Acoustic Materials and Metamaterials

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Lecture - 34
Introduction to Sonic Crystals

Welcome to lecture number 34 in the series of Acoustic Materials and Metamaterials. So, this is an important lecture because today we will begin our discussion on a new type of acoustic metamaterial, which is called as the sonic crystals. In fact, sonic crystals are one of the first acoustic metamaterials to be discovered sorry invented and studied. So, they were invented and studied and it derives from the phenomenon of photonic crystals which were actually developed for electromagnetic waves.

So, from here onward and towards the last lectures we will be studying about sonic crystals and their mode of operation, then we will solve some problems related to that. So let us begin this lecture. So, the first lecture is just an Introduction to Sonic Crystals.

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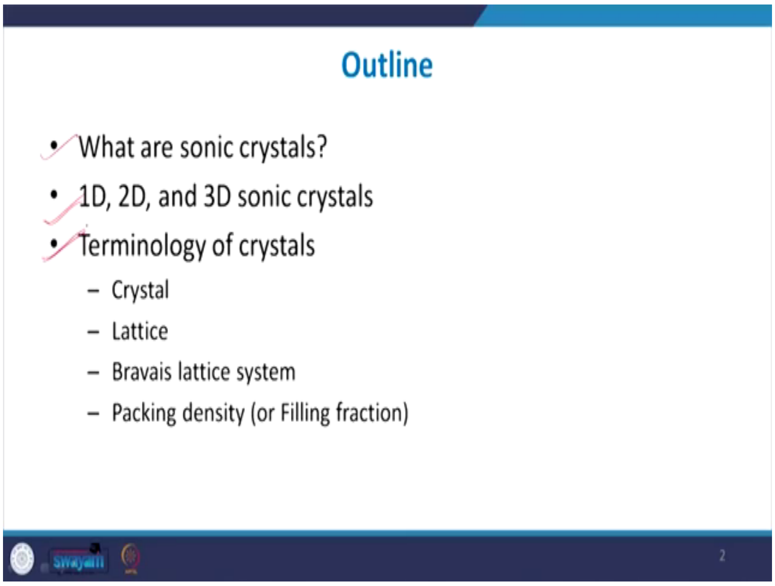

So, what we will study here is that, we will study what do you mean by sonic crystals. So, we will study what is meant by sonic crystals, then we will study the different types of 1 dimensional, 2 dimensional and 3 dimensional sonic crystals and then before we go and delve into how what is the fundamental physics behind the operation of sonic crystals, we need to gain some knowledge about some background concepts on crystal the crystallography or simply the crystals or artificial crystals and then some knowledge also on band gap formation and local resonance.

So, in this lecture we will begin with some of the background knowledge on crystals. So, that we will. So, this is what will do in this particular lecture here.

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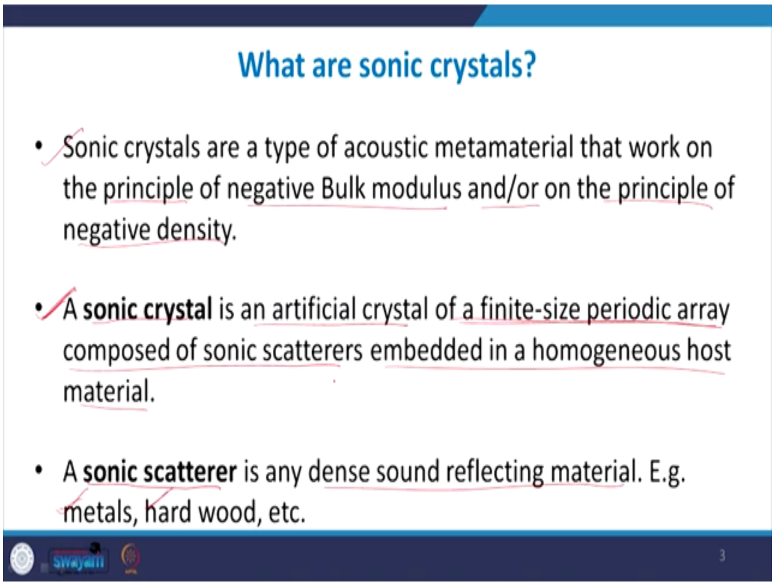

So, first of all what are sonic crystals? So, the way they are defined is that in the previous lectures when I had introduced to you what is acoustic metamaterials, I told you that there are various kinds of acoustic metamaterials and they are primarily classified on the basis of how they operate. So, you can have a material with a negative density. So, it becomes a negative density acoustic metamaterial and one example of that was the membrane type acoustic metamaterials which we had been discussing in the last 5 lectures.

And then a an acoustic metamaterial, it can also manipulate sound if it has a negative bulk modulus. So, either it is a negative b or a negative rho both of them they render the speed of sound to be imaginary. So, the propagation vector comes out to be imaginary and the region of frequencies or the range of frequencies within which either b is 0 sorry either b is less than 0 or either rho is less than 0.

So, in that range of frequencies always where rho effective becomes where rho effective becomes 0 or b effective becomes less than 0. So, whenever either of the two quantities become negative, then in that case what we get is we get in an imaginary propagation vector. So, there is no plane propagating wave. So, the propagation stops at these frequencies.

So, sonic crystals are one example where b becomes less than 0 and sometimes some sonic crystals can also be double negative metamaterial. So, this is another type of acoustic metamaterial where both b and rho are 0 sorry both b and rho are less than 0. So, when both b and rho together become less than 0 in that case what we get is, we get a very sharp bending of sound or we get a negative refractive index. So, that is yet another property. So, the sonic crystals by the definition you will see that here in these crystals either sometimes b becomes less than 0 or in some kind of cases both b and rho becomes less than 0.

So, they can act when b is less than when one of the quantities is less than 0, then it can act as a sound attenuating material. So, it can block the sound waves [when]ever whenever either one of the two parameter becomes less than 0. But when simultaneously both the parameters become less than 0, then in that case the same sonic crystals can act as strong they can act as bending the sound waves, they can bend the sound waves sharply. So, let us see here. So, what I have discussed here is that, sonic crystals they are a type of acoustic metamaterials, they work on the principle of either negative bulk modulus and or the principle of negative density.

So, you will have you will encounter both types of examples, one where either b is less than 0 or the other example where both b and rhos together are less than 0. So, now, how do we define this sonic crystal? As you can see the name has crystal word in it and sonic. So, it is some kind of crystal used for sound manipulation.

So, the weight is defined is that, it is an artificial crystal of a finite size periodic array. So, it has a finite size periodic array which is composed of sonic scatterers embedded in a homogeneous host material. So, you will have a homogeneous host material, which in general is a fluid medium and within which you have some sonic scatterers placed in a periodic arrangement.

And what do you mean by a sonic scatterer? So, any material which can reflect the sounds completely. So, ideally complete reflection is not possible, but there are materials which can at least reflect most of the sounds more than 0.9 percent could be the reflection coefficient. So, when you have such hard materials which can reflect the sounds such kind of materials, they act as scatterers for sound. So, here in this definition the sonic scatterer is any dense sound reflecting material. So, it can be metals, hardwoods etcetera.

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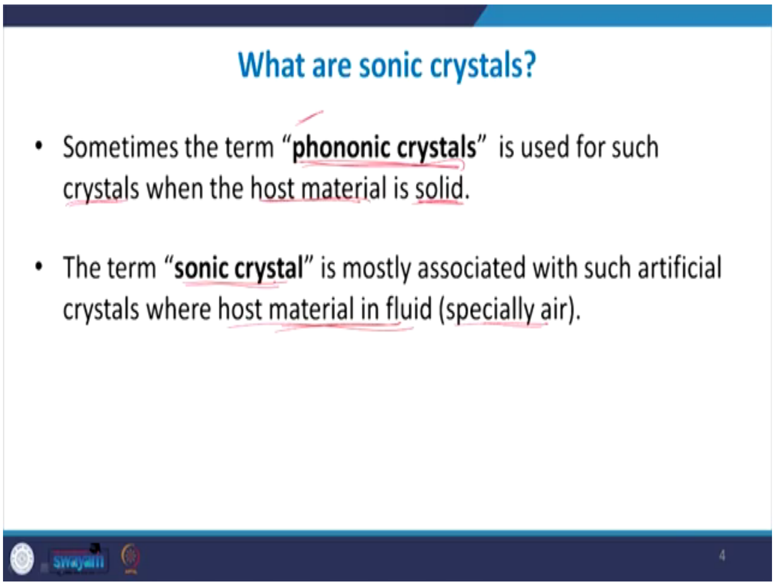

Sometimes the term phononic crystal is also used for sonic crystals. So, I will make a distinction here. So, because this is a booming field of research and difference and different group of scientists across different parts of the world they have come up with different terminology.

So, suppose a certain group of scientist in a in one particular part of the world tends to follow sonic crystals terminology, some other group tends to follow phononic crystals terminology. To just make the difference between the two clear the phononic crystals which is another term used sometime synonymously with sonic crystals is actually when these crystals. So, the same crystals the definition which is given here that is an artificial crystal of finite size periodic array, composed of sonic scatterers embedded in some homogeneous host materials.

So, you have some homogeneous medium and you have sonic scatterers placed in a finite size periodic array. When the host material for this becomes solid we use the term as phononic crystals, when the host material for this becomes air or fluid we use the term as sonic crystals.

So, in most of our discussion. So, in this particular course most of the discussion will be limited to sonic crystals, that is when we have sonic scatterers or the scatterers of sound placed within some fluid medium in general which will be air. Now, based on the direction of periodicity these sonic crystals can be classified as 1 dimensional, 2 dimensional and 3 dimensional.

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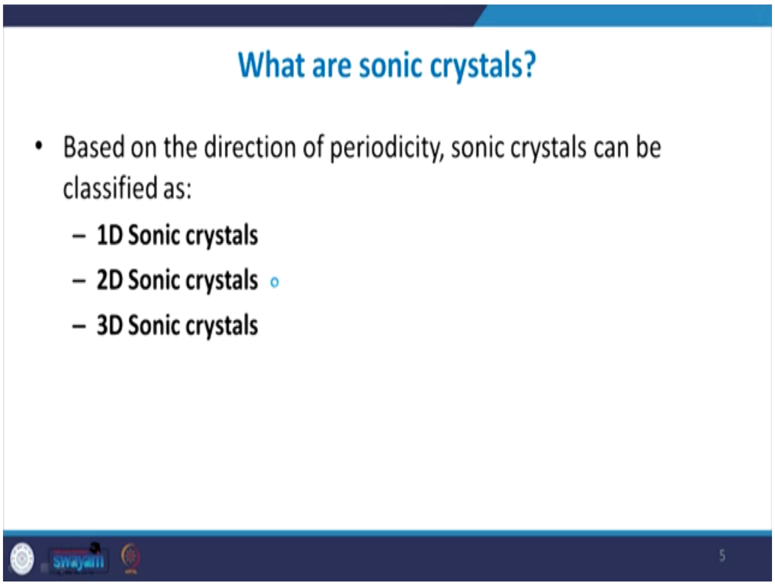

So, let us see some figures which will make you understand what is meant by a sonic crystal and what could be the various directions of periodicity.

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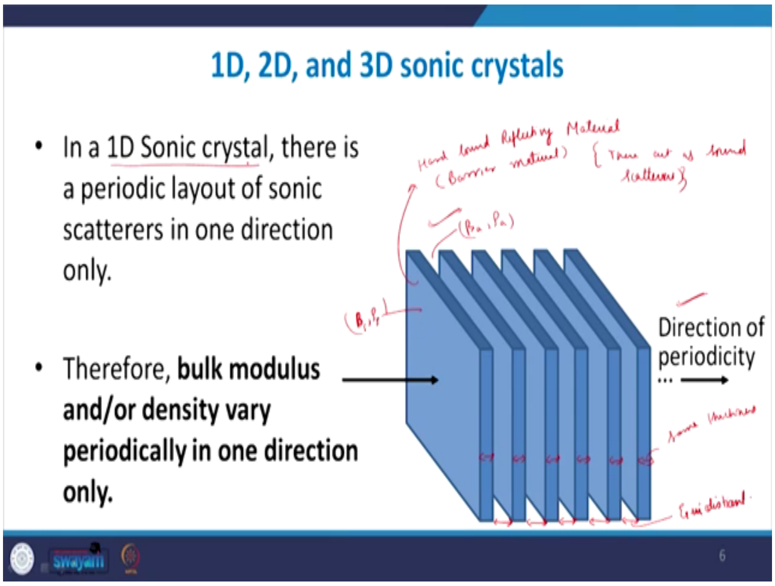

So, here the first kind of sonic crystals can be created let us say we have some barrier material. So, if you see in this figure here, all of this is some hard sound reflecting material. So, they are hard sound reflecting material or they are simply you can say a strong barrier material. So, when you use such they act as scatterer. So, these act as sound scatterers. So, when sound is incident on these materials it is reflected back. So, they are scattering the sound waves.

So, these kind of such barrier materials or scatterers they can be placed like this one by one in a periodic manner. So, you have this, so periodic arrangement would what is meant by periodic arrangement is that, this distance, this distance and so, on. So, the distance between the scatter is the same. So, these are all equi distant. So, here what you see is that, this periodicity that is when you go into this direction. So, the direction normal to the surface of the barrier material. So, in this normal direction what you see is that, suddenly you see that the medium property is changing periodically.

So, what is happening here is that let us say the scatterer has its own beta B and rho. So, this is the B and the rho values for the scatterer and this is the B and the rho value for the air or the medium which we are using. So, once you go in this direction what you see is that periodically this B and rho value keeps changing and it happens only in one dimension. So, in this particular; in this particular direction you get a periodic variation in the properties, but when you look into other orthogonal dimension. So, let us say you look into the direction vertical. So, along the vertical there is no periodic variation.

If you choose any particular point it will stay the constant throughout the y axis if this becomes x axis. Similarly along the z direction also there is no periodic variation, the periodic variation is only across this direction which is given in this figure. So, this becomes the direction of periodicity which is the direction where the properties they vary periodically.

So, as you go along this direction you will first get B and rho s and after a certain distance suddenly it will change to B a and rho a and after certain distance it will again change to B s and rho s and this pattern will keep repeating itself because all these distances and these values are same.

So, the thickness of the material is the same everywhere and the distance between the material is the same. So, this is also same thickness. So, the material thickness is the same as well as the distance or the spacing between the material is the same. So, you get a periodic variation. So, this becomes a typical one dimensional sonic crystal.

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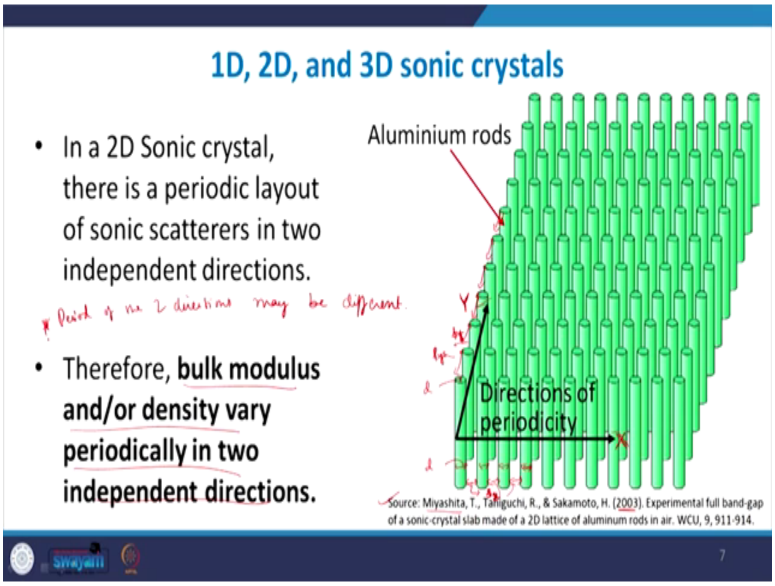

What is meant by a 2 dimensional sonic crystal? You will get the same variation periodically, but now in two independent orthogonal directions. So, if you see in this figure here. So, this is one of the most famous sonic crystals arrangement that was proposed by Miyashita et al in 2003. So, this is taken from their paper.

So, here you have a square lattice; so here this is one direction of periodicity and this is the another direction of periodicity. So, needless to say here that the diameter; so this is like a periodic arrangement of aluminium rods in air. So, needless to say that all this aluminium rods or aluminium cylinders, the diameter of these aluminium rods will be the same the material and the composition will be the same and the spacing between them should be the same. So, when the spacing is the same, so again here now two things you have to note here. So, let us say the spacing along let us say that this is the x axis and this is the y axis and there is a periodic.

So, when you go along the x axis what you see is that, the B rho values they are changing periodically because the material is changing periodically. So, in this case the spacing is the same as well as the thickness is the same. So, let us say this is some value d and this is some value s s one or the spacing along the x direction. So, this is the same all of this is going to be the same, but its not necessarily when you go along the y direction.

So, even in the y direction you can have the radius will be the same it is a circular cylinder. So, the diameter will be d, but the spacing between them that is the spacing here it could be some value s y. So, the main point is that along this direction all the spacing has to be the same, but it could be different from the spacing for the other orthogonal direction. So, in that case we get two independent directions where we have periodic variation and the period could be same or different because the spacing is because in one direction the spacing is a certain value let us say along the x direction the rods they are spaced let us say about 100 centimeters apart and along the y directions they are spaced about 120 centimeters apart.

Even then they act as that would be classified as a 2 dimensional sonic crystal, but the period in the two directions are going to be different because the spacing has become different. So, the period or the periodic variation, the period will be what? It is the distance along which the pattern is repeating or the wave material properties they are repeating and that distance would depend on what is the diameter of the rod and the spacing between the rod.

So, we get two independent directions for periodic variation and the period may be different. So, this is what I have told. So, an additional point is that the period of the two directions maybe different; they can be different, they can be same whatever be the case

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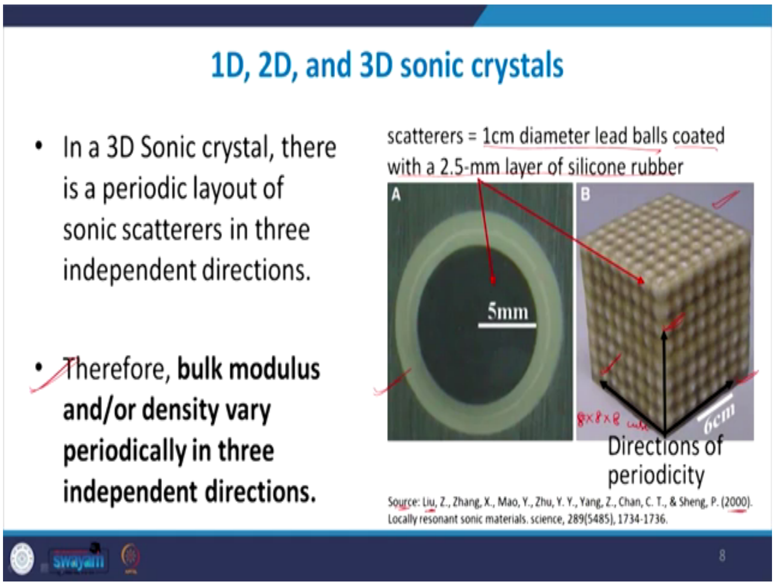

. In the same way we can define a 3D sonic crystal. So, this again this figure that I have taken is from Liu at el. So, these authors they had proposed a sonic crystal in 2000 which is a 3D sonic crystal. So, what does it compose of? So, it is an 8 cross 8 cross 8 cube, 8; 8. So, it has 8 spheres along x axis, 8 spheres along the y axis and 8 spheres along the z axis.

So, the three directions. So, periodicity here are given to you, these are the x y they correspond to the x y and the z axis of the Cartesian coordinate system and here the individual scatterers that were used for this particular year this particular sonic crystal. So, what was the. So, in the previous case the scatterers used by the authors were the it was the aluminium cylinders, solid aluminium cylinders or aluminium rods. In this case it is lead ball, so you have a one centimeter diameter lead balls which are coated on the top with 2.5 millimeters layer of silicon rubber. So, this is the kind of individual scatterer that we are using and they are periodically arranged in all the 3 dimensions to form this kind of a cube.

So, again if you go along this x direction what you will see is that, first you pass through the sphere and then comes some air gap then sphere then air gap then sphere and air gap. So, in the same way you pass along the y axis you get the same pattern and z axis you get the same pattern. So, in these three directions you are getting periodic variation in the property, but depending on what is the spacing and what is the arrangement like in the three orthogonal directions the period maybe same, it may be different it does not matter. So, this is another example of a 3D sonic crystal.

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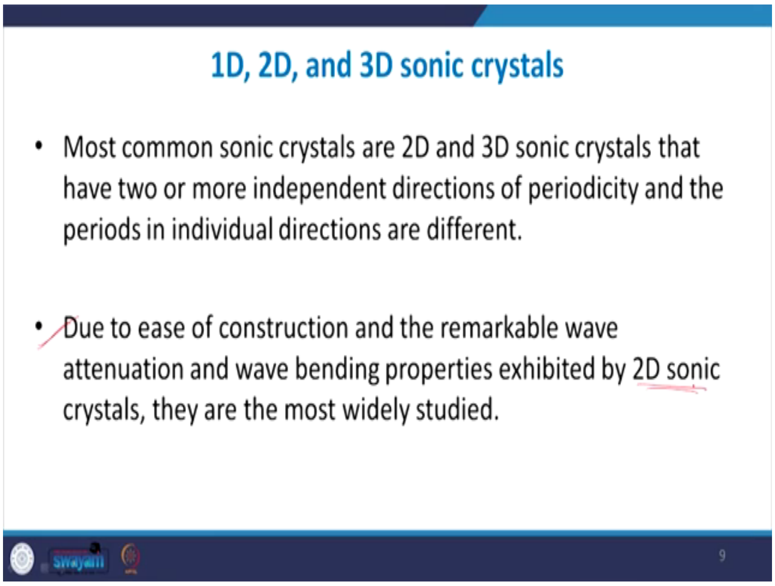

So, now that we have seen what is meant by a 1D or 2D or 3D sonic crystal. So, it depends upon the number of independent directions across which we get a periodic variation in the property of the material or a periodic variation in the b and the rho values. But the most commonly used sonic crystals are actually 2D and 3D sonic crystals and therefore, we will be limiting our discussions to 2D and 3D sonic crystals and most importantly to 2D sonic crystals because they are easier to construct and they can give some remarkable properties when it comes to sound attenuation and sound bending.

So, in the beginning of the lecture I explained to you that they can be double negative or they can have a negative b. So, in case of a negative b they are trying to attenuate the sound because when either b or rho, one of the parameter becomes negative, then k vector becomes imaginary and what we get is no propagation of sound. In the same way if we have a double negative material. So, sort of what it is doing is that if we compare it with an if with a double negative or double negative electromagnetic material, then in that case they have a negative refractive index.

So, in the same way the overall negative refractive index is achieved which means that the sound waves they turn very sharply or they bend very sharply when they come into contact with such double negative sonic crystals.

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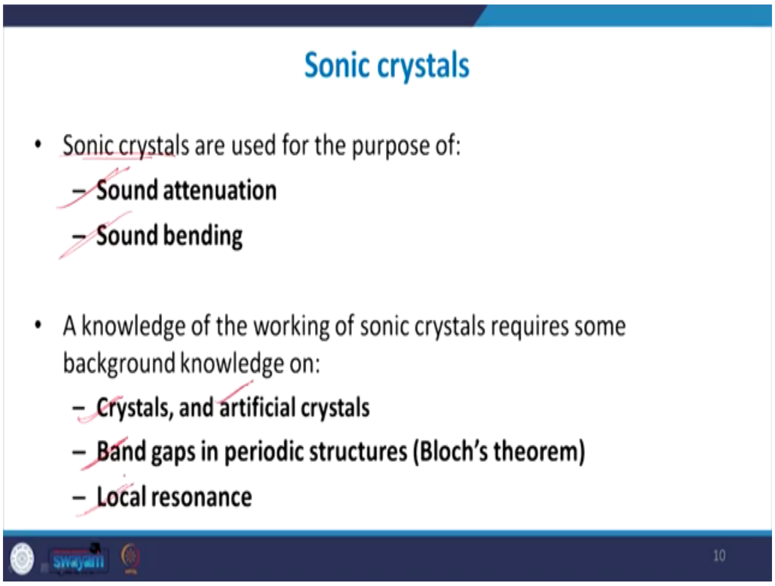

So, they can be used for both these purposes attenuation and bending and to discuss about them. So, to discuss what is the mode of operation and to gain a better knowledge, first of all we need to have some background knowledge on what is meant by crystals and artificial crystals.

So, some background knowledge on what is crystals, what are the various terminologies within a crystals, how is the wave propagation studied within crystals and then some knowledge on band gaps and the knowledge on local resonance. So, when you have this knowledge and then we can together put it together and very easily study how the waves propagate within a sonic crystal and how does the sonic crystal operate.

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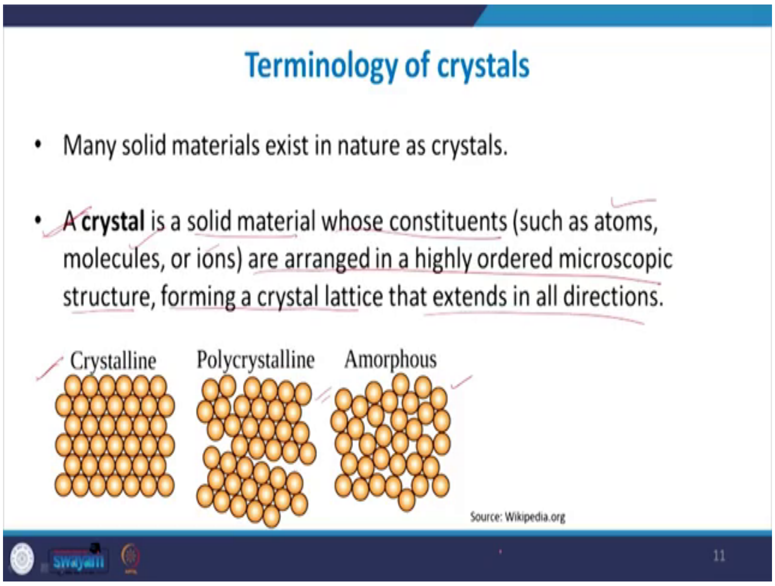

So, let us begin with crystals and artificial crystals so, here. Now, in solid state physics or when you study about materials you would have heard that usually a material can exist either as amorphous or as crystal. Amorphous is more like a powdery irregular form whereas, crystal there it has a more sharp and periodic nature and the crystals they are more they are more they have a more well defined shape if to say.

So, the material exists in the form of some structures which are which have a very rigid and well defined shape. So, what do you mean by crystal? It is a solid material whose constituents such as you can have the atoms, molecules, ions whatever. That is making of the material, there are arranged in a very highly ordered microscopic structure forming a crystal lattice that extends in all direction.

So, when the when the building blocks of the material which can be atoms, molecules or ions they are haphazardly arranged we get amorphous then a little bit of arrangement polycrystalline and finally, crystalline means a highly ordered arrangement of these highly ordered arrangement of let us say let us call it building blocks. So, instead of calling it because it depends on material to material.

Some material the building blocks are ions for example, sodium chloride N a plus and C l minus; similarly in some materials it is the elemental carbon; carbon carbon bonding existing as graphine or diamond like that. So, there the building block is atom and in most of the other cases it is a molecule.

So, instead of calling it atoms or what or molecules or ions, I will just use the term building blocks. So, the building blocks or the units of the materials there are arranged in a highly ordered fashion in a crystalline form. And what is meant by a lattice? So, if this is the crystal then this arrangement or this geometry is called as lattice.

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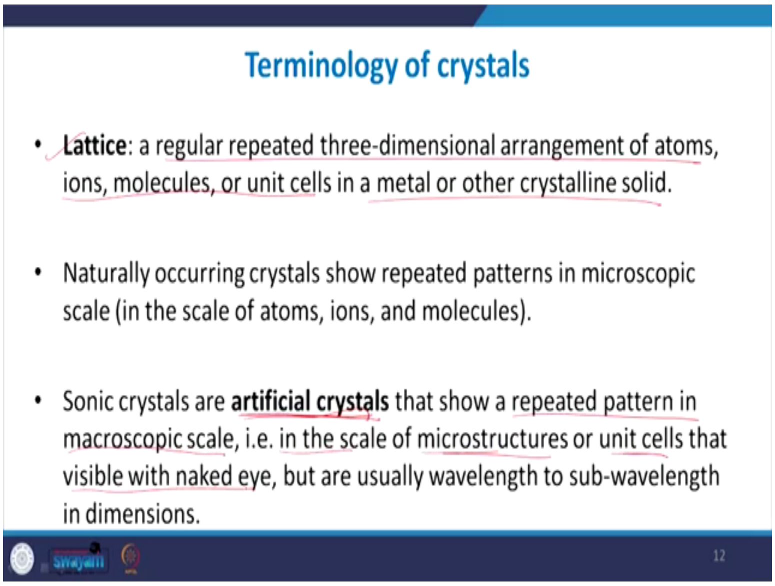

So, it is a regular repeated three dimensional arrangement of atoms, ions, molecules or unit cells which make up the solid. So, that arrange that three dimensional pattern of arrangement arrangement of these individual building block becomes the lattice. So, the naturally occurring crystals now you have to see that the naturally occurring crystals, they exist, here the repeat the repetition happens or the arrangement happens on a microscopic scale, but not, but for artificial crystals you can have some individual unit cells. So, and acoustic metamaterial can be thought of as an artificial crystal.

So, we have some individual building block which is a unit cell and then it is being arranged in a periodic fashion. So, overall it is forming a sort of a crystal lattice. So, this is what this is where the word artificial crystal comes in. So, there it is such kind of materials which have a repeated pattern in a macroscopic scale, i.e in the scale of the microstructures or unit cells. So, instead of now talking about atoms we have more larger; more larger structures which can actually be visible with naked eye.

So, you can see them and these structures get repeated periodically and they are arranged either in one dimension or two dimension or three dimension in a periodic array and together that material can be sort of thought as a artificial crystal. So, a sonic crystal is an artificial crystal in fact, most of the metamaterials which involve arrangement of unit cells in a periodic fashion, they can be thought of as an artificial crystal.

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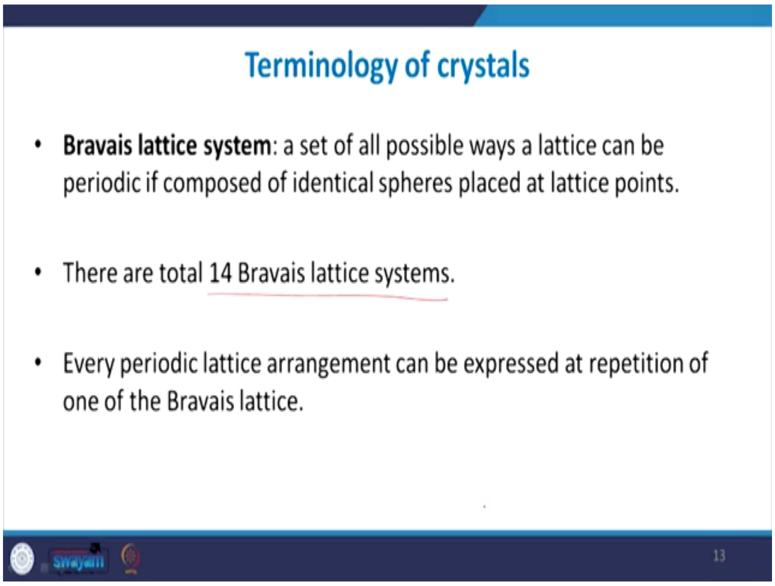

Another terminology that I will be discussing is the Bravais lattice system. So, this is like a set of all possible ways a lattice can be periodic if composed of identical spheres placed at lattice point. So, whatever is the structure if you replace that structure and you just place identical spheres on the lattice points, then what are the various shapes that it can take so, that a crystal can be formed.

And 14 such systems have been identified and they are shown here. So, the entire crystal can be generated when such Bravais lattice they are repeated. So, this is the various forms which exist and these are shown in this figure here.

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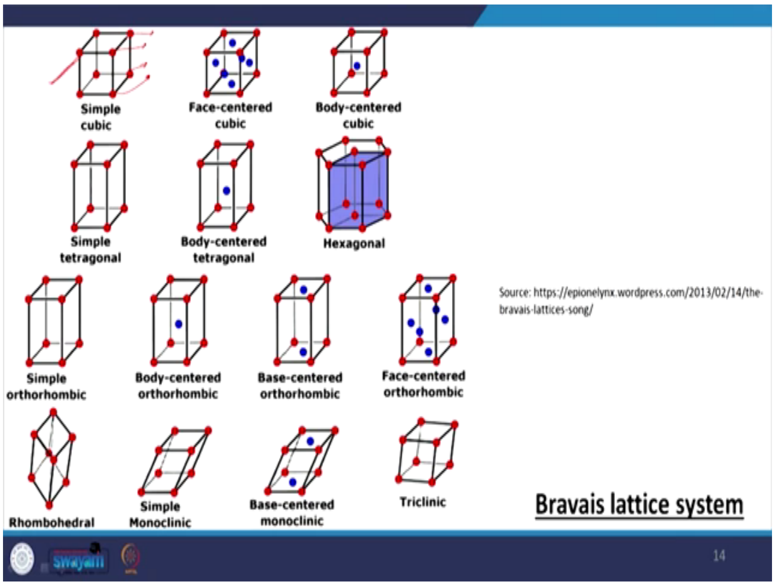

So, what you get here is that, you have a simple cubic a face centered cubic. So, simple cubic means that let us say we have the parameters a b and c.

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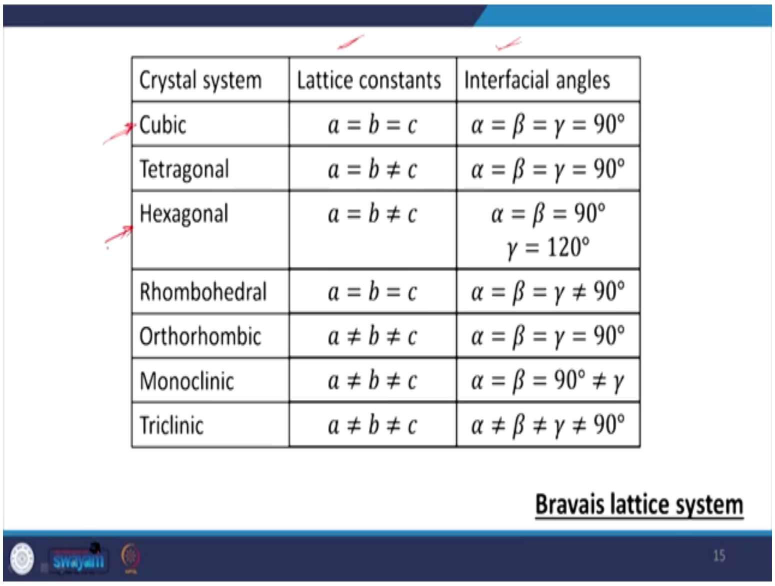

Crystal system	Lattice constants	Interfacial angles
Cubic	$a = b = c$	$\alpha = \beta = \gamma = 90°$
Tetragonal	$a = b \neq c$	$\alpha = \beta = \gamma = 90°$
Hexagonal	$a = b \neq c$	$\alpha = \beta = 90°$ $\gamma = 120°$
Rhombohedral	$a = b = c$	$\alpha = \beta = \gamma \neq 90°$
Orthorhombic	$a \neq b \neq c$	$\alpha = \beta = \gamma = 90°$
Monoclinic	$a \neq b \neq c$	$\alpha = \beta = 90° \neq \gamma$
Triclinic	$a \neq b \neq c$	$\alpha \neq \beta \neq \gamma \neq 90°$

Bravais lattice system

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So, I am not going to go into the detail of this because every student has studied solid materials and materials in their class 10th, 11th and 12th and even in the b-tech. So, they have already the knowledge of this what is meant by a simple cubic structure or a face centered cubic or a body centered cubic.

So, this is the different forms of standard arrangements of a particular crystal lattice. So, when you repeat this particular structure. So, you will have some infinite such cubes which are stacked, adjacent on top or below. So, they are repeated on all the three directions then you get a crystal; similarly you can repeat this structure in all the three directions and you get another structure. So, using these standard kind of shapes various form of crystals can be created and this is just a review of what these means like, but the point of interest here is the cubic structure and the hexagonal structure.

Because the overall point of these lectures is to study about sonic crystals and sonic crystals in general, they either are arranged in a square lattice or in a hexagonal lattice either in 1D or either in 2D or 3D. So, they either take this cubic form or hexagonal form. So, this is more of our interest and as we go on an study further about the arrangement of sonic crystals, we will realize that what is the particular importance of studying this Bravais lattice.

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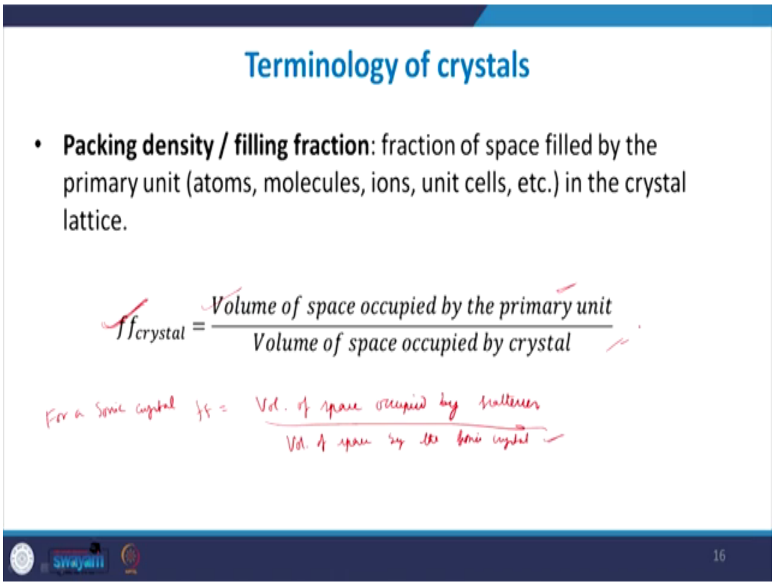

So, for example, here the individual lattice points here the space they can actually represent the location of sonic scatterers. And the other is the space the spacing between them. So, here every you can have the sonic scatterers and the other thing becomes the spacing or the host material. So, this is how this scatterers can be arranged; they can be arranged in a cubic form or they can be arranged in a hexagonal form. The last terminology in studying about crystals is packing density or filling fraction. So, as we know that what do the crystals compose of? They compose of some building blocks which are arranged in a periodic fashion and in a sonic crystal which means it is composed of some sonic scatterer which is arranged in some periodic fashion.

Then the filling fraction is defined as, what is the space occupied by that particular repeating unit. So, what is the fraction of the space occupied by it? Compared to the total space of the material. So, the filling fraction or the packing density is given as the volume of space occupied by the primary unit. This is this primary unit in terms of lattice becomes a lattice point or in terms of sonic crystal becomes the sonic scatterers. So, if we have for a sonic scatterer the same definition so, for a sonic crystal the definition of f f crystal will be the filling fraction will become volume of space occupied by the scatterers divided by the total volume of space by the material by the sonic crystal.

So, sonic crystal will compose of both the scatterer and the host medium. So, the total volume of space will be the volume of the scatterers plus the volume of the host material. So, denominator is that term total volume and the numerator is the volume of space occupied by scatterers.

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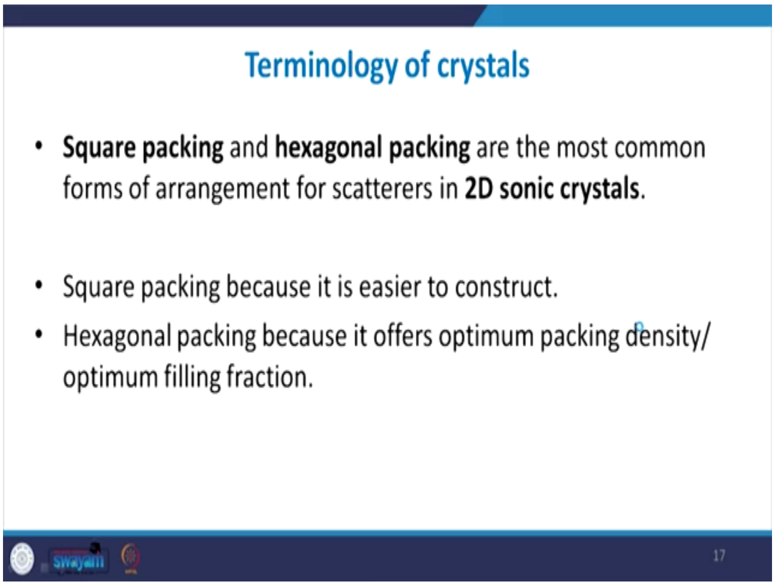

So, this is the definition of filling fraction or packing density. Now, usually the as I told to you cubic and hexagonal systems are more of interest which means, now we will be studying more about 2D and 3D sonic crystals, but most generally about 2D sonic crystals.

So, in a 2D sonic crystal the cubic system can be thought of in the two dimension as a square packing and in the two dimension the hexagonal kind of a arrangement can be thought of as a hexagonal packing. So, these two forms of packing are most common for sonic crystals and square packing it is easier to construct and hexagonal packing why is that preferred? Hexagonal packing is preferred because it has a much higher filling fraction. So, within this small volume you can contain more sonic scatterers while having the same spacing between them.

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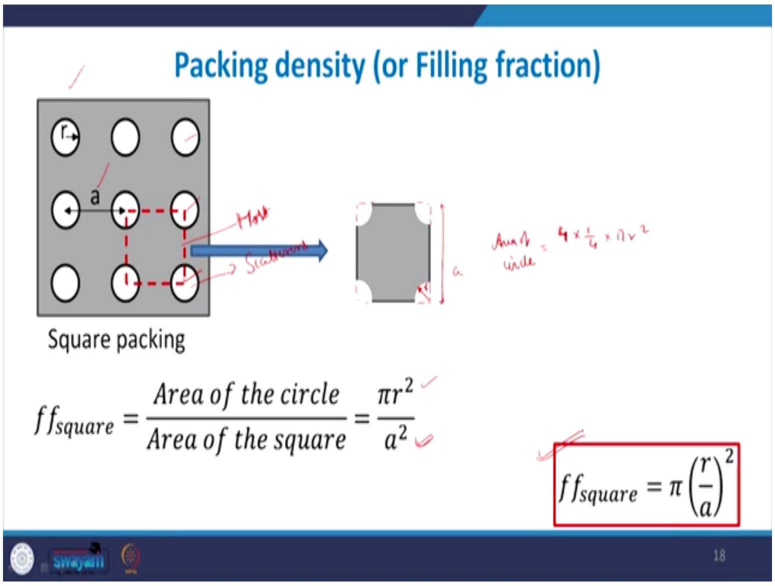

So, if I show it here, let us say these are these individuals sonic scatterers and this is the arrangement in which they are being arranged and this is the host material. So, this is the host material and these are the scatterers ok. So, in that case and the spacing between the two scatterers is given to be a and the radius of the scatterer is given to be r. So, we have the r and the a value.

So, let us take a repeating unit here. So, this is the repeating unit and let us see what happens here, we have a square. So, this repeating unit is actually a kind of a square and within the square. So, the side of the square is a and within the square we have one fourth, one fourth and one fourth of circles on all the four corners where the radius is given by r.

So, what will be the filling fraction? It will be what is the volume of space occupied by the scatterers which is the total area occupied by the circles divided by the total area of the square.

And what is the area of the circles? How many circles do we have? We have four one fourth circles, every circle is one fourth. So, we have 4 such things. So, this becomes the that area of circles. So, we have 4 into1 by 4 pi r square, so which comes out to be pi r square and the area of the square is a square. So, the filling fraction that we get for a square packing is pi into r by a whole square.

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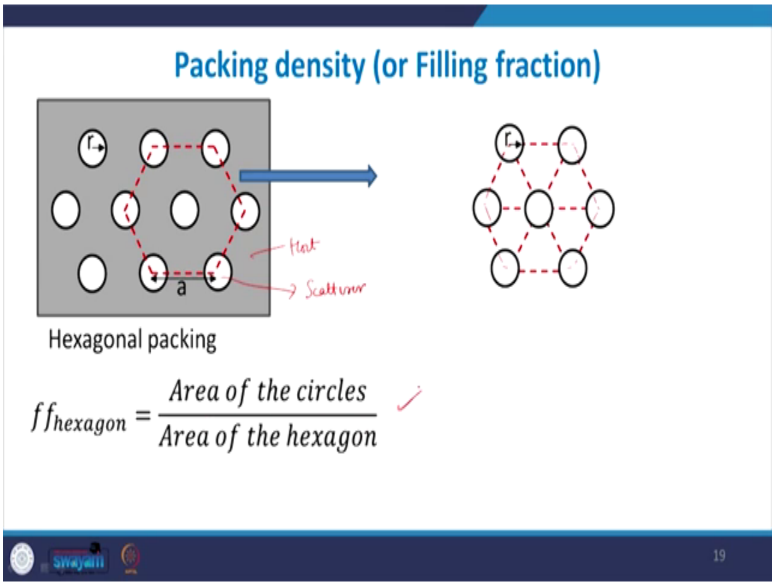

Similarly, if we have a hexagonal packing, if we take a repeating in the unit here. So, we have the. So, here we have the same r and the same a, so the radius of the scatterer is the same and the spacing between the scatterer is the same. So, here this is the host material and this is the scatterer and they have the same radius and spacing. So, let us find out the filling fraction. So, we take a repeating unit here.

So, here again the filling fraction will be whatever. So, if this is the particular repeating unit this thing, this hexagon is repeating itself and it is creating the entire arrangement. So, within this repeating unit whatever is the volume of the scatterer is whatever the volume is whatever is the area occupied by the circles. And similarly the total area, the total space occupied by the material will be the area of the hexagon. So, area of circles by the area of hexagons.

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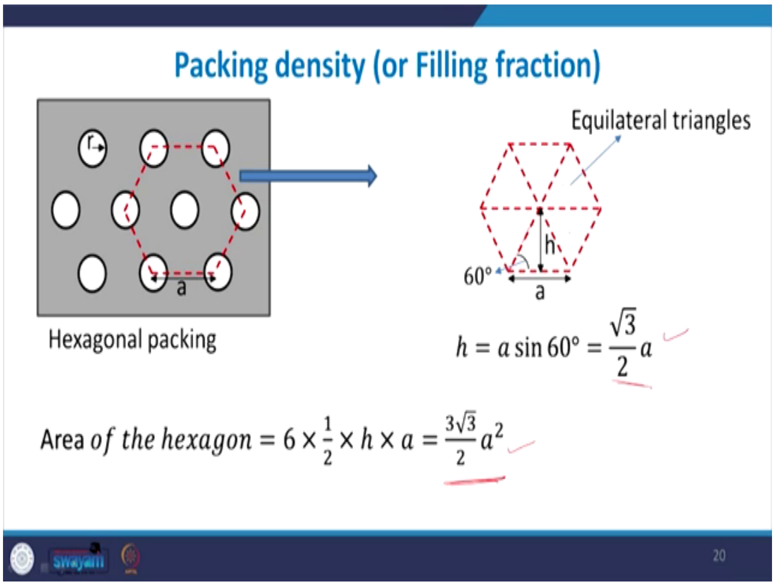

So, let us first calculate what is the area occupied by the total repeating unit which is going to be the area of the hexagon. So, here the side is a and hexagon can be thought of it can be divided like this. So, all of these triangles are equilateral triangles with 60 degree angle. So, if this is a which is the spacing, then this height h for once particular triangle becomes a sin 30, sin 60 sorry, so it is a times. So, in this particular triangles.

So, if you take this triangle here this bigger triangle this is the hypotenuse and this is the opposite angle. So, this h becomes a times sin of 60 which is root 3 by 2 a. So, the area of the hexagon will be what? It will be 6 times because all these triangles they are similar triangles they are and they are equivalent triangles and they are all equilateral. So, 6 such triangles are there.

So, we have 1, 2, 3, 4, 5,6, so 6 multiplied by the area of the triangle which is half into the height into the base, so half into h into a. So, h is this particular value here, which you which if you put here what you get is 6 into 1 by 2 is 3, 3 into root 3 by 2 a square. So, this comes out to be the area occupied by the material.

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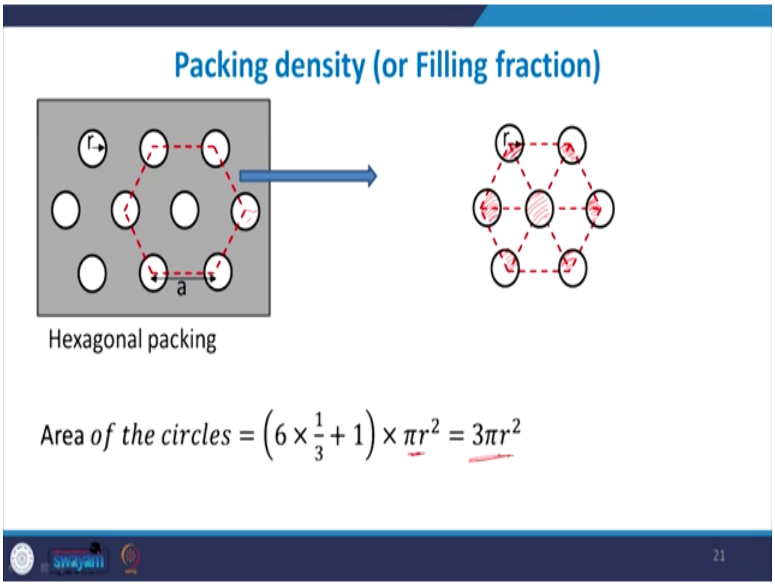

Now, let us find out what is the area occupied by the scatterer within this unit. Now, we have 6 such circles on the corner. So, first of all there is one circle at the center. So, this volume is what we are trying to find out this area sorry. So, this is the area which we are trying to find out.

So, what will be this? This will be pi r square and then all of these sections if you see here, all these sections are actually one third. So, if you have a line here this and this are going to be same all of this is 120 degree. So, it is one third of a circle. So, 6 into 1 by 3 plus 1 total number of effective number of circles multiplied by pi r square. So, this is the total area occupied by the scatterers.

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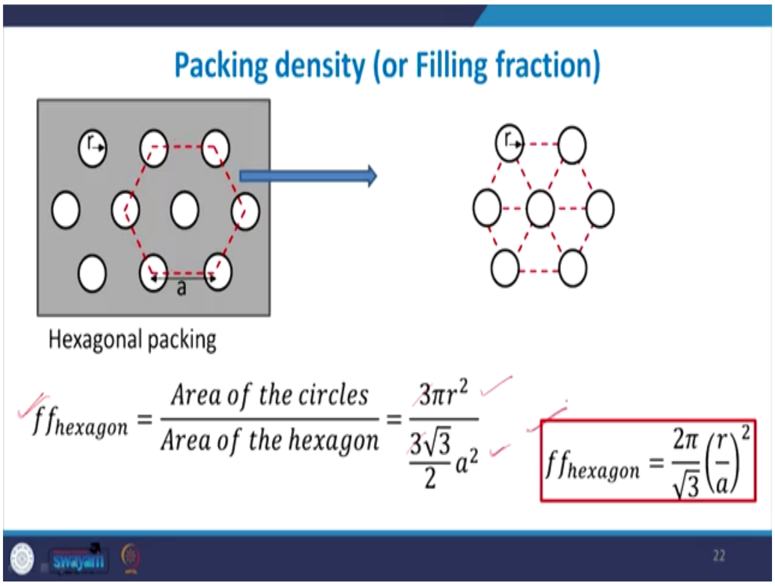

So, the filling fraction will then become the area occupied by scatterers divided by the area of the total material which if you see it becomes. So, this and this cancels out, 2 pi by root 3 r by a whole square. So, this is the filling fraction of hexagon.

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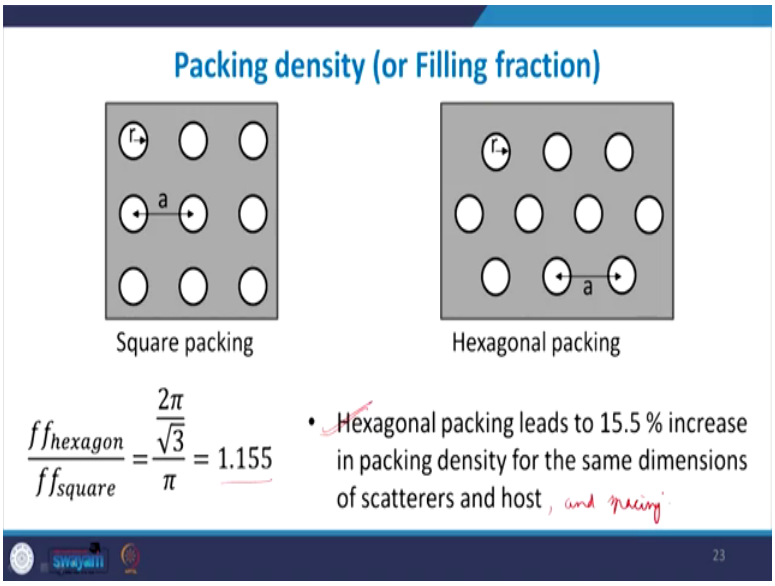

So, this is the filling fraction of hexagon and for the square it is pi into r by a whole square. So, if you compare the two filling fractions. So, if you compare the two quantities here what you get is, the filling fraction of hexagon is almost 1.155 times the square. So, which means that in a hexagonal packing, it can lead to a 15.5 percent increase in the packing density for the same dimension of scatterers and for the same dimension of the host and the spacing; so that is what you get and spacing.

So, with the same spacing and with the same type of scatterers used even then you get a much better filling fraction using a hexagonal packing and that is why this particular arrangement is preferred for arranging two dimensional sonic crystals.

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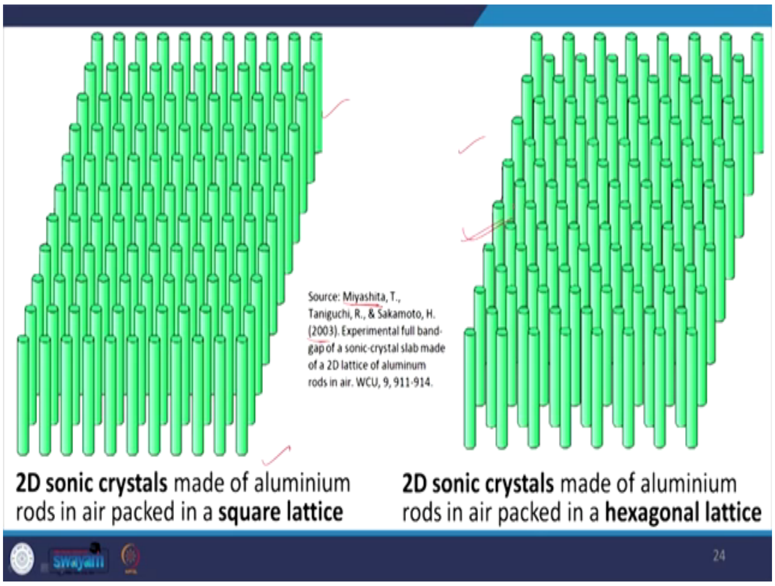

So, I will just close this lecture with this figure here. So, what you see is that this is a 2D sonic crystal made of aluminium rods. So, again the same paper Miyashita at el 2003 is used. So, they show two different arrangement this is a square arrange square packing and this is a hexagonal packing. So, we will continue our discussion on crystals in the next class.

Thank you.