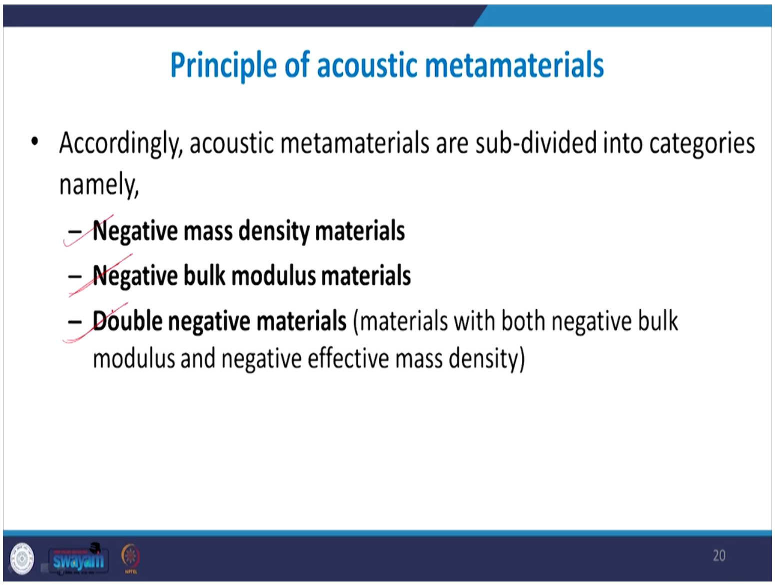

And accordingly, the acoustic metamaterials, they are divided into the categories namely negative mass density material, negative bulk modulus material and double negative materials were both negative modulus and negative mass density can be attained simultaneously. So, in this particular course, we will go and discuss about a particular example called a membrane type metamaterial that is a negative mass density material and then, we will also study about sonic crystals which are negative bulk modulus materials.

But we will not study about double negative materials because of the time constraint and the fact that these materials have been very recently discovered and they are still in the very initial stage of research. So, let us go through a few problems before I close this lecture and that will help you gain a better understanding of what we studied in today's lecture and the previous lecture.

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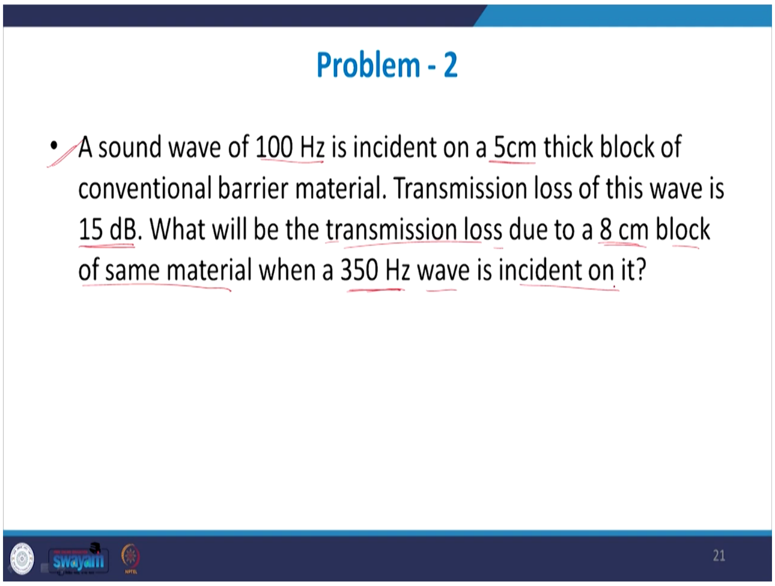

So, let us solve the first question here or the second question for this week. It is a sound wave of a 100 Hertz is incident on a 5 centimeter thick block of conventional barrier material. The transmission loss of this wave is 15 decibels. What will be the transmission loss due to a 8 centimeter block of the same material when a 350 Hertz wave is incident on it?

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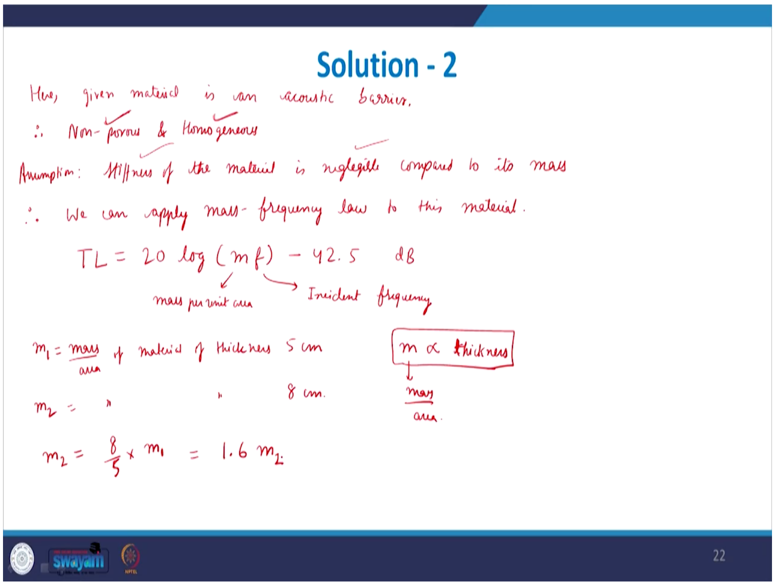

So, this is a question on mass frequency law. So, we can solve it like this. So, here given material is an acoustic barrier material. So, it acts as an acoustic barrier. Therefore, it is non porous and homogeneous in nature. Now, let us begin with the assumption that stiffness of the material is negligible compared to its mass; so now, with this assumption which usually holds true for all barrier materials. This particular material now satisfies all the conditions to obey the mass frequency law. So, you have non porous material, homogeneous in nature, stiffness can be neglected.

So, therefore, we can apply mass-frequency law to this material. So, what we get is that from the mass frequency law, the transmission loss is 20 log m f minus 42.5 in decibels. Here, this is the mass per unit area and this is the incident frequency. So, in that case, now if suppose and m 1 is the mass per unit area of the material of thickness 5 centimeters and m 2 is the mass per unit area of the material of thickness 8 centimeters; then, mass because we are using the same material.

So, the mass will be directly proportional to the thickness; because it is density into volume. Density is same and area of the exposed because we are taking per unit area. So, area becomes same. So, it is directly dependent upon the thickness of the material. So, the m 2 for the 8 centimeter block can be written as 8 by 5 times of m 1 because mass per unit area is proportional to thickness. So, it will be 1.6 times of m 1. So, with this knowledge let us solve the two conditions.

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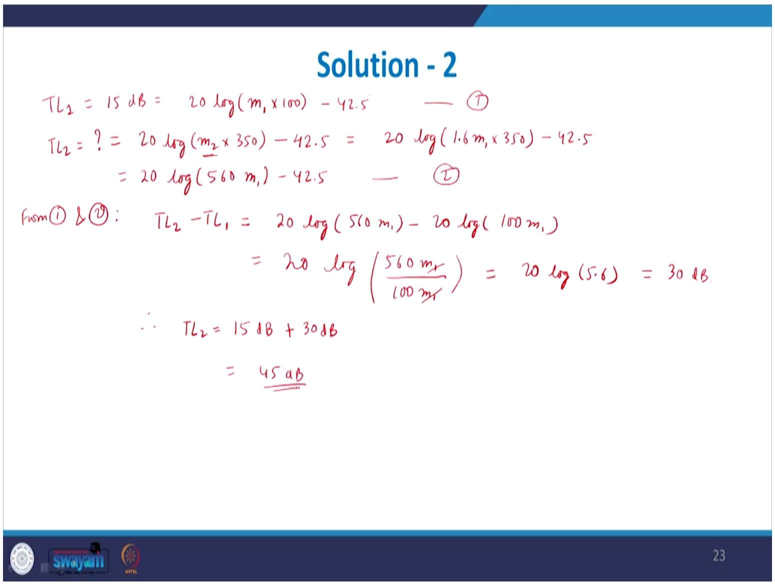

So, it is given to you that in the first condition. We solve it here in the first condition, the transmission loss is 15 decibels which corresponds to 20 log and m 1, the frequency is a 100

Hertz minus 42.5 in decibels and T L 2 is the unknown here which is 20 log m 2 times the incident wave frequency being 350 Hertz. So, we are replacing f by 100 and 350 minus 42.5.

So, this is the T L 2 2. So, this can be written as 20 log. We can replace this m 2 as 20 log of 1.6 times of m 1 into 350 minus 42.5. So, this becomes our new equation. 1.6 times of 350 becomes 560 times of in m 1 minus 42.5. So, we have got one equation for a T L 1 and one equation for T L 2. So, from one and two, T L 2 minus T L 1 can be written as you subtract this expression from this expression; this constant cancels out.

So, overall, we are left with is 20 log 560 m 1 minus 20 log of 100 m 1. So, which is going to be 20 log, by the property of log it is 560 m 1 by 100 m 1. So, which is 20 log of 5.6, which comes out to be approximately 30 decibels. Therefore, T L 2 will be T L 1 which is 15 decibels plus 30 decibels.

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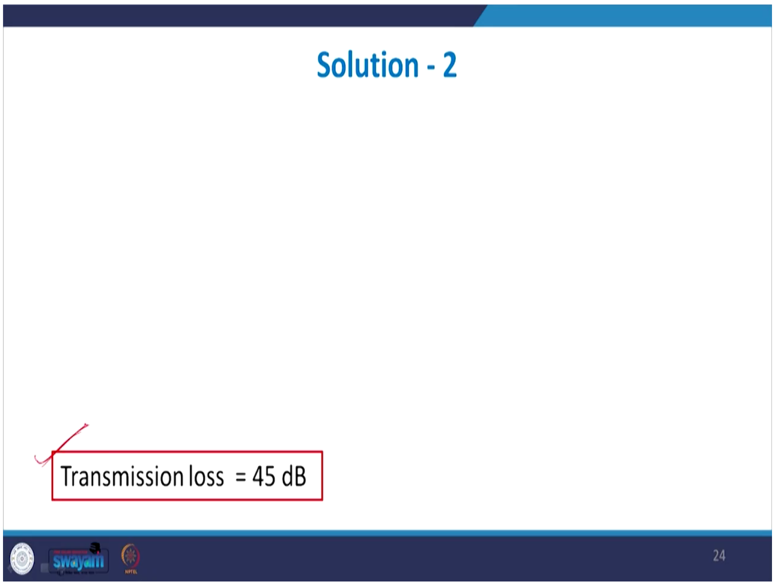

So, difference between the two is 30 decibels here. So, this is 45 decibels.

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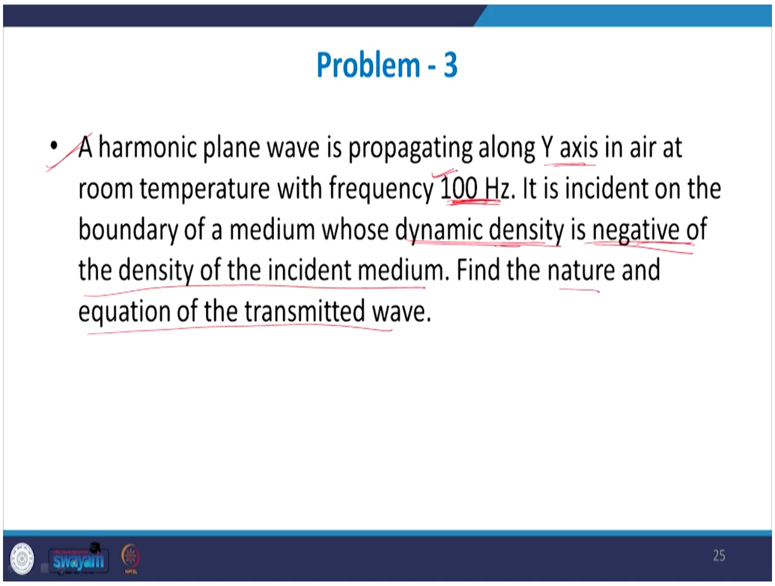

So, that is the solution to this problem. So, let us quickly solve another problem and this problem will give you a more in depth on what happens when the speed of sound becomes negative. So, here it is given that a harmonic plane wave is propagating along positive y axis in air at room temperature with frequency 100 Hertz. It is incident on a boundary of a medium whose dynamic density is negative of the density of the incident medium. So, here rho becomes just the negative value of what it was originally. You have to find the nature and the equation of the transmitted wave.

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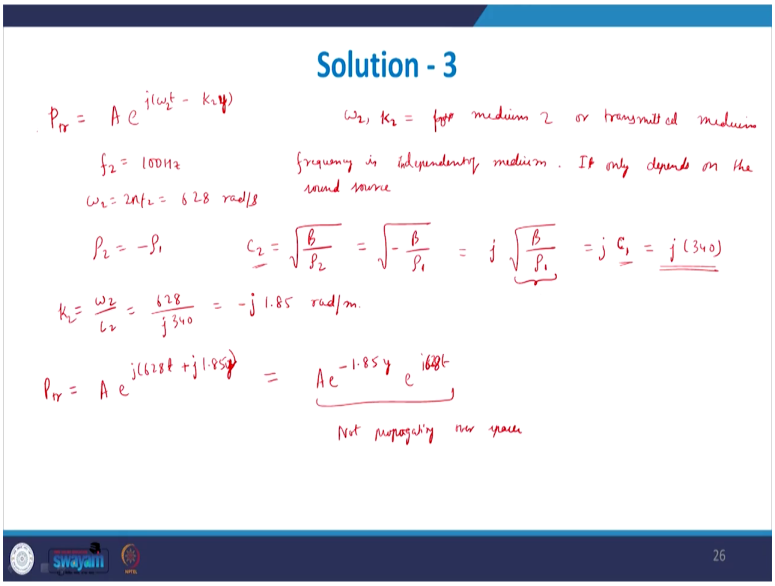

So, as you see here let us directly begin with what happens to transmitted wave. The overall p transmitted if you represent it in this general form should be something like this omega 2 minus k 2 x, where omega 2 and k 2 are the angular or for medium 2 or the transmitted readings. So, these are angular frequencies and the propagation vector for the transmitted medium. Now, f 2 will be 100 Hertz because the f 1 was given to be 100 Hertz.

So, it the wave was incident at 100 Hertz and f as we know that frequency is independent of medium. It only depends on source on the sound source and remains the same irrespective of what medium it is propagating in. So, f 2 will also be 100 Hertz. So, omega 2 becomes 2 pi of f 2 which becomes 628 radians per second. And c 2; let us find out c 2. So, c 2 is given to be minus of c 1 and what is c 1? It is for the air at room temperature. So, c 1 is minus 3. c 1 is

340 meters per second. Sorry, it is given that the density becomes negative. So, rho 2 becomes negative.

So, rho 2 is becoming minus of rho 1 then c 2 will be under root of B by rho 2 which will be under root of minus B by rho 1 and this becomes j times of under root B by rho 1 and this is given to be c 1. So, it is j times c of 1 which is for air at room temperature, it is 340 meters per second. So, this is the value of rho 2 c sorry c to c 2 is given by this value. So, we get c 2 as this. So, k 2 becomes omega 2 by c 2. So, it becomes 628 divided by j 340 which is minus j of 1.85 radians per meter.

So, if you put these values into this first equation, the transmitted wave equation will be A e to the power j 628 t minus plus j times of 1.85 times of x. Is it propagating over y axis? So, this will be y; this will be y. So, this is the equation. If you solve it further, this is what you get. So, what we get is a wave that is not propagating. It does not propagate over space, only vary sinusoidal with time; does not propagate over space.

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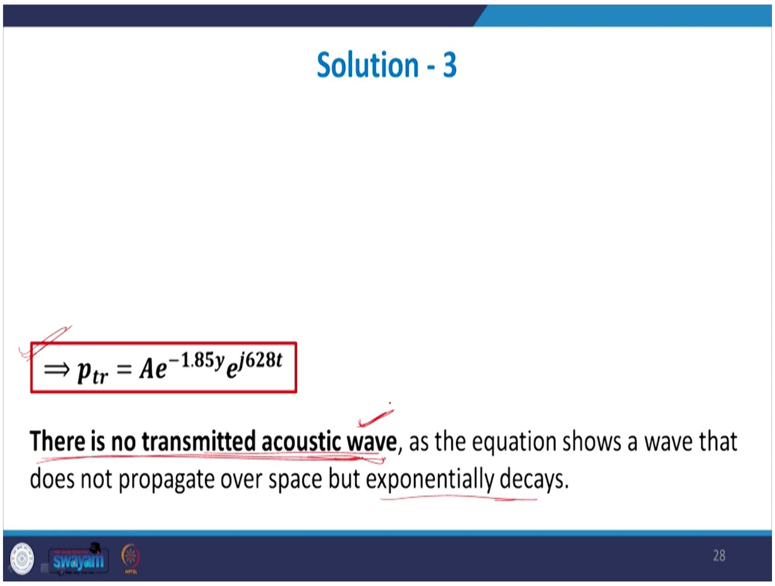

So, the nature of the transmitted wave is given by this is the equation we have obtained. So, we back here this omega is 628. So, the overall equation is given by this. So, what we get is that for in such a case, there will be no transmitted acoustic wave because we shows that the wave does not propagate with over space; it just exponential decays. So, overall and acoustic wave means a propagating wave here.

Because only the propagating wave or whenever the pressure fluctuations, they propagate over space and they reach our ear that is when we can hear it. So, acoustic wave means propagating wave and because it is not a propagating wave, we get no transmission.

So, with this I would like to close the lecture.

Thank you.