Acoustic Materials and Metamaterials
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Lecture – 36
Principle of Working of Sonic Crystals-1

Welcome to the last week of this course on Acoustic Materials and Metamaterials and today is lecture number 36. So, so far we have been discussing about Sonic Crystals. And, before we begin to understand how do the sonic crystals work, I went through the fundamentals of the Crystal Theory, because that is important in understanding, why there is a sound attenuation in sonic crystals.

So, we studied about some of the concepts like, what are the lattice vectors, what is a direct lattice, what is a reciprocal lattice and then what is a Brillouin zone and what is an irreducible Brillouin zone.

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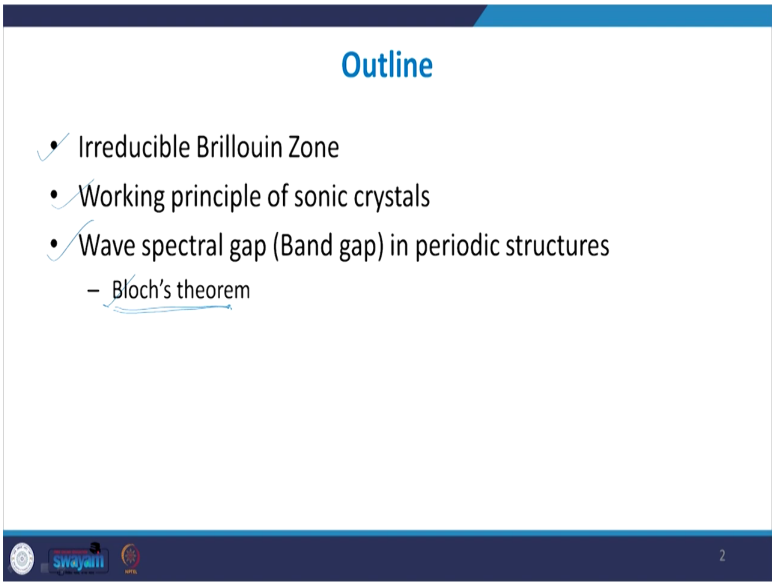

So, in this particular lecture we will continue with the discussion on the irreducible Brillouin zone and then based on that I will tell you, what is the working principle of sonic crystals and then sonic crystals they work on two particular principles. So, in this lecture we will discuss the first principle which is the wave spectral gap or the band gap in periodic structures. And, then we will go through a special theorem called as the Bloch's theorem.

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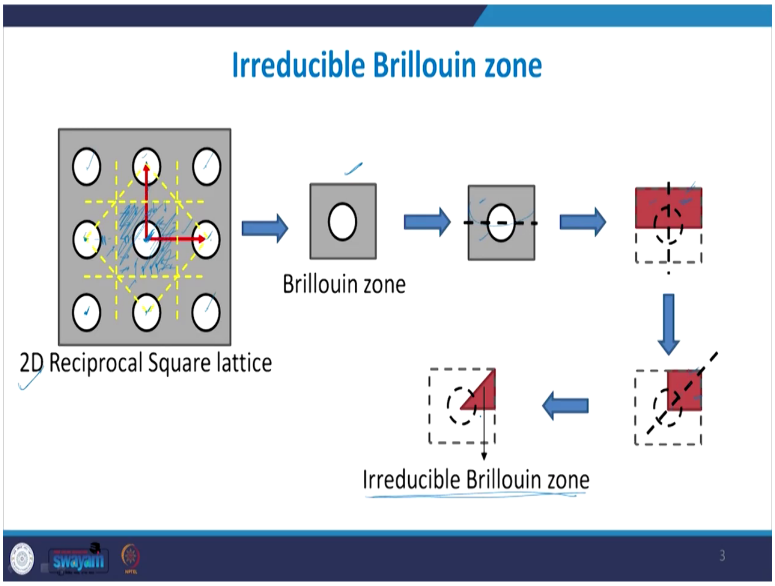

So, based on the discussion on the last class about irreducible Brillouin zone, we know that when we have a reciprocal lattice, then we can take primary unit a primary repeating unit. So, we can select one lattice point and find out the volume of space that is closer to that lattice point compared to any other adjacent lattice points. So, when you get that that volume of space becomes the Brillouin zone and that is the unique space.

And, if we know the information of wave propagation in that space we can predict the wave propagation in the entire crystal, but the Brillouin zone itself can have some symmetry. So, when all these symmetry is removed the nonsymmetrical basic unit left is called as the irreducible Brillouin zone. So, let us discuss some examples here. So, a 2 D reciprocal square lattice is given. So, now, we know that for a square lattice the reciprocal itself will be a square lattice. So, this is the reciprocal lattice and is the lattice point about which we have to define a Brillouin zone and these are the various lattice vectors.

So, how do you begin? So, let me say that the core is adjacent points next to this or this, this, this, this, this. So, all these are the adjacent points next to this particular lattice point. So, let us say find out the volume of find out that is a area within which the points are closer to this one compared to this particular point.

So, if we draw a line here in the midpoint between the 2,.so this is the midpoint. So, this is the line joining the 2 centers and this is the midpoint. So, if you draw a line here then all the points which lie within this line will be closer to this lattice point compared to this lattice point..

Similarly, let us see what happens for this one for this particular lattice point. Again, if we draw a line here, which is at the midpoint between the 2 lattice points then all the and then all the points which lie in this direction, nearer to this between the prime lattice point about which we are defining the Brillouin zone and the line. Then, all these points they will be closer to this compared to this.

And, the same way we can continue and we can keep drawing lines for every adjacent point. So, for this one, this is the line and all the points that are in this direction are closer to this compared to this one. So, they are closer to the prime lattice point. Similarly, for this one again the Brillion zone must contain the points within this zone which is a square. Now, let us also cross check with the other diagonal lattice points.

So, if we say what should be the points which are closer to this one, the primary lattice point about which we are defining the Brillouin zone and there closer to this one compare to it is diagonal neighbor. So, if we draw up again a line, that is the perpendicular bisector of the line joining the center. So, all these lines are the perpendicular bisector between the 2 centers. Then all of this point that is within this will be a part of the Brillouin zone and so on we can continue for this point, for this point, for this point.

So, for every point one by one we are drawing what is that zone of area, where the zone of area, which is closer where the points they are closer to the primary lattice vector compared to the to the adjacent neighbor. So, when we reduce this we get this volume of space. So, this volume of space equals to all that area. So, here it is a 2 dimensional lattice. So, I am taking area, when it was a 3 dimension the same concept applies to the volume.

So, that area or the volume within which that is closer to this particular lattice point compared to any other adjacent point. So, this becomes the Brillouin zone. So, this is how we have found the BZ. Now, the Brillouin zone itself has symmetry. So, let us say if we take our axis here, then the top portion is symmetrical to the bottom portion. So, which means we can we can only find the solution in this space the top space and replicate it in the bottom and we can get the whole solution.

So, we can reduce this space further. Now, we have a reduced space, does it have a symmetry about this vertical axis, if you see? Yes there is a symmetry about the vertical axis. Then you can again only find the solution in one of these portions let us say this one and remove this. So, once you get the solution within this portion this particular section. Then, you can first reflect it about the vertical axis and then the total solution can be reflected about the x axis to get the full solution, the full wave solution.

Now, this is the zone we have left with do we have any further symmetry? So, if you see if you draw a line here this one. Then this portion can be reflected about this xy plane that is 45 degrees. So, xy axis and they are symmetrical to each other. So, the final zone I am left with I can choose either one of it. So, this is the final zone that I am left with. So, this becomes the irreducible Brillouin zone, which means the Brillouin zone, which cannot be reduced further into more such sections of symmetry.

So, as you can see you cannot do a section here this cannot be done why? Because see the Brillouin zone and irreducible Brillouin zone by their definition must have the center at the lattice. So, they must have the lattice point and this one does not contain a lattice point. So, this is not symmetrical about these 2 sections and so on.

So, if I just erase this thing again it just to explain to you. So, finally, what we are left with? We are left with this particular portion, which is the finest or the smallest area, where if we find the wave solution we can keep replicating it using various symmetrical operations to create the solution for the entire crystal. Let us see another example to make this concept clear.

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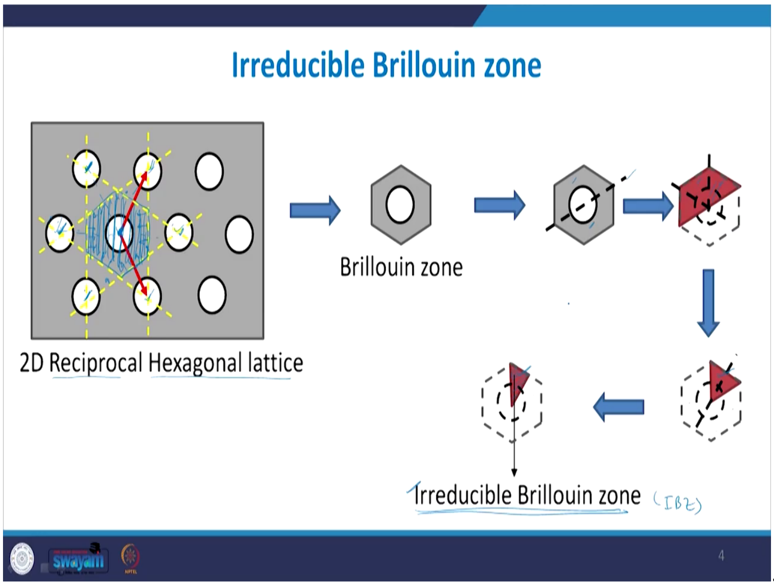

So, let us say now we have a 2 dimensional; reciprocal lattice for a reciprocal horizontal lattice. So, this is their horizontally, so sort of spaced and let us say this is the so, you can for when you are defining a Brillouin zone, you can choose any one lattice point. Because, it is sort of a very large array not infinite, but a large a very large array and there are lots of lattice points.

So, you can choose any one lattice point and then define the Brillouin zone about that lattice point. So, here I am choosing this particular one and these are the primary lattice vectors. So, now, I have to find that region of area, where the points they are closest to this one compared to all it is adjacent neighbors. So, these are the adjacent neighbors and we have to find the points which is closest to this one compared to the next neighbors. So, let us start with one of it.

So, if I start with this one again I am drawing a line which is a perpendicular bisector of that this line joining the centers between the two. So, this will pass because this is a regular hexagon. So, this will pass through the centers of the hexagon these 2 hexagons because of the regular hexagons. So, all the points that lie nearer at this end of the line all these points they will be closer to this one compared to this one.

Similarly, let us say now I have I want to look at one of the other ones. So, let us say I want to look at this one. So, again I draw the same with the same concept I draw a perpendicular bisector along the line joining the centers for the 2 points under consideration. And, all the points within this will be closer to this particular point compared to this one the outside one. And, similarly for this one what will be the line like? It will be like this and these should be the points.

Similarly, for this one again all the points lying on this side of the line would be closer to this particular central lattice point compared to the outside one. Similarly, now we take this one then perpendicular bisector for this center will pass through this one and this these should be the points. And, finally, so, all these it has 6 adjacent neighbors, so one by one for every adjacent lattice point we are finding out we are joining that we are joining the lattice point about which we have to define the zone.

So, we are joining the center of that lattice point with the center of it is nearest neighbor, we are drawing a perpendicular bisector, then all the points on the on the others on the inner side of the perpendicular bisector will be the points, which lie closer to the lattice point about, which we have to define the Brillouin zone. So, so on and so, forth finally, the last neighbor I have considered. So, one by one I have considered all 6 neighbors and the intersection of this is this zone here.

So, this zone is the intersection. So, all the points here they are closest to this lattice point compared to any of the other adjacent lattice points. So, this becomes my definition the Brillouin zone. So, this is what is the Brillouin zone. So, for a hexagon the Brillouin zone itself is a hexagon. Now, as you see for a square lattice what we saw was that it was on a it was again a square, but the square was of the same side with the same lattice constant, but here this is a reduced hexagon.

Now, let us see. So, now, this is a prime unit cell about which we can find some solutions and we can repeat it to get the solution for the entire lattice. So, this repeating unit can be repeated in all the directions to get for the solution for the entire lattice, but we can reduce the computation time further by reducing the zone further and further so, let us start with this particular axis. So, here this portion is symmetrical about this. So, which means we can reduce it one step further.

We can only find the solution in one of this one of these portions and then when we reflect this result about this axis we will get for the full zone. Let us see if we have more symmetry. So, let us see the axis here. So, here we have this entire zone, we can divide it into 3 equilateral triangles. So, if we choose any one equilateral triangle here, then we can reflect it about this point and then we can reflect this about this point and so on and we can if we get the answer in any one of these, let us say in this one or any one of this.

Then, we can find out then the then due to by reflecting it about the respective axis. So, using these symmetrical operations, what we can get? We can get the solution for the entire zone. So, we have reduced it into 3 symmetrical portions. So, we are retaining only one portion, so if we find the so, here as you can see if you find the solution only in this area, then we can replicate it to get for the entire Brillouin zone. Can it be reduced further? Yes, it can.

So, you draw a line about this retain one of the portion. Now, this cannot be reduced any further. So, this becomes the irreducible Brillouin zone. So, the minimum area till which we can reduce it beyond which we cannot reduce. So, now, if we know so, when we solve the; when we solve the equations for the entire crystal instead of solving the wave equation for the entire crystals, we can simply find what is the Brillouin zone, what is the irreducible Brillouin zone, this is the short form for this.

So, what is the IBZ? When we find the IBZ, we only find the solution about this and then we keep reflecting it using some symmetrical operations and we can get for the entire Brillouin zone. And, this means when can be further translated and translated about the different lattice vectors to get the answer for the entire crystal.

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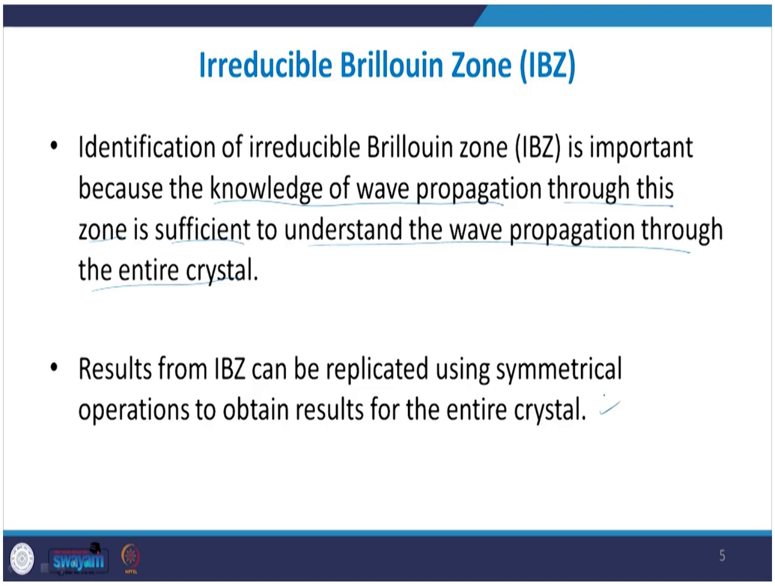

So, that was the purpose of using an IBZ, which is because the knowledge of wave propagation through this IBZ is sufficient enough to understand the wave propagation through the entire crystals. Thus, we just by knowing what is the kind of wave propagation happening within this small zone we can know what is the wave propagation through the entire crystal?.And, we can replicate the results using some symmetrical operations.

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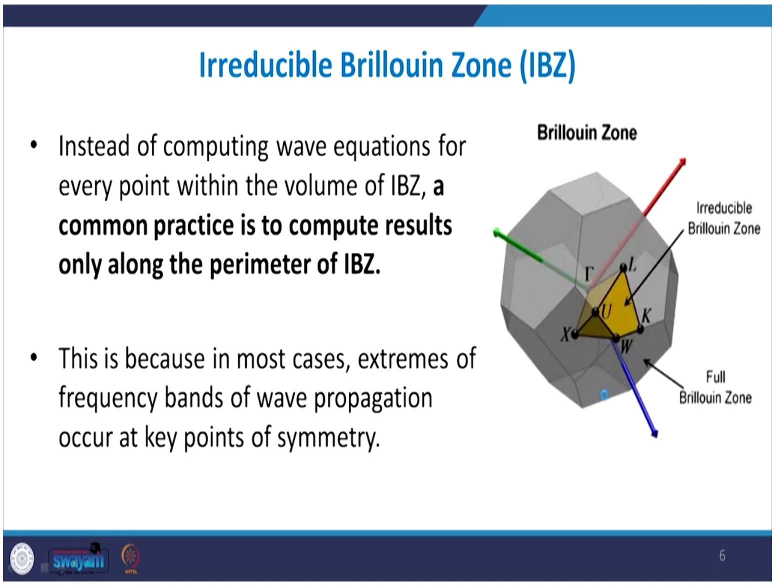

So, this is a 2D view this is a sorry this is a 3D view this is a 3D crystal and then there is some irreducible Brillouin zone obtained, we can only obtain the results here and then we can replicate to get the knowledge for the entire crystal. So, this effectively reduces the computation time a lot. Now, what has been found empirically and out of lots and lots of experiments is that, that the extremes of the frequency bands, they only occur at the key points of symmetry of the Brillouin zone.

So, if we can reduce the computation even further. So, instead of finding the solution for every point within the full volume of the IBZ, we can just go along the perimeter of IBZ and calculate the values at the parameter points and that will give us the all the extreme. So, because the extremities they are occurring at these along the parameter or the key points of symmetry. So, just translating the parameter the parameter can give you a good enough idea of how the wave is propagating and you can find out what are the frequencies within which there is no wave propagation.

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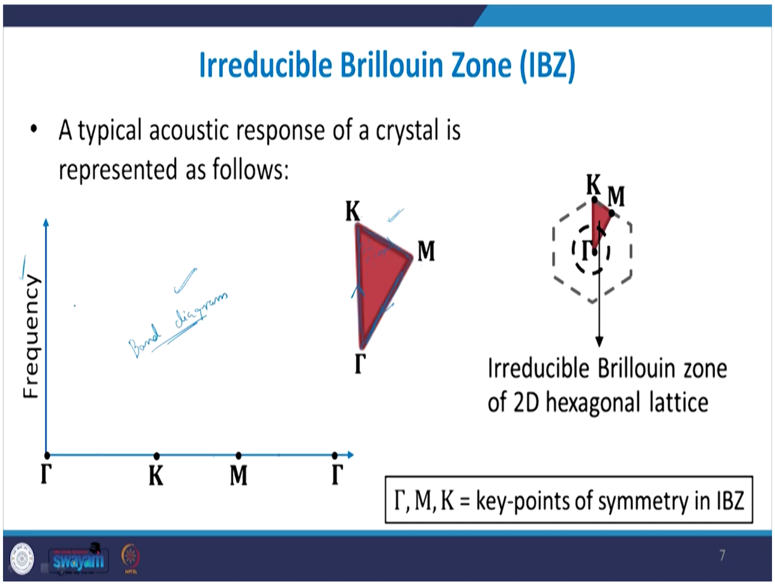

So, when we do this; so the further reduction can be let us say we have the hexagonal lattice and this is the irreducible Brillouin zone we obtained. So, instead of finding the solutions for every point within this area, we can just find the solutions within the perimeter. So, only along the perimeter points if you find the solution, then we can get a good enough idea of what is happening in the entire crystal.

So, here so, the wave frequency response for sonic crystals is represented is that, we represent vertical axis becomes the frequency and this becomes the perimeter of the IBZ. So, here the direction which I have started is tau. So, from tau to K, K to M M to tau, so, this is how the entire perimeter has been translated and represented here in a scaled manner. So, this tau M will be proportional to this distance K M will be proportional to this distance and M T will be proportional to this. So, as you can see these 2 points are closer so, they are closer here.

So, it is proportionately divided and a linear scale is created. So, my idea of telling you about the Brillouin zones and the irreducible Brillouin zone was to make you understand that, in any periodic structure be it a sonic crystal or a natural crystal. So, both artificial and natural crystals, you can simply find how the frequency is propagating. So, for that first of all to find as I told you that, the actual information is in the is the actual wave is as a variation of time and space.

But, if you want to find out the frequency component or the frequency how the frequencies propagate, then first of all you represent the constant in the reciprocal space, that will give you that what are the various frequencies. So, instead of space and time now you get in frequency and wave numbers. And, then after that so, once you have represented in reciprocal space, then you can find the smallest element of symmetry and along that parameter you can find out, what is the wave propagation like and this will give you a idea of what how the propagation is taking place in the crystals.

And, what are the directions where the wave propagation is highest for which frequencies it is highest and for which frequencies there are no wave propagation. So, this is a typical band diagram. So, a typical band diagram this is the vertical and the horizontal axis. So, this is how the sonic crystals are represented.

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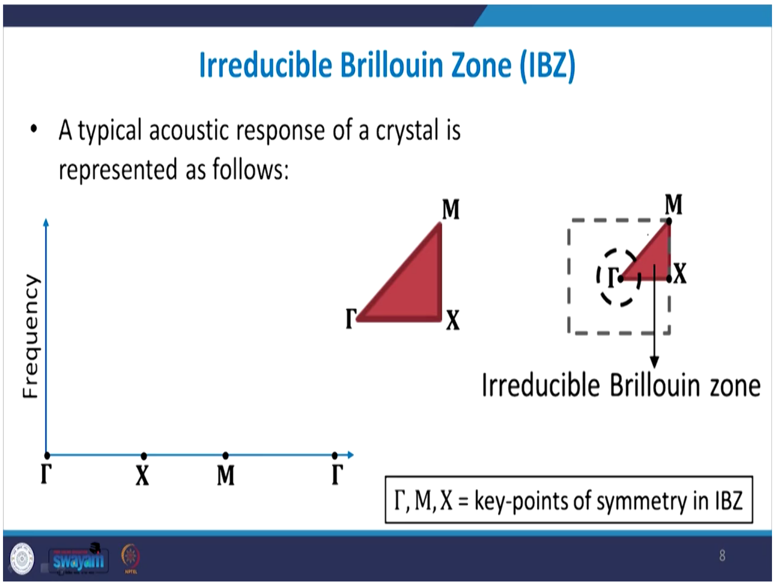

Similarly, for if we have a square lattice. So, say let us say we have sonic scatterers arranged in a square lattice, this becomes the IBZ we can represent it. So, the horizontal axis will become here. So, here you can; you can go in any direction, whether you go in this direction or the other direction the information is going to be the same. So, you translate throughout the parameter and you represent it, in this is the perimeter and this is the frequency ok.

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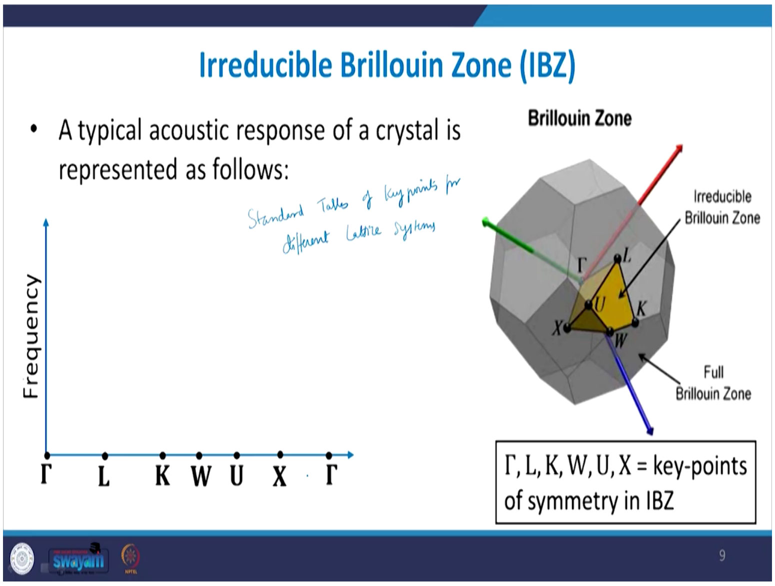

And, similarly for this 3 D lattice again you can have the frequency and then you can represent the perimeter for this zone. So, here you can see that, these are the key point's tau L K W U and X and I would like to point out here that for ease in calculations. Now, we have some standard crystals like for example, for sonic crystals usually a square lattice or a hexagonal lattice is used.

So, for every such lattice you can there are some standard tables of key points for different lattices. So, if you know what is the lattice structure like and what are the lattice constants like, or what is the distance between the different lattice points, then there are some standard tables available to you where you can you will already know that this is a complicate, this is a lattice, and this is a particular crystal type for this, this is the reciprocal lattice, then for this reciprocal lattice what will be the key points?

So, you can find so, this values can also be obtained from the standard table. So, once you know these values you will know these distances and you can plot it in the horizontal axis. So, that is how the data is represented. Obviously, in this particular course you will not be asked or you will you will not be learning how to exactly use those tables and draw those diagrams. This is just to give you an idea of how the band diagrams are made formed various sonic crystals and what do they represent.

So, now, we have covered this. So, let us go into what is the principle of working of the sonic crystals. And, as I said before sonic crystals they used either for attenuating sound or for bending of the sound waves, where they act as a very strong reflector and they can reflect the sound waves and they work on 2 main principles. The first principle is the classical wave spectral gap in structures with periodic variations in elastic properties.

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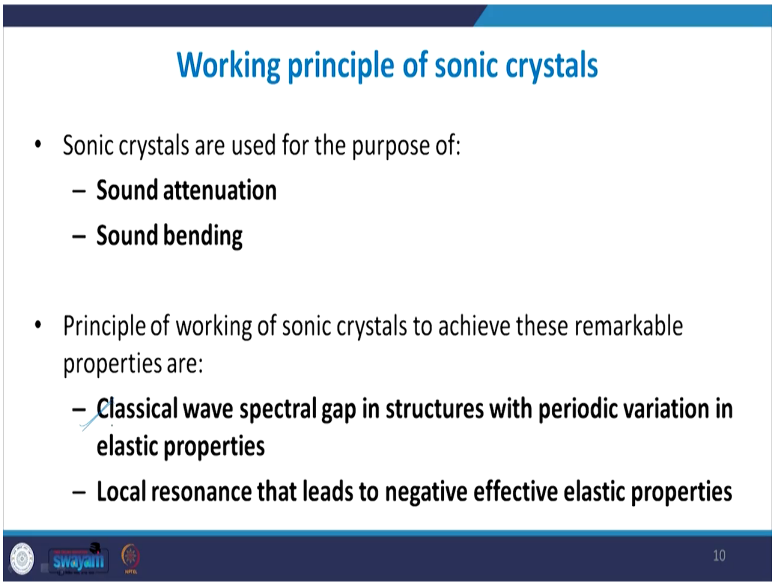

So, as I told you sonic crystals are what they are periodic array of sonic scatters in some fluid medium. So, they behave as if they are a periodic structure and they are elastic properties which means, they are bulk modulus and density keep varying periodically in the different directions.

So, in such kind of structures ah typical frequency gap is created and we will study about this principle in today's lecture. The second principle about which they work is that at certain frequencies local resonance can be set up, which can lead to negative effective elastic properties.

And, we already know what happens when we have a negative density or a negative bulk modulus the sound attenuation happens or the propagation stops. So, these are the two principles. So, let us go through the first principle which is the wave spectral gap or the formation of band gap in periodic structures.

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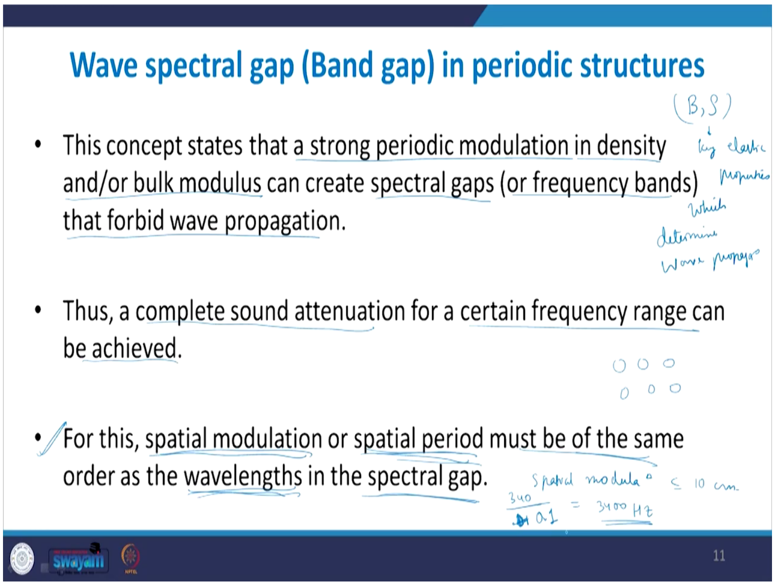

So, in this concept so, what does it state is that, if you have a strong periodic modulation in either the density or the bulk modulus. So, we know that for acoustic waves this bulk modulus and the density are the key elastic properties, which determine wave propagation.

So, if we are able to create a structure where this B value. Either B value a row value or sometimes both, they vary periodically in the structure, then that structure can create some spectral gaps of frequency bands and in those frequency bands no wave propagates to the structure. So, a complete sound attenuation way at a certain frequency range can be achieved and for this to happen.

One of the key condition and I will explain to you why this happens, we will study about Bloch's theorem next, but for this concept to be valid that in the periodic structures due to the periodicity or the periodic modulation of B or rho there is a we get some frequency bands within which no wave propagation is taking place. But for this concept to be through the spatial modulation or the spatial period, it must be of the same order of magnitude as the wavelength in the spectral gaps.

So, this is and this will become more clear later, when we study about the limitations of the sonic crystals, but what it means is that let us say we have a sonic crystal. Let us say the it is made up of aluminum cylinders and the diameter is 5 centimeters. And, they are spaced apart. So, the distance between the 2 centre's is let us say 10 centimeters.

So, here all the dimensions are like or the order of 5 centimeters, 10 centimeters and so on. So, in that case this kind of array of sonic crystal will only be able to reduce the, it will only work. So, here a gap between ah frequency gap can only be created in the order of it is wavelength. So, let us say if the spatial modulation, which is which means that the dimension of the diameter of this aluminum cylinder and the spacing between the aluminum cylinder. So, we have some 2 D aluminum cylinders arranged in a certain way and they are of the order of 10 centimeters.

So, what is the frequency corresponding to a wavelength of 110 centimeter? It is going to be 340 divided by 0.1 sorry 0.0 it will be 0.1, so it will be 3400 Hertz. So, which means that this structure is no good for frequencies of the order less than 3400 Hertz. So, the gap will only be created when the wavelength is of the same order as the dimensions of periodicity within the arrangement of the sonic crystal. Now, how does this happen, why do periodic modulation lead to this frequency gap? So, this is explained by a theorem called as the Bloch's theorem.

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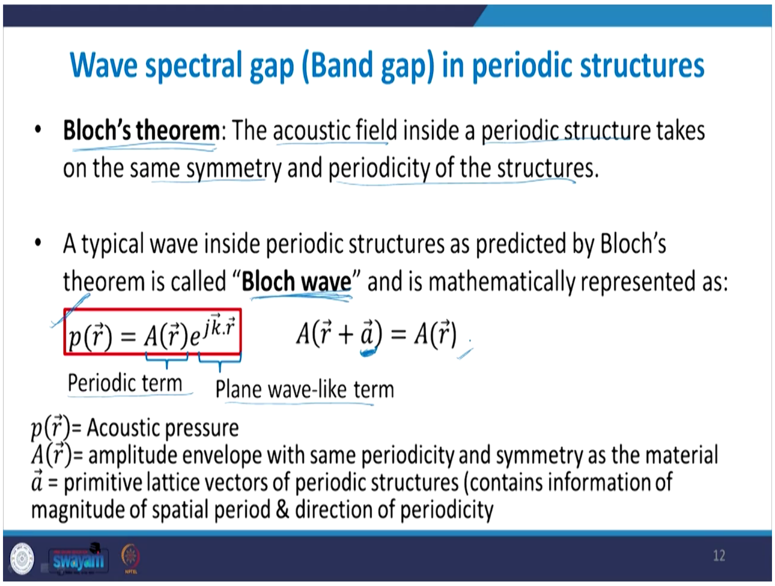

So, in the Bloch's theorem what happens? So, here the acoustic this theorem states and the derivation of this theorem is out of course, for this particular it is out of scope in this particular course. So, I am directly stating what is the Bloch's theorem and it is taken from electromagnetic theory. So, here whatever acoustic field is generated in a periodic structure, it will take the same symmetry and periodicity as that of the structure itself.

So, if we have some periodic structure, then the waves that are created in these periodic structures are also called as the Bloch waves, because they are governed by in Bloch's theorem. And, the typical wave form is this is the typical wave form in the equation of this, which means that. So, here the acoustic pressure is some function of amplitude into e to the power j k into r. So, this is the term which is similar to a plane wave like term, but the amplitude term is a periodic term. So, amplitude values periodically.

So, let us say we have a sonic crystal and the lattice constant or the distance between, this is the lattice vector; the lattice vector can give you the idea of two different quantities. The first one is that if you take the mod of lattice vector, what you will get is the magnitude of the distance between any 2 scatterers. So, the distance between any 2 adjacent centers of the scatterers. So, that will be the magnitude. And, the direction of the lattice vector gives you the direction in which there is periodicity.

So, you get both the magnitude between the you can say the magnitude of the spacing between the scatterers and the spacing here being the distance between the centers of the scatterer, and it can also give you what is the direction of periodicity. So, when so, this amplitude is a function of this lattice constant. So, which means that it keeps repeating with the same value so, this is a periodic variation.

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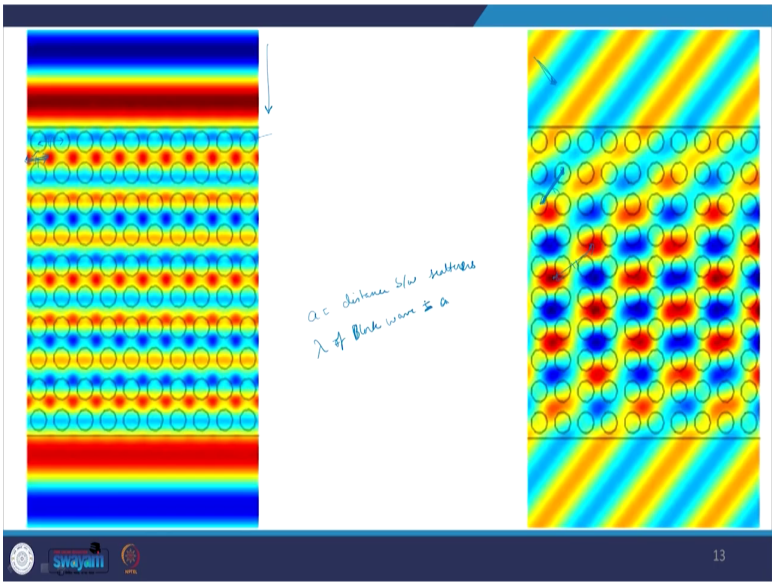

So, if I give you 2 such diagrams. So, let us say a plane wave front is incident and this is the arrangement of the sonic crystals. So, what has been found this is a simulation study and it is found that, the wave pattern it has similar nature or similar periodicity as the periodicity of the crystals. So, as you can see the distance between this and this will be same as the distance between this and this. So, this and this will be same, so that is what is meant by it. And, similarly here we have a plane wave front incident in this direction. So, in this direction this is the periodicity. So, they this is the; this is the periodicity or this is the value after which the structure is repeating itself in this direction.

Then as you see here in the typical wave pattern this distance and this distance are typically going to be the same. So, almost the wave will take the same type of periodicity or the same type of wave length as the distance between the scatterers. So, if a equals to distance between scatterers in a direction, then the lambda in that direction will be the lambda of the Bloch wave created in that structure will be same as whatever is the periodicity or the distance between the scatterers.

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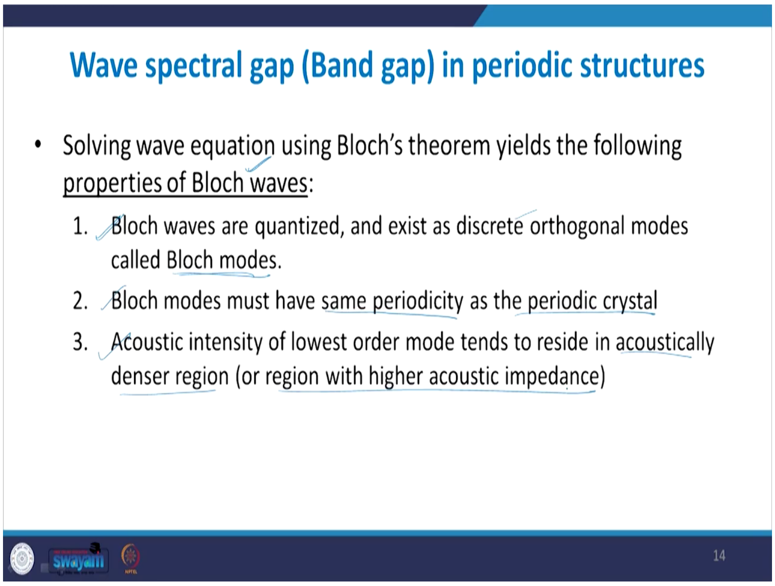

So, some of the properties that of these Bloch waves are that first of all these Bloch waves are quantized and they exist as discrete orthogonal modes, and this called as Bloch modes. And, they must have the same periodicity as the periodic crystal this is what I already discussed and another property is that the acoustic intensity of the lowest order mode, it must reside in the acoustically denser region or the region with higher acoustic impedance. So, based on these properties how do we say that there will be a band gap?

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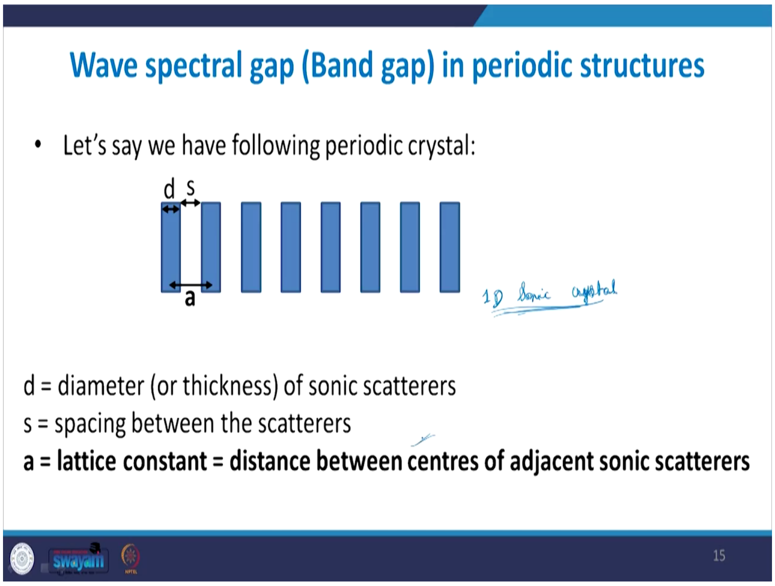

So, let us say we have a periodic this is a one dimensional sonic crystal. So, this is a 1D sonic crystal, this thing is a 1D sonic crystal and this is the thickness or the diameter for a sonic scatter, this is the spacing and here the lattice constant is actually the distance between the centers of the 2 adjacent scatterers, this is the lattice constant the weight is defined.

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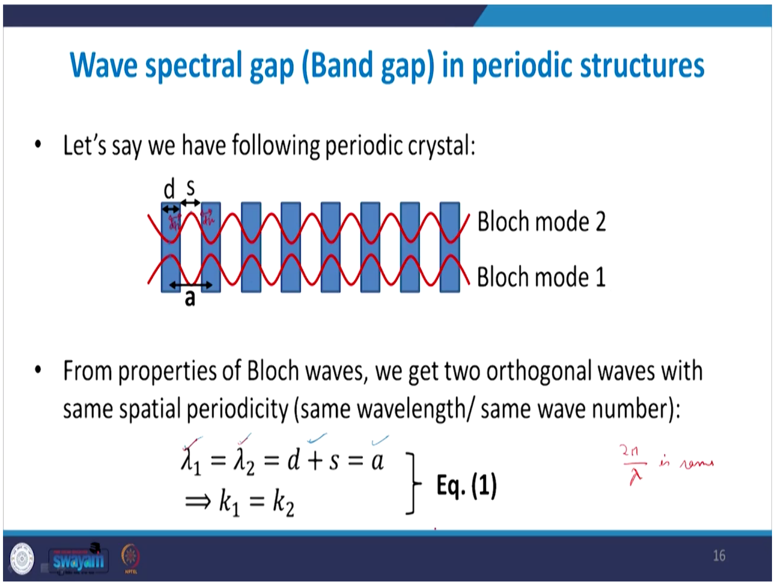

So, let us say the mode one is created now we know that the property of the Bloch wave is that the lowest order mode or the mode 1, it must have it is it must decide the nodes and antinodes of which or the maximum of it must reside in the acoustically denser region. So, what you will see here is that, the points are in the scatterers. So, the maxima occurs at the center of the scatterers. Then, we have a Bloch mode 2 and because the 2 modes are going to be orthogonal to each other. So, this here minima happens here and maxima happens in the lighter region.

Now, if you see here, then the then any wave that is created in this structure it will have the same wavelength; the same the same spatial periodicity as the periodicity of this structure. So, the wavelength will be the same for both the waves and it will be given by the periodicity of the structure. So, if you see here the structure repeats itself, after every a units, which can also be given as. If, this is d by 2; this is d by 2.

So, d by 2 plus s plus d by 2 which gives you capital, which gives you d plus s ok. This is so, this is d by 2, this is d by 2. So, d by 2 plus s plus D by 2 is the distance after which the periodicity repeats or the pattern repeats. So, the wavelengths they will be same as this particular thing. So, the wave lengths of these Bloch waves will be same. So, which means that 2 pi by lambda is same because lambda is same, so the wave number is going to be same.

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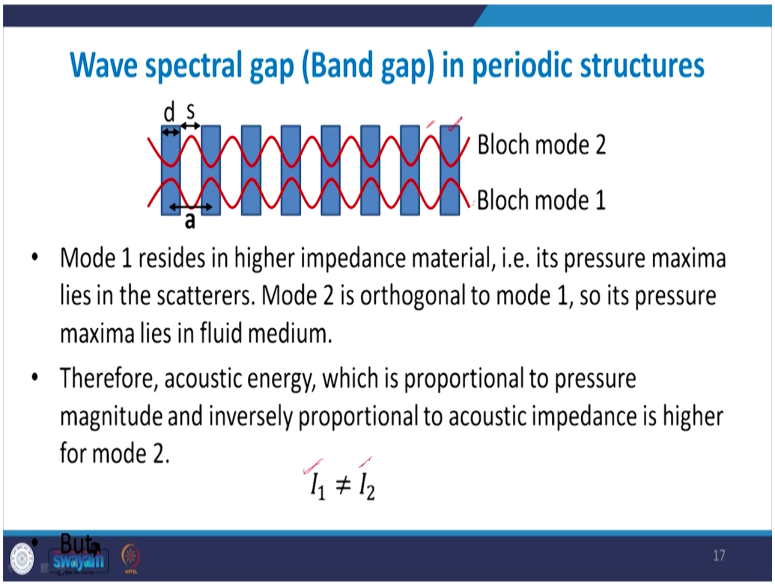

Now, as you see I already explained to you the why maxima happens in here dense region and here in the thinner region. Now, we know that the acoustic energy it is proportional to the amplitude of the wave, but it is inversely proportional to the impedance. And, here the scatterer has a much higher acoustic impedance compared to the thin fluid medium. So, as you can see here p is maximum, but in that case the impedance is very very high.

So, p square so, in that case the overall acoustic intensity for this one the energy will be lower compared to the energy of the higher mode. So, therefore, the intensity of the 2 modes are going to be different.

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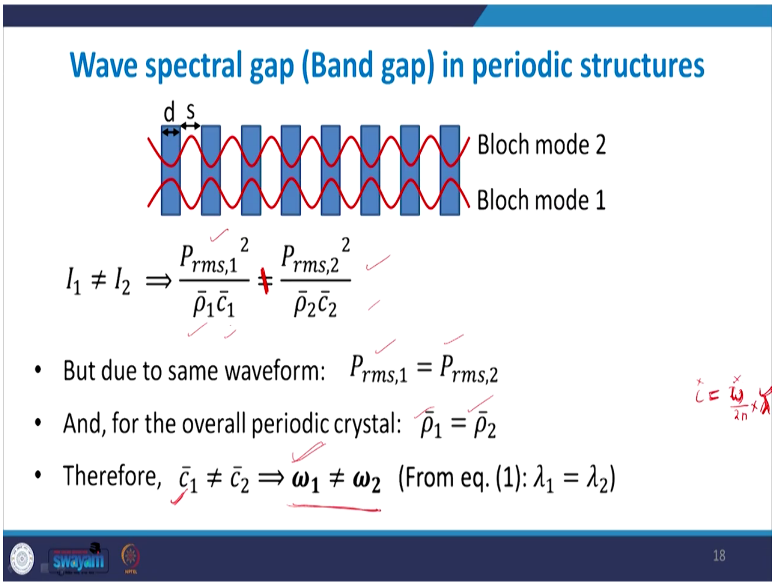

So, when the intensity of the 2 modes are different. So, which means that if we take, if we average out we takes the average rms quantities, then it can be represented as P rms 1 square by rho 1 c 1, here these are the average density and the average speed of sound. So, these are not going to be same.

So, when they are not same which means that, now for the because it is the same wave and it has the same wave form the P rms is the same. And, because for this if you take this arrangement or the crystal and you find out the average density. So, both waves are occurring in the same periodic structure. So, the average density is also the same, but the energy modes are different. So, what it means is that; that this is same this is same which means that, the speed of sound in the 2 for the 2 waves must be different.

So, the speed of sound of the 2 waves must be different, which means the frequency of the 2 sounds must be different, why because c is equal to f into lambda f is omega by 2 pi into lambda frequency into wavelength lambda is same, but c is different. So, which means omega must be different.

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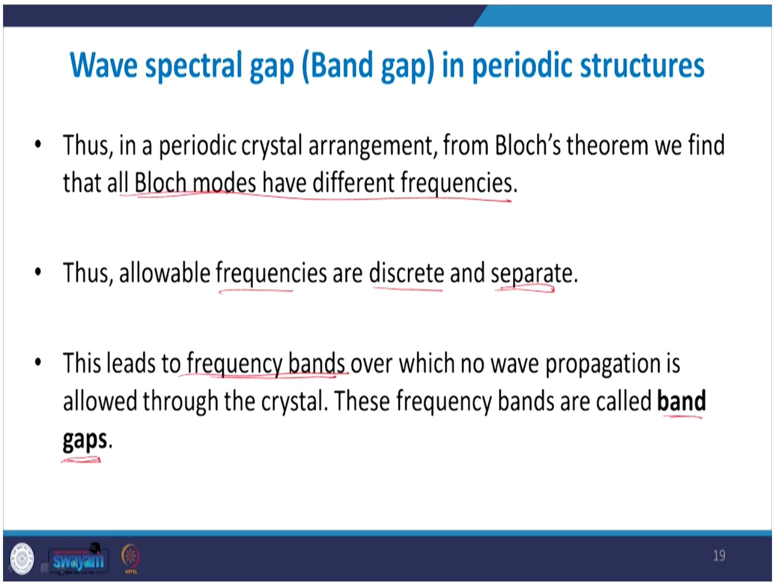

So, what we get here is that for this for all this theorem to be valid and the properties to be valid all the Bloch modes they must have different frequencies. So, whenever any wave is created the frequency of one will always be different than the frequency of the second wave and so on.

So, the frequencies are always discrete and separate and hence there is always some gap. And, these gaps in the frequencies are called as the frequency bands and it is those small gaps over which there is no wave propagation and that is called as the band gaps. So, now, that you have understood the principle of band gap we will study about local resonance in the next class.

Thank you.