Acoustic Materials and Metamaterials
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Lecture – 38
Tutorial on Sonic Crystals

Welcome to lecture 38 and in this lecture we will do a Tutorial on Sonic Crystals; so, we get a better understanding. So, some sort numerical will be there for sonic crystals topic. So, let us begin with the first problem for this particular week.

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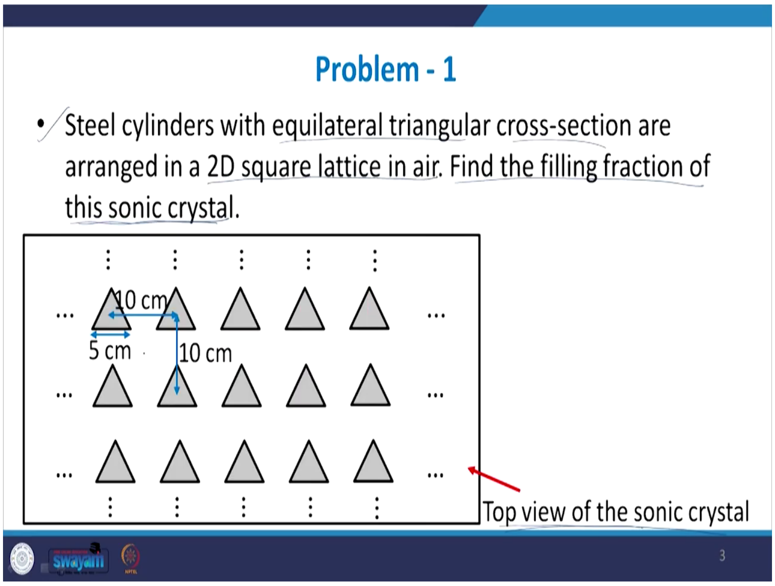

So, here we are given steel cylinders are being arranged; so, we have a steel cylinder. So, this gives our top view of the sonic crystal. So, otherwise they are cylinders and they have an equilateral triangular cross section. So, this is the cross section of these steel cylinders, they are arranged in a 2D square lattice format. So, which means that this distance here is 10 centimeter and this distance here is 10 centimeter.

So, the lattice constant which is equal to the distance between these centers is the same on the vertical and the horizontal direction and we have to find what is the filling fraction of this and sonic crystal. So, this is a pure question of geometry. So, let us solve this. So, if we take a primary repeating unit; so, we can find the primary repeating unit of this cell as this particular square here. So, I am taking this one. So, anything within this square if repeated can create the entire crystal. So, this is the primary repeating unit I have taken, otherwise you can also take this unit.

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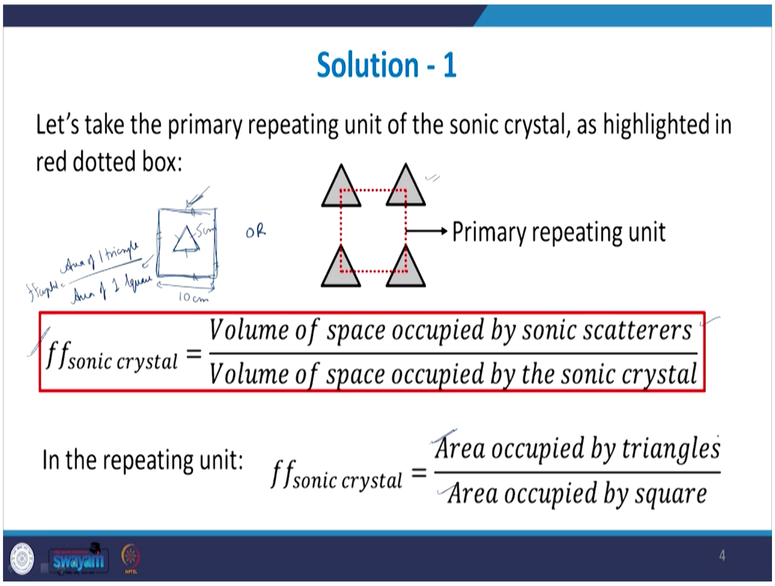

So, either this can be taken or even this unit can be taken, you can find out sort of the Wigner-Seitz cell which will give you this quantity. So, this can be taken or this can be taken.

For this one; obviously, the filling fraction can easily be found, it will be the filling fraction for sonic crystal is given by volume of space occupied by these scatterers divided by the volume of space which is occupied by the overall material. So, if you can take one primitive cell; so, it can be this or this.

So, in that how does it how can it be represented? It can be represented as whatever; so, here there is no such change in the other third dimension, only in the top view its changing. So, we can reduce it to area; so, we have area occupied by triangles divided by area occupied by the square. So, from this one what you get is that this will be area of 1 triangle divided by area of 1 square. And, we know already what is the side of the square and the triangle, the side of the square is 10 centimeter and the side of the triangle is 5 centimeters.

So, this is 10 centimeter square all sides are same and this is an equilateral triangle of 5 centimeters. So, using this you can find out what is the filling fraction. So, very easily it can be found. So, I am going to explain you the more difficult one. So, if its good if you choose the repeating unit as directly the Wigner-Seitz cell. So, it is very easy to calculate the filling fraction, but what if you took this are the unit cell?.

So, there can be many such unit cells, but only one Wigner-Seitz cell. So, let us see if you chose this one then also you will arrive at the same conclusion. So, what you will get is that; so, this is the area occupied by the triangular portion divided by the area of the square.

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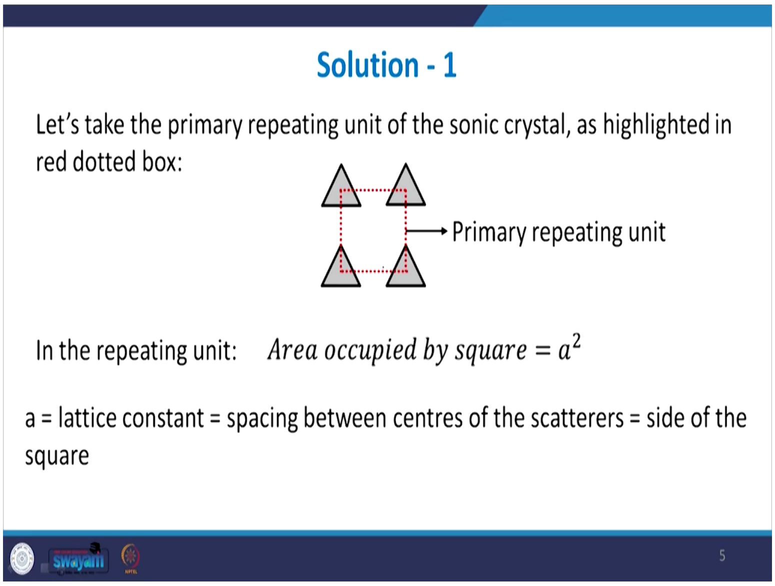

So, here area of the square is let us say a is the lattice constant which is the distance between the centers so, that becomes the side of the square. So, it becomes a square.

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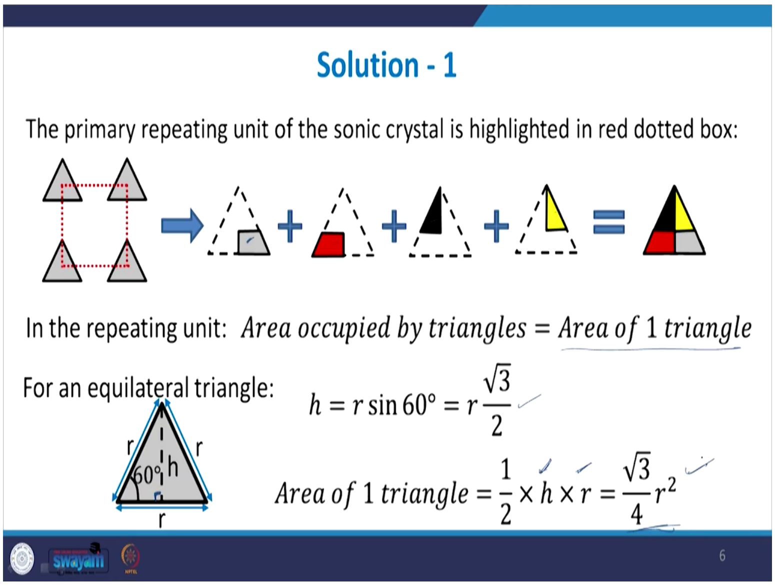

Now, if you see what is the overall area occupied by the triangle. So, here you have this, this is the first portion, second portion, third and 4th. When you join all these portions together it gives you the full triangle, this is an equilateral triangle it is passing through its centroid. So, this becomes same as the area of 1 full triangle. So, choosing either of the unit cell will give you the same results, you can either choose the Wigner-Seitz cell or you can choose a different unit cell.

So, any one repeating unit you take and you can then create your solution. So, let us find out what is the area of 1 equilateral triangle. So, let us say r is the edge length of the cross section of the scatterer. So, this is a cross section of the scatterer, let us say the edge length is r and it is so, equilateral triangle. So, here the height of the equilateral triangle is h. So, this is 90 degree and this is 60 degree.

So, what does it become? By the trigonometry relations what we get is h will be r sin theta so, it will be r into root 3 by 2 and the area of the triangle will be half into the height into the half into whatever is the height into the length of the base and the bases r here, h can be represented as r into root 3 by 2. So, using this expression what you get is root 3 by 4 r square, this becomes the area of the triangular section.

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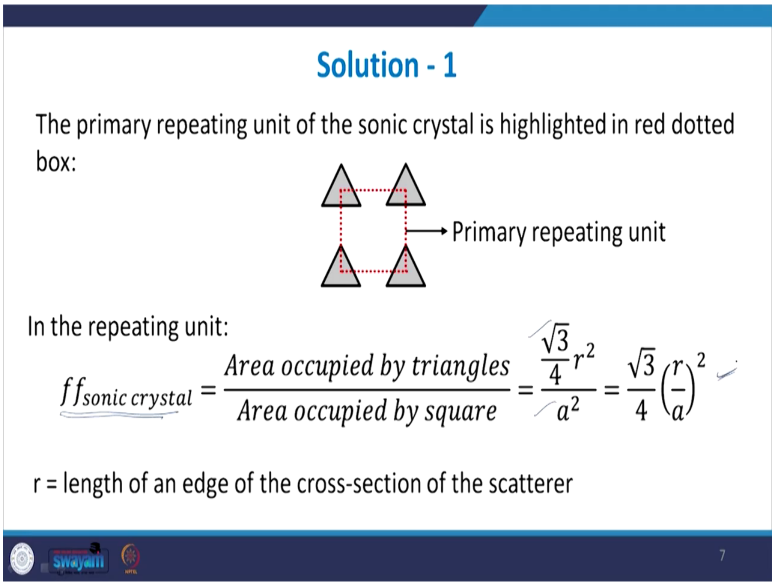

So, the overall filling fraction for the sonic crystal is what? It is the area occupied by the triangle divided by area occupied by the square within the repeating unit. So, when you do this, this is the expression you end up with; here r being the length of the edge of the cross section of the scatterer.

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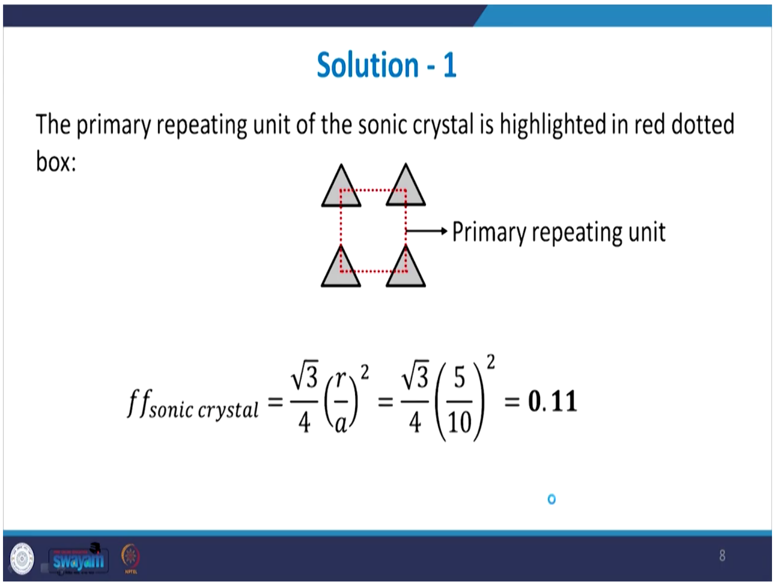

So, this is what you have end up with and we know that this r is given to us is 5 and a is 10. So, if I go back to the question just one more time. So, here this is 5 centimeters, the side length of a triangle and for the square it is 10 centimeters. So, when you put that value here in this expression it is root 3 by 4 into 5 by 10 whole square. So, this is what you get; so, this becomes the filling fraction. So, this is the end result.

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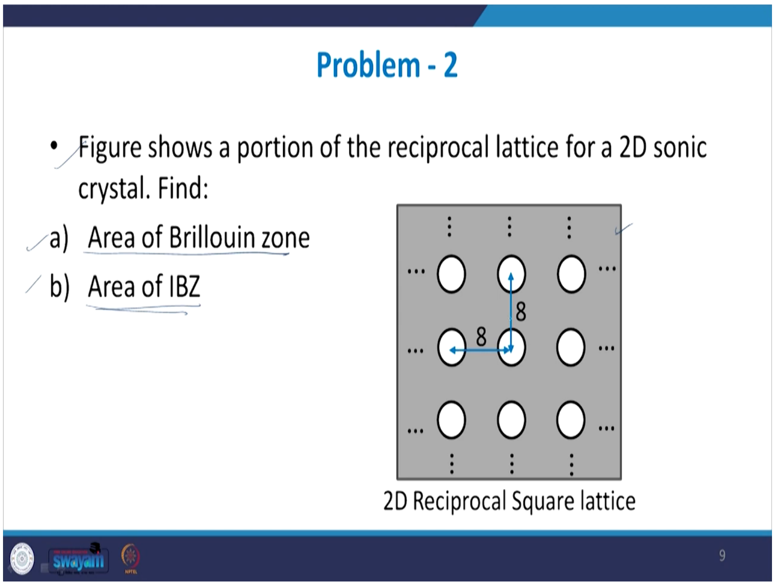

So, this was a question, now let us talk of another question here. So, here this is the question that is given to us is, here this figure shows a portion of a reciprocal lattice. So, previously we had a direct lattice; so, it was simply the what is the top view of that arrangement of sonic crystal. Now, we have a reciprocal lattice and that is given to us and as you said only a small portion is given here and the same pattern continues in all the direction.

So, it is a large square lattice, I have to find what is that what is the area of the Brillouin zone and what is the area of the IBZ. So, this is also a very easy and straight forward question. So, if you know the concept of Brillouin zone and IBZ, you can find out. So, let us say this is the reciprocal lattice which is given to us and the lattice constants in both the direction are given as 8 and 8.

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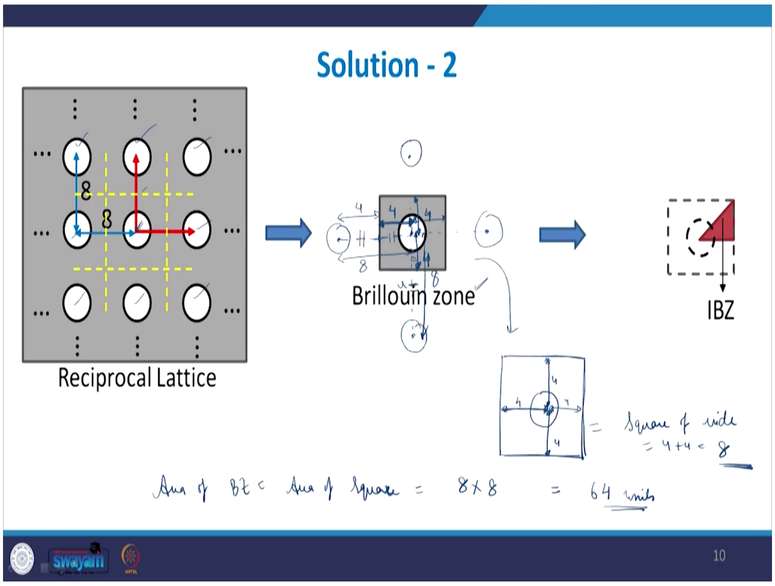

So, that is why it is a square lattice. So, you can find the Brillouin zone just the way I have explained a couple of lectures before. So, you can simply draw perpendicular bisectors; so, for this particular thing these are the neighbors. So, these are the these are those 8 neighbors surrounding this unit, this lattice, this particular lattice point. So, at this point we have these 8 neighbors, for every neighbor you can see what will be those points which are closer to actually this point compared to its neighbor. And, similarly you can keep drawing and drawing and you will end up with a square.

So, this was the same as this already has been repeated in the previous lecture so, we directly draw the Brillouin zone for the square lattice. So, this comes out to be the Brillouin zone. So, here as you can see let us find out what is the length. So, this is 8 and 8. So, what will be the length of this particular square? So, here you have the way you have drawn is this is the next neighbor, similarly this is the next neighbor, this is the next neighbor and this is the next neighbor. And, this line is sort of bisecting these lines joining the centre.

So, this length should be same as this length. So, these two lengths are same and now we know that this total length is 8 centimeters right; so, this is half of this length. So, this will be 4 centimeters and this thing from this to the centre here, this length will be 4 centimeters and in the same way this where this line is also drawn as bisecting these two lines. So, if this total length here also the total length is 8 centimeter; so, this will be 4 centimeter and this length here will be 4 centimeters.

And so on, with the same logic this length will also come out to be 4 centimeters and this upper length will also come out to be 4 centimeters. So, what you get here is that; so, this is the Brillouin zone and this is the IBZ. So, I will just draw the Brillouin zone here first. So, the Brillouin zone that you obtain is like a lattice point and a volume of space surrounding it, to create a square kind of area and the length here is 4 centimeter, 4 centimeter and 4 centimeter and this is also 4 centimeter and 4 centimeter.

So, this becomes a square of side equals to 4 plus 4 which is equal to 8 centimeter. So, it is the same side as the value of the lattice constant. So, in that case what will be the area of this Brillouin zone? It will simply be the area of this square which is going to be 8 multiplied by 8 centimeter square which is going to be. So, here I have written centimeter, but then I realize that no units has been provided to us; so, let us not put the units.

So, I have removed the units from every place. So, it is just some units; could be meters, could be centimeters; so, some same units choice has been given so, this is what we get. So, it becomes 8 into 8 which becomes 64 units and now let us find out what is the area of the IBZ or the Irreducible Brillouin Zone.

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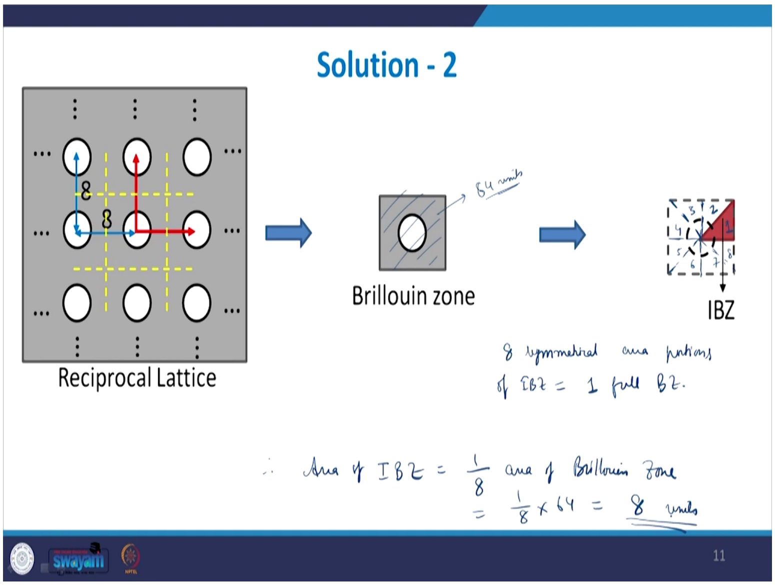

So, what we see here is that, this was the square and the area of this is 84 64 units is the total area of the square. Now, to reduce this further and remove all the symmetry an IBZ has been drawn and if you can see here, if you draw these lines like these so, this is how I have drawn the lines. So obviously, then this is 1 IBZ and this portion is symmetrical to this above portion. So, this is the second one and this portion is symmetrical to this one, this is symmetrical to this one, this is symmetrical to this, this and this.

So, we get 8 symmetrical area portions of the IBZ will make 1 full Brillouin zone. So, if we repeat this IBZ 8 times the entire Brillouin zone can be created. Therefore, the area of the IBZ will be what? It will be 1 by 8 times the area of the Brillouin zone which would be 1 by 8 into the area which is 64 which is again 8 units. So, it was a simple and straightforward question provided you know how to create Brillouin zone and irreducible Brillouin zone. So, let me just directly go to the solution.

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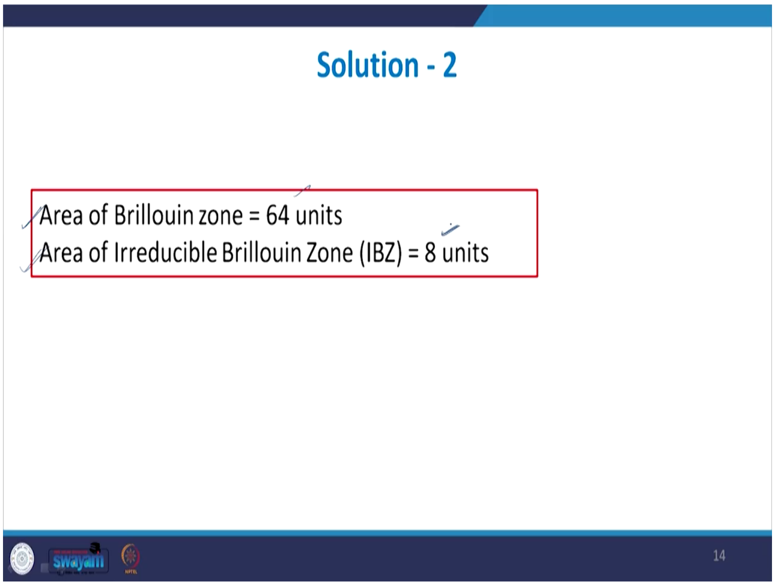

So, this is the solution: area for the Brillouin zone is 64 units and area for the irreducible Brillouin zone is 8 units. So, in this particular tutorial I am only solving the portions of sonic crystal up to the band gaps. And, then in the next class we will study a few some of the other concepts related to sonic crystal and I will solve one design question. So, for this particular class we will only solve till the Bloch's theorem.

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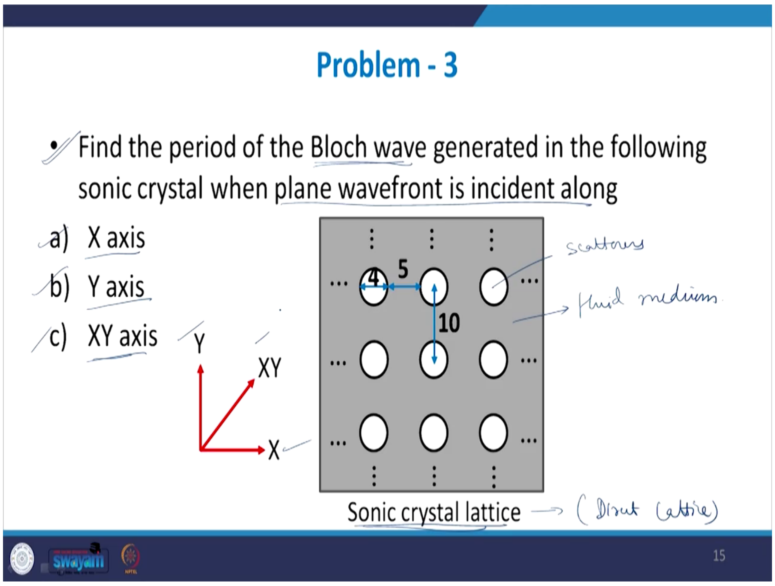

So, the third question here is given is for Bloch's theorem. So, let us say this is the question here. So, again we have some sonic crystal lattice, now this is actually a direct lattice, this is not a reciprocal lattice and I have to what; so, this is the sort of arrangement. So, here these are the various scatterers and this is the fluid medium and this is how the sonic scatterers are arranged or some metals, the metal rods are arranged.

So, what I have to find is that what is the period of the Bloch wave, that is generated or the period of the overall acoustic wave. Bloch wave is simply an acoustic wave which is governed by the Bloch's theorem. And, now we have also found out that not the wave is not only governed by the Bloch's theorem, but it is also governed by the fact whether b and rho become negative at certain frequencies. So, the overall wave can be called as a superposition of the Bloch's theorem.

And, the wave governed by a wave governed by Bloch's theorem and the wave governed by the local resonance theorem that will give you the complete wave. But, if you only take Bloch wave into account or the wave that is typically governed by Bloch's theorem, then what I have to find is what is the period of the acoustic wave that is generated when a plane wave front is incident along the X axis Y axis and the XY axis. So, three different directions are given to us, these three directions along these three directions.

So, when the wave is incident along X what will be the period of the wave? If it is incident along Y what will be the period and so on.

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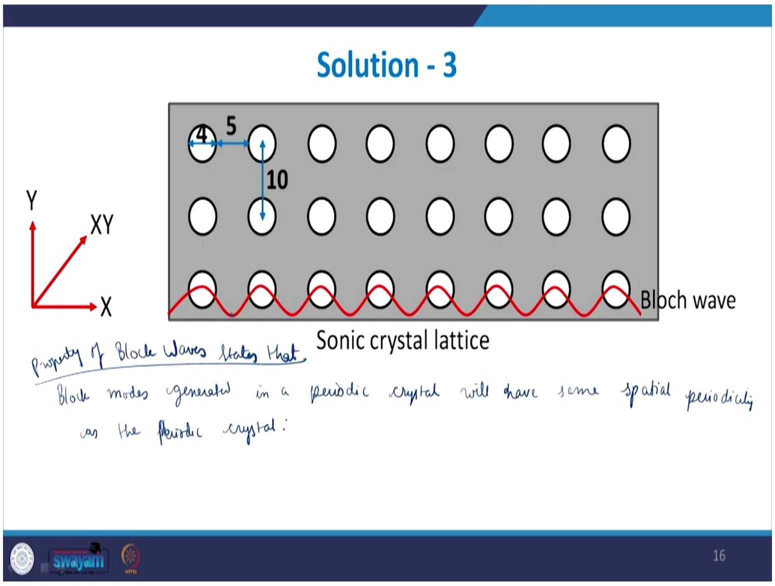

So, let us solve it here. So, here this shows a typical Bloch wave where its a first mode Bloch wave where we have the maxima occurring at the center of the scatterer and the minima occurring at the center of the spacing between the scatterers. Now, to solve this we will use one property of the Bloch wave which says that all the Bloch modes; so, the property of the Bloch wave states that the Bloch modes that are created or generated in a periodic crystal, let us say here in this case this becomes a sonic crystal, it will have same spatial periodicity.

So, the wave will repeat is repeat its pattern after the same spatial distance as the pattern in which the structure is repeating its. So, the distance after which; so, the same distance after which the structure repeats its pattern, in the same distance the wave will also repeat its pattern.

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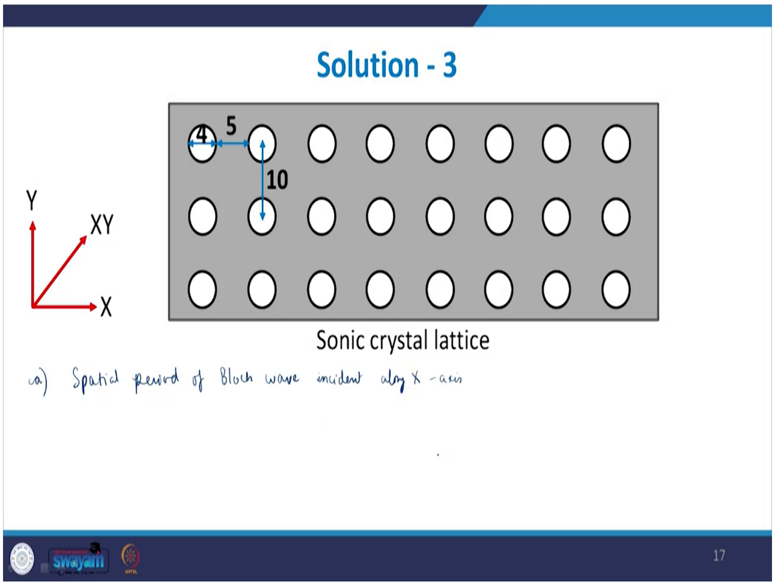

So, the periodicity of the wave and the periodicity of the structure will be the same. So, with this property let us now solve what is the period for the three directions. So, part a I have to find is what is the period when the wave is incident along the X axis, then the spatial period of the Bloch wave incident along X axis. So, when the incident wave is.

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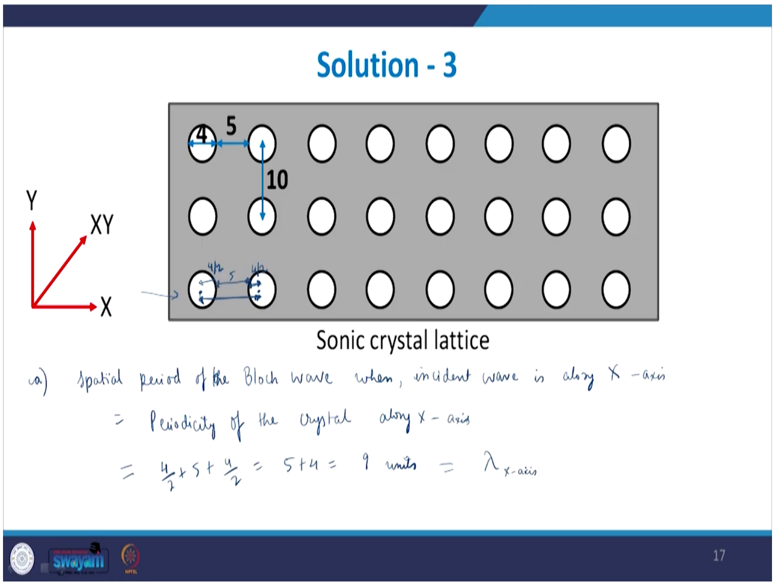

So, let me rewrite this phrase here. So, here the spatial period of the Bloch wave when incident wave, the wave that is incident on the crystal is along X axis. This period will be same as what is the periodicity of the crystal along X axis, plane wave front is incident here as we know and a plane wave front will create harmonic plane waves inside the crystal. And, the direction of propagation will be the same provided this crystal does not create bending.

So, in that case whatever is the periodicity of the crystal along X axis will give you the periodicity of the wave when the incidence happens along X axis. And, what is the periodicity of the crystal along the X axis? So, if you see here the periodicity what is the distance after which this particular arrangement repeats the pattern? So, what is the distance here? This is going to be 4 by 2, this itself is going to be so, it is a radius; so, diameter is 4.

So, this will be this distance will be 4 by 2, this will be 4 by 2 and this will be 5, this is what is given to us. So, the total periodicity or the length after which the pattern repeats is going to be 4 by 2 plus 5 plus 4 by 2 which is going to be 9 units. So, this is this length which we had to find which we found. So, this is the so, this is equals to the lambda at X axis. Now, similarly we can solve for the other two directions.

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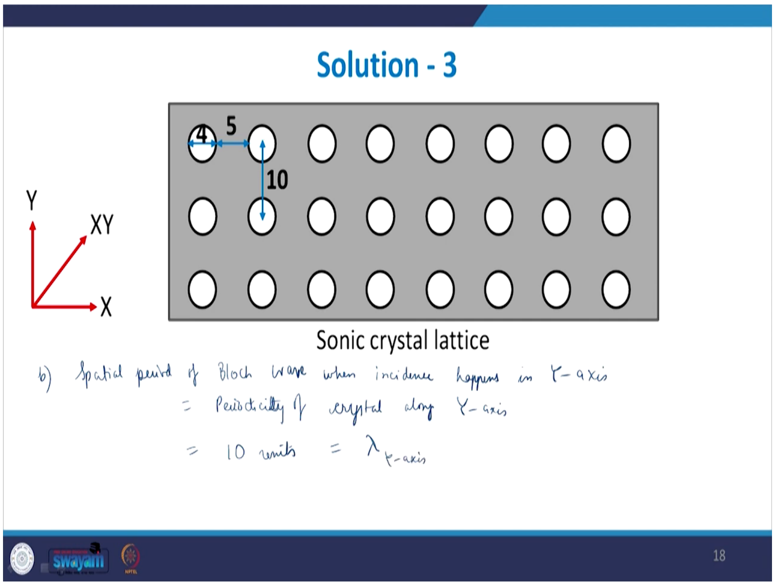

So, I will not explain more; so, directly I will write the solution. So, spatial periodicity; now let us find out what is the length after which the pattern repeats itself in the vertical direction this is directly given to you that is 10 units. So, this 10 units will give us the wave length along the Y axis, now the third one is along the XY axis.

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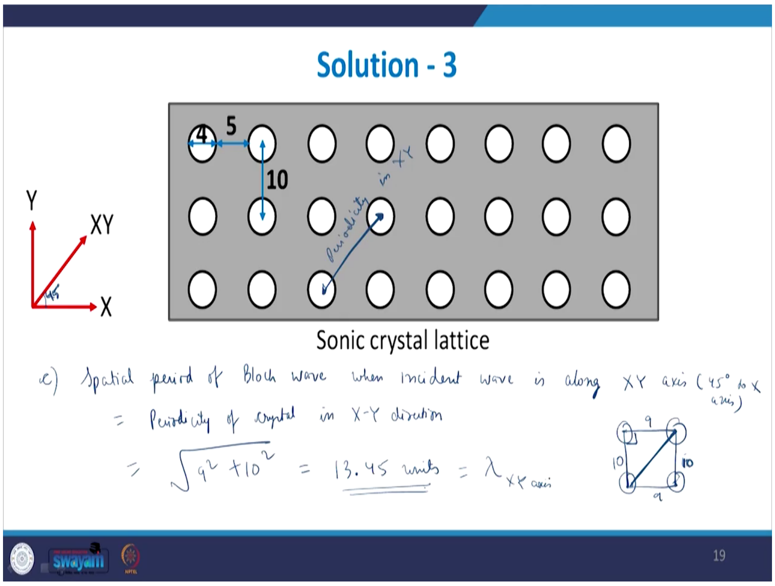

So, again what so, what I can write here is that this XY axis is actually 45 degrees to this horizontal axis here; so, this is the called as the XY axis. So, it will be equal to what is the period or the periodicity of the crystal in this particular direction and how do we solve it? So, here let me take this particular case here. So, this is how the arrangement is and this distance here is 9 and this distance is 10; so, I will just write 10 here.

So, this is like a this is the kind of the distances between the two, in the two directions and this is the and this is going to be the pattern. So, this is the distance which we have to find, this is the periodicity in XY. So, this is the length after which the pattern will repeat along this direction. So, it is going to be very easily found using the Pythagoras theorem. So, this will be under root of 9 square plus 10 square, this will give you the length of the diagonal.

So, this comes out to be 13.45 units. So, you get the length after which the wave is repeating. So, this will become the wave length along the XY axis provided the incidence happens purely along the way XY axis.

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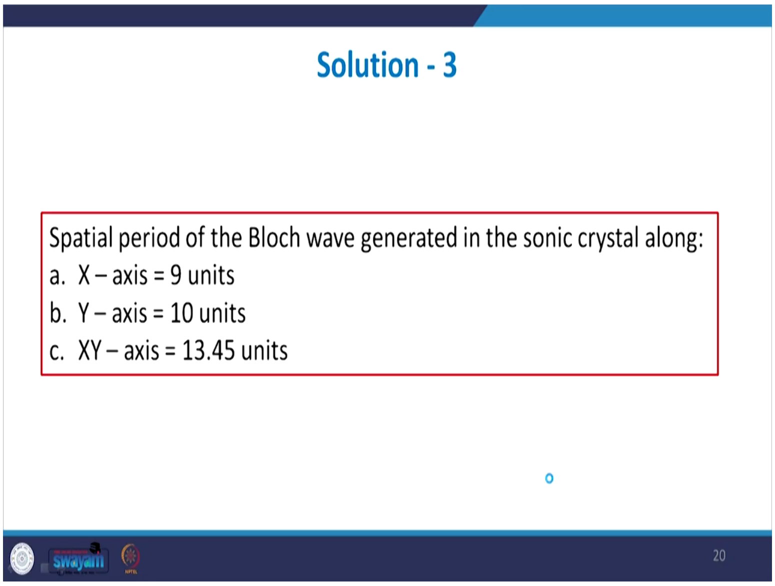

So, this is to summarize the results: spatial period of the Bloch wave in the sonic crystal along X axis is this 9 units, Y axis is 10 units and XY is 13.45 units.

So, with this I would like to end this particular lecture and see you for the next lecture.

Thank you.