Acoustic Materials and Metamaterials

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Lecture – 26
History of Acoustic Metamaterials

Welcome to lecture 26 in the series of Acoustic Materials and Metamaterials. So, we have already done 2 lectures on introduction to Acoustic Metamaterials. So, we know we have got a brief understanding on what these materials are and what are they composed of? In today's lecture we will go through some historical development and study why the meta materials came about and how they came about.

So, let us go through the lectures. So, the history of metamaterials it began with the invention of the negative index materials. So, this acoustic metamaterials came first from, the metamaterials discovered for, the domain of electromagnetic waves. So, what were these negative index materials?

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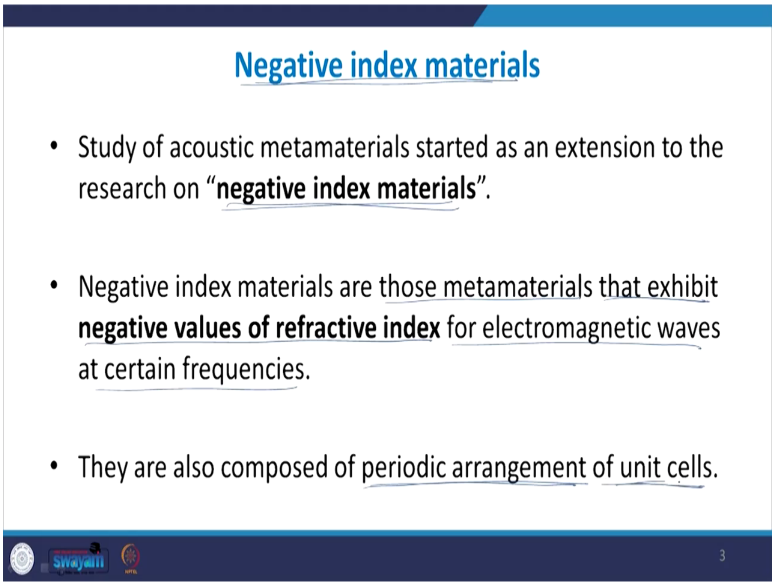

Negative index materials

- Study of acoustic metamaterials started as an extension to the research on "**negative index materials**".
- Negative index materials are those metamaterials that exhibit **negative values of refractive index** for electromagnetic waves at certain frequencies.
- They are also composed of periodic arrangement of unit cells.

So, these were a type of materials that were used to manipulate electo magnetic waves and these were those metamaterials, these are those metamaterials that exhibit negative values of refractive index for electromagnetic waves at certain frequencies. So, usually most of the classic medium it has a positive refractive index. So, in any in for example, in general in optics, most of the lenses that you use etcetera it all has positive refractive index, but certain materials could be made to have negative refractive index at certain desired frequencies.

So, the here the refractive index would be a function of the frequency and at certain frequencies the value of the refractive index will become negative. So, these materials were called as the negative index materials. So, the people who are interested to know more about this topic can; obviously, can go back in the literature for such and such materials. And, they were used to manipulate electromagnetic waves.

And, then a similar concept was borrowed to build up materials which can manipulate sound waves. So, just like acoustic metamaterials such kind of negative index materials they also have a periodic arrangement of different unit cells.

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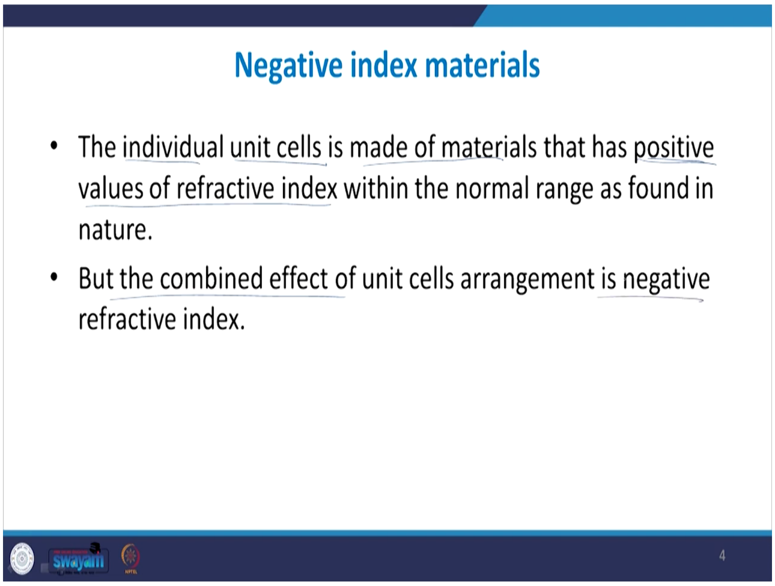

Negative index materials

- The individual unit cells is made of materials that has positive values of refractive index within the normal range as found in nature.
- But the combined effect of unit cells arrangement is negative refractive index.

And, in the same way the individual unit cells they are composed of conventional materials. So, unit cells itself they have positive value of refractive index, within the normal range as found in the additional materials, but the combined effect of a collection or arrangement of such unit cells will be, that it is able to bend the waves very sharply in reverse direction or it attains a negative refractive index. So, this is the combined effect becomes negative refractive index.

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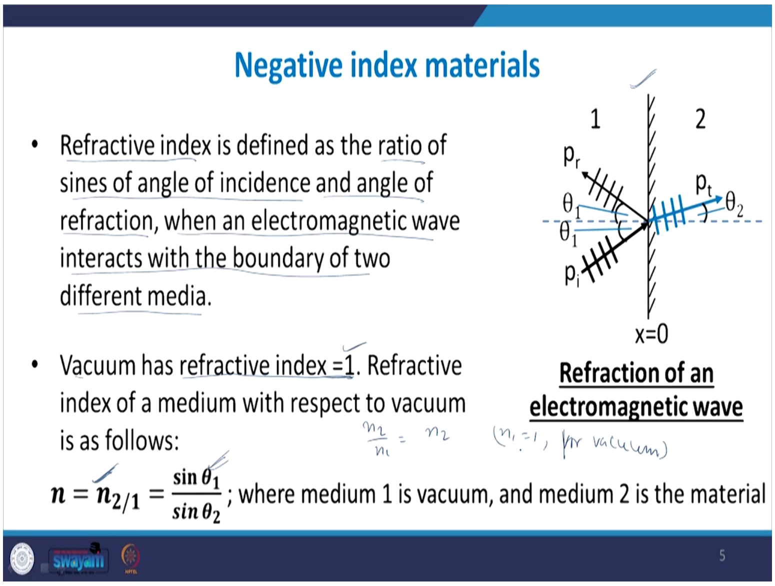

Negative index materials

- Refractive index is defined as the ratio of sines of angle of incidence and angle of refraction, when an electromagnetic wave interacts with the boundary of two different media.
- Vacuum has refractive index =1. Refractive index of a medium with respect to vacuum is as follows:

$\frac{n_2}{n_1} = n_2$ ($n_1 = 1$, for vacuum)

$$n = n_{2/1} = \frac{\sin\theta_1}{\sin\theta_2}$$; where medium 1 is vacuum, and medium 2 is the material

Refraction of an electromagnetic wave

So, to show you to go briefly through the theory of this the refractive index is defined as ratio of the sine of angle of incidence and the sine of angle of refraction, when an electromagnetic wave interacts with the boundary of the two different media. So, let us see if we see this figure here. This is medium 1 2 and sin interaction is taking place at the boundary, this is theta 1 is the angle of incidence theta 2 is the angle of refraction, then the refractive index of medium 2 with respect to medium 1 will be sin theta 1 by theta 2.

And most of the times so, this refractive index is usually a ratio or a comparison of medium to 1 medium with another medium. And, most of the times there is an absolute value of refractive index where, the medium 1 is taken as vacuum. So, when the medium 1 becomes vacuum and the refractive index of vacuum is 1. So, if n is equal to so, if n 1 is equal to 1. Then n 2 by n 1 will be same as n 2, because n 1 is equal to 1 for vacuum.

So, when vacuum when the first medium is taken as vacuum then the value of refractive index that you get for medium becomes it is absolute refractive index, which is given by the same thing sin theta 1 by theta 2.

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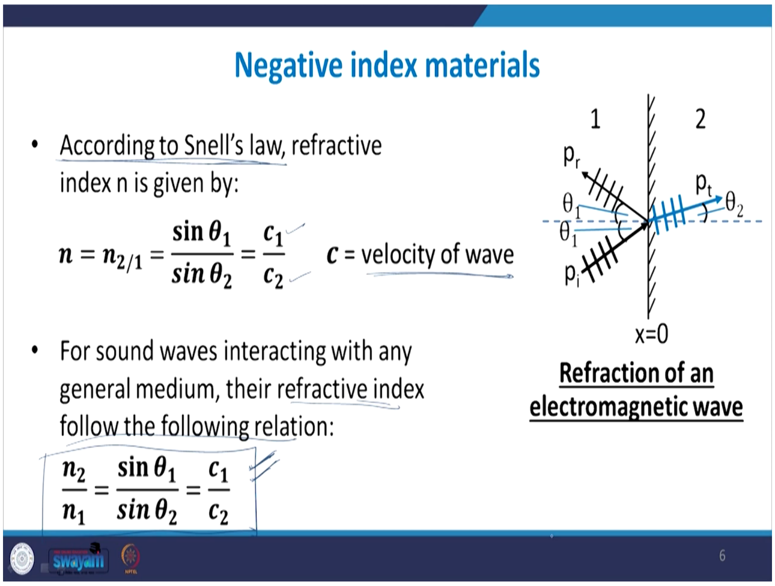

Negative index materials

- According to Snell's law, refractive index n is given by:

$$n = n_{2/1} = \frac{\sin\theta_1}{\sin\theta_2} = \frac{c_1}{c_2}$$

c = velocity of wave

- For sound waves interacting with any general medium, their refractive index follow the following relation:

$$\boxed{\frac{n_2}{n_1} = \frac{\sin\theta_1}{\sin\theta_2} = \frac{c_1}{c_2}}$$

Refraction of an electromagnetic wave

Now, according to Snell's law so, every interaction it follows the Snell's law. So, by Snell's law sine theta 1 by sine theta 2 is c 1 by c 2, where c is the velocity of the wave in the respective media. So, when the sound waves they interact with any general medium in that case their refractive index again I am restating. So, this is the overall expression n 2 by n 1 will be n 2 by n 1 or simply the refractive index of 2 with respect to 1 will be sin theta 1 by sin theta 2, which will be c 1 by c 2.

So, this is the definition of refractive index and how it is related to the speed of sound in the two different media. So, refractive index ratio is the reverse of the speed of sound ratio in the two media. And, I have explained this concept to you again previously in the lecture on acoustic metamaterials.

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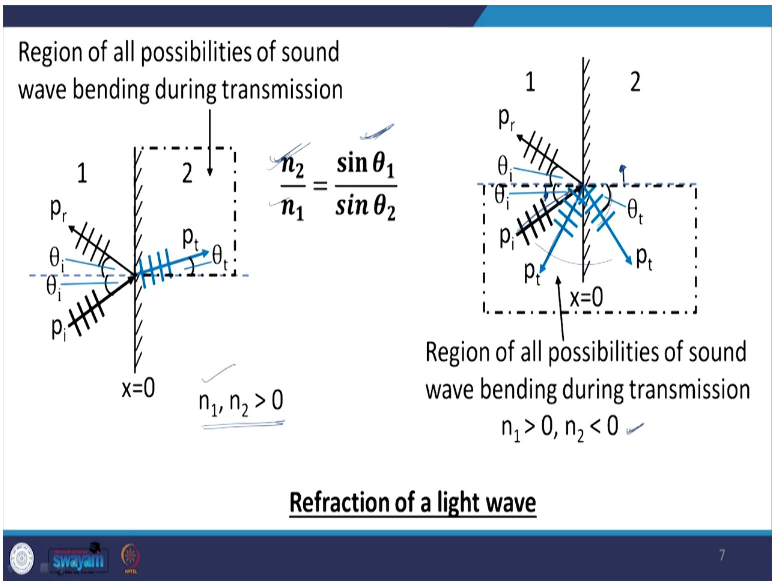

Region of all possibilities of sound wave bending during transmission

$$\frac{n_2}{n_1} = \frac{\sin\theta_1}{\sin\theta_2}$$

$n_1, n_2 > 0$

Region of all possibilities of sound wave bending during transmission

$n_1 > 0, n_2 < 0$

Refraction of a light wave

If you refer to the previous 2 lectures you can find this explanation there. So, the same explanation goes here also. So, let us say interaction is taking place and both medium 1 and 2 they have positive refractive index. So, in that case this is the equation. So, this is positive this is positive and this theta 1 it varies from 0 to 90 degrees.

So, theta 2 is limited and it can only vary within this domain, but if 1 of the refractive index becomes negative for the second medium. So, suddenly the sound the electromagnetic wave is propagating it hits the boundary of a medium, which has a negative refractive index.

So, in that case from this expression theta the theta t or theta 2, it will have a negative value. So, negative value of sin means it will bend somewhere along this zone. So, this is taken as the positive direction for theta t, this is taken as the positive direction for theta i. So, this is the convention that has been being followed theta i this is the directions of positive the theta i, this is a direction for negative theta t. Sorry positive theta t. So, theta t negative means the will bend towards this. So, this is the region of all the positive possible bendings that can take place.

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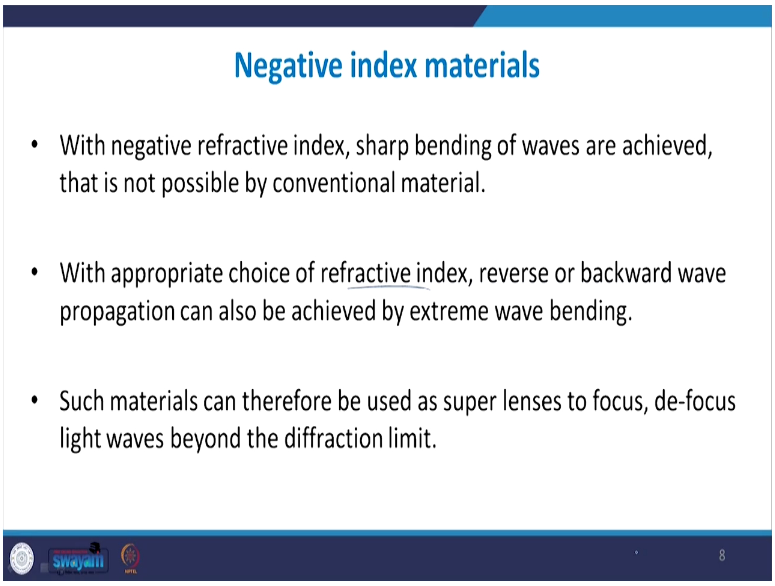

Negative index materials

- With negative refractive index, sharp bending of waves are achieved, that is not possible by conventional material.
- With appropriate choice of refractive index, reverse or backward wave propagation can also be achieved by extreme wave bending.
- Such materials can therefore be used as super lenses to focus, de-focus light waves beyond the diffraction limit.

In such a case sharp bending can be obtained and with appropriate choice sometimes you can also have reverse bending. So, if this is of extremely high negative value, then this theta t can cross this particular region and it can go here. So, effectively what is happening is that, clear your you are bombarding this material from a with the wave front in this direction, but it is not able to enter the material it simply comes back.

So, if the appropriate choice of n 2 is taken some negative n 2 is taken then theta t will be so, large, that it would not be able to enter the material, but simply bend around the material and come back. So, that can be done and therefore, such materials they are heavily used to make some super lenses where, very sharp bending of these electromagnetic wave such as light waves can be obtained.

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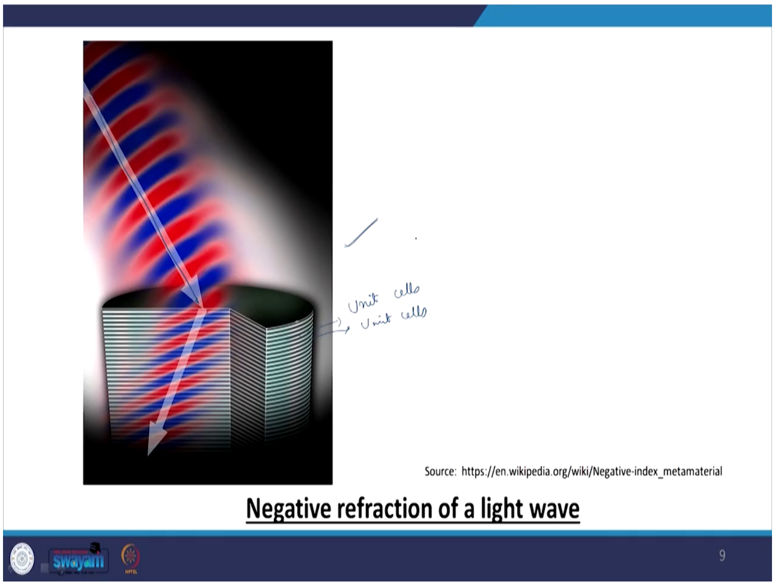

Source: https://en.wikipedia.org/wiki/Negative-index_metamaterial

Negative refraction of a light wave

So, this figure shows the kind of sharp bending from a negative index materials. So, this is a periodic this is a periodic layer of unit cells. So, these are the various unit cells here. So, when the wavefront is incident from this direction here, then it is not able to enter the material here it simply bends away and reverse it is direction and goes back to the other medium. So, very sharp bending is obtained ok.

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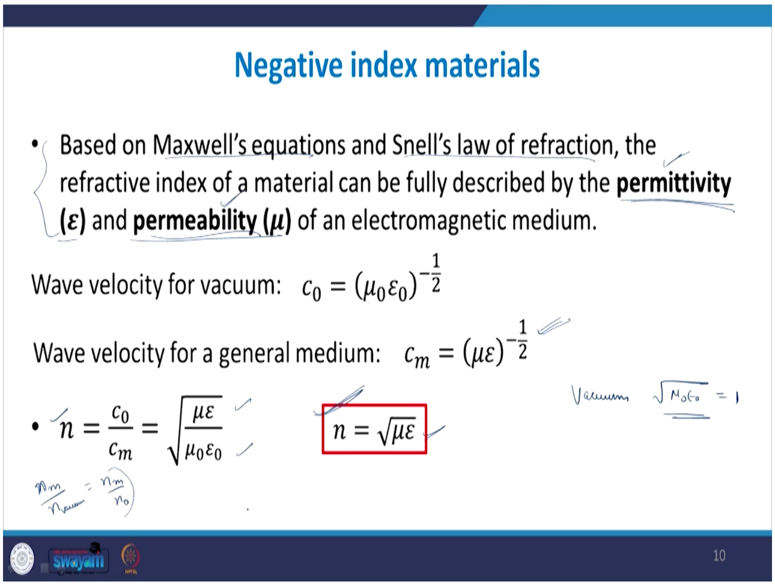

Negative index materials

- Based on Maxwell's equations and Snell's law of refraction, the refractive index of a material can be fully described by the **permittivity** (ε) and **permeability** (μ) of an electromagnetic medium.

Wave velocity for vacuum: $c_0 = (\mu_0\varepsilon_0)^{-\frac{1}{2}}$

Wave velocity for a general medium: $c_m = (\mu\varepsilon)^{-\frac{1}{2}}$

Vacuum $\sqrt{\mu_0\varepsilon_0} = 1$

- $n = \frac{c_0}{c_m} = \sqrt{\frac{\mu\varepsilon}{\mu_0\varepsilon_0}}$

$\boxed{n = \sqrt{\mu\varepsilon}}$

$\frac{n_m}{n_{vacuum}} = \frac{n_m}{n_0}$

Now, that we know that, what is the significance of having a negative refractive index. It can help in manipulating the electromagnetic waves, in a very strong way not that conventional materials cannot do. So, because this is not a course on electromagnetics or the course of course, on acoustic optics. So, this that is why discussing about electric field and magnetic field is not within a domain. So, I will directly give you what how this particular concept can be applied to acoustic, how it came to be applied to the acoustic domain.

So, from the electromagnetic theory based on Maxwell's equation and Snell's law of refraction, there are two critical quantities for critical parameters of a medium, which is called as the permittivity and the permeability which is e and mu. So, these are the 2 important properties of a medium which define what will be it is refractive index. And, the velocity is given by the velocity of the medium is given by mu e to the power minus 1 by 2 this is the velocity of the medium. And, we know that the absolute value of refractive index will be what is it is the inverse of the ratio of the speed of sound.

So, absolute value is simply the n of the medium by the n in the vacuum, which is equal to n m by n naught. So, this is absolute value n and will be c naught by cm, it will be the reverse of the velocity ratios in the 2 median. So, this overall n can be written as f mu into e.

So, the refractive index can be written as root over of the relative permeability, because vacuum in vacuum this under root of mu naught e naught is equal to 1. And, that is why n simply becomes under root of mu into e from this expression here. So, now, we get is that this mu n e are the 2 critical parameters, this is the permeability and the permitivity that can control the refractive index of a medium. And, now we want to obtain a negative index material. So, how do you obtain this negative index material?

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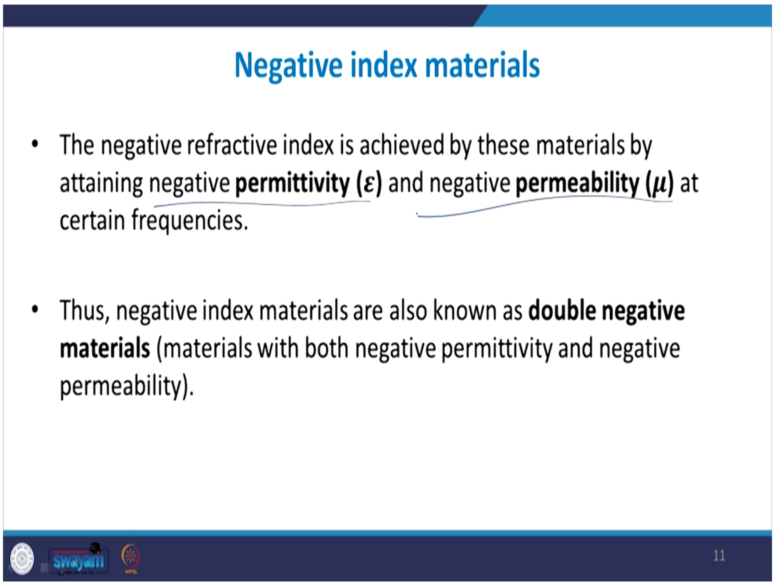

Negative index materials

- The negative refractive index is achieved by these materials by attaining negative **permittivity (ε)** and negative **permeability (μ)** at certain frequencies.
- Thus, negative index materials are also known as **double negative materials** (materials with both negative permittivity and negative permeability).

The way to achieve this is make this e value and the mu value simultaneously negative. And, therefore, such material sometimes are also called as double negative materials because both mu and e are made negative at the same time.

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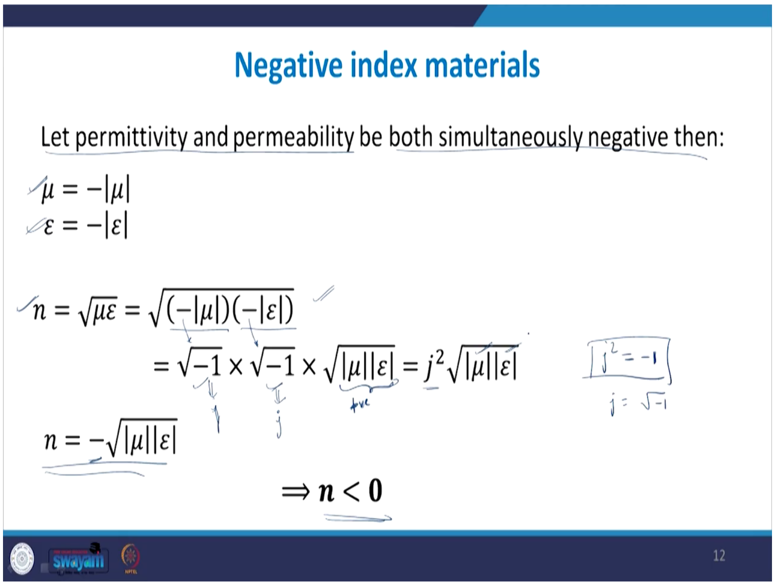

Negative index materials

Let permittivity and permeability be both simultaneously negative then:

$\mu = -|\mu|$

$\varepsilon = -|\varepsilon|$

$$n = \sqrt{\mu\varepsilon} = \sqrt{(-|\mu|)(-|\varepsilon|)}$$

$$= \underset{j}{\sqrt{-1}} \times \underset{j}{\sqrt{-1}} \times \underbrace{\sqrt{|\mu||\varepsilon|}}_{+ve} = j^2\sqrt{|\mu||\varepsilon|}$$

$\boxed{j^2 = -1}$

$j = \sqrt{-1}$

$$n = -\sqrt{|\mu||\varepsilon|}$$

$$\Rightarrow n < 0$$

So, if at certain frequencies both these values become negative what happens is. So, let us say both permittivity and permeability become simultaneously negative at certain frequencies, then this mu value is a negative value. So, it can be written as minus of it is positive value. So, this is some negative value which I am writing with a. So, let us say it was 5 minus 5. This is minus and the absolute value of this which is 5 is also written as minus into it is absolute value. And, the refractive index is root over of mu into e so, this can be written as this expression. So, if you separate these two. So, this becomes under root of mod of mu into mod e.

So, this is a positive quantity inside the square root and then you take this minus 1 factor out and this minus 1 factor out. So, you have 2 minus 1 under root. So, you have root of minus 1 from this one and you have root of minus 1 here. So, this is the breaking down of this expression. So, this is going to be a real number. But, this is going to be j, this will be this is going to be j which is an imaginary quantity this is going to be j j into j is j square. So, j square into root of a positive number.

So, overall this and j square is what, j square is equal to minus 1, because j is the root of minus 1. So, this value then becomes negative under root of mu into e. So, when both mu and e are simultaneously made negative you get a negative value of refractive index and that is why double negative materials came into existence.

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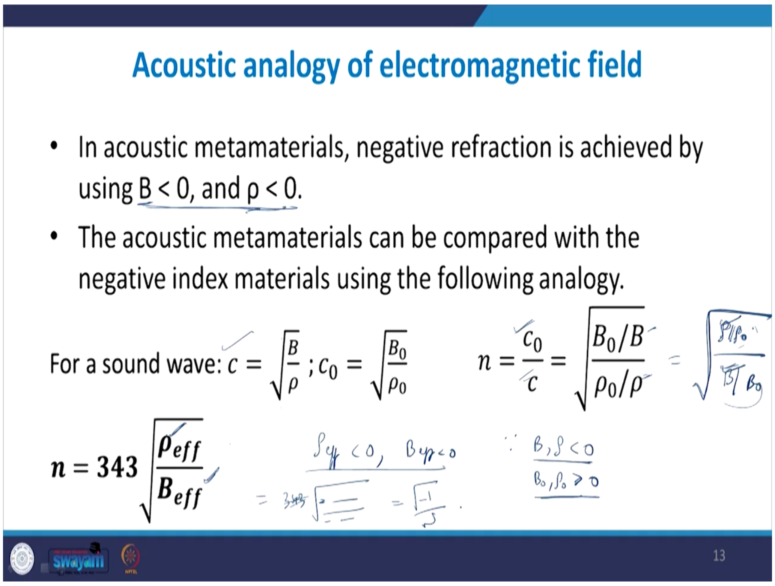

Acoustic analogy of electromagnetic field

- In acoustic metamaterials, negative refraction is achieved by using B < 0, and ρ < 0.
- The acoustic metamaterials can be compared with the negative index materials using the following analogy.

For a sound wave: $c = \sqrt{\frac{B}{\rho}}$; $c_0 = \sqrt{\frac{B_0}{\rho_0}}$ $\quad n = \frac{c_0}{c} = \sqrt{\frac{B_0/B}{\rho_0/\rho}} = \sqrt{\frac{\rho/\rho_0}{B/B_0}}$

$$n = 343\sqrt{\frac{\rho_{eff}}{B_{eff}}}$$

$\rho_{eff} < 0, \ B_{eff} < 0$

$= 343\sqrt{\frac{-}{-}} = \sqrt{\frac{-1}{-}}$.

$\because B, \rho < 0$

$B_0, \rho_0 > 0$

So, the same philosophy is then applied to acoustic materials. So, let us give you an analogy between the different parameters in electric in electromagnetics and acoustics. So, just like we had mu and e and we manipulated the value of mu and e to get a negative refractive index. In the same way for acoustic metamaterials, the critical parameters are B and rho which is the bulk modulus and the density effective mass density.

So, when they are simultaneously made as negative, then c will be c is given as under root of B by rho and c naught is given by under root of beta by rho naught so, anyways the refractive index will be the reverse of this velocity value, which can be written as some relative quantity. So, it will be rho this you can write as under root of rho by rho naught by B by B naught. So, this becomes some density with this is like a relative density with respect to air.

So, some relative density with respect to air and some relative bulk modulus with respect to air. So, this is how it can be written as the refractive index. So, when both rho and this value and this value are becoming negative. So, when this and this become negative. So, the numerator will be negative the denominator will be negative both of this rho effective will then become negative. B effective will become negative, because B and rho are negative, but B naught and rho naught for the air are positive.

So, this is what we get? So, again using the same thing which we get is 343. So, we can show some negative times of some number and some negative times of some number, you can take out this factor. So, what you so, basically what I am trying to say is that in acoustic materials the double negative does not exist experimentally, but theoretically it can be made. So, usually refractive index can be changed or manipulated using this B and rho values.

And, we already and I have already discussed with you in the previous class what happens, when rho becomes negative and B is positive, and what happens when B becomes negative, but rho is positive. So, if either one of them becomes negative, then we get an imaginary speed of sound and imaginary propagation vector.

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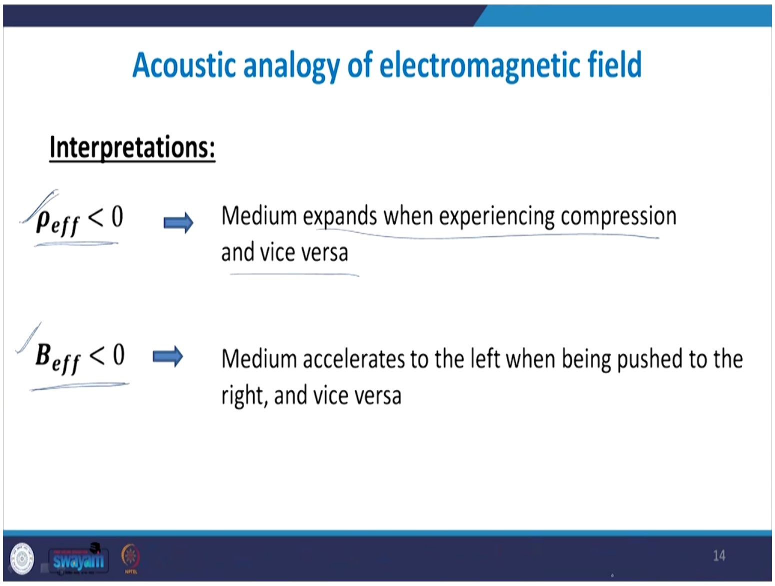

Acoustic analogy of electromagnetic field

Interpretations:

$\rho_{eff} < 0$ ⇨ Medium expands when experiencing compression and vice versa

$B_{eff} < 0$ ⇨ Medium accelerates to the left when being pushed to the right, and vice versa

And, how do we interpret these two quantities here. If, suppose some medium has negative effective density, which means that the medium is expanding, when it is experiencing some compressive force and the medium is contracting when it is experiencing some tensile or tensile force. And, if B effective is less than 0 means it is a sort of a medium which accelerates to the left, when it is being pushed to the right and vice versa.

Now, you have to take this into consideration that none of the naturally occurring materials will actually have a negative rho effective or A negative B effective. So, none of the materials which found traditionally they have a negative value of B and rho, but when they are arranged in the form of unit cells and they are arranged periodically the combined effect comes out to be negative B or a negative rho. So, let us so, this was the history of how the concept of negative index material was applied to acoustic metamaterials. Now, based on this some of the early acoustic metamaterials were proposed.

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The first acoustic metamaterials

- Concept of metamaterials and acoustic metamaterials was proposed by Victor Veselago in 1967.
- It took almost 33 years for first fabrications and experimental realization of acoustic metamaterials.

Victor Georgievich Veselago

Source: Wikipedia.org

So, here you see here is that the concept of this metamaterial and specially acoustic metamaterial was proposed by a scientist called Victor Georgievich Veselago in 1967. And, it was proposed analytically so, there was just a theoretical concept that such kind of materials can exist, but it is an irony that it was an idea much ahead of it is time. So, almost 33 years later the first acoustic metamaterial was actually made and tested experimentally. So, almost 33 years after it is first proposal did we see an experimental verification?

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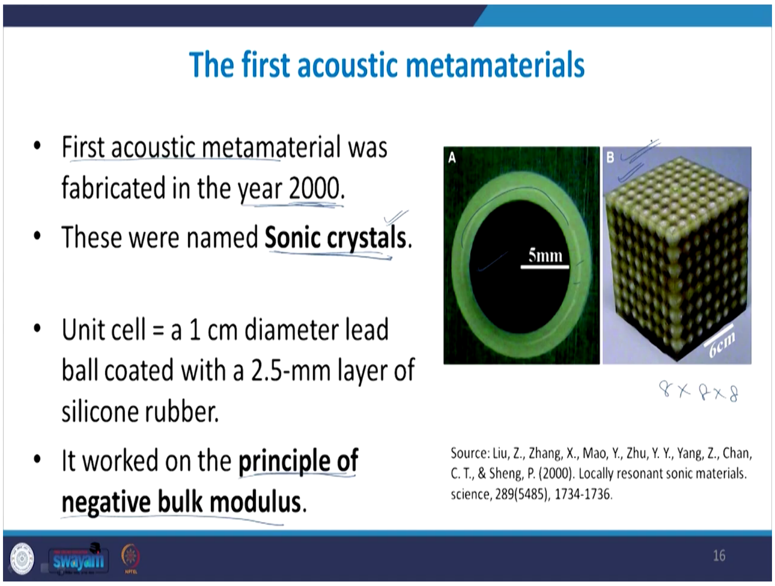

The first acoustic metamaterials

- First acoustic metamaterial was fabricated in the year 2000.
- These were named **Sonic crystals**.
- Unit cell = a 1 cm diameter lead ball coated with a 2.5-mm layer of silicone rubber.
- It worked on the **principle of negative bulk modulus**.

Source: Liu, Z., Zhang, X., Mao, Y., Zhu, Y. Y., Yang, Z., Chan, C. T., & Sheng, P. (2000). Locally resonant sonic materials. science, 289(5485), 1734-1736.

So, it came in the year 2000 and the first acoustic meta material, which is fabricated was called as the sonic crystals. And, we know that the meta the acoustic metamaterials they can either be of type of negative density, or they can be the type of negative bulk modulus, or then there can be double negative materials. So, this sonic crystal which was proposed was a better acoustic metamaterial with negative bulk modulus.

So, it worked on the principle of negative bulk modulus. So, you can see here the unit cell is actually a hard led ball, which is covered by thin rubber. So, here 1 centimeter diameter led ball was used, which is covered by 2.5 millimeters layer of silicon rubber. And, they were arranged in such kind of cube, they were put together in a cube and it was an 8 cross, 8 cross, 8 cube.

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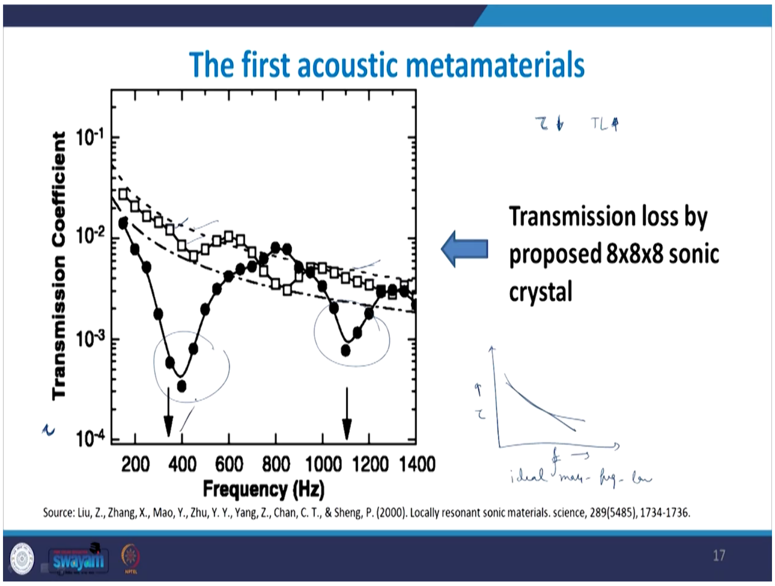

The first acoustic metamaterials

Transmission loss by proposed 8x8x8 sonic crystal

Source: Liu, Z., Zhang, X., Mao, Y., Zhu, Y. Y., Yang, Z., Chan, C. T., & Sheng, P. (2000). Locally resonant sonic materials. science, 289(5485), 1734-1736.

So, this was the kind of material proposed and the transmission coefficient. So, this was experimentally made and then sound wave was impinged on it. And, the spectrum and the other vent was noted and the transmission loss was computed. So, what was the transmission coefficient and this is not transmission and this is tau the value of tau. So, higher the value, if higher so, a lower value of tau means less transmission which means higher transmission loss.

So, as you can see here in this particular material, it does not follow the mass frequency law, because according to mass frequency law, ideal mass, frequency law, if this is tau and this is f, then as frequency increases the transmission should go down. Because, because; obviously, the performance will improve. So, this should be like the curve or something like that a uniform decline, but here as you can see, this the traditional materials they follow this decline curve, but this particular material here.

So, this was what the when the individual balls they were their transmission coefficient was noted. It was found to obey this traditional mass frequency law, but the combined effect was this. So, although it did follow the mass frequency law, but there were 2 areas where a major dip was observed.

So, as you can see even at a very low suddenly you are getting a very high dip compared to at high frequencies. So, what it means is that, even at this low frequency of 400 Hertz, the material can be can provide a very heavy transmission loss suddenly at a low frequency. So, this was observed. So, this is the region where it showed exceptional values.

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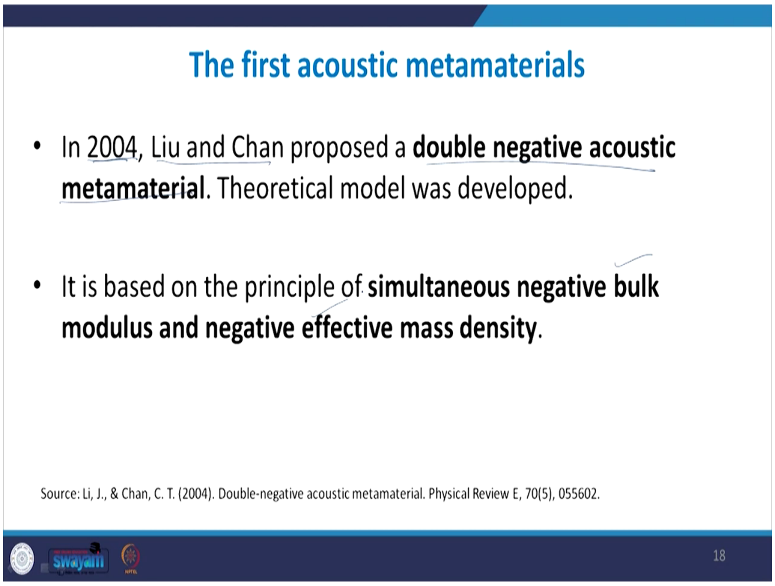

The first acoustic metamaterials

- In 2004, Liu and Chan proposed a **double negative acoustic metamaterial**. Theoretical model was developed.
- It is based on the principle of **simultaneous negative bulk modulus and negative effective mass density**.

Source: Li, J., & Chan, C. T. (2004). Double-negative acoustic metamaterial. Physical Review E, 70(5), 055602.

So, the second form of the major acoustic metamaterial was then proposed in the year 2004. And, for all of this I have given you some sources in which you can study as an additional reading, if you are further interested to study about them, but I will cover the sonic crystals, in our later lectures, in a detailed manner.

So, the second material was in 2004, it was proposed by Liu and Chan this was a double negative acoustic metamaterial. So, it was theoretically proposed which will have a negative bulk modulus and negative effective mass density, but so, far experimental verification is still not concrete.

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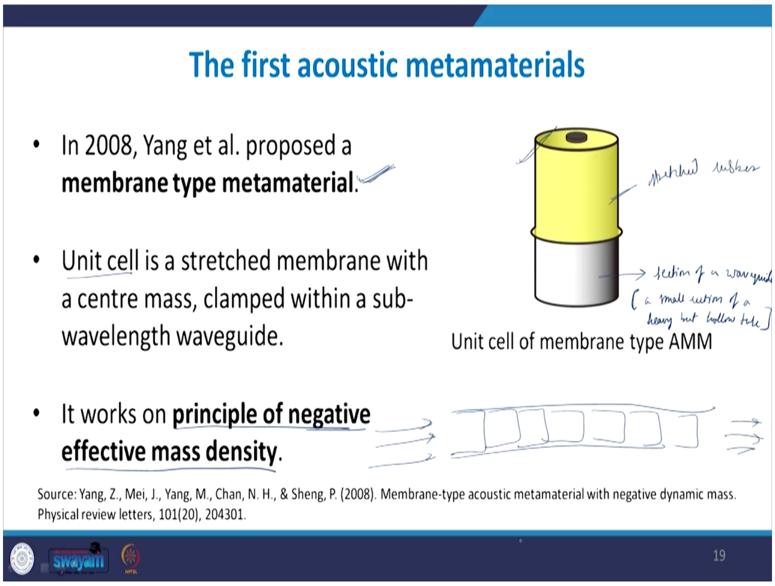

The first acoustic metamaterials

- In 2008, Yang et al. proposed a **membrane type metamaterial.**
- Unit cell is a stretched membrane with a centre mass, clamped within a sub-wavelength waveguide.
- It works on **principle of negative effective mass density.**

Unit cell of membrane type AMM

Source: Yang, Z., Mei, J., Yang, M., Chan, N. H., & Sheng, P. (2008). Membrane-type acoustic metamaterial with negative dynamic mass. Physical review letters, 101(20), 204301.

Then finally, in 2018 the first complete model of a membrane type metamaterial came into being. And, after this lecture we will begin with a discussion of this membrane type metamaterial in detail. So, here the unit cell is what we have a small waveguide or a small diameter sub wavelength diameter tube and in that you have some stretched membrane.

For example, you can have a stretched rubber attached on the top like this. So, this is the stretched. So, here a stretched rubber was used, but any stretched elastic membrane can be used and this is a section of a waveguide, or in other words this is a small section of a heavy, but hollow tube. So, tube made of a heavy material, but hollow inside. So, this is a hollow tube and an elastic rubber or an elastic membrane is attached on the top and this particular metamaterial it works on the principle of negative effective mass density.

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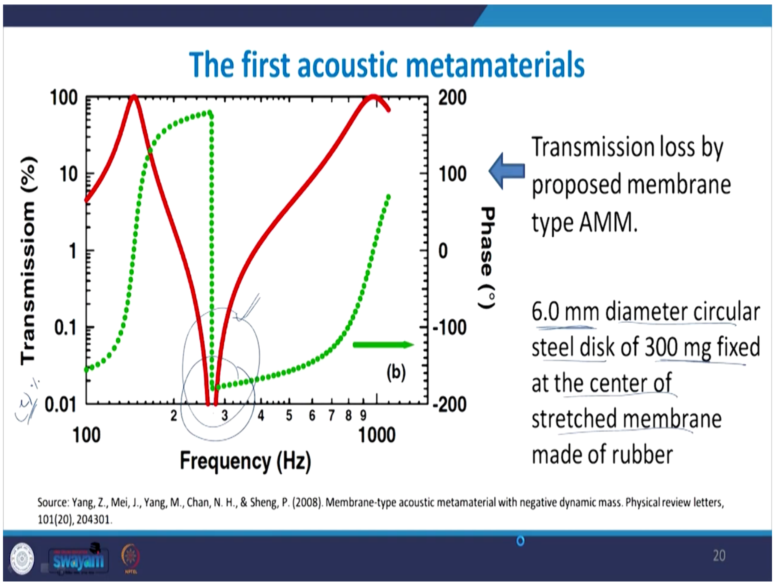

The first acoustic metamaterials

Transmission loss by proposed membrane type AMM.

6.0 mm diameter circular steel disk of 300 mg fixed at the center of stretched membrane made of rubber

Source: Yang, Z., Mei, J., Yang, M., Chan, N. H., & Sheng, P. (2008). Membrane-type acoustic metamaterial with negative dynamic mass. Physical review letters, 101(20), 204301.

So, the transmission loss that is observed for this kind of material as you can see here is that, this is again tau expressed in as percentage, to transmission expressed as percentage. So, here also what you see is that at some low frequency region suddenly you observe a very sharp dip, which is not expected, because at low frequency is the transmission loss is going to be less for a traditional material, but the heavy dip is observed at these values.

So, this is where the material is breaking the mass frequency law, it is showing you exceptional transmission loss or a very heavy noise control even at low frequency. So, this is the kind of material that was used 6 millimeter diameter circular steel disk 300 milligrams of fixed mass at the centre of the membrane. So, this is a mass attached here.

So, this was the unit cell and it was arranged periodically in this series manner. So, a long waveguide can be having this unit cell. And, some sound was impinged from this end and it was noted in the other end and the heavy transmission loss was seen. Similarly, in 2019 a very famous experimental work was conducted by Zhang Yin and Fang.

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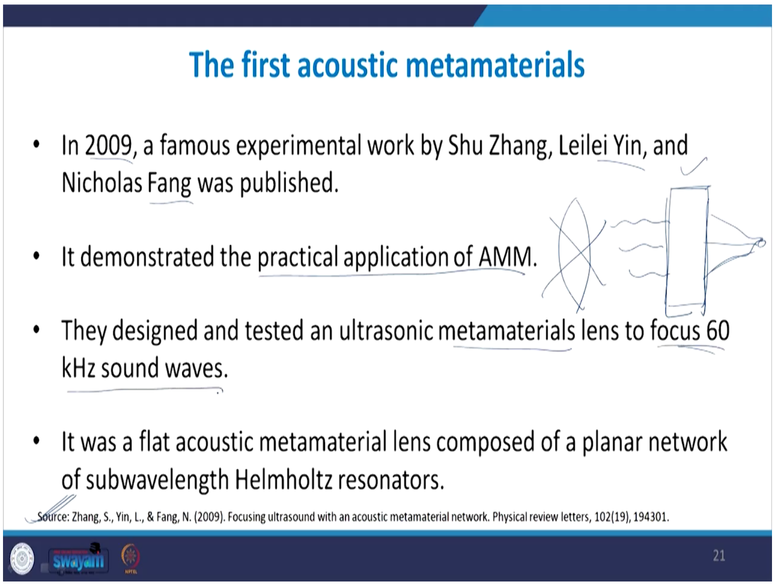

The first acoustic metamaterials

- In 2009, a famous experimental work by Shu Zhang, Leilei Yin, and Nicholas Fang was published.
- It demonstrated the practical application of AMM.
- They designed and tested an ultrasonic metamaterials lens to focus 60 kHz sound waves.
- It was a flat acoustic metamaterial lens composed of a planar network of subwavelength Helmholtz resonators.

Source: Zhang, S., Yin, L., & Fang, N. (2009). Focusing ultrasound with an acoustic metamaterial network. Physical review letters, 102(19), 194301.

All the sources are given here, you can study these papers separately. If you are interested to do some further reading on how the what is the progress in the development of the metamaterials in the last 20 30 years. So, here what they demonstrated was the practical application of bending due to a metamaterial.

So, what they designed was they designed a flat layer of material. So, usually a lens is of this form, but this is not what they used they designed a flat layer of material, which was able to focus the sound waves by achieving very sharp bending.

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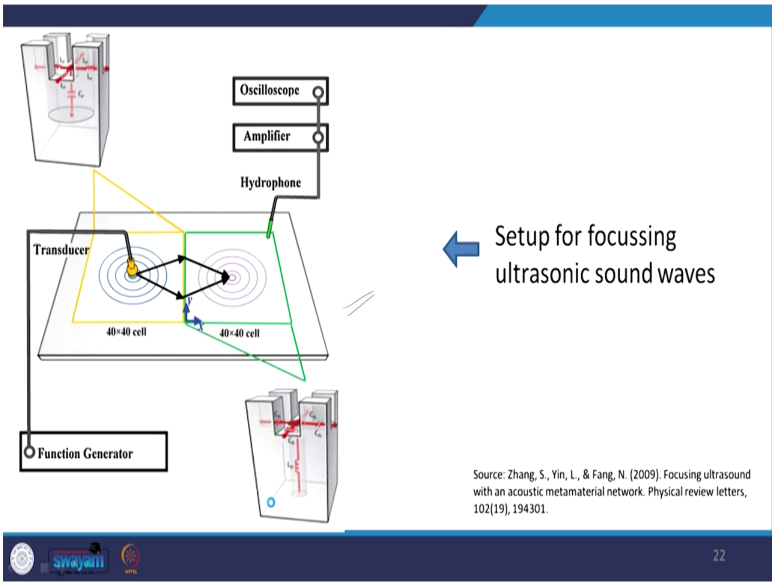

Setup for focussing ultrasonic sound waves

Source: Zhang, S., Yin, L., & Fang, N. (2009). Focusing ultrasound with an acoustic metamaterial network. Physical review letters, 102(19), 194301.

So, this was the kind of equipment that was used and here they demonstrated how this particular metamaterial was could be used to focus 60 kilo Hertz of sound waves. So, this shows a setup used and they shows a typical graph.

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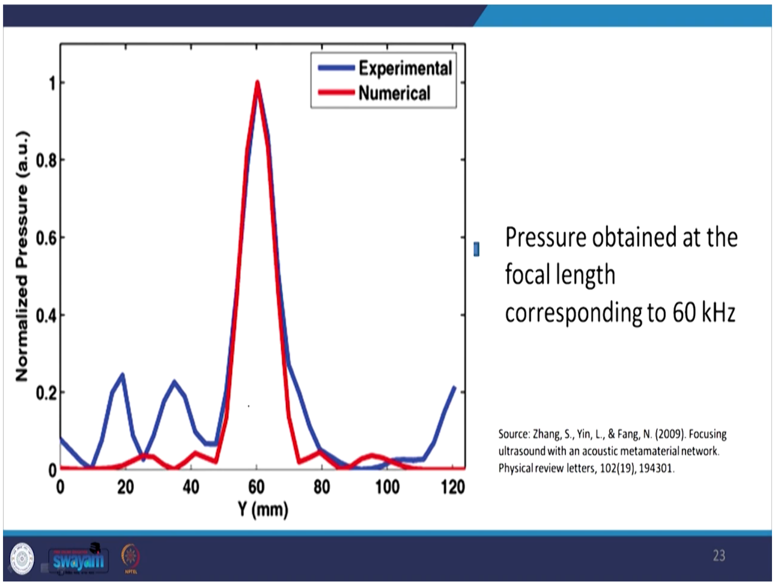

- Pressure obtained at the focal length corresponding to 60 kHz

Source: Zhang, S., Yin, L., & Fang, N. (2009). Focusing ultrasound with an acoustic metamaterial network. Physical review letters, 102(19), 194301.

So, here this is a graph this is the graph of the focusing of the 60 kilo Hertz wave. So, as you can see this is the wave is being focused here is only 60 kilo Hertz.

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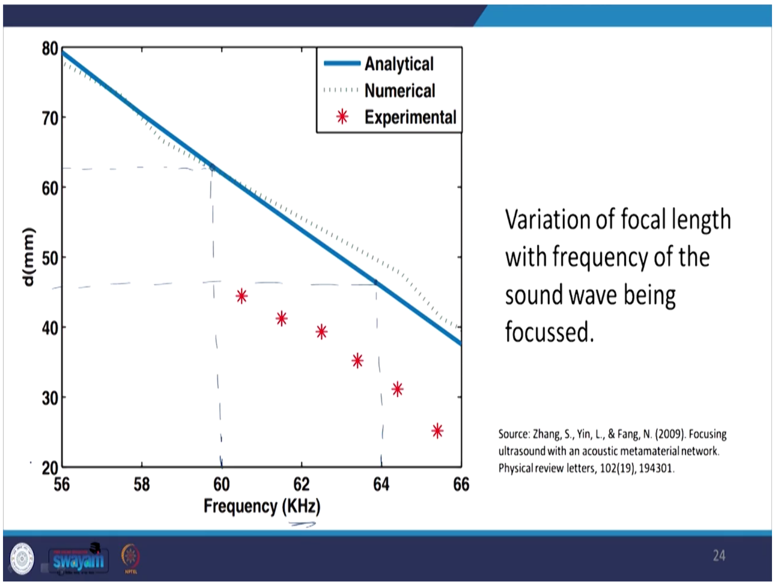

Variation of focal length with frequency of the sound wave being focussed.

Source: Zhang, S., Yin, L., & Fang, N. (2009). Focusing ultrasound with an acoustic metamaterial network. Physical review letters, 102(19), 194301.

And this is the wavelength. Now, depending upon where the focusing so, what is the focal length? So, depending on the frequency the focal length used to vary. So, different frequency waves could be bent together and could be focused at a point at different distances. So, this shows the variation of the focal length with the frequency. So, what it means is that a 60. So, this is in kilo Hertz. So, which means that a 60 kilo Hertz wave will be focused at a distance of 60.60, let us say about approximately 62 millimeters away from the layer of material.

Similarly, it will be able to focus a 64 kilo Hertz wave at a distance of so, all this is just so, at a distance of approximately 45 or 46 millimeters away from the layer of material. So, the kind of lens can be designed which can be used to bend the sound waves, bend the sound waves, very sharply and focus them at a particular point. So, with this I would like to end this theoretical lecture on the various progression and the history historical development of acoustic metamaterials.

Thank you.