Acoustic Materials and Metamaterials
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Lecture - 29
Membrane Type Acoustic Metamaterials – 2

Hello and welcome to the lecture 29 on the series on Acoustic Materials and Metamaterials. I am Prof. Sneha Singh from the Department Mechanical and Industrial Engineering, IIT Roorkee. So, this is our second lecture on Membrane Type Acoustic Metamaterials. And in the last class we studied that what is meant by a membrane type metamaterial and what are the 2 different types of units cells proposed. And in this class we will study about the effective mass density of one type of unit cell.

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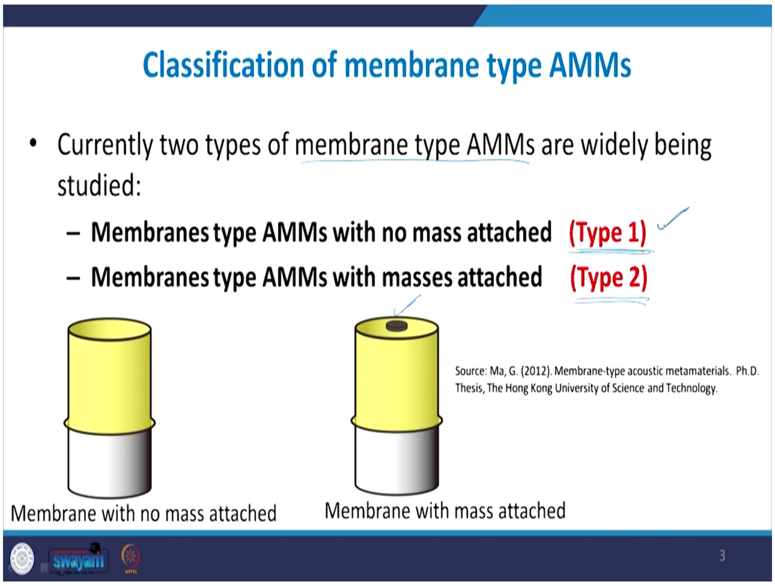

So, to sum up from the previous lecture we saw that, the membrane type AMMAs they are widely being studied and they can be till now 2 main types of units cells have been proposed, where one you have you have a wave guide and a stretched membrane that is loaded on the waveguide and then the second one is that you have a wave guide and you have a stretched membrane with some mass attached on the top. So, in this particular class we will focus on type 1.

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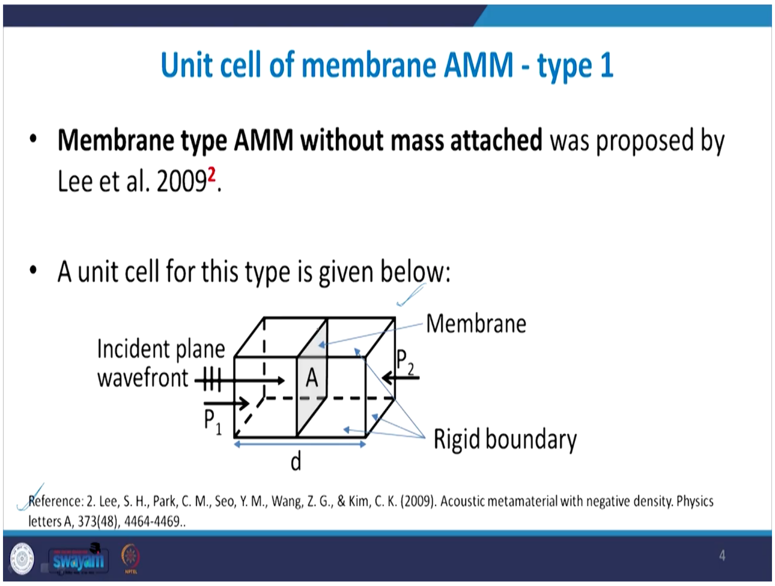

So, the type 1 was proposed by Lee et al 2009 and this is the difference for the author. So, this unit cell type is given below. So, here you have a sub wavelength wave guide. So, here all the dimensions of this unit cell needs to be in sub wavelengths.

So, whatever is your target lambda, the dimensions of the unit cell need to be much smaller than the lambda that you are trying to target, the wavelength. So, here you have the stretched membrane and in this particular case, it is being loaded with some fluid medium. I am taking here air, but you can use any fluid medium, it can be loaded with water or any other fluid medium.

So, there is some fluid medium let us say air inside this wave guide and a stretched membrane in between and the pressure acting on the left and the right side the average pressure is P1 P2 and the plane wave front is incident on this, this being the stretched membrane.

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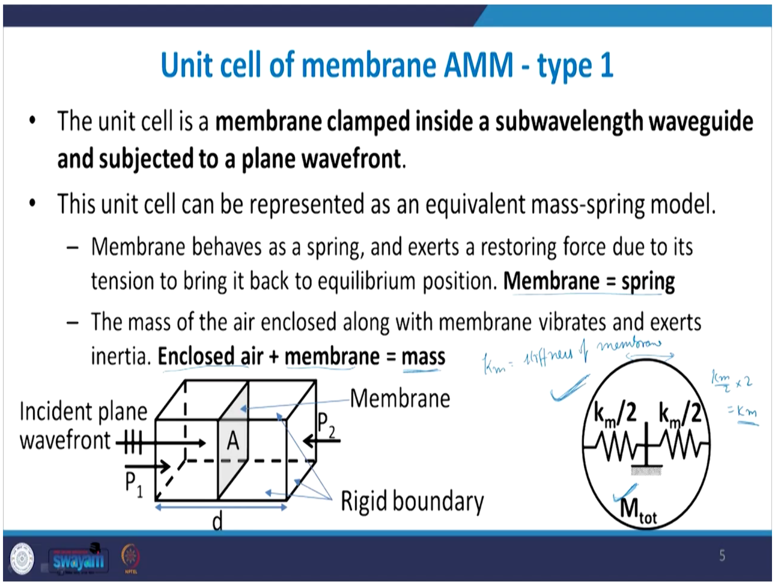

So, here for this particular unit cell this acts like a typical mass spring oscillator. So, in this case you have a thought stretched membrane. So, let us say if you give some displacement to this membrane.

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So, let us see you have some membrane and you are giving some displacement then it might try to deflect. But, due to that tension of the membrane or we can also call it as a stiffness of the structure due to this it tries to oppose any deflection from the equilibrium position.

So, once you stretch the membrane there will be an opposing force which will try to bring it back to its equilibrium position. And so on if you stretch it from the other end again some force will act due to the tension of the membrane, it will try to bring it back to the equilibrium position. So, whatever be the transverse displacement there will be a force acting on it which will try to bring it back to its equilibrium position.

So, in this sense you can say that this membrane is like the spring element this is trying to restore, it is acting it has a stiffness and it is opposing the displacement using a restoring force and whenever the membrane vibrate. So, when the plane wave front is incident and some vibrations are generated, as the membrane vibrates the same oscillation pattern is followed by the air particles inside the waveguide.

So, all of them the membrane and the air particles together they move in unisons, so, they oscillate back and forth and their displacement function would be the same. So, the mass element here becomes the mass of the air inside the waveguide plus the mass of the membrane. So, if you see here, this is the equivalent mass spring model. So, membrane being this spring and the enclosed here plus the membrane mass being the total mass of this system.

So, this is M total which is the mass of membrane plus the mass of the air contained within the waveguide or the air contained within the unit cell and then you have this spring element. So, if suppose the stiffness of the membrane, k m is suppose the stiffness of the membrane, then you can say that this is oscillating to and fro this particular thing is oscillation to and fro like this. So, I this can be split into 2 equivalent springs and the total stiffness will be k m by 2 into 2 which will be the total stiffness of the membrane.

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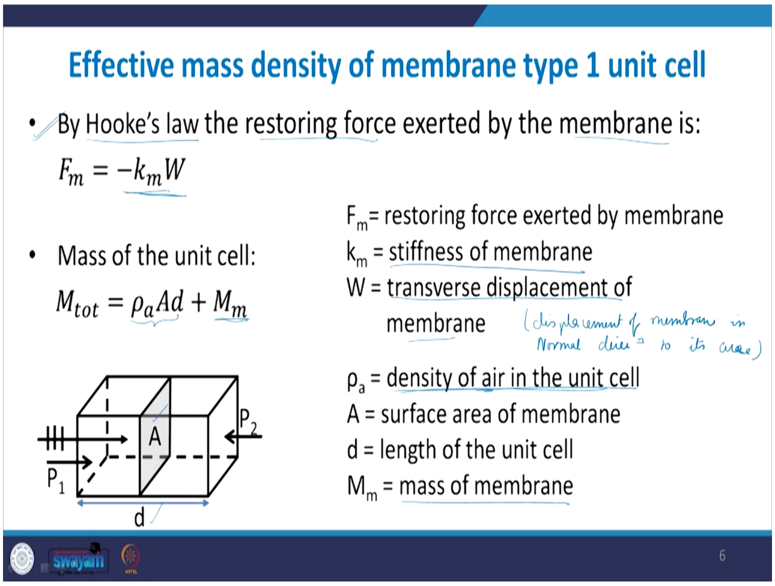

So, this equal stiffness has been created. So, this is the mass spring model of the structure. So, let us solve what will be the effective mass density for this particular unit cell. So, here if you use Hooke's law for in the membrane, so, which means that whatever be the force applied, the restoring force will be equal to the stiffness of the membrane multiplied by the displacement and it will act in the opposite direction to the displacement.

So, by Hooke's law if you see here, the restoring force that is exerted by the membrane is simply minus km into W, where W is the transverse displacement of the membrane. So, which means, the displacement of the membrane in normal direction to its area. So, at equilibrium position if the area is along this direction then the transverse displacement will be along this direction. So, here W is the transverse displacement and this is the this is the equivalent spring constant of the membrane or you can say simply the stiffness of the membrane here and the total mass of this unit cell is given by if say, let us see let this M m be the mass of the membrane plus the mass of the air contained in unit cell.

So, here I have taken the fluid medium as the air. So, the density of the air in the unit cell let us say its rho a; then rho a being the density of the air multiplied by the volume of the unit cell which is area of the surface area of the membrane multiplied by the length of the unit cell d. So, this becomes the expression for the mass.

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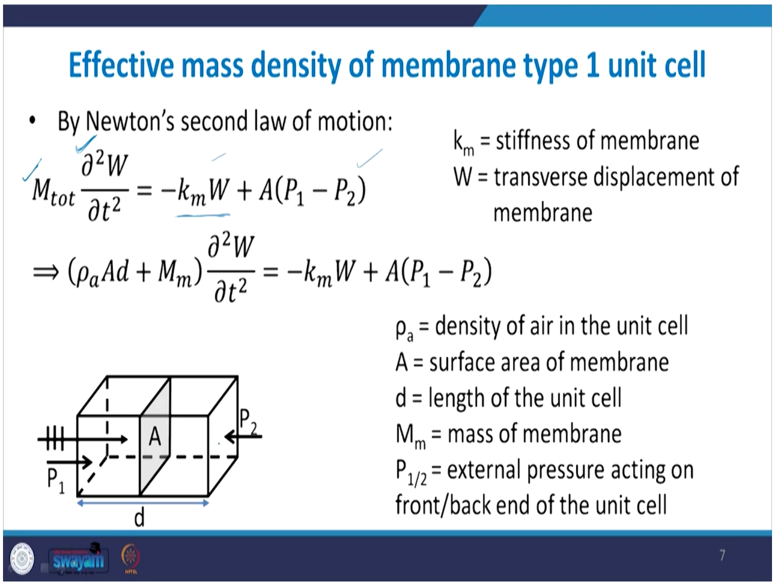

So, now we apply Newton's second law of motion into the complete unit cell. So, which means, the net force applied will be equal to mass into acceleration. So, if we do that then the total mass into the acceleration and here the entire unit cell is moving with the acceleration del square W by del t square; because the air particles and membrane they are in unison, they are moving or oscillating back and forth with the same displacement function.

So, you get mass multiplied by acceleration is equal to the net force acting and we know that the restoring force acts in the direction opposite to the displacement. So, we have minus km into W and then we have the force due to the pressure gradient or difference in the pressure. So, it is P1 minus P2 into A.

Let us divide the entire thing. So, this M total that we had calculated previously is given by this expression. So, we replace M total by this expression here. So, this is what we get.

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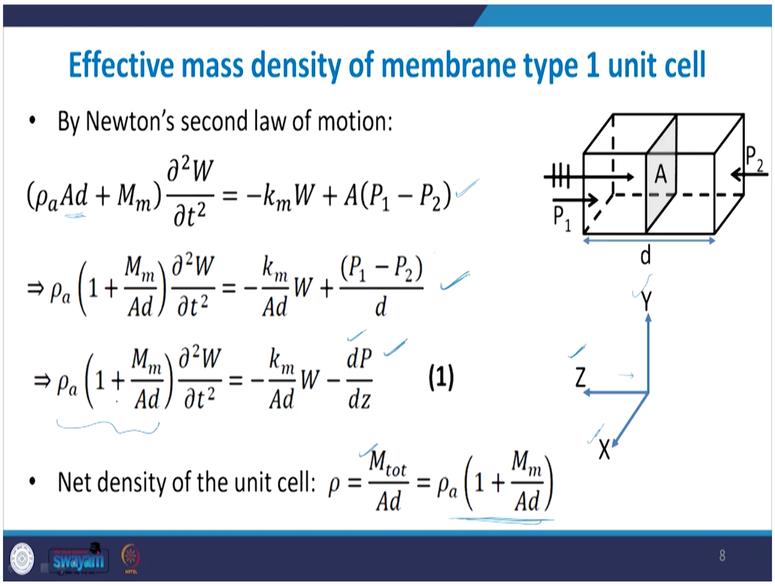

So, this is the expression we are getting using Newton's second law of motion. Now let us divide both left and right hand side by the total volume of the unit cell. So, when you divide by total volume which is the total volume of the unit cell is A into d. So, you are dividing by A d this factor.

So, this is the expression you get at the end. So, this becomes the expression when you divide everything by the volume of the unit cell. So, that is the expression we have got and this P1 minus P2 by d can simply be the written as the pressure gradient, which is the change in pressure divided by considering that the pressure gradient remains uniform throughout this unit cell. Then the pressure gradient can be given by the difference in the pressure divided by the linear distance between the 2 pressure points.

So, this becomes the pressure gradient here. So, here the convention that I have used is X axis Y axis and therefore, the Z axis comes here and the pressure gradient I am calculating is P1 minus P2 which is opposite to the Z axis. So, this expression becomes minus of d P by dz ok. Now the net density of the unit cell is what? The net density of the unit cell will be the total mass of the unit cell by the total volume of the unit cell. So, M total by A d.

So, if you look at this expression here, this was what? This was the total mass divided by A d. So, this becomes this we can simply replaced by a common rho which is the density of the unit cell.

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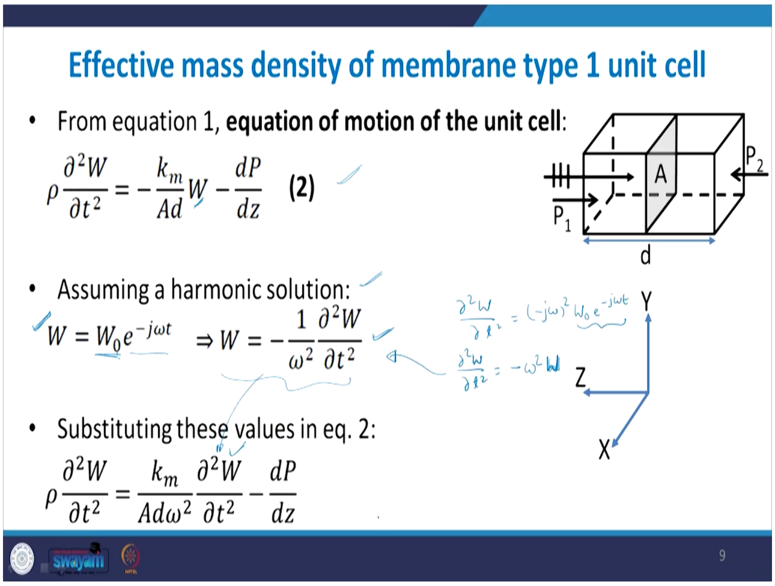

So, using this rho value here, what we end up with is this equation. Now, because these are acoustic processes and all the deflections are going to be much smaller so, for these acoustic processes with within the small transverse displacements, we can assume the function to be harmonic in nature. So, assuming this harmonic solution, we get this W can be some harmonic solution which means it could be some amplitude W naught into e to the power j minus j omega t. So, if this is a harmonic solution here, so, when you double differentiated.

So, if a double differentiate this thing what do you get? You get minus j omega whole square into W naught e to the power minus j omega t. So, del square W by del t square comes out to be minus j square is minus 1; so, what do you get is overall minus 1 into omega square W, this expression becomes W.

So, del square omega del square W by del t square is minus omega square W. So, omega sorry omega square W. So, W becomes minus 1 by omega square del square W by del t square. So, if you substitute this value here, so, W can be written in terms of this quantity then here everything can now be replaced and written as a double derivative of W.

So, it becomes rho del square W by del t square is equal to it is k m by A d; k m by A d and this W becomes minus of 1 omega square. So, minus minus becomes plus here. So, it is 1 by omega square del square W by del t square. So, this W has been replaced with this expression here minus del P by del z.

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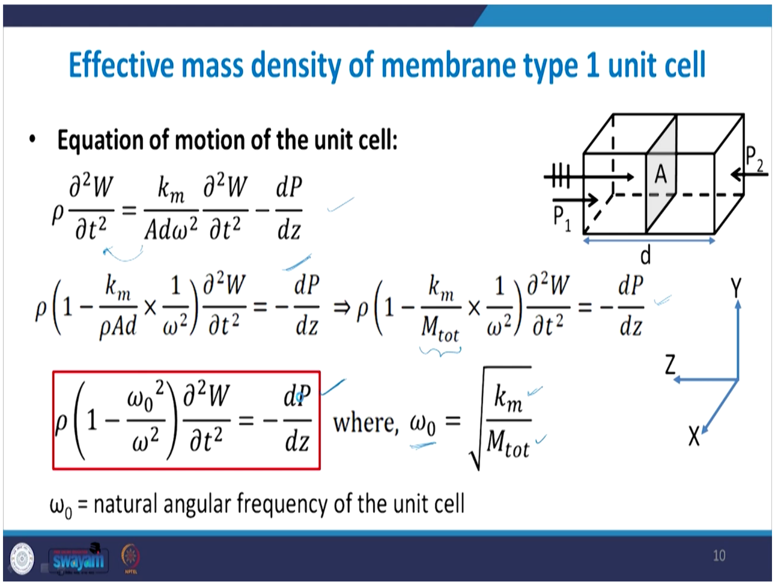

So, this was the expression we obtained. Now, let us if we bring this expression to the other end and take this del square W by del t square is common, so, this is what we end up with. It is we have taken rho and del square W by del t square. So, with this becomes 1 minus k m by rho into A into d into 1 by omega square and this becomes minus del P by del z. So, this is the equation that we are getting overall. So, here in this particular unit cell, if you remember that the membrane was the spring and the total mass which is membrane plus the enclosed here was the total mass of the oscillator.

So, if so, in that case for that particular oscillator what would be the angular frequency? The natural angular frequency will be under root of the stiffness by mass. So, this becomes under root of the stiffness by the total mass. So, this is the natural angular frequency of the unit cell. So, we replace this particular quantity here by omega naught square. So, this is the ultimate expression we are getting ok. So, this is to remember this expression, now let us see how to represent minus dP by dz.

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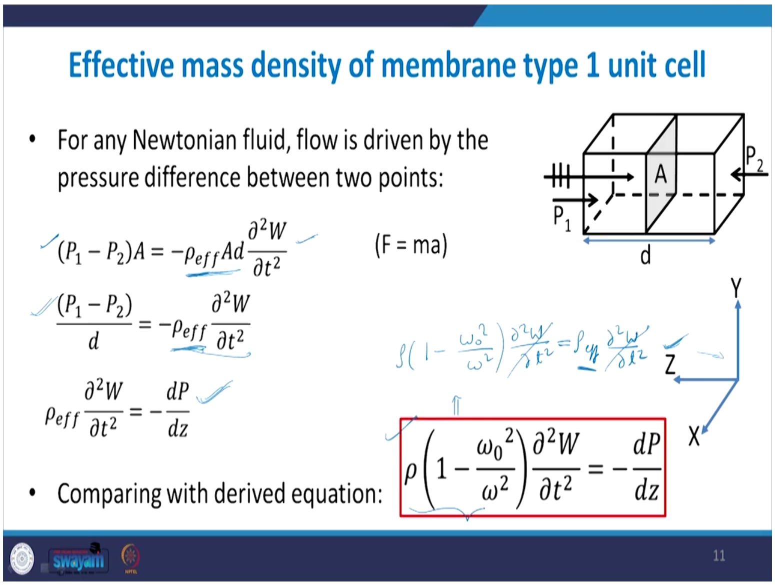

Now, if you have any Newtonian fluid, so, any fluid which follows the classical laws of Newton's physics or the Newton's laws then in that case it is the difference in the pressure between any 2 points which acts as a driving force for flowing the fluid.

So, the fluid flows due to the difference in the pressure. So, the pressure gradient is actually making the fluid flow. And in that case by definition this total force P1 minus P2 into A is simply minus again minus here the minus sign is taken because of the convention of Z, Z we have taken in this direction. So, p minus P2 is in the opposite direction that is why a minus sign is come here.

So, this entire force is equal to the density into the volume which is the total mass into the acceleration. So, for a fluid medium considering the entire thing as a fluid medium we get this expression. So, this is the definition of effective density for a Newtonian fluid. So, when you solve this expression P1 minus P2 is equal to this which means that this thing is minus dP by dz.

And from the previous equation this was our previous equation. So, rho into 1 minus omega naught square by omega square into del square W by del t square is equal to minus dP by dz. So, using this previous equation what we get? We replace this dP by dz by rho effective. So, which what we get here is essentially, this will give us rho into 1 minus omega naught square by omega square del square W by del t square is equal to rho effective into del square W by del t square if we take this value from here into this. So, this gets subtracted, so rho effective comes out to be this particular expression.

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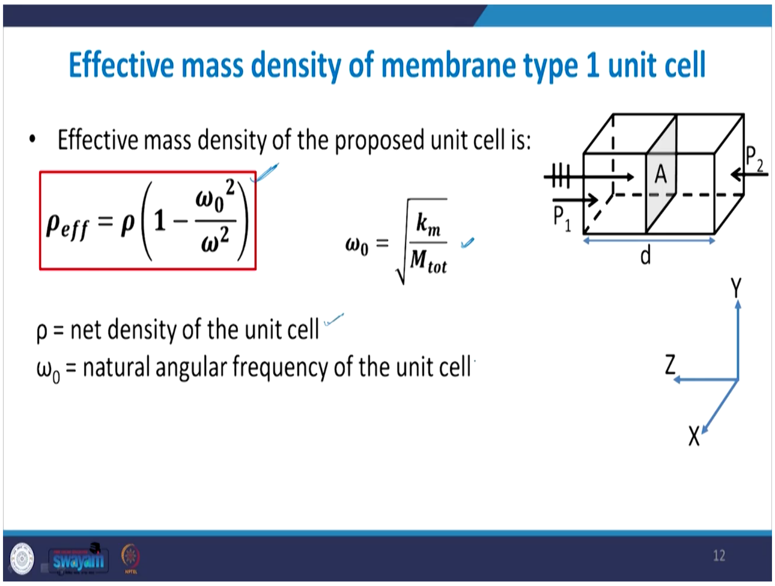

So, this becomes our effective mass density of the proposed unit cell, where omega naught is the under root of k m by M total, the natural frequency of this unit cell. So, here rho is the net density of the unit cell omega naught is the natural angle frequency.

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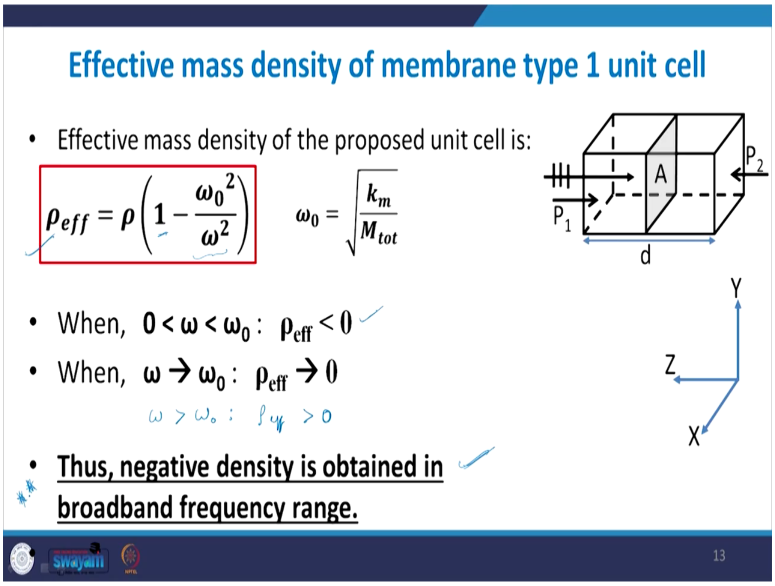

So, let us look back at this particular equation. It is also called as the (Refer Time: 15:53) form of equation. So, here now we know that the acoustic metamaterials they operate on the principles of either negative effective mass density or the negative bulk modulus and this particular membrane type meta acoustic metamaterial, it is working on the principle of negative density. This is a negative density acoustic metamaterial.

So, in the regions of negative density it will become a complete blocker of sound, no sound waves can propagate. So, let us see here. So, now, we have rho effective given here. So, if you see this expression, this is 1 and this quantity has to be always less than 1 for the entire expression to be positive. So, which means omega should always be when omega is greater than omega naught then rho effective becomes greater than, so when omega is greater than omega naught rho effective becomes greater than ze0o.

But whenever this omega is less than omega naught, so, between 0 to omega naught this rho effective is always negative. So, this is an important very important finding. So, in all are conventional materials what we saw was that absorption a very high absorption cannot be obtained at low frequencies and especially and even if it is obtain at low frequencies the magnitude is low as well as the capacity to block the sound.

So, for a traditional barrier material they only perform well at high frequencies at the low frequencies they are not able to completely block the sounds because of the traditional mass frequency law. But here what we see is that within this range of frequency starting from 0 till the critical till the natural frequency of the unit cell for this entire region the density becomes negative and in that case it does not allow the sound waves to propagate. So, it is breaking the mass law. Now we get a broad band low frequency sound blocking or sound reduction.

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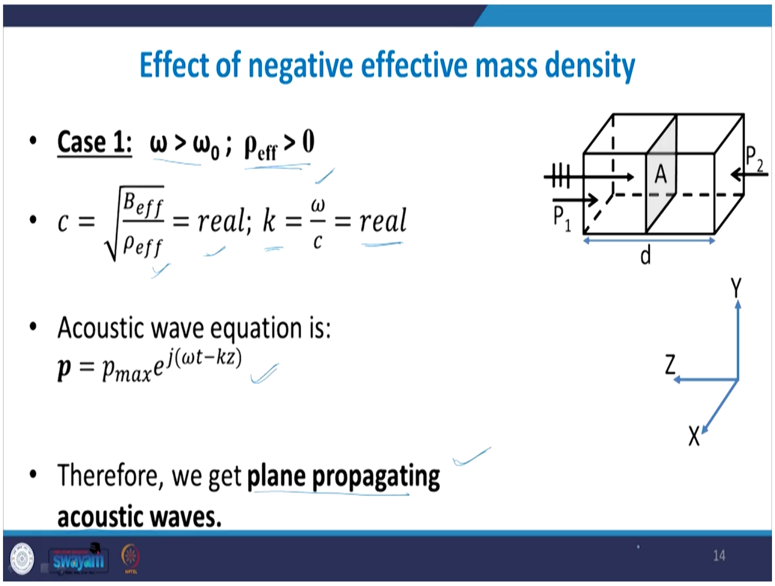

So, let us consider this case by case here. So, case 1 when omega is greater than omega naught rho effective is greater than 0, in that case the c which is the speed of the sound which is under root of be effective by rho effective will be a real quantity.

So, the speed of wave propagation is real, the propagation vector itself is going to be real which is omega by c. So, overall acoustic wave equation is a plane propagating acoustic wave equation. So, we get wave propagations whenever rho effective is positive.

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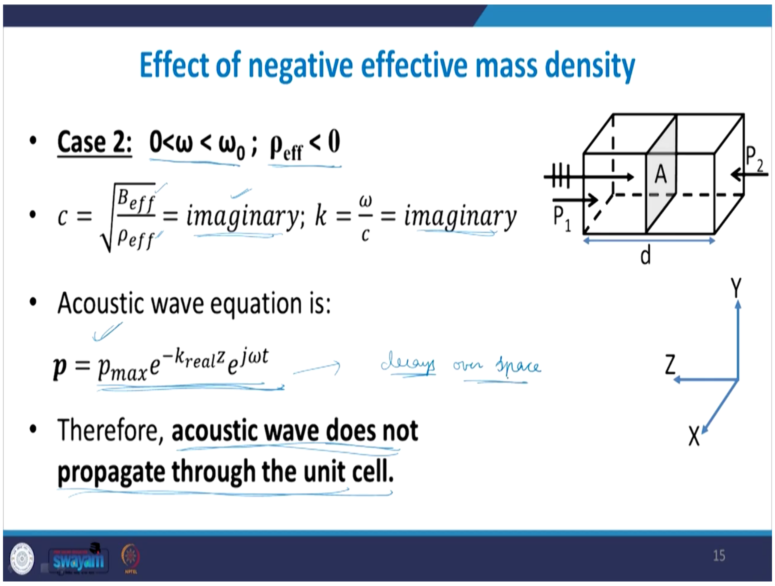

However, in this broad region from 0 to omega naught rho effective becomes negative. So, when rho effective becomes negative. So, when rho effective becomes negative and B is positive. So, B is positive this is negative we get under root of some negative quantity. So, this is an imaginary number, k also comes out to be imaginary and if you go back to the lecture on the introduction to acoustic metamaterials we have already solved what happens when rho effective becomes negative.

So, we solved case by case what happens if either rho becomes negative or either B becomes negative. In both cases the propagation vector is purely imaginary which means the wave does not propagate and this was the form of wave equation. If you solve it this is the form of wave equation you get. So, this means that this is like a decaying wave, it is not a propagating wave and we know that the human ear, sound to a human ear is actually the pressure fluctuations which reach the human ear and if there are no fluctuations if it is not a fluctuating wave it is just decaying in that case it is not perceived a sound.

So, it is not carried forward in space. So, this propagation does not take place, the wave does not propagate through the unit cell because now it is not a propagating wave it becomes a decaying wave, decays over space does not propagate over space.

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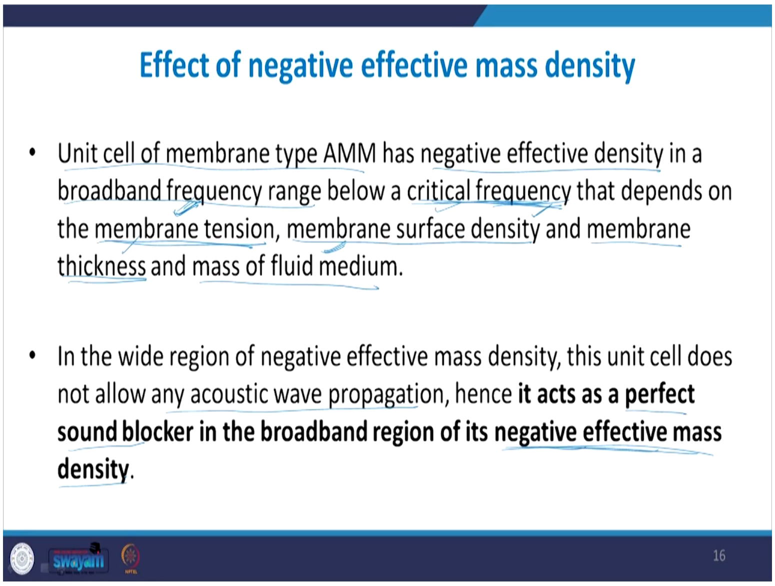

So, this is the overall conclusion that the unit cell of the membrane type AMM has negative effective density in a broad band frequency range below its critical frequency and this range which is what is the cutoff frequency below which we will have negative density, this range will depend upon as you know it depends on omega naught because still omega naught we will have negative density and omega naught is under root of k m by M totals.

So, which means it will depend on the stiffness of the membrane and the stiffness is given by the membrane tension; the most tensed the membrane is the most stiffness it will have and then it will also depend upon the mass of the membrane, so, which effectively means it will depend upon the membranes surface density and the membrane thickness.

So, it will depend upon these quantities: tension, surface density and membrane thickness as well as the mass of the fluid medium which is enclosed within this unit cell. And in this wider region this unit cell does not allow any acoustic wave propagation, this acts as a perfect sound blocker or in this perfect sound blocker and this region of negative effective mass density.

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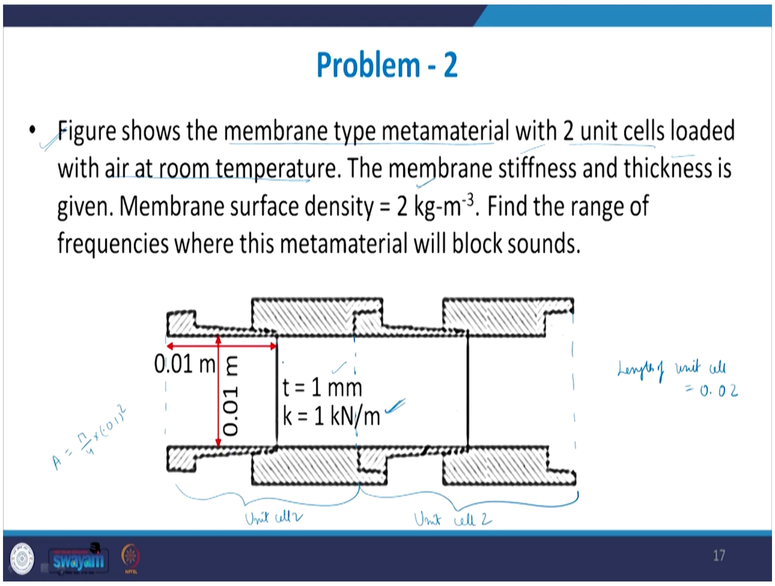

So, that was an advantage of using this metamaterial that now we have a material which can offer which can offer to blog the sounds completely and do not allow any more acoustic wave propagation in a broad region from 0 till its natural frequency. So, let us solve a problem to see how it works in practical life.

So, here a problem given to you is that you have a membrane type metamaterial with 2 unit cells which are loaded with air at room temperature, so that is the fluid medium. The stiff the membrane stiffness and thickness is given here. Thickness is 1 millimeters and the stiffness is 1000 Newton's per meter and the surface density of the membrane is 2 kg per meter square. So, the membrane all the membrane properties are given to you, the density thickness and the stiffness. So, you have to find what will be the range of frequencies where this metamaterial will completely block the sound. So, let us solve this.

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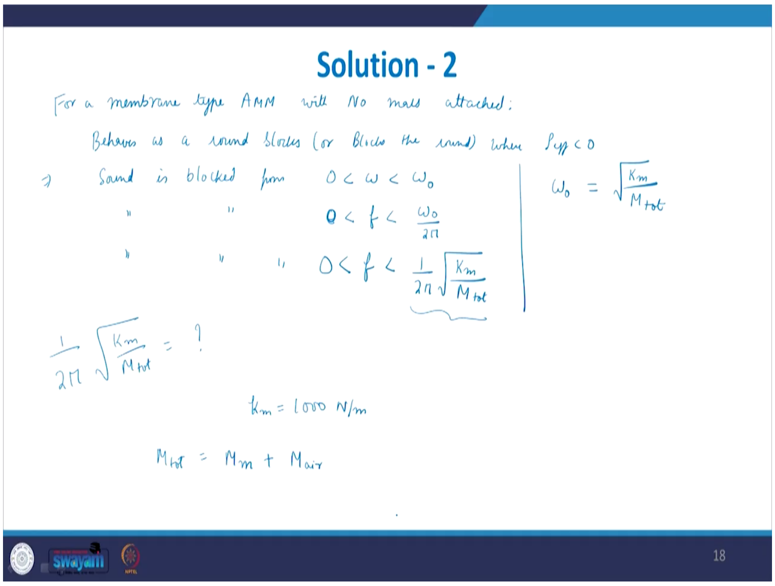

So, here for a membrane type acoustic metamaterial with no mass attached, it behaves as a sound blocker or it blocks the sound, the sound in the region of in the region of where the density becomes negative. So, this means that sound is blocked from 0 to omega naught.

So, in terms of frequency we can say that the sound is blocked from a frequency of a frequency whenever the frequency is between the values 0 till its natural frequency which is given by omega naught by 2 pi ok. Now, we know that omega naught is omega naught is what it is under root of k m by M total. So, we can say that the range we are looking for the range where the sound is blocked is going to be 1 by 2 pi under root of k m by M total.

Now, let us find this values. What is this value? Now, k m is already given to you the stiffness is given here if we can see is this value and all the other things are also mentioned.

So, let us go one by one. So, km is 1000 Newton's per meter. So, I am writing everything in SI unit and then the M total will be what? M total will be mass of the membrane plus mass of the air.

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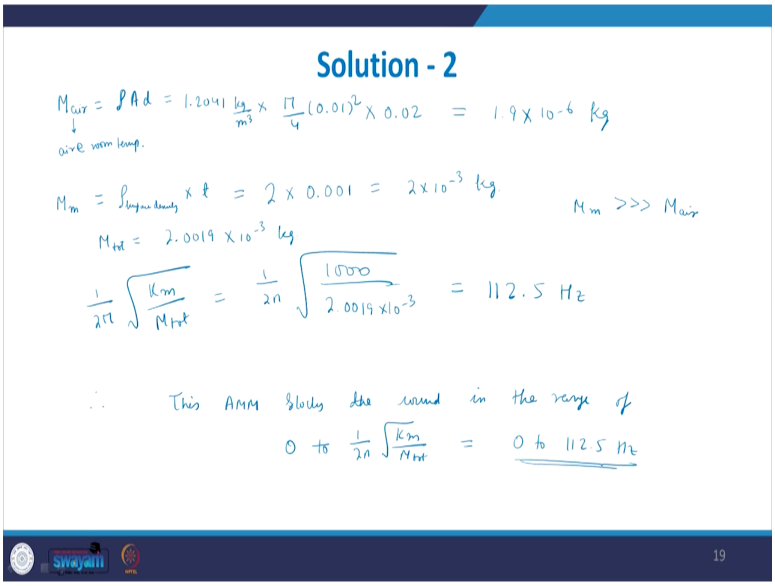

So, let us first find out what is the mass of air. So, if you go here mass of air will be the density of the air at room temperature. So, it is given that its loaded with air at room temperature multiplied by the volume into the length of the unit cell sorry the area of the membrane into the length of unit cell. So, this is the value of the density of air at room temperature, air is at room temperature. So, this becomes the value of the density and we multiply it with the length and the area, so let us see what is the length and the area here.

So, you can see the diameter is 0.01. So, pi by area will be pi by 4 into 0.01 whole square and the length of the unit cell now here you have to look carefully is that this is the here there are 2 unit cells one by one.

So, this entire thing from here till here this becomes 1 unit cell. So, this is unit cell 1 and then the same the same pattern is repeated till here. So, this is unit cell 2. So, 2 units cell side by side and this distance is given as 0.01. So, the length of unit cell becomes 0.02, its double of this distance because its being repeated here assuming that the variables remain constant.

So, what we get here is that a can be written as pi by 4 into 0.01 whole square multiplied by the length which is 0.02. So, the value that you get by solving this would be 1.9 minus 6 kgs putting the various units. Now, the mass of the membrane let us calculate that. We know it will be the surface density multiplied by the thickness and the surface density is given to us as 2 kgs per meter square.

So, mass will be 2 kgs per meter square multiplied by the thickness which is point which is 1 millimeter. So, it is 0.001, everything in SI unit. So, this becomes the net mass of the membrane. So, as you can see here mass of membrane is much much greater than the mass of air in this case. Anyways the total M becomes 2.0019 into 10 to the power minus 3 kgs. So, now, we have the value of M total in k total.

So, we can find out this value. This comes out to be which is approximately this Hertz. Therefore, this AMM or Acoustic Metamaterial blocks the sound in the range of 0 to 1 by 2 pi under root of k m by M total. So, that was already established in the previous slide, so this becomes 0 to 112.5 Hertz.

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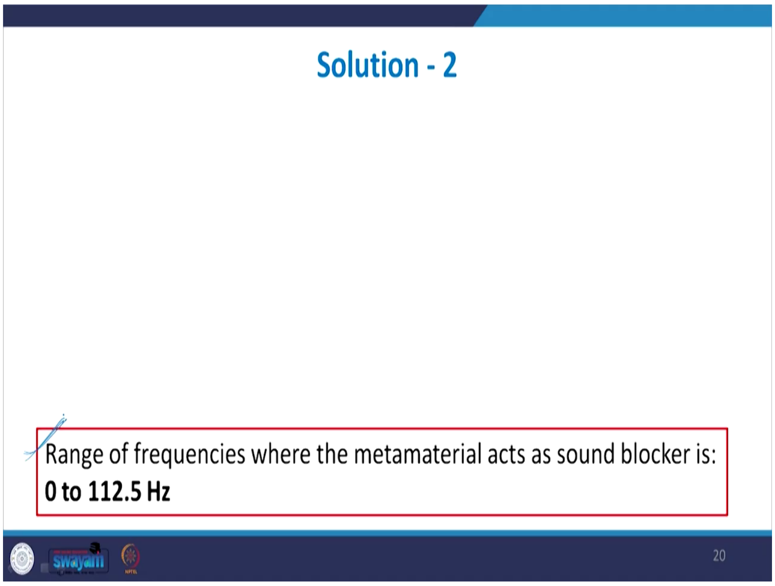

So, this is the solution. So, this shows that we just a simple example where a membrane has been stretched with a particular and with a particular thickness and density and how using just the membrane just manipulating these membrane properties we are able to get a broad band range well the material becomes a perfect barrier material and so on. So, we will discuss about the second type of a unit cell in our second lecture.

So, thank you for listening.