Acoustic Materials and Metamaterials

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Lecture – 30
Membrane Type Acoustic Metamaterials - 3

Hello and welcome to lecture 30 of the series on Acoustic Materials and Metamaterials and today is the last lecture of week 6 and the 3rd lecture on Membrane Type Acoustic Metamaterials. So, in the last class we studied about the expression for effective mass density of the unit cell type 1, where we have a stretched membrane inside a waveguide and there is no mass attached to it.

And what we found was that it can block sounds at a broadband range between 0 till its natural frequency. So, over a broadband range of low frequencies it can be a perfect barrier material. So, as you can see it is advantageous over all the traditional barrier materials as it is able to break the mass frequency law and give you a complete control at low frequencies.

In today's class we will begin our discussion on the second type of unit cell which is when we have a stretched membrane with the mass attached to it, we will see what is it is vibration response and what is the expression for its effective mass density.

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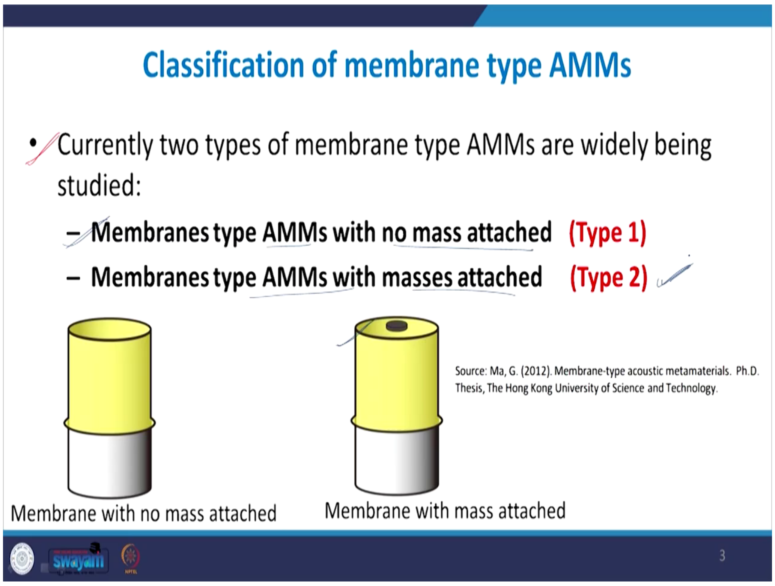

So, here just a revision quick revision again of the, so, as you can see here. So, you have 2 types of unit cells: the first one is the membrane type AMM with no mass attached and the other one is with mass attached and this is what we will discuss today in this particular lecture.

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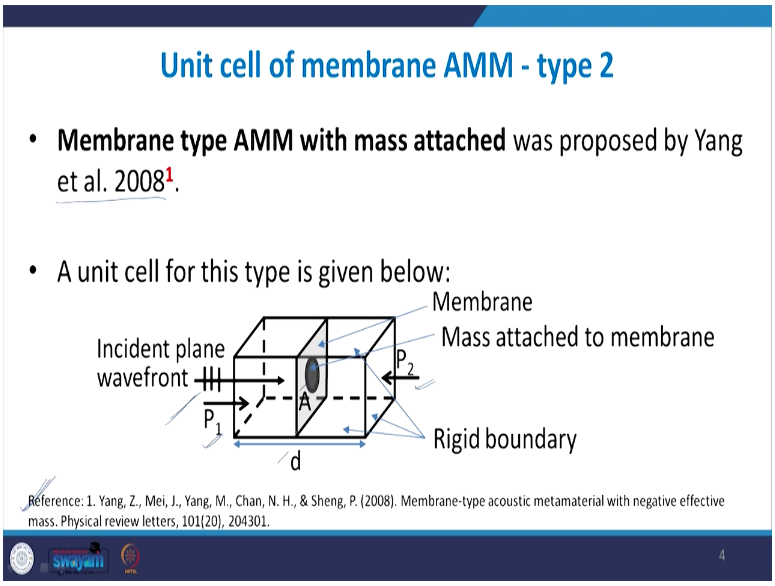

So, this type of unit cell was proposed by Yang et al. 22008, this is the reference paper which you can refer to where this kind of unit cell is proposed. So, what you look at is it is the same as the previous one. So, you have a sub wavelength waveguide with the air enclosed inside and a stretched membrane inside this waveguide and there is some central mass attached to the membrane and some plane wave front is incident and P1 and P2, the uniform pressure at both the ends of the unit cell are given to us. Membrane area, density all and the length of the unit cell these dimensions are known to us.

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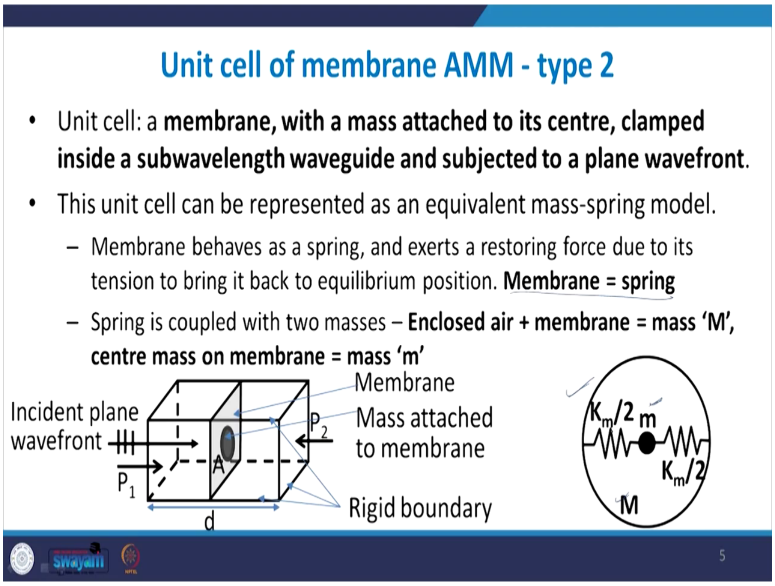

So, the unit cell is effectively it is a membrane with a mass attached and at the center and it is clamped inside a waveguide and subjected to a plane wave front. So, this just like the previous unit cell, this can also be represented by an equivalent mass spring model. So, in this case what happens is that here when the vibration when the air vibrates, the membrane is going to be the spring element because that is the only element.

So, whenever being stretched, it tries to oppose its transverse displacement and there and exert a restoring force in the direction opposite to its displacement which is given by this stiffness. So, the stiffness is given by simply the tension that is acting on the membrane, we all this stretched membrane. So, spring element is the membrane and, but here we have 2 different masses. First of all as the membrane is vibrating, it is the vibration of the membrane which leads to the oxidation of the air particles.

So, actually the other way around when a plane wave front is incident then the air particles they will oscillate they will touch the membrane and the membrane will then vibrate and the entire thing is going to vibrate in you unison. So, you will have the same displacement function for the air particles which are oscillating back and forth as well as for the membrane. So, together they can be considered as one mass capital M, but then this additional mass attached at the center which is a smaller mass.

Now, if the membrane let us say is vibrating. So, if we have a if I can show you visually, let us say this is a stretched membrane and it vibrates like this and you have some thick mass attached. So, even when this mass is attached, the membrane might vibrate at its original mode, but the mass can have a different mood altogether. So, this can vibrate like this whereas, due to the mass the vibration pattern of the mass portion could be different.

So, membrane does not vibrate in the unison with the mass. So, membrane has a different vibration pattern and the center mass can have a different vibration patterns. So, this central mass becomes the second mass. So, if you look here in this particular figure. Now the same diagram for the previous unit cell, we replace it with 2 different masses; the outer mass is the mass due to the membrane and the air and the small one is the mass of the is the central mass which is attached to the membrane.

So, these 2 masses are there and they are joined together by this spring element. So, whenever which is the restoring force acting on both of them.

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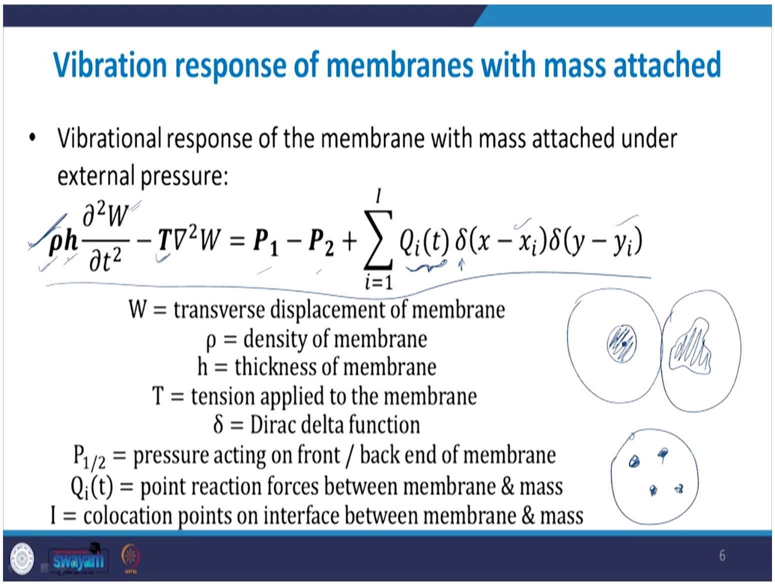

Now, the vibration response of this particular, so, whenever you have a membrane with some mass attached on it then the vibration response of that membrane is given by this particular expression here, if you can see. So, the derivation of this expression is obviously, not something within the scope of this course, you can do you can do that as a you can try it at home if you want to, but it is not a part of the course.

And we already had discussed about what is the vibration response with just a stretched membrane and what we found was that in that case the vibration depends upon some external characteristics like the external pressure apply the external pressure on the membrane. The external tension of the membrane or the amount of stretching that is being done on the membrane and the internal properties of the membrane which is density and the thickness of the membrane.

In this case we have the vibration response will not only depend on the membrane properties, but also on the mass properties. So, in this particular equation here you can see, this is the vibration response. So, what are the quantities on which it depends? Here W is the transverse displacement rho is the membrane density this is the membrane thickness, T is the membrane tension P1 and P2 these are the external pressures being applied on the front and the back end of the membrane and this particular thing is the point reaction force. So, here the mass is considered as a continuum of many such point masses.

So, here the mass can be of any shape, so, let us say it can be a circular shape, irregular shaped or you can have a membrane. So, this is a membrane with some mass here or you can have a membrane with some mass like this or you can have a membrane with masses at different locations some mass here some mass here and so on. So, all of this total mass can be considered as a sum of these point masses. So, this Q i t is the point reaction force between the membrane and the mass.

So, if you consider any small elemental area like this then it will due to its density because that mass has got certain the mass has the because of the mass or its the kg. So, because of its density it will exert some reaction force on the membrane where it is being attached.

It will exert a normal reaction force which is given by Q i t and this is the dirac delta function for all the such for the x and the y locations these are the x and y locations for these point masses. So, here these are the collocation points which is the interface between membrane and mass.

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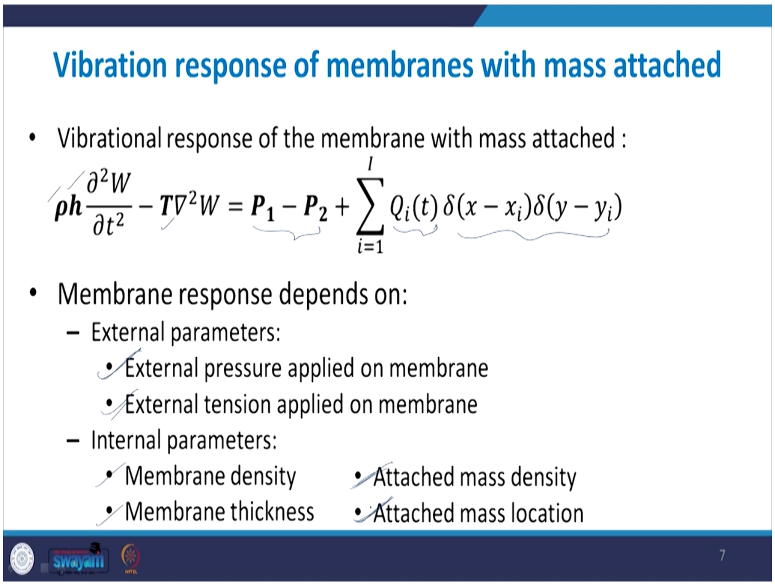

So, overall what you get is that this response it depends upon some external parameters like the external pressure applied which is this, the external tension applied which is this and some internal parameters which is the membrane density thickness and the attached mass density, now this particular point reaction. So, this is the reaction force due to every point mass within the total mass. So, we have we are considering this total mass as a collection or a continuum of many point masses.

So, every point mass will exert some reaction force and that will depend upon what is its density because the normal reaction depends upon the mass. So, it will depend on mass density and this value will depend upon how the mass is distributed; so, what are the x i and the y i values. So, it will depend upon the location of the mass with respect to the membrane. So, these are the various parameters on which this vibration response will depend.

So, when we solve for the unit cell what we will see is that the frequencies within which it acts as blocking the sound or as reflecting the sound, it depends both on the membrane property and the mass property.

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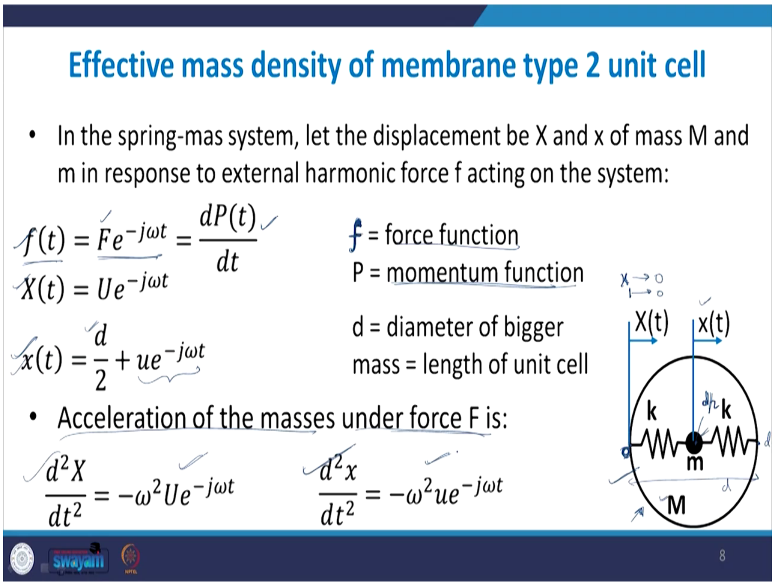

So, let us start with the derivation for effective mass density for the type 2 unit cell. So, in this case let us say, now let us see we are taking this equivalent mass spring model for the unit cell. So, let capital X t bt displacement for the bigger mass M and small x t be the displacement for this smaller mass small m. If we take the origin here, so, this is where X begins. So, X or the displacement begins from here this is the point where this begins.

So, this is at 0 and if d is the total length of the unit cell then this point is at d by 2 at its equilibrium position and this point is at d. So, at equilibrium position this is at 0 and mass m is at d by 2. So, now let us say this system is now subjected. So, we had this unit cell and suddenly some external excitation is given to it. So, some external harmonic excitation is given and suddenly the masses they start to oxalate because of the excitation.

Then we can see we can write the excitation as a harmonic function F into e to the power j omega t; so, where this is the force amplitude. Now by the Newton's law again force is the rate of change in momentum. So, the force function is going to be the rate of change in the momentum function. So, it will be d by dt of Pt; this is the force and the momentum function respectively.

The small f is the force function and similarly the displacement function for the 2 masses can be given by again harmonic solutions because we are assuming it is acoustic process, all the fluctuations are very very small, therefore, it can assume a we can say we can take a harmonic solution. So, Xt becomes some amplitude into e to the power minus j omega t and small x t becomes again a harmonic solution plus d by 2 because we are taking the origin here at 0 and at the equilibrium this mass is always at d by 2.

So, we are measuring the displacement from d by 2 what is its displacement. So, d by 2 plus whatever is its displacement function. So, this becomes x t and capital X t. So, now, that we have this harmonic solution that is then the acceleration of the masses under this particular force acting on it will be del square Xx by del t square and del square small x by del t square. So, you double derivate it with respect to time, this is the expression you end up with.

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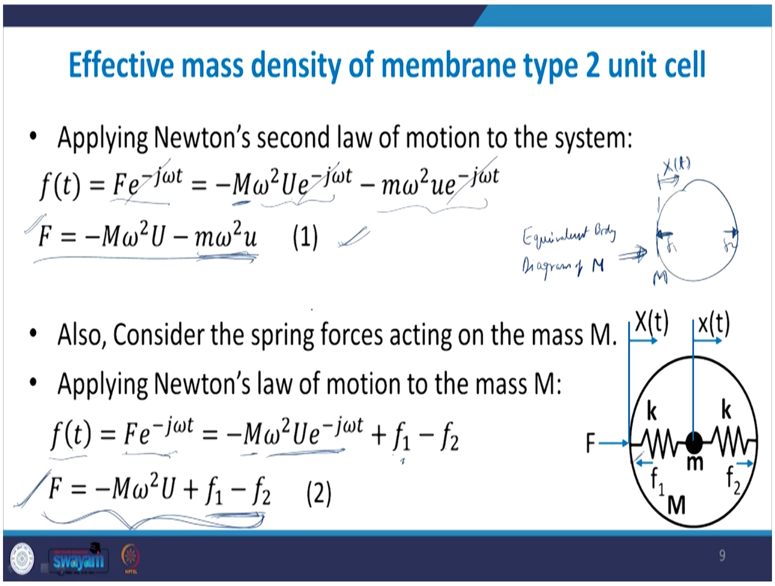

So, now you apply the Newton's second law of motion to this particular system which means that the net force acting will be equal to the mass into acceleration. So, we have the net force acting is equal to what are the 2 masses? Capital M; so, capital M and its respective acceleration. So, we have already found the expression for acceleration of mass capital M and the acceleration of the mass small m. So, we multiply this. So, we will get mass into acceleration plus the small m into its acceleration.

So, this is the particular expression you are getting. So, let us remove this common factor here, the sinusoidal variation and just compare their amplitudes. So, assuming that everything is beginning from the time t equals to 0 there is no phase difference then we just cut this constant out and just compare their amplitude. So, F becomes minus M omega square u in minus small m omega square small u. So, this is our 1st equation.

Now, let us see what are the forces acting on the mass capital M. So, right now we derive this expression by considering both the masses. Now let us draw an equivalent body diagram of just the capital mass M. So, let us just observe this bigger mass M. So, if you draw the equivalent body diagram and you replace this portion you will get is mass M which is moving with some displacement X t ok.

And we are only observing this and not the smaller mass. So, if you remove this thing which means that if you remove this then there will be a force acting due to this spring and the force acting due to this other spring which is given by f 1 and f 2.

So, this becomes the equivalent body diagram of capital M. So, now, we are just focusing on mass capital M then in that case if you apply the Newton's law to this particular system by removing the other parts what you get is the net force is acting on the outside of the mass M and it is this which exerts the force and then capital mass M and small mass m both they start to vibrate.

So, here the force acting will be whatever is the mass into the acceleration and then the total force is this is the total force acting in this direction capital F and f 1 and f 2 these are acting in the this is acting in the opposite direction f 1 and in the same direction as f 2.

So, this is F, the net force acting will be F minus f 1 plus f 2. So, F minus f 1 plus f 2 you bring this quantity here. So, this becomes the net expression. So, this becomes the expression here as you considering the initial point time t equals to 0, the sinusoidal where variation is cancelled out and this is the expression we get, where f 1 and f 2 are the two spring forces acting on the mass capital M.

So, from this what we get is that we already had an expression when we considered the entire system together and then we got some expression for force when we considered only the mass M and remove the system and replaced it with the equivalent spring forces. So, these are the two expressions. If you compare the two what you get is this thing should be same as this thing.

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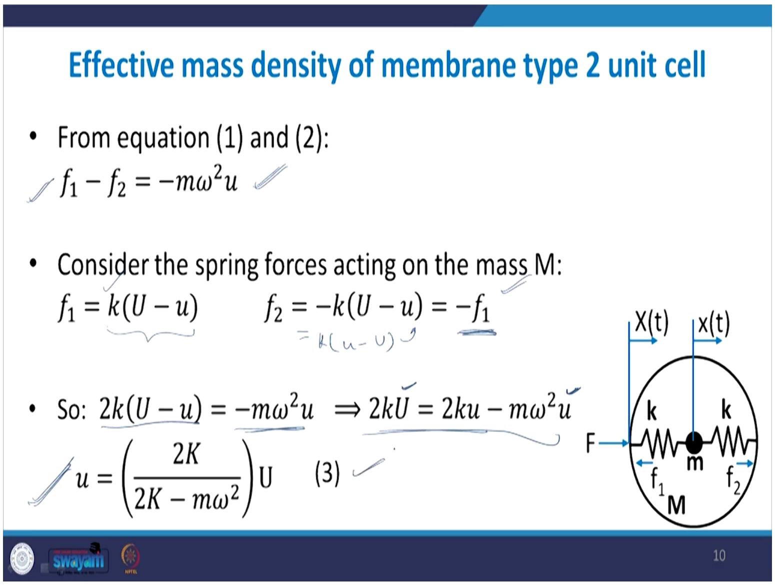

So, from 1 and 2 this becomes what we get here, this expression becoming this. Now, let us consider what is these spring forces which are acting on the mass capital M. So, f 1 as you can see will be k times whatever is the deformation of the spring. So, stiffness multiplied by in del x or the deformation. And what will be the deformation? The net deformation will be capital X minus small x will give you the net deformation in this spring.

So, now only considering the amplitude and writing it we get k into capital U minus small u and then we have so, both the springs have the same stiffness. So, the force will be k into x into del x or the deformations. So, the deformation here is capital U minus small u for f 2 here. Now if you consider here f 2, this is x and this displacement is x. So, here in this case it is small x minus capital X.

So, it becomes this becomes k times of u minus capital U or minus k of u minus small u. So, you as you see here from this symmetry because both of them they have the same stiffness, but the direction in which the force is represented is opposite to each other. So, the magnitude comes out to be equal and opposite. So, if the magnitude comes out to be the same and the forces they are equal and opposite in nature.

Because we have the same stiffness and the system is symmetric on both the ends. So, now, that we have these 2 and we have this expression here. So, f 1 minus f 2 will be what? It will be twice of f 1 because f 2 is minus of f 1. So, what we get here is 2 k into capital U minus small u is equal to minus m omega square into u.

So, if you break it down further this is what you get: 2 k U equals to 2 k small u minus m omega square into small u. So, you get a relationship between the displacement amplitude of the smaller mass and the displacement amplitude of the bigger mass which is this expression here; 2 k by this expression. So, this is what we have obtained.

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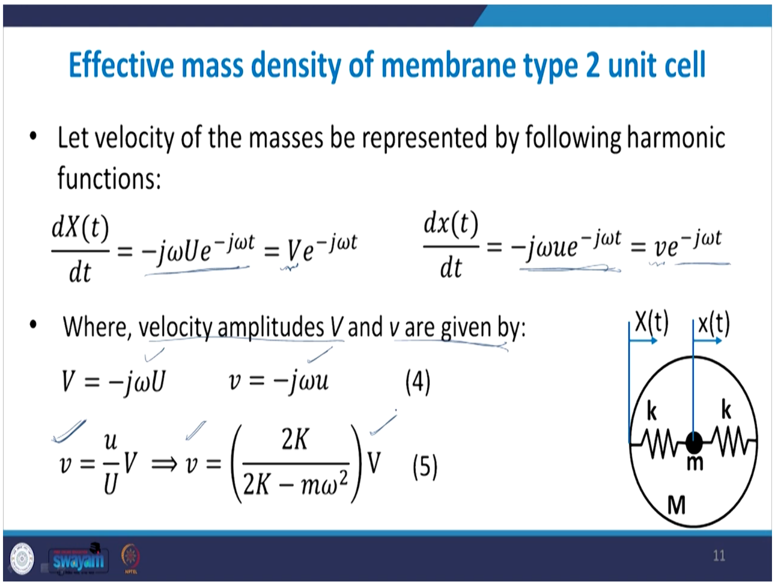

Now, let us find out the velocity functions. So, we have X we differentiate it once with respect to time. So, this becomes the first velocity function for the bigger mass and for the second mass this is the velocity function which we can represent as some amplitude into e to the power minus j omega t and some velocity amplitude into again e to the power minus j omega t.

And here the respective velocity amplitudes for the 2 masses will be as you can see it is minus j omega capital U and small v is minus j omega small u and we already know what is the so, if you divide the 2 expressions here what you get is V will be u.

So, V is going to be small u by capital U times of V. So, the velocity the velocity amplitudes of the 2 masses are in the same ratio as the displacement amplitude of the 2 masses and we already know what is the ratio here; this is the ratio. So, we can replace this u by capital U by this expression. So, this is how the 2 velocities are related to each other. So, we have got a relation between the velocity amplitude of the 2 masses and the displacement amplitude of the 2 masses.

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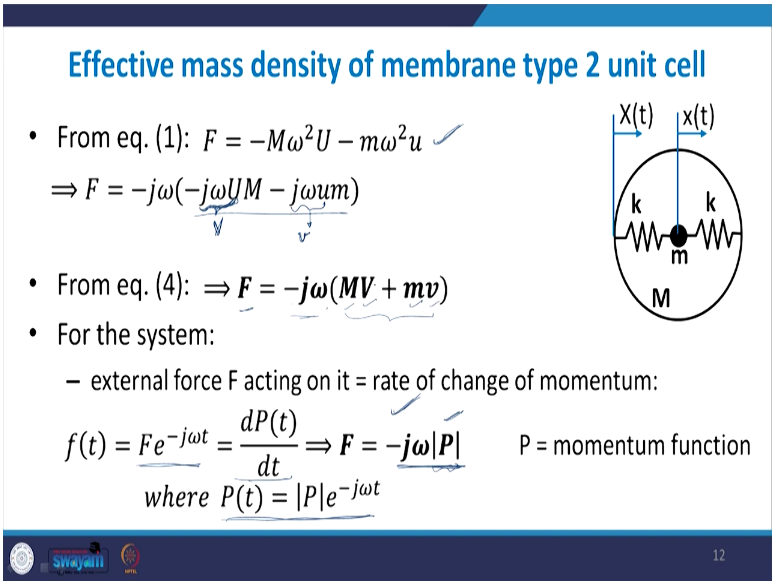

Now, again getting back to the very first equation which was force is equal to mass into acceleration, when it was applied on both the masses. This was the expression we had got the very first expression. Then in terms of velocity we can write this as if we take minus j omega constant this becomes minus j omega U into M minus j omega small u into m.

And this is going to be capital V and this expression becomes small v. So, j omega into U becomes v velocity amplitude. So, F can be written as minus j omega then the bigger mass into its velocity plus the smaller mass into its respective velocity and we also know that force acting is equal to the rate of change of momentum.

So, this was the case F is equal to the rate of change of momentum. So, if suppose P t, momentum itself is a harmonic function, so, if you do d by dt of P t what you get is minus j omega into the momentum amplitude. So, the force amplitude is minus j omega into the momentum amplitude. So, this is the expression you have got and you know that F is equal to minus j omega into its momentum.

And F is also given by minus j omega into the summation of mass with their respective velocities. So, what you get is momentum is equal to be the masses multiplied by their respective velocities.

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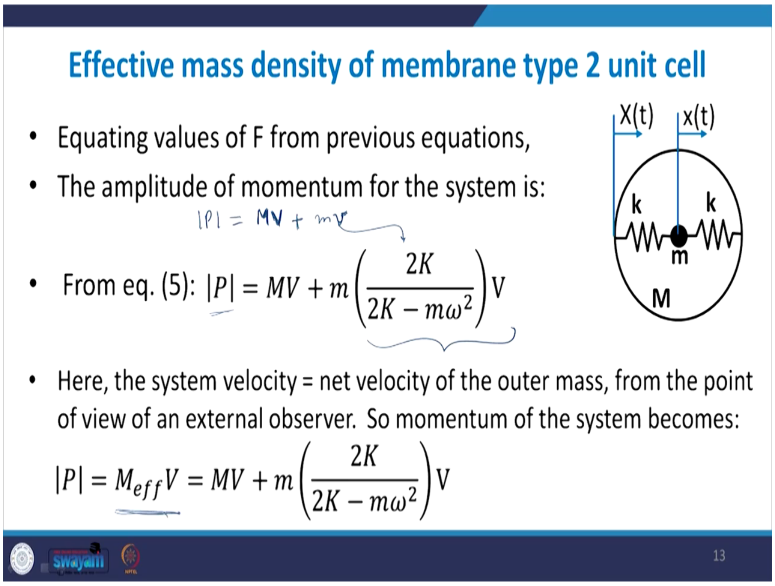

Now, in the same expression, so, this is what we got: P is equal to capital M into capital V plus small m into small v and which is justified because momentum is actually the sum of the momentum of both the masses which is going to be mass into their respective velocity. Now, however, for our outside observer they are only able to observe the external mass. So, what, so, if somebody is from the outside took to them the unit cell what it means is that some force is being applied to the external mass and which leads to some oxidation of the external mass.

So, if you take this point of observer here. So, let us write both everything as in terms of the capital V; it is good to write everything in terms of capital V and we know that we know the relationship between small v and capital V. So, small v is given by this expression here. So, this becomes the expression of small v. So, we have capital M capital V plus small m times this entire thing into V because we already obtained the relationship between the 2 velocity amplitudes equation 5.

So, we have applied the equation 5 here and this is the expression we are getting for the momentum and now the momentum can simply be written as mass effective into the capital velocity because the observer is external to the unit cell. So, the entire effective mass multiplied by the overall velocity of the unit cell and the overall velocity of the unit cell will be capital V. So, this will be the expression.

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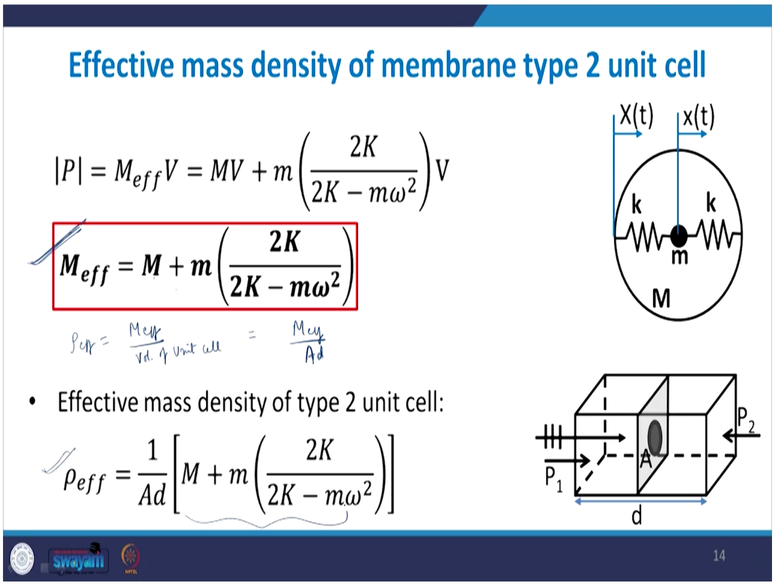

So, ultimately what we get is effective mass of the unit cell can be given by capital M plus small m times of 2 k by 2 k minus m omega square. So, this is a complicated expression that we finally obtain for the effective mass of this unit cell. So, what will be the effective density? You simply divide the total expression by the volume of the unit cell. So, mass by the volume will give you the effective density.

So, effective density becomes so, rho effective is the mass effective by the volume of the unit cell which is M effective by A into d. So, rho effective is going to be M effective by A d. So, you divide both these ends by A d. So, what you get is rho effective becomes 1 by A d times this whole expression.

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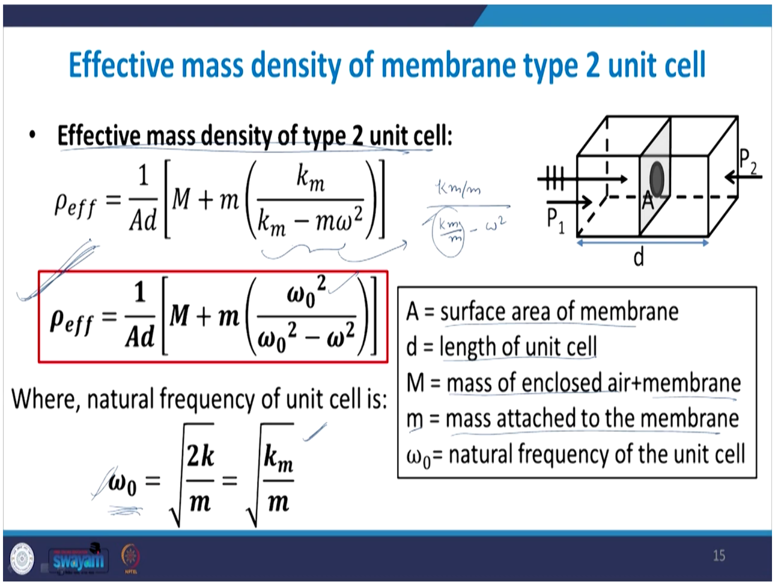

So, we have got an expression for the effective mass density of the type 2 unit cell. So, here A is the surface area of the membrane, d is the length of the unit cell, capital M means the mass of the enclosed air plus the membrane. So, both air and membrane together what is their mass and small m is the mass attached to the membrane.

Now, if we take this expression here and let us see we replace this we divide both top end and bottom end by capital M so, by small m. So, what we get is if we divide this by small m, so, this is m by m. So, we are dividing both these ends by we are dividing both them by small m. So, what we get is this can be written as this expression can be written as k m by m k m by m minus omega square.

So, we are dividing both numerator and denominator by small m and this is what we replace it as under root of k m by m is what we call as the natural frequency of the unit cell. So, here, the natural frequency of the unit cell is under root of the stiffness of the membrane divided by the central mass.

So, only central mass is taken into account. So, this particular expression is what we call as the natural angular frequency of the unit cell. So, if we replace this value here, so, this becomes omega naught square this also becomes omega naught square and this becomes omega square.

So, first you divide by small m on both numerator and denominator in this expression and then replace it with this value. So, this is the ultimate form you are getting for effective mass density.

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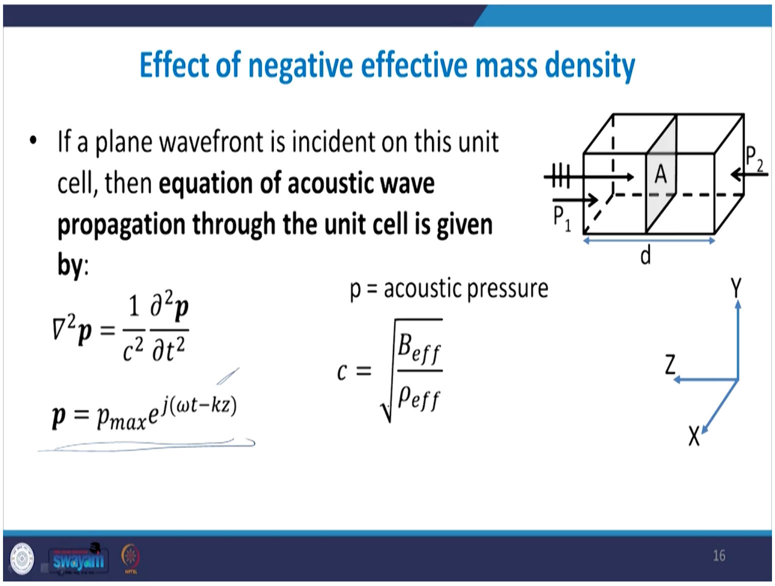

Now, we know that when a plane wave front is incident then the it is given by this equation here each of the p max into e to the power minus j omega t minus k times of z and c is equal to b by rho.

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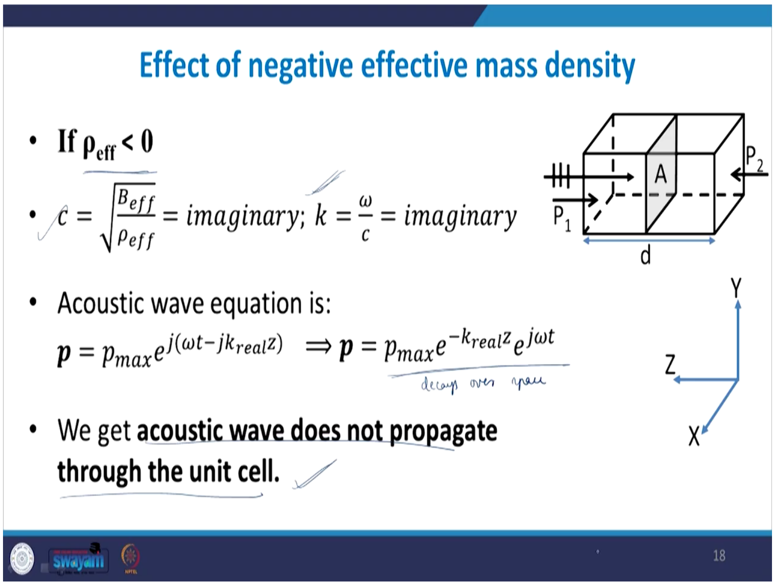

So, whenever the rho effective is positive which means c which is under root of b by rho will be real propagation will be real and we will get a plane propagating wave and similarly when rho effective is negative we will get an imaginary c an imaginary k and we will get a decaying wave. So, a wave that decays over space. So, it is not fluctuate it is not sinusoidally varying with space, it only sinusoidally varies with time and quickly exponentially decays over space. So, which means that it is not it is a non propagating wave over space.

So, whenever we have rho effective less than 0, the acoustic wave will not propagate through the unit cell. So, this is the same thing the same principle again and again. We are just finding what is the region where this effective mass density becomes less than 0 because it is in that region that the propagation vector will be imaginary and there will be no waves flowing through the unit cell or propagating through the unit cell.

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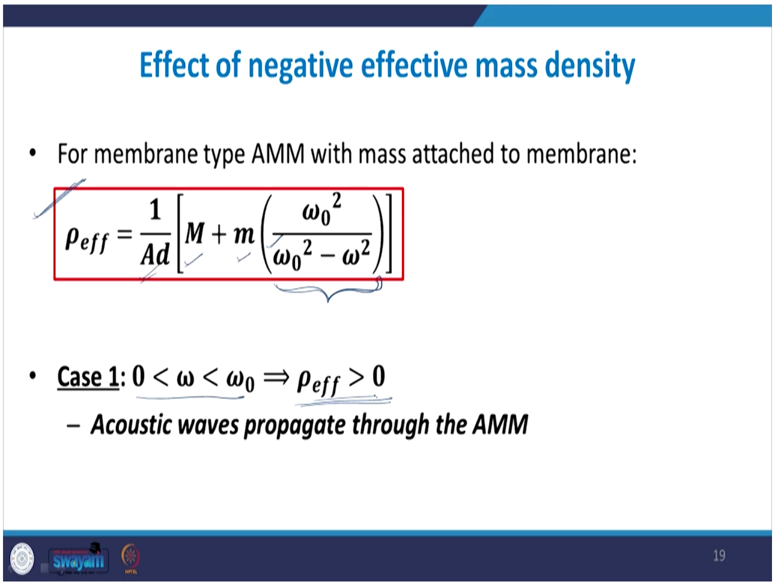

So, this was an expression for rho effective. So, if you see here, this is positive this is positive and this is positive. So, when this becomes positive that means, rho effective is going to be positive. So, if everything is positive, so, whenever omega is smaller than omega naught; so, which means this quantity is going to be positive here the denominator. So, the overall thing will always be positive. So, in this first case whenever omega is 0 to omega naught we have positive density which means that acoustic waves they will propagate. Let us consider a case 2.

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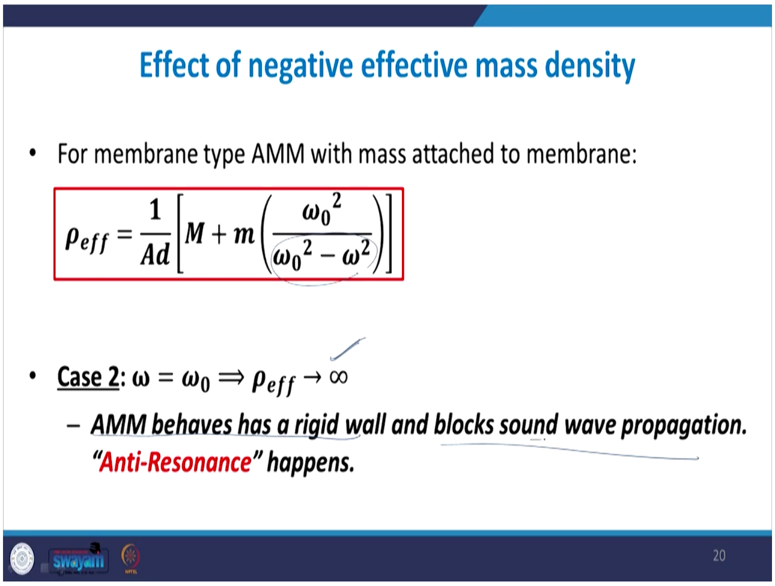

In the case 2, so, till omega to omega naught waves propagate through the AMM, but as soon as omega tends to or approaches omega naught in that case this quantity becomes 0. So, the overall rho effective tends to infinity because this expression tends to infinity. So, what happens? As soon as the target frequency reaches the natural frequency of the system, so, here instead of getting a resonance we are getting an anti resonance, the overall the mass density is becoming a almost infinity. So, which means that the AMM behaves as a rigid wall and then it blocks the sound wave propagation.

So, we consider 2 cases till now. First when the target frequency is less than the natural frequency in that case the density is positive and the waves they can propagate through the metamaterial but as soon as the target frequency approaches a natural frequency you suddenly have an anti resonance and you will have a sharp dip in the transmission loss which means that so, they a sharp rise in the transmission loss which means that the waves will suddenly stop propagating at the natural frequency.

So, we will continue our discussion on this type 2 in our next lecture.

Thank you.