Acoustic Materials and Metamaterials
Prof. Sneha Singh
Department of Mechanical and Industrial Engineering
Indian Institute of Technology, Roorkee

Lecture – 33
Tutorial on Membrane Type AMM

Hello and welcome to lecture number 33 in this series on Acoustic Materials and Metamaterials. So, today is the last lecture on a Membrane Type Acoustic Metamaterials and this is a tutorial session. So, we will solve a few problems related to the two types of unit cells that we have studied, so that you can get a more better understanding of how to design such kind of membrane type metamaterials.

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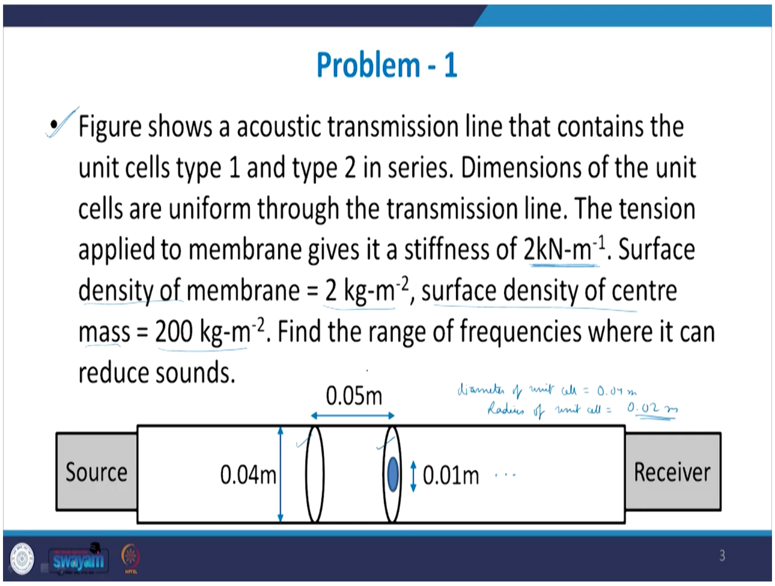

So, the first numerical if you see here, the problem that is given to us is; you have a figure is shown here and you have an acoustic transmission line, which contains two types of unit cells.

It has a unit cell type 1 which is given here and the unit cell type 2 that is given here and they are connected in series. And the dimensions of the unit cells are uniform throughout the transmission line.

So, you, also the way it is given is that, so this is like a continuation. So, you have first unit cell 1, unit cell 2, then unit cell 1, then another unit cell 2. So, alternately these two types of units cells they are connected together into a long acoustic transmission line or an acoustic wave guide. So, over here the things that are given to us is that, the tension that is applied to the membrane it gives it stiffness and that is given by this quantity. So, it is 2 k Newton's per meter or 2000 Newton's per meter. The surface density of the membrane is given as 2 kgs per meter, square surface density of the center mass is given as 200 kg per meter square.

So, you have to find out here, what is the range of frequencies within which it can act to control noise or it can act to reduce the sound? So, we will start with this problem here. Now we know that for both unit cells the range of frequencies where they will reduce the sound is actually the range where the effective rho; that is the effective mass density becomes negative, at is that range within which suddenly the wave propagation stops, because the propagation vector becomes imaginary. So, let us first deal with unit cell type 1.

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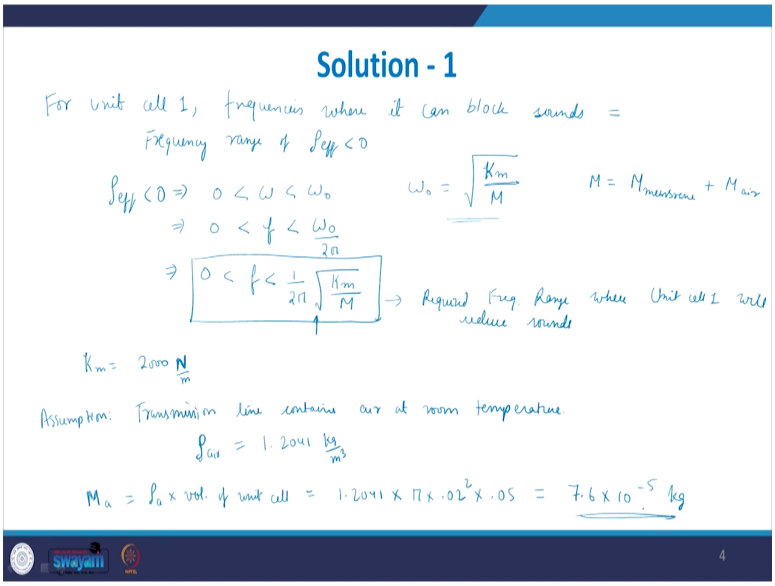

So, for unit cell 1, the frequencies where it can block sounds is equal to the frequency range of rho effective being negative. So, this and we know that rho effective is negative. So, last class, in the last few lectures we have studied that, for the first type where there is no mass attached; then the rho effective is negative where the angular frequency is between omega to omega naught, where omega naught is the natural frequency of the unit cell that is given by the stiffness. It is given by the, you can say the effective stiffness of the membrane; although membrane is not very stiff, but due to the tension applied it gives some stiffness to the membrane. So, this K m divided by capital M; where capital M is the mass of the membrane plus the mass of the enclosed air. So, this is given to us. So, let us find out what is this range.

So, if you do this, then what will be the range in the frequency scale? In the linear frequency scale the range will be 0 to omega naught by 2 pi. So, the range that we are finding out is between 0 to omega naught is going to be this value here. So, we get 1 upon 2 pi times under root of K m by capital M. So, this is the range of frequencies that we have to find. This is the required frequency range, where unit cell 1 will reduce sounds, will reduce or control sound. So, let us find out this value here this value. So, K m is given to us as 2000 Newton's per meter, ok. And capital M, let us find out what is capital M. So, let us find out first what is the mass of the membrane?

So, the mass of the and the mass of the air; now air here you can see that, the condition for loading is not given. So, we assume it is being loaded with. So, that is the general assumption that the transmission line contains air as the medium at room temperature. So, at room temperature what happens is that. So, at room temperature what you get is the rho of air.

So, these are the values which are easily [avail/available] available to you, you can look up into the book. So, you look books or standard tables even online, so the density and the speed of sound value. So, the rho and the c values of air at different temperatures are already pre calculated and available at various sources. So, rho of air at room temperature is 1.2041 kg per meter cube. So, this is the value which we will be using.

So, let us now calculate what is the mass of air? It is going to be this rho of air multiplied by the volume of the unit cell, which is going to be 1.2041 multiplied by. So, here you have area into the length of the unit cell. So, what is the area here it is this is the diameter. So, diameter d is equal to, the diameter of unit cell is given to us as 0.04. So, radius becomes half of this which is 0.02 meters, right. And the length is 0.05. So, let us use this value here. So, what you get is diameter pi. So, the area becomes pi r square which is 0.02 whole square multiplied by the length which is 0.05.

So, everything is in SI unit. So, the value that we find is 7.6 into 10 to the power minus 5 kg's. So, that is the first value.

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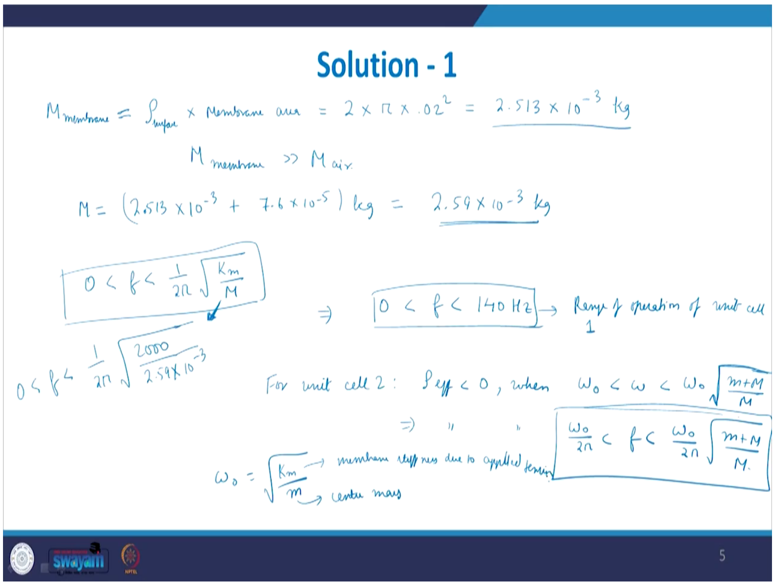

Let us find the second value which is the mass of the membrane. And the mass of the membrane again you have the surface density is given to you. So, surface density of the membrane is what is the total mass per unit area of that particular membrane? So, you can take the total mass will be surface density. So, this is the surface density multiplied by the membrane area.

So, this we can write. So, the surface density given to us is 2 kg's per meter square multiplied by the area; everything I am writing in SI units and the area is the same as the area of the unit cell. So, the total mass that you get is going to be 2.513 into 10 to the power minus 3 kg's. So, as you can see here, mass of membrane is much greater than mass of air. In general mass of membrane is greater, and sometimes you can approximate the total mass enclosed into just being the mass of membrane. So, but in this case let us take both the mass we added together.

So, the total mass then M becomes 2.513 this quantity in kg's. So, the total mass that we are getting over here is 2.59 into 10 to the power minus 3 kg's.

So, this is the mass here. So, let us now find out what is that value for the frequency. So, the frequency was the range that we have to find is between this to this, that is the required range. So, 1 upon 2 pi under root of 2000 this is this value is that what we are calculating now is 2000 divided by 2.59. So, frequency lies between 0 to this particular value. So, everything I am putting in SI units. So, the frequency range we end up with is 0 and closed to 140 hertz. So, this is the range of operation of unit cell 1.

Now for unit cell 2; this is the cell where you have if you look back into the question. So, this is the type 2 where you have membrane with some dense mass attached on the top of membrane. So, in that case, the rho effective becomes 0. So, for unit cell 2 rho effective becomes 0, when this omega lies between omega naught to omega naught under root of this expression here. So, we have already derived these expressions, we have already learned in the previous lectures what is the region of the negative density; so this is the region of the negative density here. So, you can now calculate this value.

So, in this case this implies that, rho effective becomes 0 when the frequency lies between omega naught by 2 pi to omega naught by 2 pi times under root of m plus capital M by M. So, that is the range we have to find and omega naught is given by under root of K m by small m, this is the center mass attached to the membrane and this is the membrane stiffness ; membrane stiffness due to tension, due to applied tension, ok.

So, let us calculate these two values to find out what is the frequency range for the second case. So, for the type 2, let us first calculate. So, we can write this equation here, this particular frequency..

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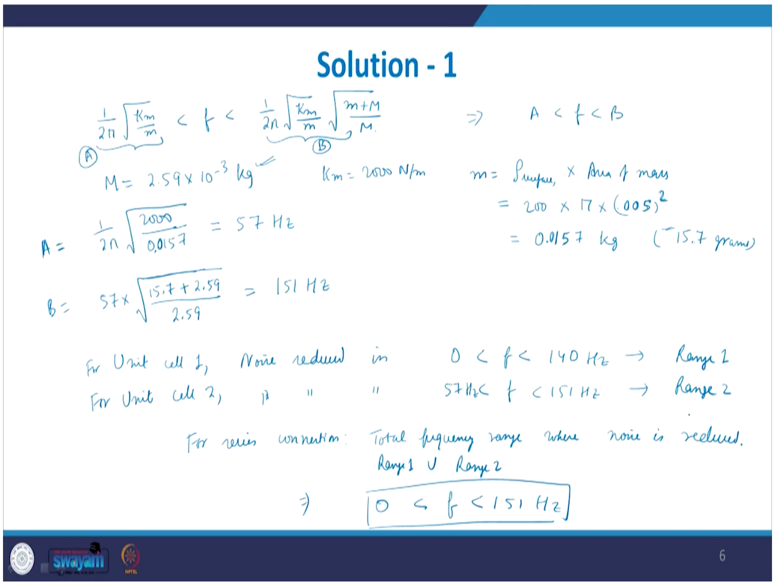

If you put the value of omega naught into this equation what we get is; the overall frequency should lie between this value here to this value by this.

Now, because of a unit cell is the same. So, the membrane is the same and the enclosed air volume is also the same. So, capital M is the same as the previous case which is this quantity; this we have already calculated in the previous case for type 1. So, that is capital M and K m is the same quantity which is 2000 Newton’s per meter. So, omega, so this first thing here; so let us call this as quantity A and this as quantity B. So, this implies frequency will lie between this quantity A and B; and A we calculate as 1 upon 2 pi under root of 2000 and the small m is what it is given to be. So, small m here the value is given to us.

So, here for the center mass some surface density is provided that is 200 kg’s per meter square and the diameter is given to you. So, diameter is given. So, how do you calculate the center mass? This becomes the surface density of the center mass multiplied by its area, area of that mass. So, what do you get is 200; everything in SI units 200 kg's per meter square multiplied by.

So, what you do is, you multiply it by the area and the diameter is 0.01. So, the radius will be 0.005. So, pi r square that will be it is area. So, the total mass then comes out to be 0.157 kg's or you can say 15.7 grams; but we will be using this SI unit kg for the calculation. So, we put the value of M here. So, the quantity A then becomes 0.157. So, it is 1 upon 2 pi under root of 2000 by point; sorry this a value comes out to be 0.0157 that is why it is 15.7 grams. So, this becomes 0.0157.

So, when you calculate this quantity what you get is; it is coming out to be somewhere approximately 57 hertz. And the value B you can calculate, it will be this quantity multiplied by this; so it will be 57 hertz multiplied by small m is going to be. Let us do it in, let us take both the numerator and denominator in grams and see; because that will make our calculations easier, we can reduce some powers. So, it will be 15.7 plus the value of capital M was given to was calculated as this. So, it is 2.59 grams, so we have taken 10 to the power minus 3 and removed it divided by 2.59.

So, what do you get this value becomes approximately when you calculate it becomes as 151 hertz. So, now, the range of so, for unit cell 1, noise reduced in between 0 to 140 hertz that we had calculated earlier and for unit cell 2 the noise is reduced between 57 hertz to 151 hertz. Now because they are connected in series; so for series connection, you will simply combine the two frequency ranges. The total frequency range where noise is reduced will be range 1 union range 2. So, this will give us from 0 to 151 hertz. Now this is the range 1 and this is the range 2.

So, to think of it in this way, you have one unit cell and then you have another and so on. So, when the sound passes through the first one; then 0 to 140 hertz are already reduced. Then it passes through the second one, then further reduction between 57 to 151 hertz. So, overall reduction will be a combination of all these ranges. So, this gives us the value which we were looking for. So, that is the answer.

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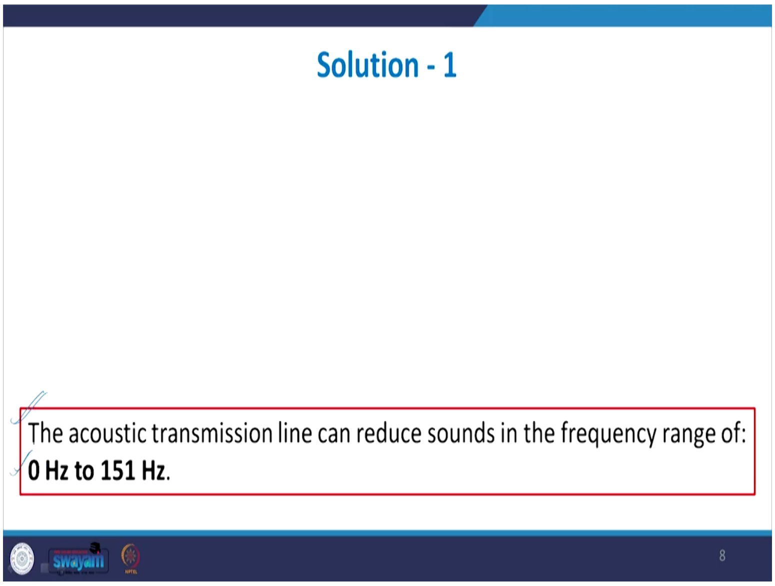

So, the acoustic transmission line can reduce sounds in the frequency range of 0 to 151 hertz, ok.

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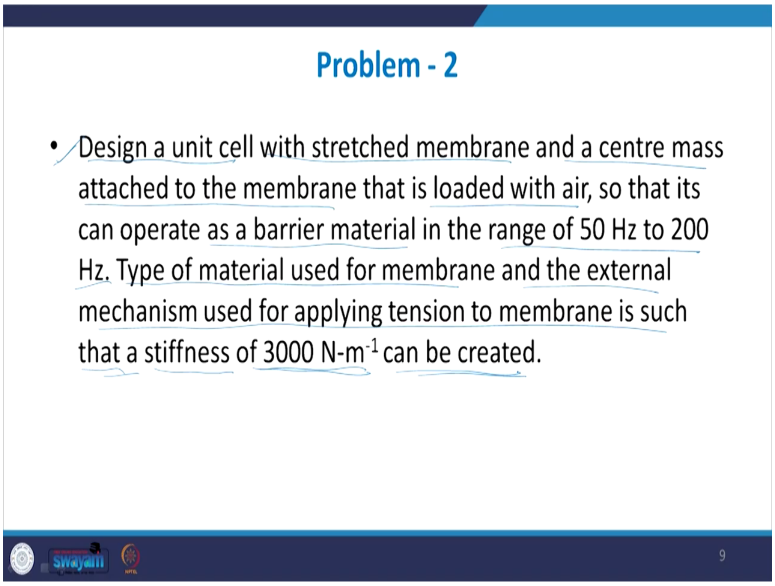

Let us study another problem that is problem 2. So, in this problem we are given design a unit cell. So, this is a design problem. So, you have, you already know what should be the range of operation, how should it reduce the sound; and then based on that you have to design the dimensions and the value of the masses etcetera. So, here what is given to us is; we have to design a unit cell with a stretched membrane and a center mass attached to the membrane. So, we have the unit cell type 2.

So, this is now loaded with air at room temperature, so that it can operate as a barrier material. So, we have to designed this particular unit cell type 2 loaded with air, so that this unit cell can act as a barrier material in the range of 50 to 200 hertz. So, we are designing it, so that it can act as a perfect barrier material and it can block the sounds between 50 to 200 hertz.

And the type of material used for membrane and the external mechanism used for applying tension to the membrane is such that; so based on what type of material you used and how much of the applied tension it can withstand. It is given that, a total stiffness of 3000 Newton's per meter can be created. So, this is a design constraint that is given that, you have to design, so that these stiffness do not exceed this value.

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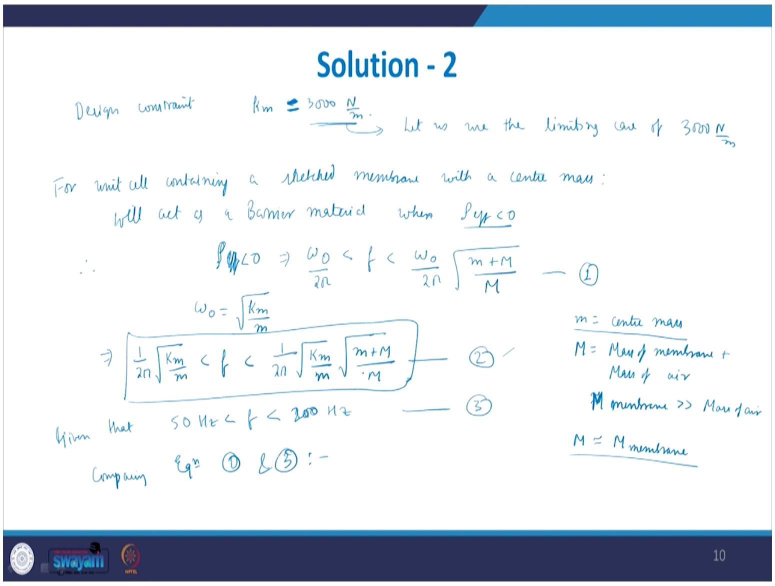

So, the stiffness, so the constraint here becomes, so here the design constraint let us say is that; the stiffness must be smaller than or equal to 3000 Newton's per meter. So, what we will now look for as a designer that, it can easily operate at 3000. So, if that is the last allowable stiffness, the nearest take that stiffness and because we know that the less stiffness you have then; because the range of the operation is directly proportional to omega which is under root of K m by the mass, so under root of stiffness by mass.

So, if you increase the stiffness, then mass has to be increased proportionately to keep that omega same. So, the more the stiffness you apply; sorry the more stiffness you apply if you want to get a bigger range with a smaller mass also. If you increase the k value, then the omega naught value will increase; because of the increase in the k value. However, if your k value is small, then in that case you will have to reduce the mass even further.

But anyways let us just to use the limiting case. So, let us use the limiting case of 3000 Newton's per meter. So, it is given that, this rather than this equality then; let us make this as equality. So, the design constraint is given that, this is the kind of stiffness which can be created by using an external mechanism to apply the tension. So, we are already given K m value.

So, now we have for type for unit cell containing a stretched to membrane with a center mass rho effective becomes less than 0, it acts. Let us just put it this way that, this particular unit cell with a stretched membrane and the center mass will act as a barrier material or it will block sounds when rho effective is less than 0. And therefore, we have to find.

So, therefore, the range in which this will be acting as a barrier material is when rho effective is 0 and rho effective is 0 implies the frequency lies somewhere between omega naught by 2 pi 2 omega naught by 2 pi under root of K m sorry under root of small m plus capital M by m. So, this is the range that we have to find. This is the range which is given to us, where m is the center mass and capital M is the mass of membrane plus the mass of air, which we can approximate. We know that, mass of membrane is much greater in magnitude than mass of air; so for ease of calculation here what we can do is we can simply approximate, this M as mass of the membrane itself.

So, m becomes the center mass small m and the capital M we are taking it as the mass of the membrane. So, let us now find out this range here that we have been given in equation 1. Now here omega naught is under root of K m by small m,. So, this is the range here. So, this ultimately you can reduce this equation even further and what you get is 1 upon 2 pi under root of K m by m small to 1 upon 2 pi under root of K m by small m. So, this becomes the entire expression.

So, this is the range of frequency of operation. So, let us give it as equation 2. Now it is given from equation 2. Now we know that from equation 2 it is given that, this range is actually between 50 hertz to 200 hertz. So, this is what is given to us and this is what is the analytical expression. So, if you comparing equation 2 and 3; if you compare the two equations, then what you get is.

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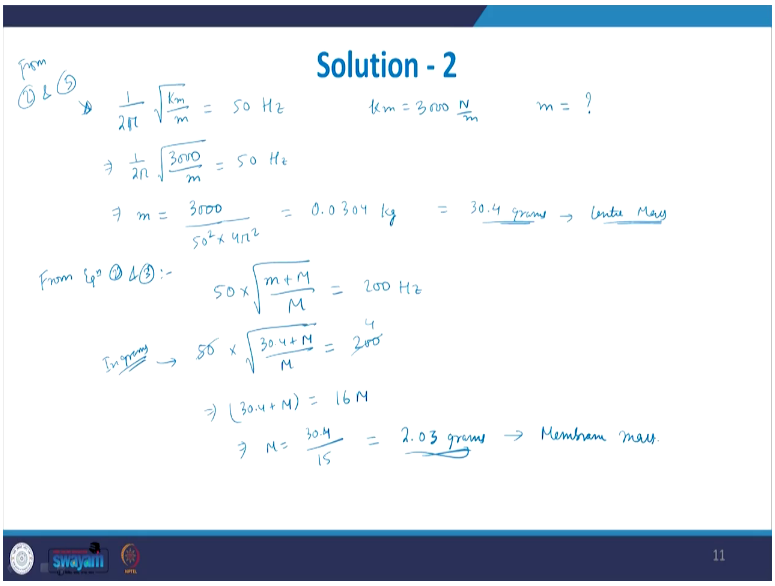

So, let us do it here. So, what you get is 1 upon 2 pi. So, from equation 2 and 3 this is what you get from equation 2 and 3; this value should be 50 hertz and we know that K m is 3000 Newton's per meter and small m is what we have to find.

So, when you put this value here what do you get is this. If you solve this equation, you squared this quantity up; this should be the answer, it is going to be this one from this particular equation. So, when you solve this and you put the values various values here, what you get is it comes out to be 0.0304 kg's. So, this is all in SI unit. So, which means around 30.4 grams is the is equal to the center mass. So, one design parameter we have obtained that is the center mass should be 30.4 grams.

Now, let us find again we compare from equation 2 and 3. So, let us compare the other end of the frequency. So, here this quantity should be equal to 200 hertz, if you see equation 2 and 3. So, this entire thing becomes 200 hertz and we already know that this is 50 hertz; 1 upon 2 pi under root of K m by m is 50 hertz. So, from 2 and 3; 50 multiplied by this quantity should give us 200 hertz, and we know m is equal to 0.0304. So, putting everything in grams; what do you get should. So, in grams, what we get is this thing plus the capital M divided by capital M should be 200. So, this comes out to be 4 here. So, we solve this equation further. So, what we get is this thing.

So, which means that M should come out to be 30.4; so here the M goes it becomes 15 M, so it is 30.4 divided by 15. So, this is what you should be getting. So, when you solve this, the answer that you get is 2.03 grams. So, this is the membrane mass,. So, that is the.

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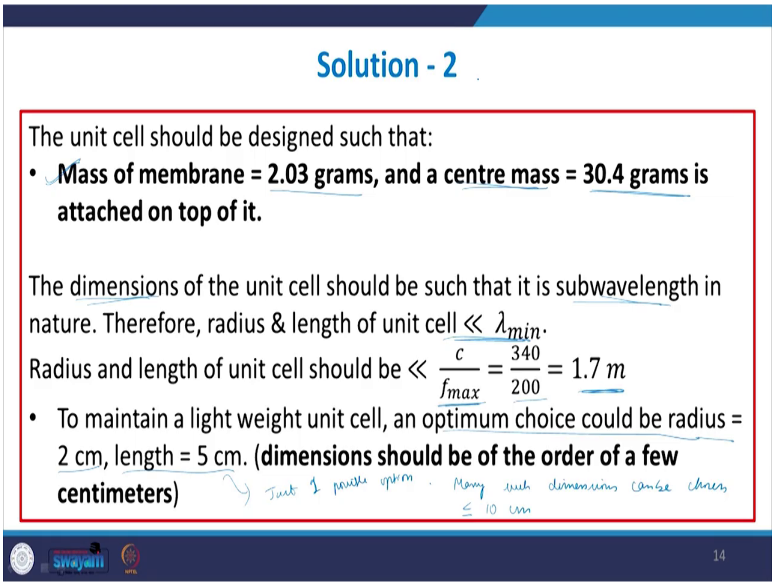

So, we have found the two design parameters which I am showing you here. So, the unit cell should be designed such that the mass of membrane is about 2.03 grams which we obtained and the center mass attached to it is about 30.4 grams, so that the frequency range of operation is 50 to 200 hertz..

Some other things to explore are that see the dimensions. So, here this is the value of the mass now; what should be the dimensions of the units cell ? There are innumerable options to make the dimensions of the unit cell; the only thing is that, the dimension should be sub wave length is in nature. So, whatever is our frequency range we are dealing with, so the minimum wavelength that we are dealing with; the dimension should be much smaller in magnitude, then the minimum wave length that it targets.

So, this is what is given here, that all the dimensions they should be sub wave length. So, smaller than much smaller than the lambda minimum, and the minimum lambda is when the frequency is the maximum. So, what is the maximum frequency of operation here; it is 200 hertz.

So, the lambda minimum becomes c divided by f max which comes out to be 1.7 meters. So, as you can see here, the minimum wave length we are dealing with; because we are it is a low frequency noise control, so sub wave length dimension is not that a problem. So, the minimum wave length that we get is 1.7 meters and the order of magnitude of the various dimensions of the unit cell would should be much smaller than this.

So, in general the practices to take the dimensions at least by 10 of lambda. So, if you do by 10 you get about one point about 17 centimeters and so on. So, let us take all the dimensions within 10 centimeters range. So, one choice, one good choice could be 2 centimeter as the radius and length as a 5 centimeter. So, this is just one option. So, this is just one possible option ; many such dimensions can be chosen with the constraint at which are less than equal to let us say 10 centimeters, something within 10 centimeter range should work fine. So, we have solved these two problems here and with this we would like I would like to end the lectures on membrane type acoustic metamaterials.

Thank you.