Acoustic Materials and Metamaterials

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Lecture – 31
Membrane Type Acoustic Metamaterials-4

Hello and welcome to lecture 31 in the series on Acoustic Materials and Metamaterials. So, we are discussing here about Membrane Type Acoustic Metamaterials and this is the lecture number 4 on this particular type of acoustic metamaterial. So, in the previous lecture we studied about what is the effective mass density of a unit cell, where we only have a stretched membrane and then we studied about what is the effective mass density of a unit cell of such metamaterial, where you have a stretched membrane with some center mass attached to it.

So, today we will continue our discussion on that and we will discuss about the expression for the effective mass density and then the region where this density becomes negative and what is its effect, which will be followed by, what is the response characteristics of such kind of membrane type acoustic metamaterials. So, let us begin our discussion here.

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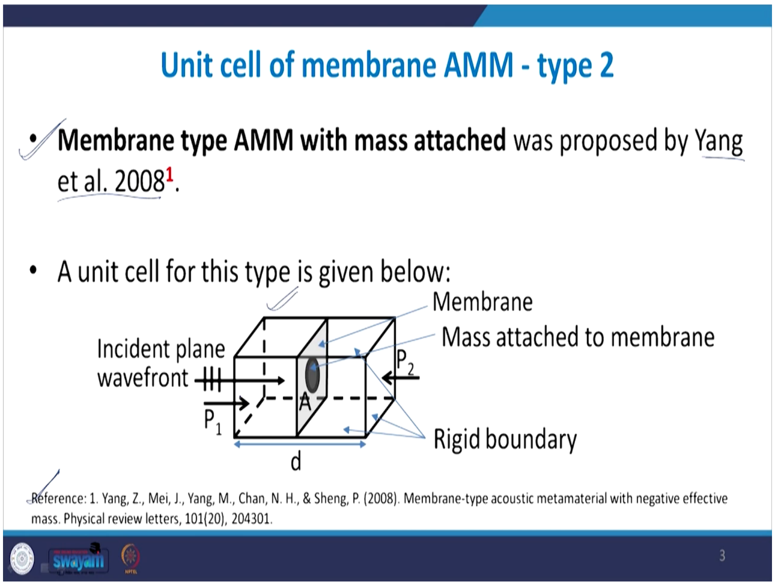

So, to quickly review this type of membrane type acoustic metamaterials, where you have a stretched membrane and there is a mass attached on top of it and everything is clamped inside a wave guide. So, this is the unit cell for this and it was proposed by Yang et al 2008 the difference is given here. So, this is the unit cell that was proposed, you have a sub wavelength wave guide, then you have stretched membrane with a mass attached which is clamped inside this and because this unit cell they will usually be connected in series and it will be a part of a long wave guide.

So, whatever wave front is incident when it passes through the long wave guide, it becomes a harmonic plane wave. So, only plane wave propagation because of the; because of the condition imposed by a wave guide only plane wave propagation takes place. So, harmonic plane wave front is incident on this unit cell. So, in the previous lecture we derived what is the effective mass density for this type of unit cell and the expression is given by this.

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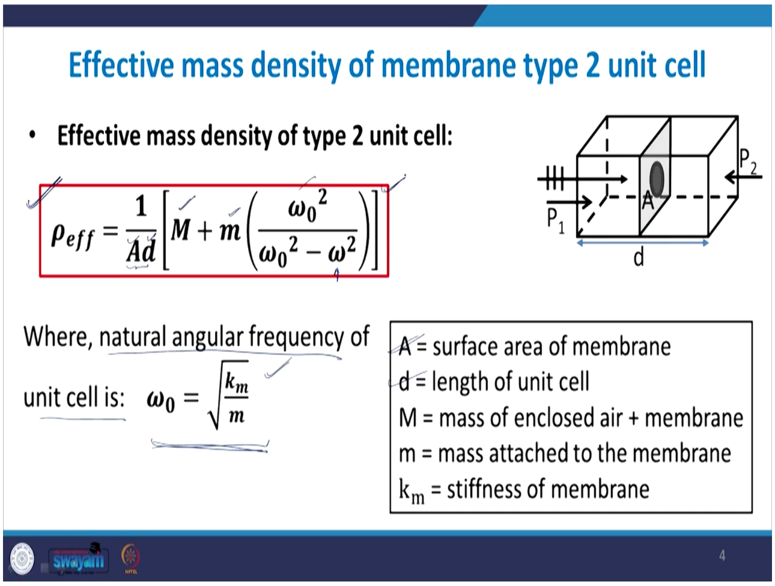

So, if you see here rho effective is this particular expression here. So, here in the here this is 1 by A into d; here A is the surface area of the membrane, d is the length of the unit cell. So, A actually is the surface area of the membrane when the membrane is not being transferred when the membrane is in equilibrium, it has not undergone any transverse displacement.

So, under the equilibrium condition the area of the membrane will be same as the area of the unit cell or the cross sectional area of the wave guide. So, this is cross sectional area of the unit cell multiplied by the length of the unit cell. So, this becomes 1 by the volume of the unit cell this quantity. And in the inside we have capital M which is the total mass of the membrane plus the enclosed air and then small m which is the small mass attached to the membrane. So, although the mass is small the value of the mass will be bigger because its a dense material that is attached to the membrane and multiplied by omega naught square by omega naught square minus omega square.

So, here omega is the incident frequency, omega naught is the natural angular frequency of this unit cell and we found that in this was under root of km by m. Where it is km is the stiffness of the membrane and the stiffness of the membrane depends on the tension applied to the membrane. So, this is the stiffness of the membrane by the mass of by the mass that is attached to the membrane.

So, as you see here, the natural frequency in this case is independent of the mass of the membrane itself because the membrane is usually a very light, it is a light thin pliable material and on top of it you have attached some dense mass. So, the overall natural frequency will be governed by the dense mass, because the membrane mass will be quite negligible compared to that. So, this was the expression for the natural angular frequency and this is the expression for the effective mass density of this unit cell.

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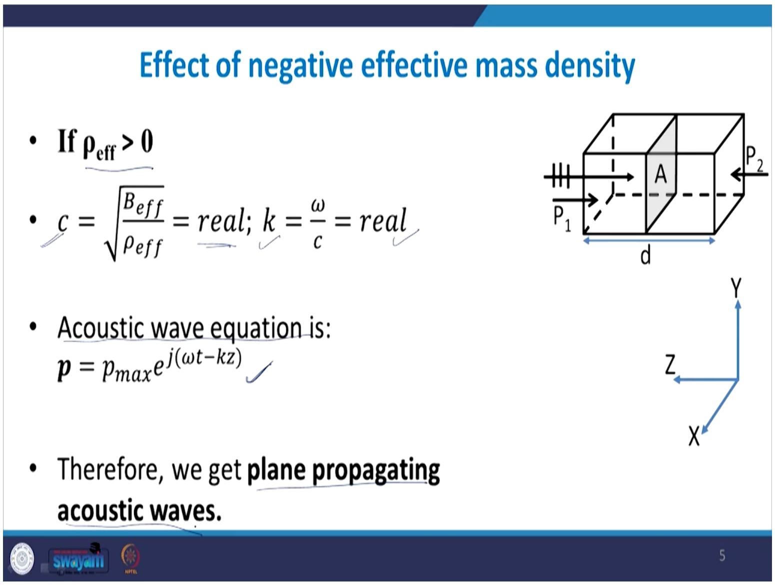

So, let us explore what happens when this density becomes negative. So, this concept has also been discussed in the lectures on acoustic metamaterials, where we discussed what happens when either the bulk modulus or the density becomes negative. So, you can refer back to those lectures also, but I am briefly discussing the effect here. So, when density is positive, then the c which is the speed of the acoustic wave will be B by rho which will be a real quantity.

So, it is square root of some positive quantity and similarly k which is omega by c will also be a real quantity and the acoustic wave equation will be a equation of a harmonic plane traveling wave or a harmonic plane propagating wave something of this form; p max into e to the power j omega t minus k times of z where z is the direction of wave propagation. So, here I have taken z because z is a direction in which the wave is propagating and x and y is the is a plane of the membrane.

So, we get a plane propagating acoustic wave whenever we have a positive density. So, when the incident wave is such that the density of the medium is positive, then the waves will propagate through the acoustic metamaterial.

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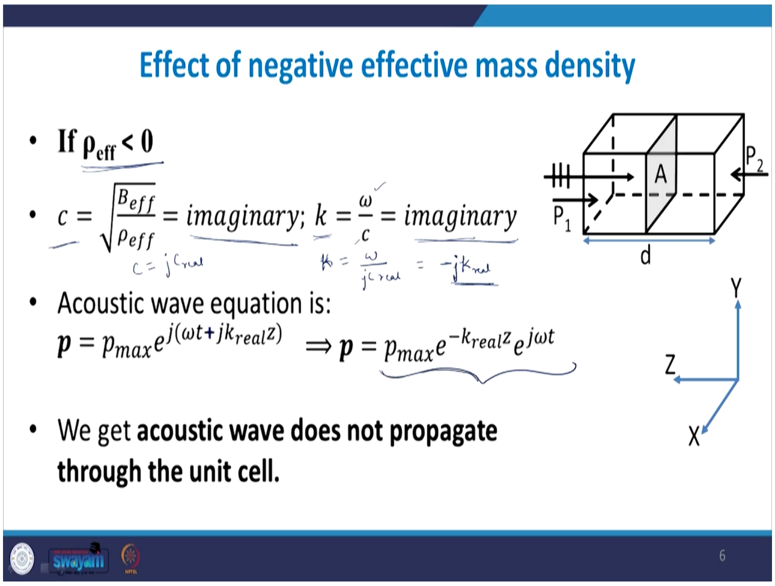

But when this density at certain frequencies becomes less than 0 then c which is equal to under root of B by rho. So, it will be under root of some negative quantities. So, this will be imaginary; k which is omega by c will also be an imaginary quantity because this is real this is imaginary. And we have solved this equation before what happens when rho or B becomes negative, c becomes some j times of c real and k becomes omega by j times of c real so, it becomes minus k times of minus j times of k real.

So, it is minus sum minus j into a real number. So, this is the value of k. So, when you put this value into this previous equation, which was the equation for a plane propagating wave, you end up with a quantity which is something like this. So, over here you end up with a quantity which looks something like this here. So, here this is plus because this was minus k z. So, minus k becomes plus j k real. So, this becomes the overall equation.

So, what you observe is that the equation that you are getting is not the equation of a propagating wave, it is the equation of a decaying wave and the sound wave or an acoustic wave is defined as a propagating wave, when the when the fluctuations or the disturbance in the medium propagates through the space, then it reach then the sound reaches from point a to point b and the listener can hear it. But here the wave decays down, it does not propagate through the material.

So, we do not get a acoustic wave propagation through the unit cell. So, this is the effect of negative mass density. So, let us now explore this expression and find what is the region where negative effective mass density occurs.

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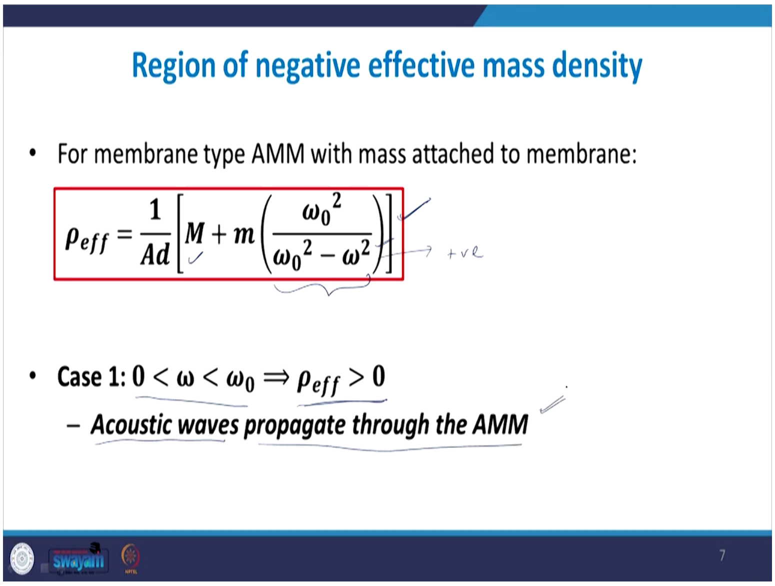

So, this is our equation rho effective. So, as you see here this quantity is positive this all of this is positive. So, this value will become negative only when this quantity here becomes negative. So, whenever omega is less than omega naught, then this denominator will be positive here. So, what we get? Denominator is positive. So, the overall quantity is going to be positive, positive plus positive will always be positive.

So, in this particular range effective density is greater than 0 and because it is greater than 0. So, we get. So, the acoustic waves will propagate through the acoustic metamaterial.

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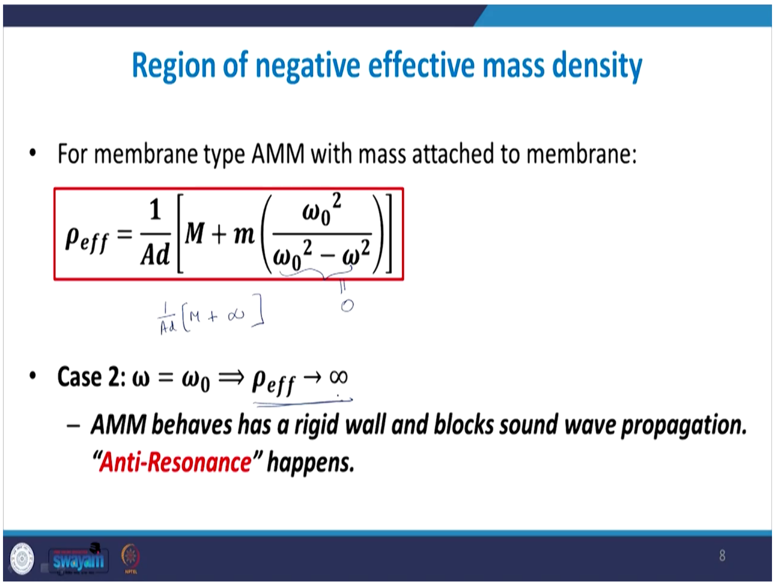

Now, let us see the case 2. So, in case 2 what happens? So, now, we discussed what happens still omega is less than omega naught. Now what will happen if omega becomes equals to omega naught? So, it becomes equals to omega naught, this denominator will then become 0.

So, when denominator becomes a 0, this becomes some quantity, this becomes 1 upon Ad into M plus some infinite value. So, this becomes 0 the overall quantity tends to infinity. So, rho effective tends to infinity. So, what do you mean by the effective mass density becoming infinity? It means that we now have the material now behaves as a very dense rigid wall and it simply blocks the sound and does not allow the waves to pass through.

So, as you see here when the incident frequency is becoming equal to the natural frequency of the cell, then instead of a resonance we get an anti-resonance. In the resonance you will have resonance leads to lead resonance is a phenomenon when a material offers minimum resistance to the flow of sound waves and hence you get large amplitude waves propagating through, but here its the other way around here the material is offering maximum resistance to the flow of sound and it is blocking the sound wave propagation. So, this is an anti resonance which happens here ok.

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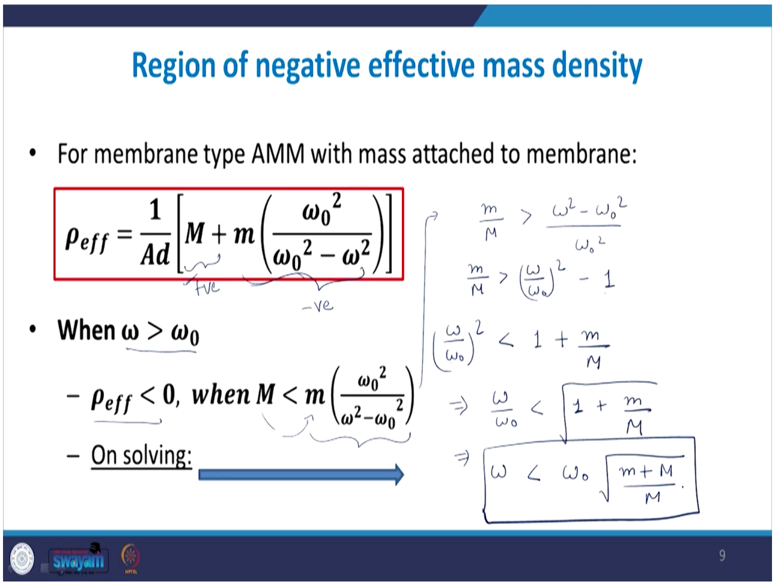

Now, what happens when omega becomes greater than omega naught? So, when omega will become greater than omega naught, this quantity will become negative, but this quantity will remain positive. So, the overall rho effective will be positive only when the magnitude of this positive quantity is going to be; is going to be smaller than the magnitude of the negative quantity.

For example, let us say the first quantity was 4 and the other was minus of 5; only then when the magnitude of this quantity and magnitude of the negative quantity are such that the magnitude of positive quantity is smaller than the magnitude of the negative quantity then the sum will be some negative quantity. So, this was just an example to show you what is meant by how will the rho effective be negative.

So, as you see here. So, when this is the case, overall rho effective will be negative when the magnitude of M which is m and the is smaller than the magnitude of this negative quantity and the magnitude of the negative quantity is going to be minus of that quantity. So, it will become this one. So, M should be smaller than small m omega naught square by omega square minus omega naught square. So, let us solve this to get what should be the value. So, if you solve this here.

So, what you get is you can take this m to the other end. So, what you get is small m by capital M should be you are taking this m here over here is going to be greater than and you take this quantity to the other end. So, you would it becomes omega square minus omega naught square divided by omega naught square. So, this is what you get if you solve this inequality, which is which means m by capital M should be omega by omega naught whole square minus 1.

Solving this further what we get is omega by omega naught whole square should be smaller than 1 plus. So, I have just taken this minus 1 here. So, it becomes m plus capital M plus 1 which is going to be greater than omega by omega naught whole square. So, let us solve it one more time. So, what we get is omega by omega naught is smaller than this value, all of it is positive. So, we take the square root. So, ultimately what we get is omega should be smaller than omega naught times and the root of m plus capital M by M.

So, this is the value which we get. So, whenever the omega satisfies this inequality, that it is greater than omega naught, but less than this quantity here, then we will get a negative density.

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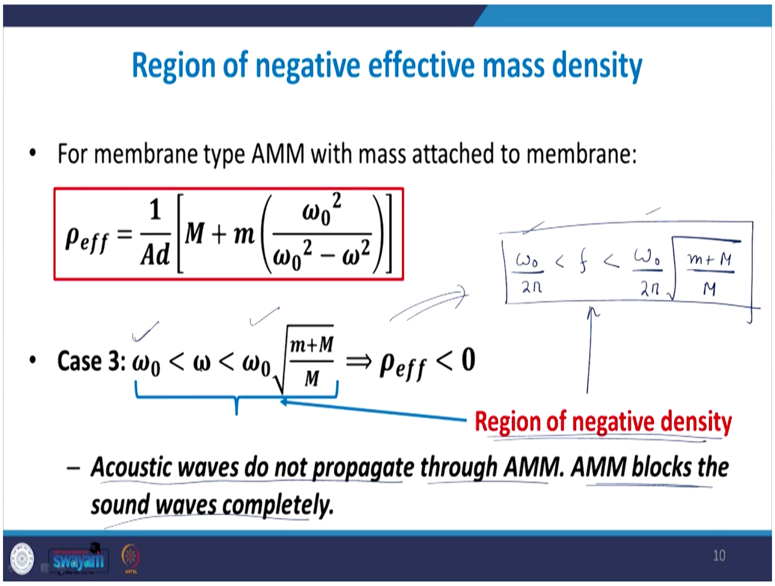

So, to reiterate this is the region, this is the range within which the incident frequency has to lie for the material to behave as a negative density material. So, this is what we call as the region of negative density from omega naught to omega naught under root m plus capital M by small m and we can write this in terms of linear frequency as f naught should be f should be smaller than. So, just writing this again here, this f should be smaller than omega naught by 2 pi because f is equal to omega by 2 pi and this will be omega naught by 2 pi under root m plus capital M by M.

So, the this is the region of negative density. So, where the angular frequency lies within this range or the linear frequency lies within this range in that case, the material behaves as a negative density material. And when the density becomes negative then we do not get any propagation. So, what happens, acoustic waves they do not propagate through the acoustic metamaterial.

So, within this region the sound gets blocked or the material behaves as a traditional it behaves as a perfect barrier material. Now let us study the case 4, what happens when omega becomes equal to omega naught under root of m plus capital M by capital M by capital M?

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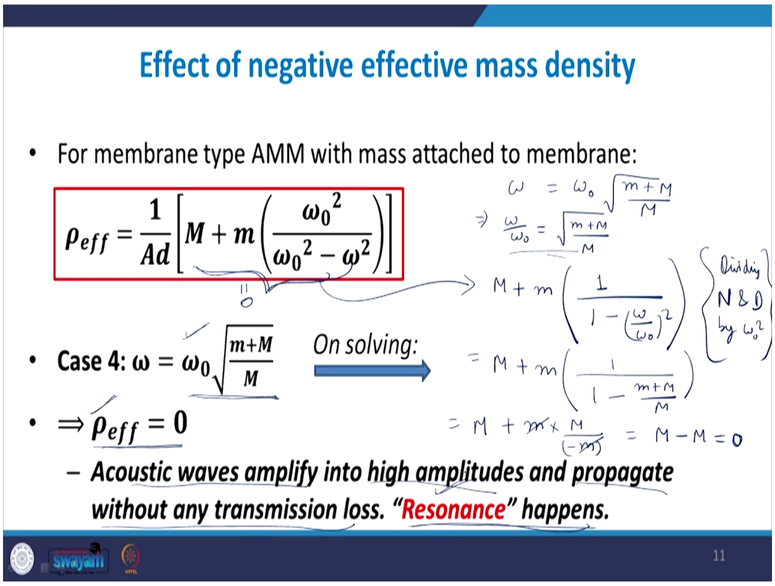

So, what do you get is that, in that case if you solve what you will get here is. So, I am solving here. So, omega equals to omega naught under root of small m plus capital M by M. So, if you put this values of which means that omega by omega naught let us rewrite this equation here. So, what you get is, this is the equality.

So, if we put this value here. So, this implies that omega by omega naught I am putting as this quantity. And what is this expression inside? This expression inside let us evaluate this expression inside. This is what? It is M plus small m and if you divide the numerator and denominator by omega naught square what we get is, 1 upon 1 minus omega by omega naught whole square.

Here we have dividing the numerator and denominator by omega naught square, this is the expression we end up with. So, let us put this value of omega by omega naught from this expression here. So, what we get is, M plus small m into 1 upon 1 minus this quantity here. If you solve further what you will get is m plus M into this will become this will become a multiplied by M divided by this m minus m plus M minus m. So, it will become minus of m. So, what you get is M minus M which is equal to 0. So, once you put this equality and you solve you get a 0 value.

So, this expression becomes this expression becomes equals to 0. So, when omega reaches this value what we get is rho effective becomes 0. So, here we get the resonance. So, here instead of getting the resonance at natural frequency, we are getting a resonance at this particular value here. So, now, the material behaves as if there is no density. So, which means that its almost like a air medium or no effective density and therefore, it offers no resistance and what we get is that any response. So, a negative density will mean that whatever be the even the smallest amount of force can excite the material and the material will accelerate.

So, which means that the particles in the material can accelerate even at the smallest excitation. So, here the wave suddenly amplify into large amplitudes and they propagate without any transmission loss. So, this is what happens resonance happens and heavy transmission takes place at this particular frequency. Now let us see the last case.

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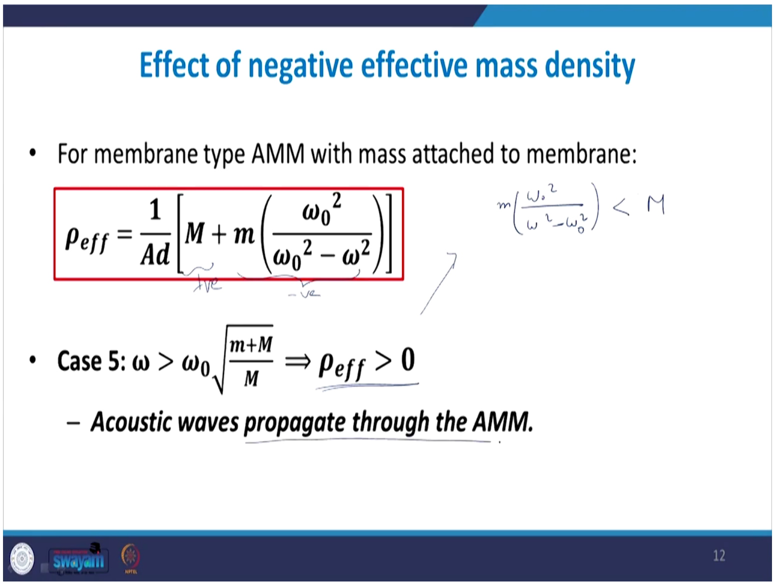

So, the last case is when omega becomes greater than omega naught into under root of m plus capital M by capital M. So, as we had studied before, we already studied the other inequality when omega was less than this and we found that in that case rho effective is negative. In the same way when omega is greater than omega naught of this, which means that this magnitude this will be smaller than this positive quantity here.

So, this will simply mean that m times of omega naught square by omega naught square minus omega square mod if you take should be or let us just erase this and write the mod by it simply means that this will. So, the mod of this value would be smaller than the mod of this value. So, overall it will be a larger positive quantity plus a smaller negative quantity. So, we will get some smaller positive quantity. So, rho effective will come out to be positive and the waves will propagate through the AMM.

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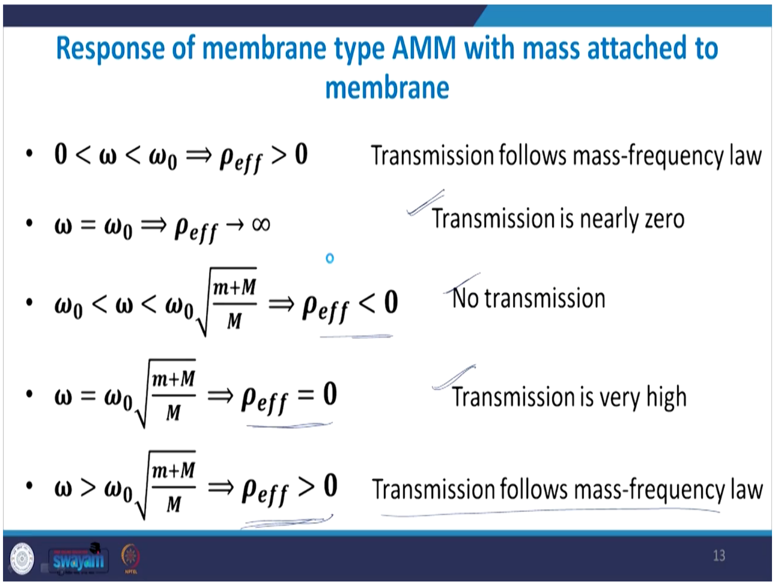

So, let us summarize the results. So, what we get is when. So, 1 by 1 we are exploring all the values of the incident frequency and seeing what happens to the negative dense, what happens to the effective density. So, initially from 0 to starting from 0 till the critical frequency omega naught, the density is positive and the transmission that takes place to the material, it follows a traditional mass frequency law. But suddenly when omega tends to omega naught.

So, in near about omega naught and equal to omega naught, the density becomes infinity anti resonance happens and the transmission is nearly 0. So, this is the region where the mass frequency law is broken and then carrying it forward from omega to omega naught to omega naught under root of m plus capital M by M, in this region density remains negative and again no transmission takes place the mass frequency law is broken. And then when it reaches this value suddenly you have a resonance and density becomes 0 and the transmission will be heavy.

Again this does not follow the traditional mass frequency law, but finally, once omega becomes much greater than this quantity, rho effective is greater than 0 and then the transmission takes place according to the mass frequency law. So, this is summing up.

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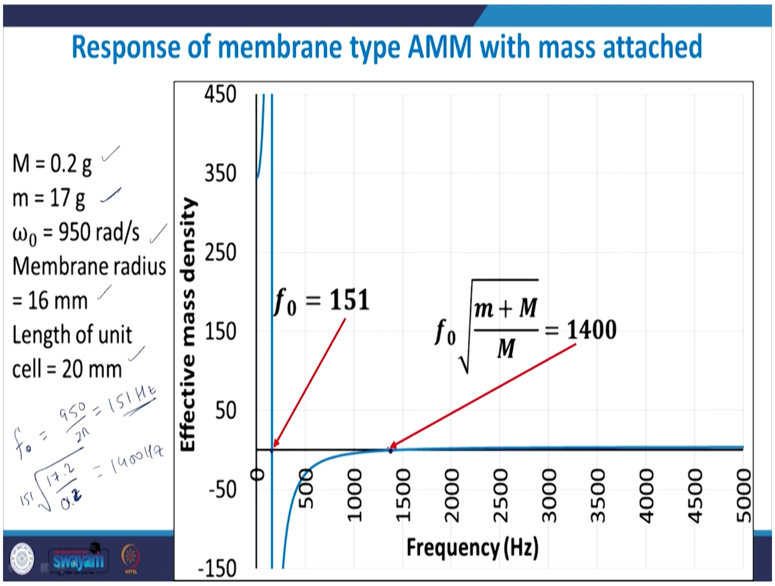

So, let us do an exercise here. So, I have plotted how the rho effective varies with the variation in the frequency. So, the parameters I have taken is that, I took a membrane its a light membrane whose mass is 0.2 grams and the smaller mass the smaller dense mass which is attached on top of the membrane, it is 17 grams. And the unit cell is such that omega naught is equal to 950 radians per second and omega naught we know is under root of km by m.

So, the stiffness of the membrane is the stiffness of the membrane or the tension in the membrane is maintained in such a way that we get the natural frequency as 950 radians per second. So, 950 radians per second means what will be its linear frequency? It will be 950 by 2 pi which comes out to be approximately 151 Hertz.

So, this is the frequency where you the which corresponds to the natural frequency of the unit cell and the radius I have taken a 16 millimeters and length of the unit cell I have taken as 20 millimeters. So, if you use this expression here, we know the radius is 16 millimeters. So, we can find out the area which will be pi into r square, similarly we know the length of the unit cell which is being used which is 20 millimeters. So, A value you can find out d value you can find out you know capital M value, you know small m value and you also know the value of omega naught.

So, you know all these constants in this expression, then you can find out how will this rho effective, you can calculate the rho effective as a function of omega. So, this was an experiment I did and this is what I have found. So, this frequency this is the pattern of the effective density as a variation of frequency f. So, what you see here is that, it initially starts with some high value a positive value and suddenly the value of effective density reaches infinity and once it reaches infinity after that it becomes negative and then beyond certain point it becomes positive, but the positive value is small.

So, if you go by this here. So, what you will see is that initially we have a positive effective density. Both this expression and this expression are positive so, we have a large positive quantity then suddenly tends to infinity at omega naught and beyond which between this region, it remains negative and suddenly when it becomes equals to 0 at this quantity and after 0 what you will see here is that, after 0 once it crosses this value.

So, you have one positive quantity minus some negative positive quantity with some negative quantity subtracted from it. So, overall value will become a smaller value. So, we will have some positive value minus some negative minus another value. So, the two value are being subtracted, but the magnitude of this is small smaller than the positive one. So, slowly it will increase. So, initially the value will be higher because you have two positive quantities being added together, but then later on the value will decrease because omega is increasing.

So, anyways this value is decreasing. So, overall some quantity and some subtraction will give you a smaller value a smaller positive value. So, this will be the trend. So, it starts from a high positive value reaches infinity, then suddenly becomes negative and after that it becomes 0 and then carry smaller into a smaller positive value and that is the trend that we observe here. So, we can see from theory also and it is clear from the graph also that because I have not drawn the graph here in the fifth full resolution, but when I tried it.

So, in the graph this exact point corresponded to 150 something. So, it was 151 which is the natural frequency and this point corresponds somewhere around 1400 and if you calculate this 151 under root of 17.2 by 17 sorry 0.2. So, putting this m and capital M value, you will get 1400 Hertz and that is what this is corresponding to.

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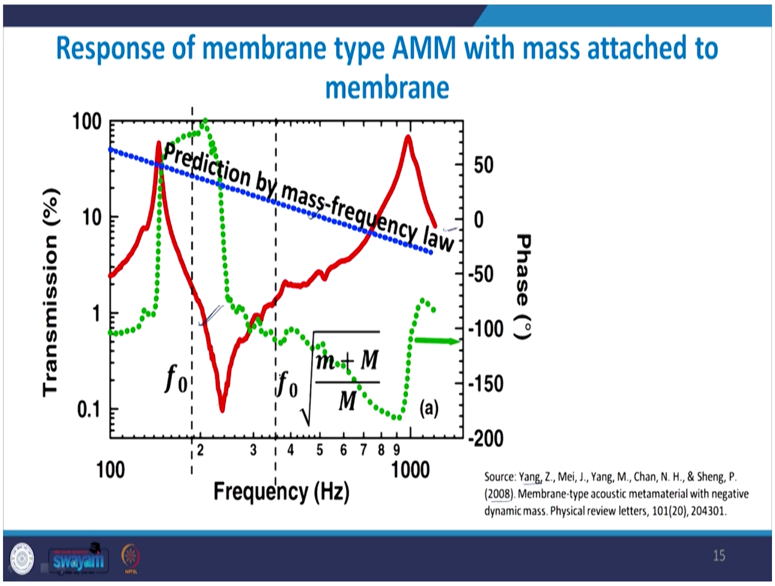

Now, I will show you some of the graphs from the actual experiments that were done. So, one experiment was carried out by Yang et al 2008 who proposed this metamaterial. So, with there a graph is shown. So, in this particular graph what this saw is that this blue line predicts. So, this is a graph between transmission versus frequency and what is transmission as percentage, it is the it is 100 multiplied by the transmission coefficient. So, what percentage?.

So, it means what percentage of the incident wave is getting transmitted. So, higher the transmission means the material is not good, its not blocking the sound, its not a good barrier material and the lesser of the transmission which means that the material is a good barrier material. So, here this is the prediction by mass frequency law, we know that by mass frequency law as the frequency increases the noise reduction increases or the or there is more transmission loss.

So, in other words as the frequency increases, the transmission reduces. So, there is a linear relationship which is given by this blue, but the actual. So, this is what the mass frequency law predicts, but the actual transmission coefficient as a transmission has been found to follow this red line here, this is the transmission. So, what you see is that it has two peaks and one sharp dip.

And this sharp dip is the region where the density becomes negative and that is why when the density is negative the propagation will stop. So, there will be a heavy dip in transmission. So, this is the region where suddenly the mass frequency log its broken.

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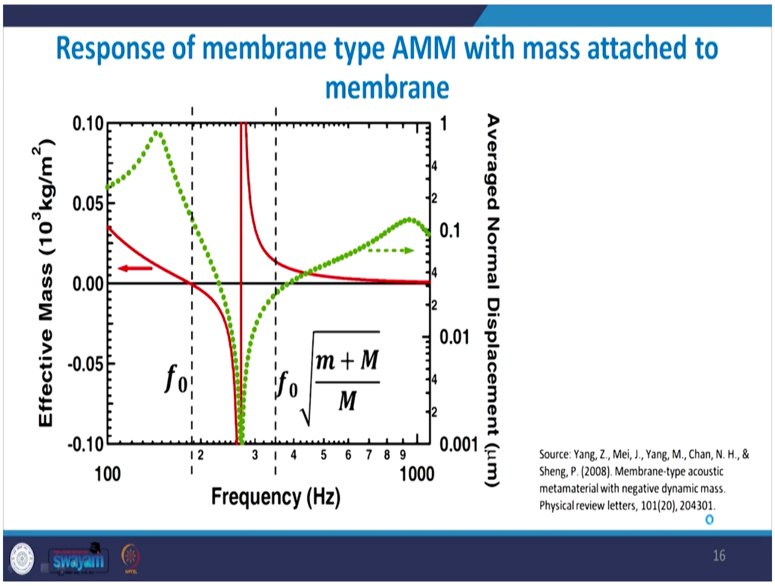

So, this is for the same material this is the graph of the effective density. So, as you can see the same point where you obtained a heavy dip in transmission, it corresponds to the point where you have the region of negative density.

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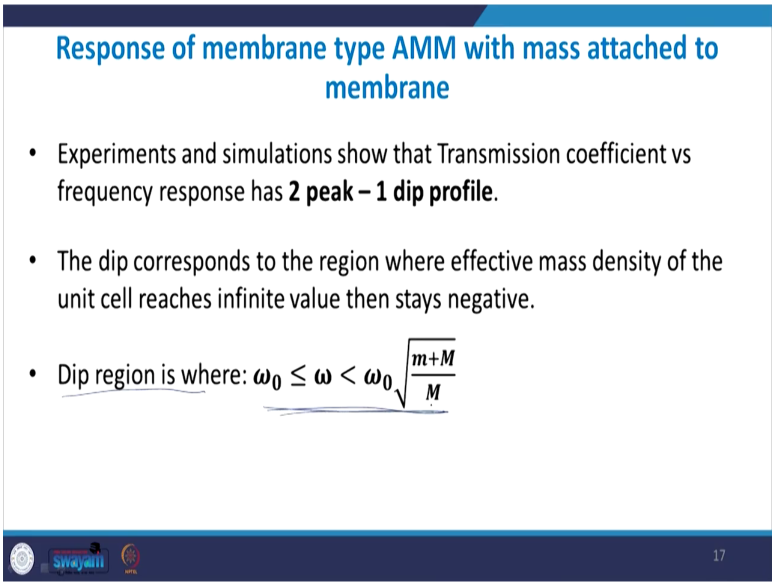

So, to summarize from both the experiments and simulations so, what we see is that, when we use this kind of metamaterial, then the transmission versus frequency response is actually a two peak one dip profile and the dip corresponds to the region where the effective density becomes negative. So, the dip region is where this is being satisfied.

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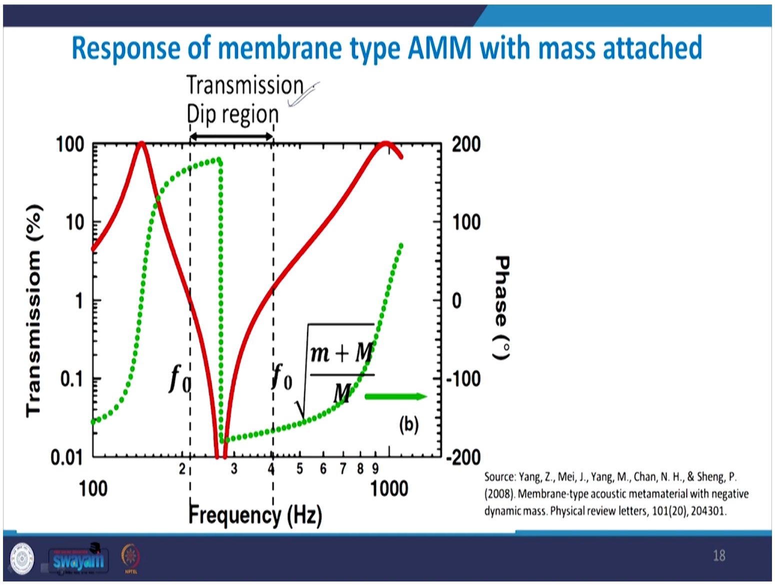

So, the same graph this becomes the transmission dip region, which is this is the transmission dip region.

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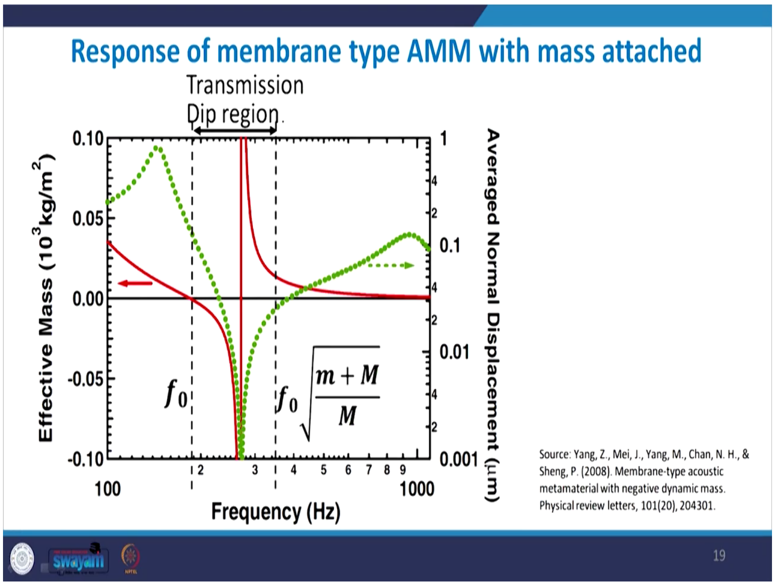

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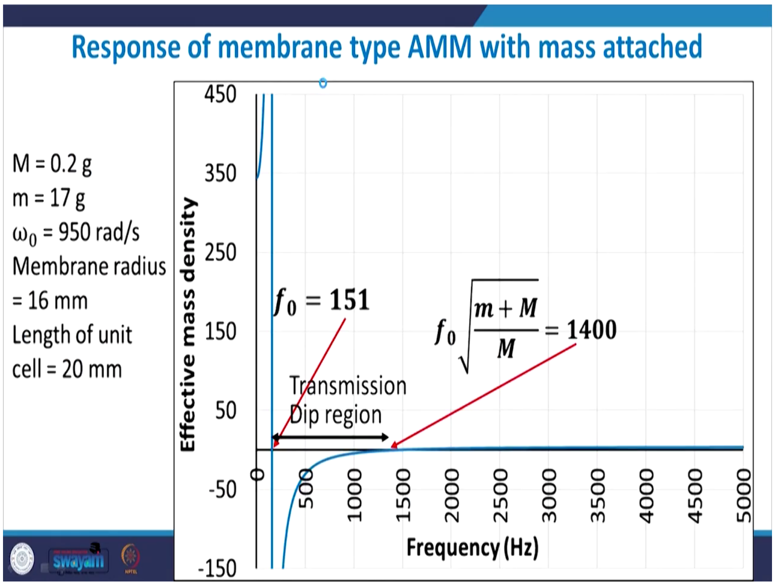

And in r graph also this region where the negative density occurs is the region of transmission dip ok.

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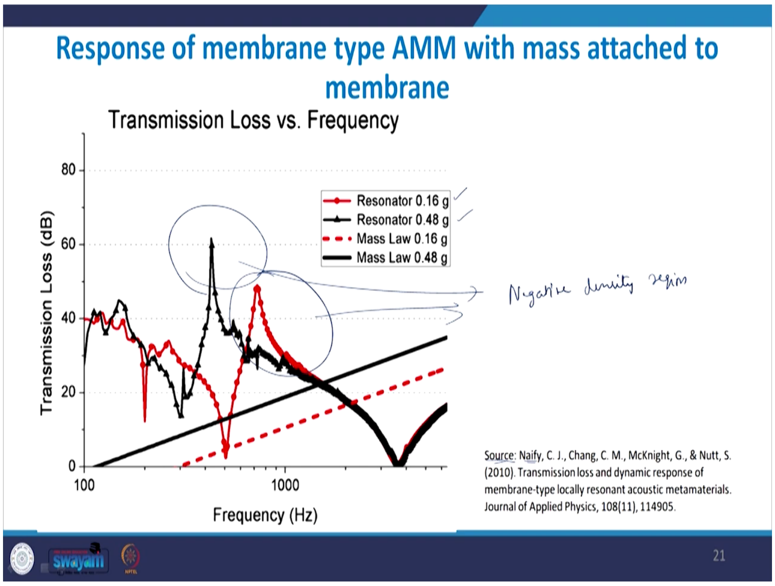

So, again this is a comparison. So, this was a this is a quite newer paper by Naify et al 2010. So, here some of the resonators were compared. So, same kind of metamaterial was prepared, but the mass that was attached to the membrane was varied. So, you have one in one case you have 0.16 grams and in the other case you have 0.48 grams and what you see is that that the profile remains the same. So, just like you had two peak one dip profile. So, if you do the transmission loss it will follow the inverse relationship because it is inverse to the transmission coefficient.

So, here you get is two peaks. So, you one two dips in one peak sort of. So, here the peak corresponds to the region of negative densities. So, I highlighted here. This is the region of negative density, similarly in this black graph the region of negative density is this region. So, in both these regions suddenly you see that you have a sharp peak. So, this one is one region similarly you get one other region like this and so on.

So, this is what you observe and erase this does not correspond to it, this is these other two sharp peaks that you get and both of these correspond to negative density region. So, the frequencies where density become negative and as you can see you will have a heavy transmission loss even at low frequencies and it has a. So, such materials they have been again tested and it has been found that the first low frequency transmission peak.

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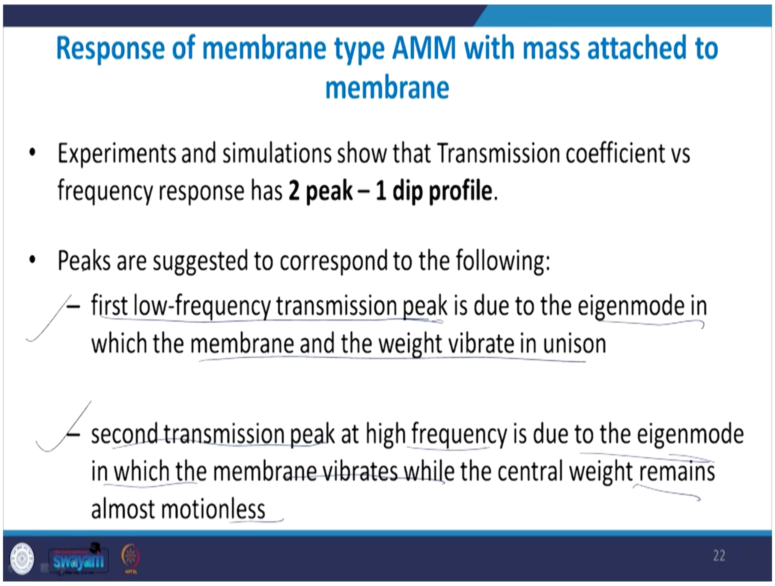

So, transmission versus frequency is a two peak one dip profile and the first peak is due to the eigenmode where membrane and the weight attached to it the both oxalate in unison, they are coupled together. And the second one corresponds to the eigenmodes where the membrane is vibrating, but the mass remains motionless. So, now, with this I would like to conclude this lecture on the membrane type acoustic metamaterials.

So, we have studied the two types of unit cells and what is the region when the density becomes negative and how does it affect the response of the overall material.

Thank you for listening.