Acoustic Materials and Metamaterials
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Lecture – 37
Principle of Working of Sonic Crystals-2

Welcome to lecture number 37 in this series on Acoustic Materials and Metamaterials. So, today we will continue our discussion on the Principle of Working of Sonic Crystals. So, so far we have studied why a periodic structure creates a frequency band gap or a gap in the frequencies where no wave propagation takes place and that is governed by the Bloch's theorem.

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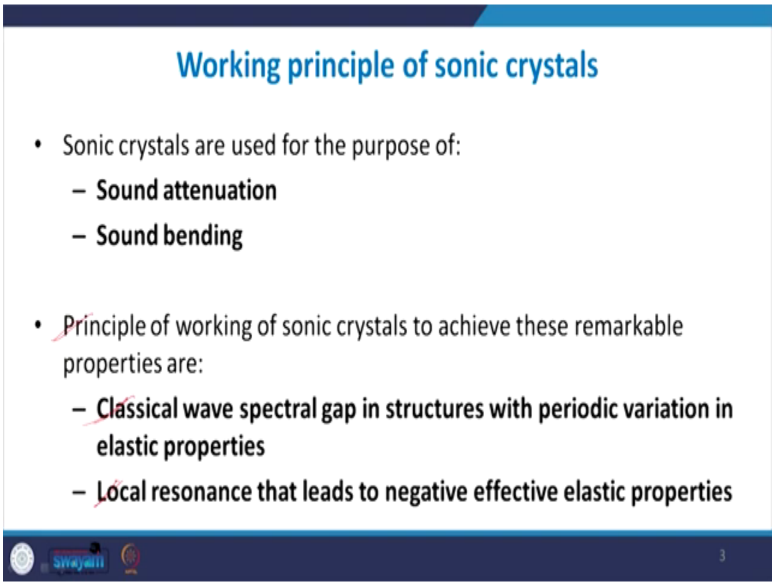

So, to reiterate the principle of working is creation of this classical wave spectral gap which we also called as band gap and then the second one is the local resonance which leads to a negative effective elastic properties.

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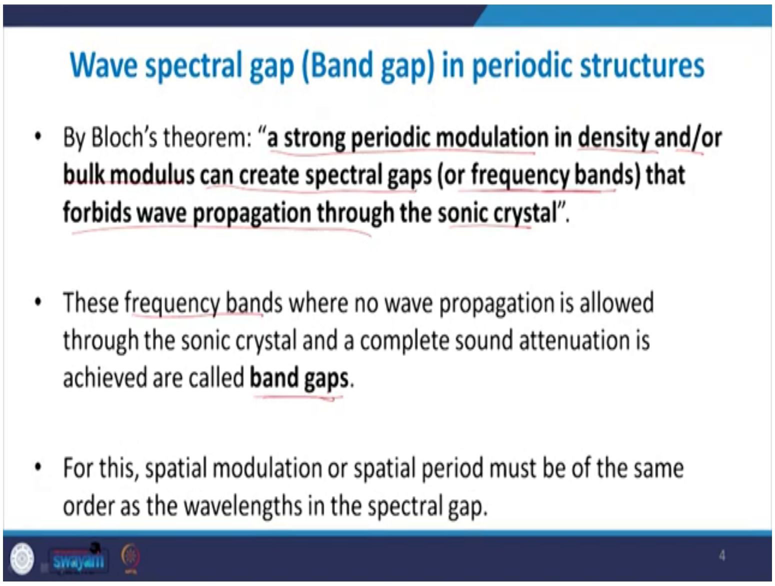

To just to summarize what is Bloch's theorem; let me it states that a strong periodic modulation either in the density or in the bulk modulus. So, in any one of the key parameters for an acoustic wave, it can create spectral gap. So, it can create some frequency bands with in which there will be no way wave propagation through the sonic crystal.

So, this is how within certain frequency ranges the sonic crystals they are able to attenuate the sounds exponentially or a heavy attenuation takes place because they are governed by the Bloch's theorem. And these frequency bands through which no wave propagation is allowed where the complete sound attenuation takes place are called as the band gaps and one important condition here is that the spatial dimensions of the sonic crystals. So, whether it is the diameter of the sonic crystal or the thickness of the sonic crystal or the spacing between the sonic crystal, the lattice constant etcetera all these dimensions they have to be of the same order as the wavelength where we want to create the band gap.

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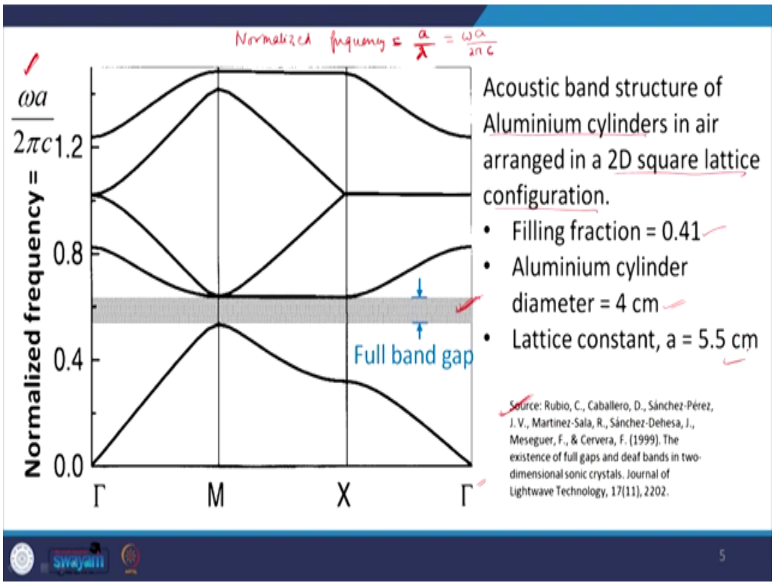

So, let us see a typical figures of what a band diagram looks like. So, this is the typical band diagram this as we; as I have already discussed is the key points of symmetry between in the IBZ of the crystal. So, these are the perimeter of the IBZ and this is this scale is a linearized frequency.

So, you can either you represented it as normal frequency as, but the most common convention is to represent it as all normalized frequency. So, normalized frequency is what normalized frequency is; a by lambda. So, whatever is the frequency like; so representing the wavelength in terms of the lattice constant, so this is the normalized frequency. So, you can replace lambda by with omega and c terms. So, what you get is it is going to be c by f; c by f an f is omega by 2 pi; so this becomes the normalized frequency. So, the vertical axis is omega a by 2 pi c and the horizontal axis is the perimeter of the IBZ.

So, in this diagram what you find is that here if you look at this shaded zone. So, between this and this frequency no wave propagation is taking place. So, you can see there are no lines or no graphs; so this is a full band gap. So, this is the gap, this is the gap in the frequency where propagation does not take place. So, when you have a look at these diagrams; you can always predict what are the frequencies within which a particular sonic crystal will attenuate sounds. So it will attenuate sounds in the frequency range where there is a band gap.

So, this is taken from this particular source and the type of sonic crystals that we have used were aluminum cylinders; so it was 2D. So, 2D crystals; so aluminum cylinders they were arranged in air; in a 2D square lattice format and in the very first lecture on sonic crystals, I gave you some figures for this. It is a same source where you had the crystals the aluminum cylinders arranged in a square lattice format. The filling fraction is 0.41 and here diameter of the cylinder is 4 centimeters and the lattice constant is 5.5 centimeters and this is the typical band diagram for this.

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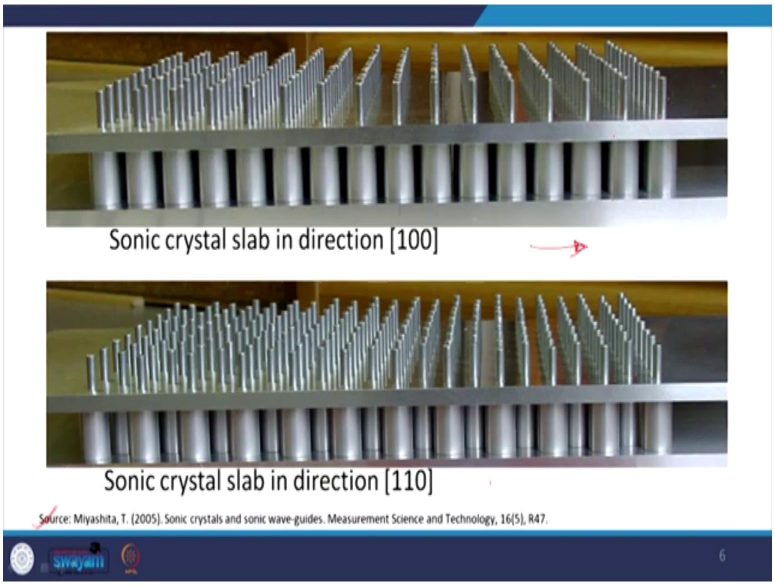

Now, I will have a look at some other kind of band diagrams. So, here this is the source I have used. So, here a sonic slab was created. So, you have a slab and within which you have this stainless steel cylinders arranged and these are the two directions. So, we have the same crystal, but once we have finding in the let us say in this 100 direction which is like a x axis direction. So, this is the direction of 10 and 110 could be diagonal direction. So, then the same crystal it is viewed from the diagonal direction; sorry 110 is going to be the vertical direction.

So, we have some arrangement of a crystal; first we have a look at what is the wave propagation along the x axis and then what is the wave propagation along the y axis because this is not a symmetrical square lattice. So, the periodicity for the two directions is different. So, therefore, the band gaps will be different; in the first case there was similar periodicity because it was a square lattice. So, we could only represent the band gap for one direction and know the same behavior will happen in the other direction also. So, here band gaps are created for both the directions.

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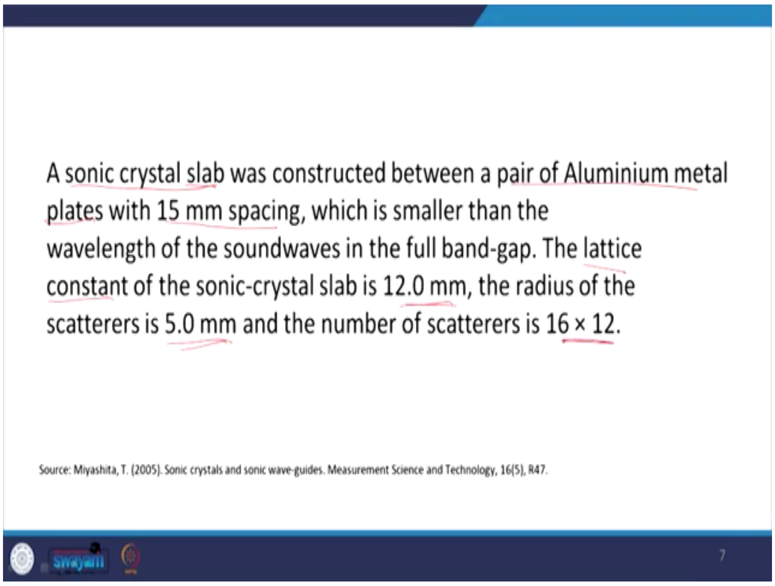

So, this is some of the specification. So, the slab was the sonic crystal slab was created between a pair of aluminum metal plates 15 millimeter spacing. So, definitely all the dimensions were kept smaller than what is the target frequency, to satisfy the Bloch's theorem. And the lattice constant is given to be 12 millimeters, the radius of the scatters is 5 millimeters and they are arranged in a 16 cross 12 format.

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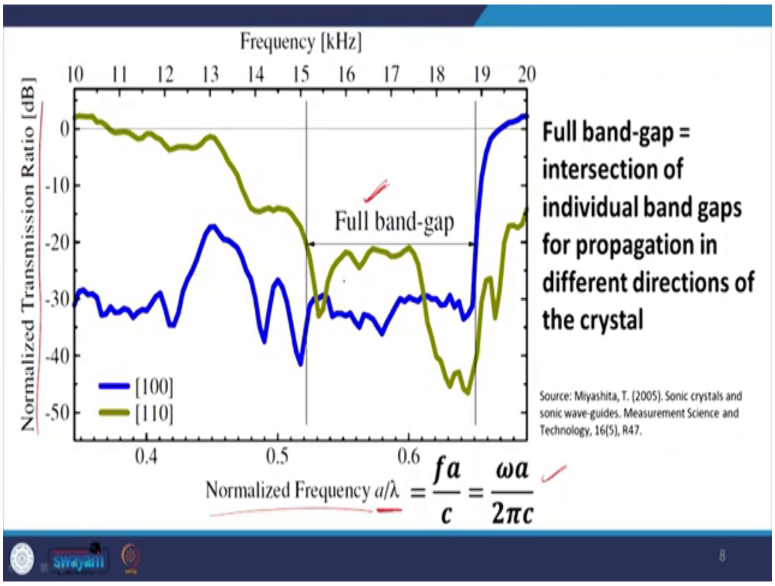

So, for the two different directions you are getting two different band diagrams and here it is represented in the reverse way. So, here what I have is the now the vertical direction gives you the normalized transmission ratio in decibels and this gives us the normalized frequency a by lambda which is omega a by 2 pi c. So, the band diagram is obtained; so; obviously, the points where this transmission ratio becomes takes a very low value will be the points where the crystal is not allowing any transmission. So, these are those points; so the region has been identified and the common region is this, so this becomes the full band gap.

So, as you see here for this crystal in different directions we are getting different band diagrams. So, we take an intersection to find out what is that intersection to find out what will be the full band gap or the gap within which in any direction you point the acoustics wave, there will be no wave propagation and this is the full band gap. So, this can be found as the region where the transmission ratio becomes reaches minima. So, this is the region where it is very less minus 20 and so, beyond that and that region is selected and the intersection is taken to get the full band gap. Now, what is the limitation of having sonic crystals like this?

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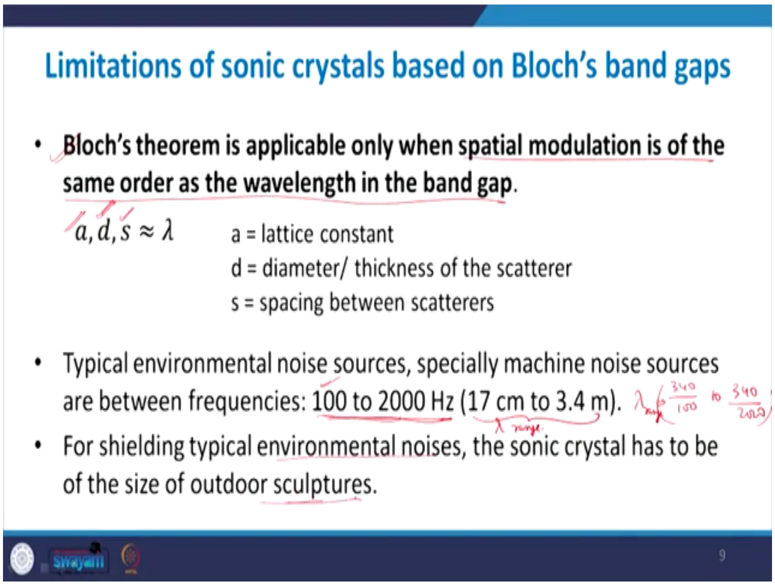

So, now we have; so here if you just create such crystals then they can give you band gaps and they can give you sound attenuation in that frequency. So, what is the' what could be the limitations likes.

So, the main limitation is that such theorem is only applicable when the spatial modulation is of the same order as the wavelength in the band gap. So, which means that all these values; the lattice constant, the thickness of the sonic scatterer: if it is spherical sonic scatterer it becomes it the diameter of the sonic scatterer, if it is a some slab like structure it becomes a thickness of the scatterer. So, all the dimensions involved as well as this spacing between these structures.

So, all these dimensions they have to be of the order of lambda where a gap is created; it cannot be created in.

So, in that case; however, we know that most of the environmental noise sources specially the machinery noise; they are low frequency noises. And as I had a explained to you in this entire course is that a main limitation of tradition materials is that; they are not able to attenuate the sounds in low frequency region. And therefore, the frequency region typically 100 to 1000 Hertz is considered to be the most critical region for noise control where it is very difficult to control noise.

But if you want to controls such noises in such low frequency ranges for example, let see we want to control the noise between 100 Hertz to 2000 Hertz; then the corresponding wavelength for these frequency. So, if you do 340 by 100 to 340 by 2000; this will be the lambda range and the range of the wavelength will be what? It will be 17 centimeter to 30 3.4 meters.

So, is this is the frequency range and this is the wavelength range. So, in that case we need to create very large structures; we cannot have minute structures, we need to create some visibles structures like sculptures, the big sculptures and certain outdoor sculptures and so on. And therefore, it cannot be very convenient to be put into machinery or into some integrate deliberate parts; if you want to control the environmental noises. So, there is a size limitation; so we need a bigger size to control a smaller low frequency noise.

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So, let us say for example, this was an outdoor sculpture here that was created with the purpose to shield environmental noise. So, as you can see here, these cylinders they were of the diameter 2.9 centimeters, this is the source of the structure and the cylinders they were fixed on a 4 meter diameter platform ok. So, huge structure was created with 2.9 centimeters; not a very huge structure here ah, it was as you can see it was a 4 meter diameter sculpture with some; thin hollow rods 2.9 centimeters in diameter and what is the kind of band gap we obtained or the band diagram we obtained?

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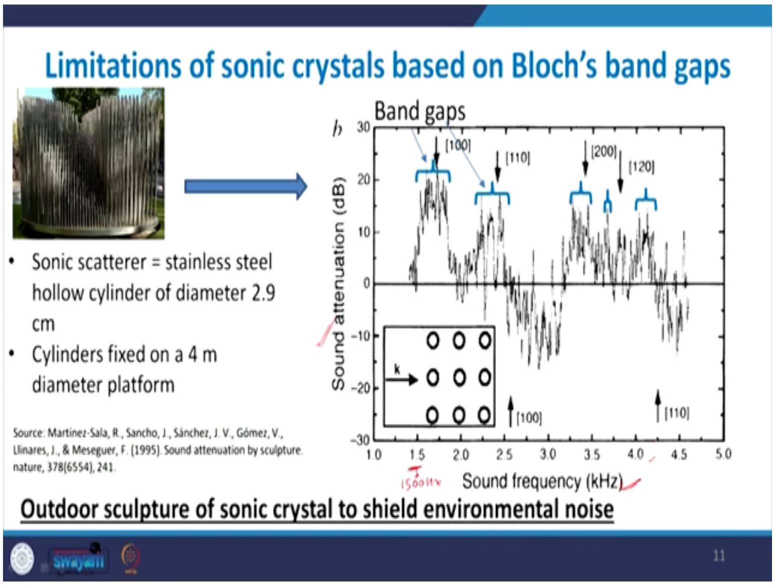

This is the typical response; so here we have the sound frequency in kilohertz and the sound attenuation in decibels.

So, what you see is that the first sound attenuation begins at 1.5 kilohertz. So, what is this value going to be a right; this means this is around 1500 Hertz; then we have something around 2000; 250 Hertz and then something around 3500 Hertz and so on. So, there is no attenuation below 1500. So, even this big sculpture is not able to reduce a sound below 1000 Hertz; it needs evens bigger sculpture to reduce such low noise.

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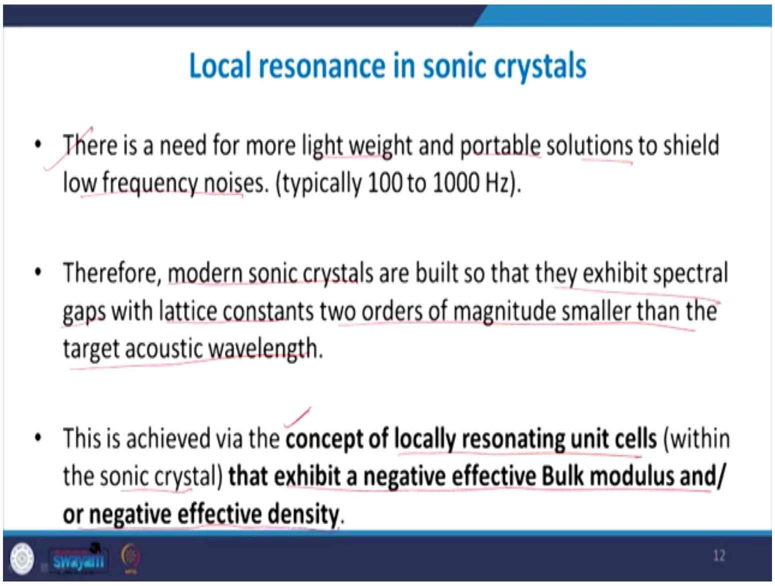

And that is why some sonic crystals have been created which operate on one additional principle. So, that will be the principle of local resonance and the creation of negative effective elastic properties.

So, let us discuss this principle the second one now. So, as we know there is a need for more light weight and portable solutions which can shield low frequency noises. So, nowadays some modern sonic crystals are been built and they exhibit spectral gaps with lattice constants even two orders of a magnitude smaller than the target acoustic wavelength. And how is this achieved? It is achieved by the concept of locally resonating unit cells within the sonic crystal and these exhibit a negative effective bulk modulus and a negative effective density.

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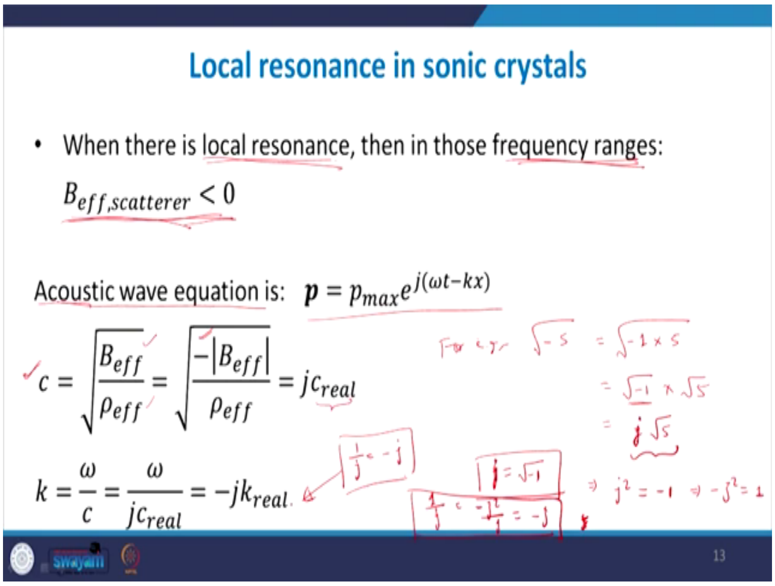

So, here now the; this is the same principle; so I will not go in to the detail again and again, but just like you studied for what happens in a negative density material. So, for example, in a membrane type material here the density, the effective density was a function of the frequency and within certain frequency ranges it took a negative value.

So, what happen when it took a negative value? Then the propagation vector became imaginary and therefore, there was no propagation in that; for that particular range where the density was negative. In the same way in the sonic crystals, when there is a local resonance in that case what we have? We have certain frequency ranges; which means we have certain frequency ranges. So, here the bulk modulus is a property of in this case here the bulk modules is the property of the incident frequency. So, under certain frequency ranges this B effective of this scatterer is negative.

So, when this is negative what happens? So, let us say this is a typical acoustic wave equation and c or the speed of sound will then be given by under root of B effective by rho effective; see it is under root of B by rho.

And since B is negative here for this scatterer; so for certain frequency range B becomes negative. So, this negative quantity can be represented as the minus sign into the; its positive value. So, for example, minus 5 can be represented as minus 1 into the positive value which is 5; so this is how I have represented this. So, you can take out this under root of minus 1. So, when you take out this under root of minus 1, this is what you are left with. So, this is just an example for example, so this quantity is what it is the j or the j is the a unit complex number, a unit imaginary number which is under root of minus 1.

So, this becomes j root of 5 which is some purely imaginary quantity. So, the same thing happens here the negative one comes out and this become j times some real quantity. So, it is become overall it becomes a purely imaginary quantity and k is omega by c. So, it becomes omega; omega being the target frequency divided by j into c real and we know that j is equal to this. So, this implies j square is equal to minus 1. So, if you do this thing here; what will be 1 upon j?

So, this means that minus of j square is equal to plus 1. So, 1 by j will be what? 1 can be represented as minus j square by j which will be minus of j. So, 1 upon j is minus of j. So, this is the very simple; this is the typical basic properties of a complex number; a basic properties of imaginary number which I am repeating here, but everybody knows this. So, 1 by j is equal to minus of j. So, if you put this property here; so, this becomes minus j times k real.

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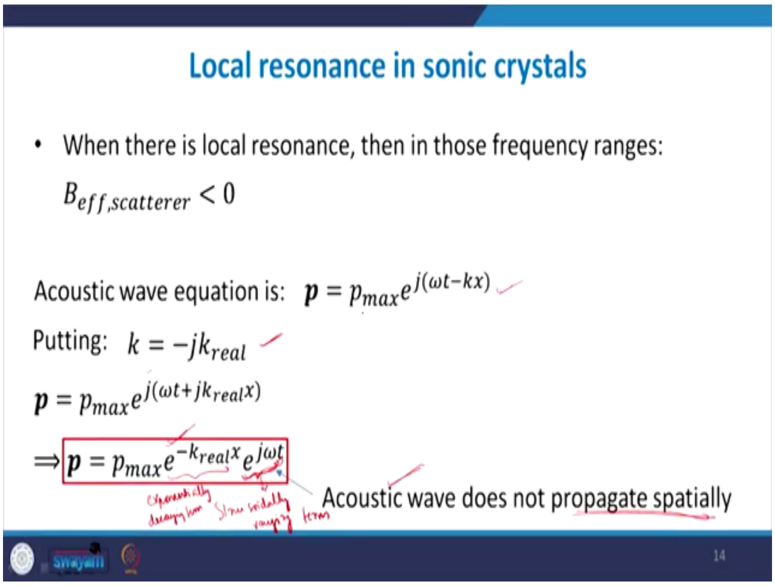

So, now when you put this value in this particular equation here; so you left with j omega t minus k; minus k becomes plus j k real putting this value and solving it what you get is; this is the equation what you get. So, j square becomes minus 1; so minus of k real, so j square becomes minus 1. So, this quantity becomes minus k real times x and then e to the power j omega t.

So, this is a sinusoidally varying term and this is a exponentially decaying term. So, this is the term which propagates over time, but this is a decaying term. So, what you get is that there is no spatial modulation or the acoustic wave does not propagates spatially because a typical propagating wave is represented in this form.

So, it should be j omega t minus k x which would be it should be a sinusoidally varying term with respect to x; which is not the case here. So, that same principle applies. So, what happens is that when there is like local resonance, then the scatterers they attain negative modulus sometimes they also attain a negative density and then because of that; there is no acoustic wave propagation.

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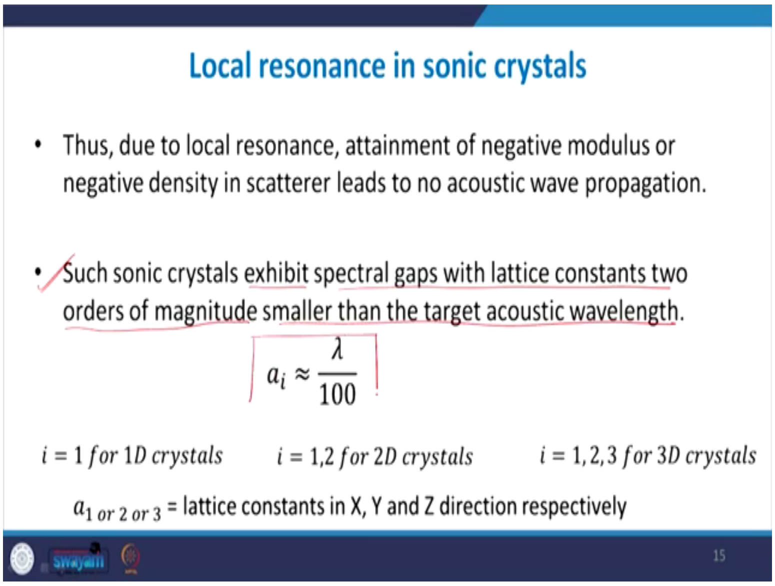

So, in such kind of sonic crystals; as I already told you some of them they exhibit spectral gaps with the lattice constants they are two orders of magnitudes smaller than the target acoustic wavelength. So, now with this additional concept e the lattice constants; so the e; so this could be; so all the all the parameters within the crystal whether it is the thickness of the material or it is the spacing between the material, it is lattice constant in the different direction. So, all of them will be an order of magnitude which is almost lambda by 100; so two orders of magnitude less.

So, now we can create even smaller structure for a low frequency noise control. So, we know that low frequency means higher lambda, but we can go divided by 100. So, we can go two orders of magnitudes smaller in dimension and yet be able to control the low frequency noise ; so this is an equation here.

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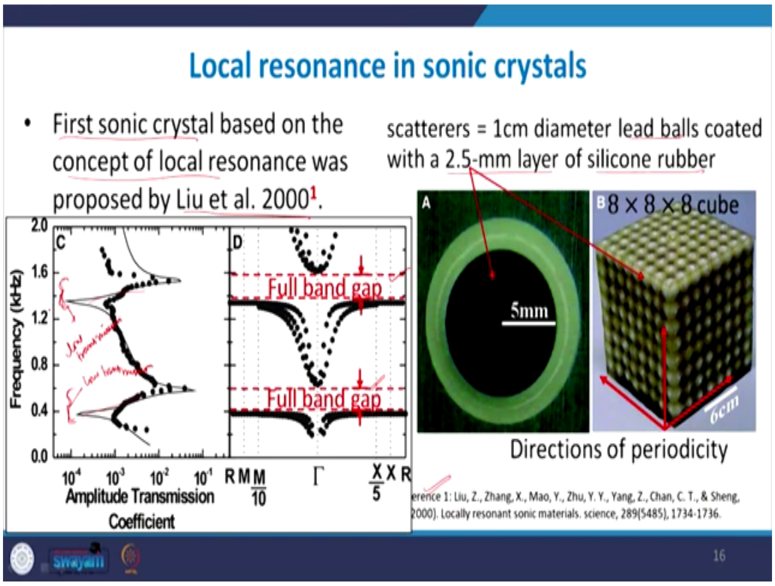

So, this is a example of the crystal which does that. So, the first sonic crystal which was based on this concept was proposed by Liu et al; this is the reference here. So, what you see is that I have already discussed this is crystal with you.

So, we had a 1 centimeter diameter lead ball which is coated with 2.5 layer of silicone rubber arranged in this cube 8 cross 8 cross 8 cube; this is the typical band diagram. So, this is the band gap here or the range of frequencies within which there is no graph or no wave propagation and this is the corresponding frequency versus acoustic transmissions, as you can see smallest transmission is happening in these regions and that is corresponding to the band gaps.

So, these are the regions where the transmission is low; the regions where transmission is low which is corresponding approximately to the band gap; low transmission ok.

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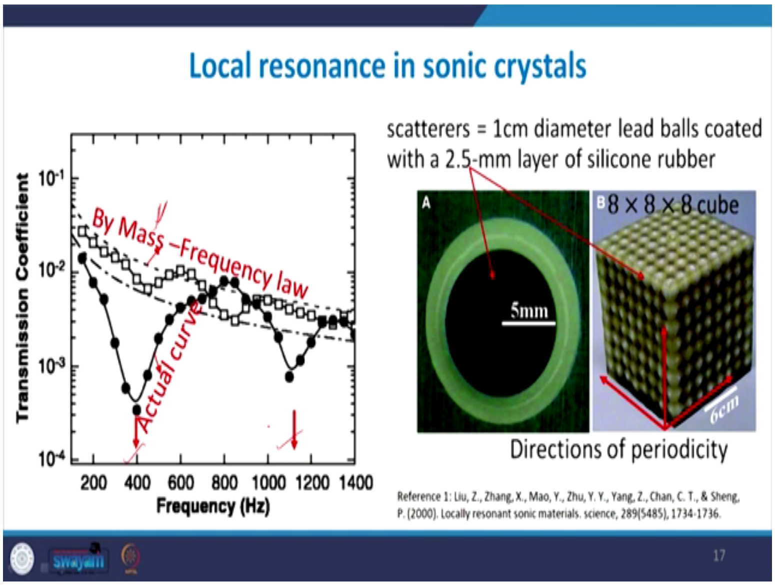

So, if you another way of representing the response of this structure is as a transmission coefficient by frequency. So, as you can see that mass frequency laws states that as the frequency increases, the noise reduction is going to increase or the transmission coefficient is going to go down because of low transmission. So, transmission coefficient should go down with the increase in the frequency.

So, this particular these kind of graphs represent what is governed by mass frequency law and this is the actual curve of the sonic crystals. So, as you can see there are two dip regions; there are two major regions within which it breaks the mass frequency law which happens here and here. And these are the regions where there is a band gap 400 and somewhere approximately around 1200 Hertz. So, these are the regions where we get a band gap.

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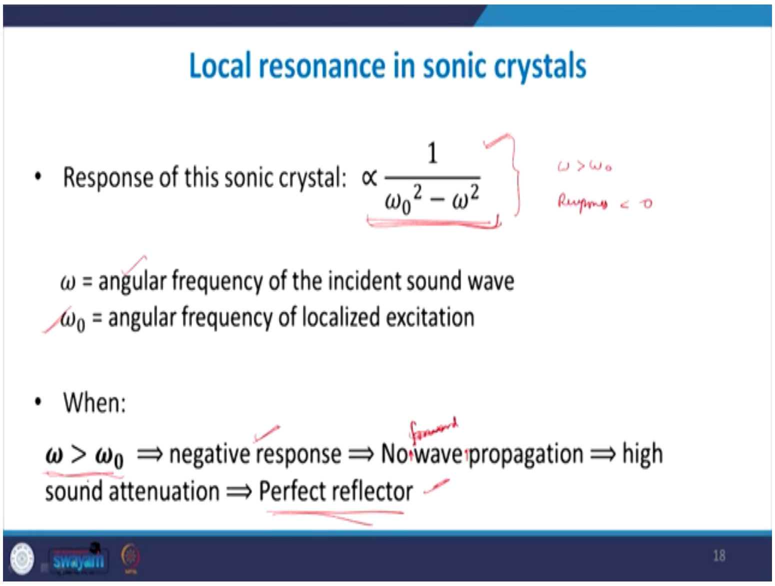

Now, the response of this particular sonic crystal was is almost directly proportional to this quantity here. So, when we solve some numerical; you will get a better idea on how the various sonic crystals work. So, here the response of this crystal is given by this expression here where this is the angular frequency of incident sound wave and this is the angular frequency of localized excitation.

So, as you can see here whenever omega becomes greater than omega naught; this expression will turn out to be negative. So, when omega is greater than omega naught, response is going to be negative. So, what do you mean by a negative response which means that you give the excitation; it does not carry forward in fact, it reflects back. So, at this region negative response which means that no further propagation, no further forward wave propagation, no wave no forward; let see I am adding the term forward here.

So, there is no forward wave propagation; thus a high sound attenuation takes place when the response is negative and it behaves as a perfect reflector. So, you can find out the zone of frequencies where; so, if you get the response function, you can find the zone where it will act as a reflector or it will act as completely bending the sound waves not allowing the waves to pass through the material. So, with this, I would like to end this lecture; this the last lecture on the principle of working of sonic crystals.

Thank you.