Acoustic Materials and Metamaterials
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Lecture – 04
Sound Wave Propagation in Fluids-III

Welcome to the 4th lecture in the series on Acoustics Material and Metamaterials, we will continue our discussion on Sound Wave Propagation in Fluids. So, last class we discussed about a general equation for a harmonic plane wave and we arrived at a equation, and then we arrived at the equation for the velocity of such wave. So, based on the equations that we studied last time we derived last time; today, let us solve some numericals based on .that.

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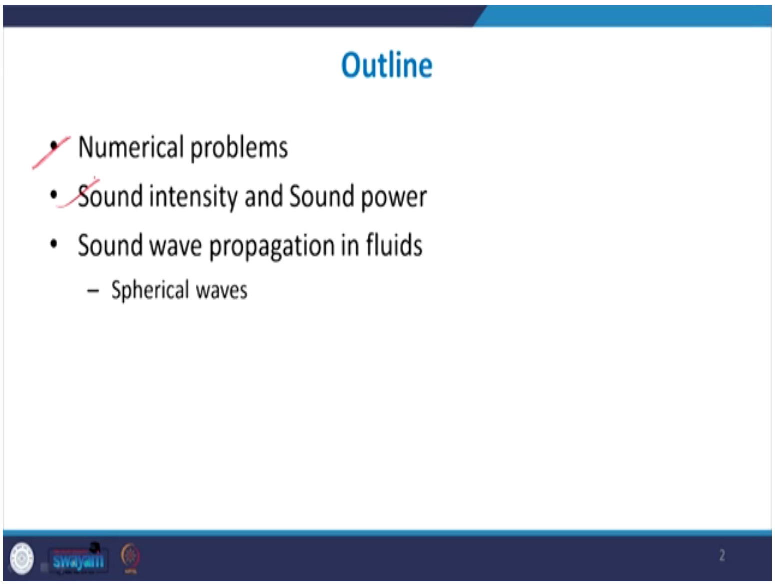

So, let us so, this is the outline for the course will solve a few numericals, then I will discuss with you two important terms which are called as sound intensity and sound power. So, these are two important quantities in acoustics and then finally, we will discuss about sound wave propagation in fluids again and the different type of wave called as the spherical waves.

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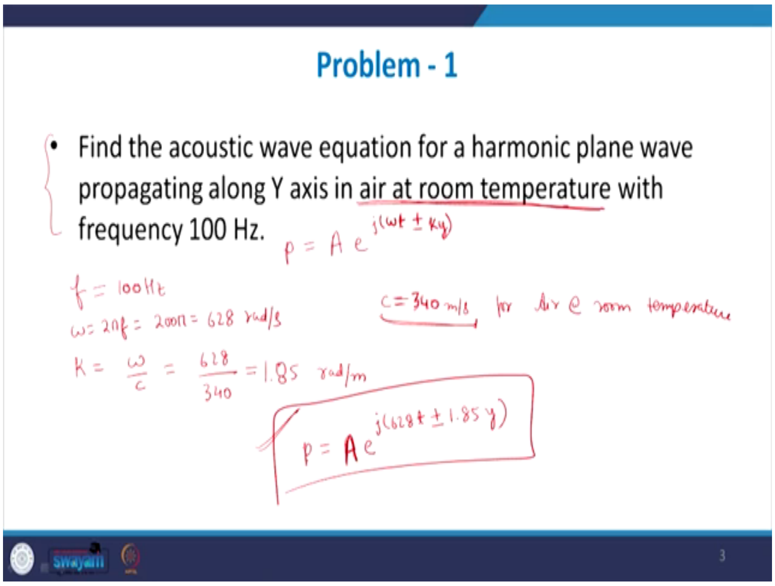

So, let us start. So, the first problem at hand is that find the acoustic wave equation for a harmonic plane wave. So, this is the question, find the acoustic wave equation for a harmonic plane wave propagating along Y axis in air at room temperature with frequency 100 Hertz. So, let us solve this.

So, let us solve this here, here f is equal to 100 Hertz. So, we know that the equation for such a wave traveling along positive Y axis if you have a harmonic wave is going to be Ae to the power j omega t plus minus k times of y. So, this is going to be the equation for a harmonic plane wave traveling along Y axis because it is not given to us whether it is traveling along positive or negative Y axis we assume a combined solution.

So, f is given to be 100 hertz. So, omega will be 2 pi f which is going to be 200 pi which comes out to be 628 radians per second and we have to find so, we need omega and we need the value of so, k we find as omega by c and it is traveling in air at room temperature, this is what is given to us. So, the value of speed of sound c is equal to 340 meters per second for air at room temperature.

So, as we know that the speed of sound is fixed for a medium. So, if you know what is this fluid medium whether it is air, molten iron, water, steam etcetera. So, what is that fluid medium? So, we can so, for every fluid medium and for a particular temperature the speed of the sound is fixed. So, when such questions are given to you usually the speed of sound will be mentioned to you, but I am here mentioning you that the speed of sound for such air is 340 meters per second.

So, we will use this value; so, what we get is 628 by 340. So, the k value comes out to be 1.85 approximately, this is going to be radians per meter. So, we have got both these values, if you put both these values the equation that we get is Ae to the power j omega is 268 times t plus minus k which is 1.8 5 times of y, no other parameter is given to us. So, we cannot find the value of this amplitude. So, we only leave the solution at this.

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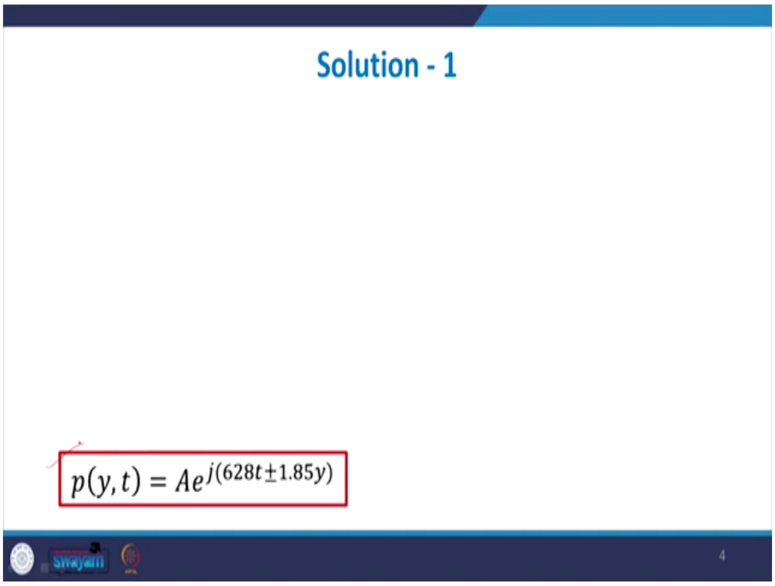

So, this is the solution we got p as a function of y, t's Ae to the power j 628 t plus minus 1.85 times of y, ok.

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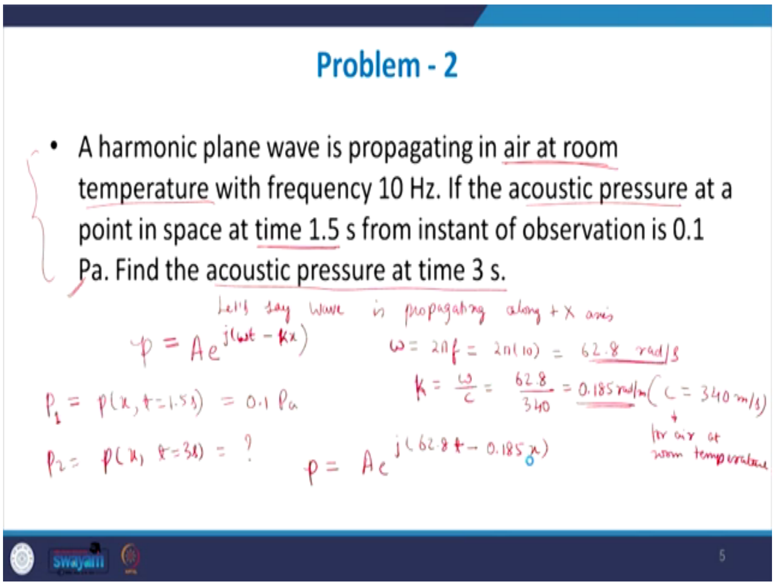

Now, let us solve another problem. A harmonic plane wave is propagating in air at room temperature. So, the same thing is given to us and the frequency is given as 10 Hertz. Now, the question also mentions a few additional details. It mentions that the acoustic pressure at a point in space.

So, here what point in space is not given to us? So, the value of x y or z is not known to us. So, we have some particular.in space and at time t equals to 1.5 seconds, we get the pressure as 0.1 Pascal's. So, we have to find the acoustic pressure at 3 seconds. So, let us assume the same function same; so, let us say here the propagation direction is not known to us. So, let us say it is propagating along let us say the wave is propagating along positive X axis.

So, you can assume any propagation direction, but anyways when you solve it will cancel out so, the direction will not matter. So, we write the equation as this omega t minus kx, or let us assume you can assume a combine solution as well and the answer will be the same, but I have assumed just one particular case. So, this is the equation I am getting. Now, it is given to us that first some instrument is measuring the acoustic pressure so, at one first measurement is at p equals to x some value of x and t equals to 1.5 seconds.

So, here omega will be found as 2 pi of f which is 2 pi into 10 Hertz so, this comes out to be 62.8 radians per second and k becomes omega by c. So, 62.8 and it is the air at room temperature I guess. So, you assume the speed as 340 meters per second it is for air at room temperature; for air at room temperature. So, you use the speed. So, the value you get is 0.185 radians per meter. So, you have found the value of k and omega with just the value of frequency.

So, if you put this equation here then this is given to you as 0.1 Pascal's and 2 2 which is equal to P at x and t equals to 3 seconds is what you have to find. So, the equation of P you are getting is Ae to the power j 62.8 times t minus 0.185 times the propagation that is x.

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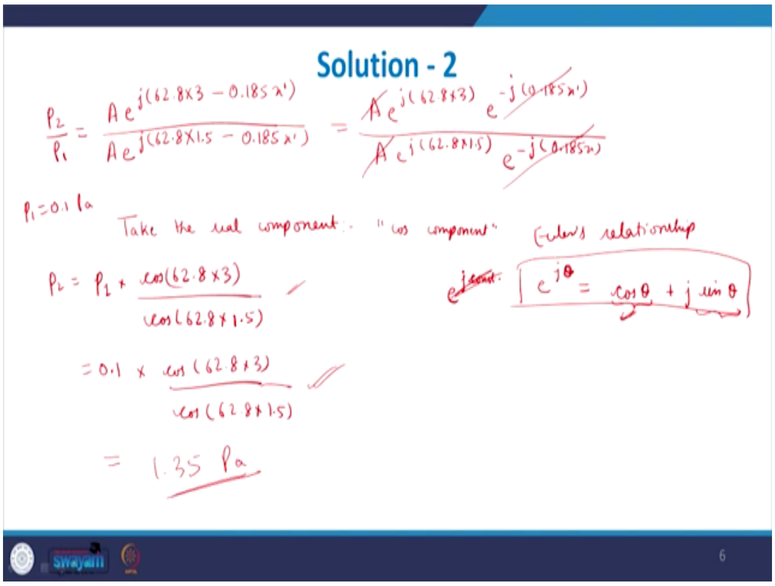

So, using this equation let us solve some more. So, P 2 by P 1 will be Ae to the power j 62.8 t minus 0.185 x divided by Ae to the power, sorry so, because we are doing P 2 so, were taking the value of P 2, and P 2 is at time equals to 3 seconds. So, now, we put the value of t as 3 and x is not known to us, but all the only thing we know is that both observations are taken at the same point. So, that value of x for both observations is same.

So, we see it is some x prime so, it becomes 0.185 times some observation point x prime and, the first observation was taken at 1.5 seconds let us go back to the question. See was at 1.5 seconds we are getting 0.1 Pascal so, we have to find what is at 3. So, this is given to us and this is at the same point x prime.

So, this cancels out, we can simply write this as Ae to the power j 62.8 times 3 into e minus j 0.185 x prime. So, we have separated the two components so, we have separated the x and the time component. So, because it is the same it cancels out, this amplitude also cancels out. So, this is what we get and P 1 is given to the 0.1 Pascal's.

So, now this is the ratio that we are getting and now we have to find the value; so, I told you that the exponential function is assumed just for the sake of simplicity, but once you arrive at the final solution you have to take the real component to get the solution. So, we take the real component so, if you take the real part of this e to the power j something. So, which will be the cos component, the cos is the real cos of this exponential function.

So, what we get is p 2 will be p 1 times cos of 62.8 multiplied by 3 divided by cos of 62.8 multiplied by 1.5 which is equal to 0. 1 times cos of this value by cos of this valuem And, why we have taken the cos component because again e to the power j of anything, any constant let us say e to the power j of theta is defined as cos of theta plus j times sin of theta, this is how it is defined this these are called as the Euler's relationships.

So, the exponential; the complex exponential function is defined as a sum of real cosine plus an imaginary sine part and for the sake of simplicity this is how we solve it. So, once we have solved we have only taken the real part to get our answer; so, we have taken this cost component here. So, when you solve this thing the answer that you will be getting is 1.35 Pascal's, this p will be 1.35 Pascal's when you solve this particular equation; the previous equation here this will come out to be 1.35 Pascal's.

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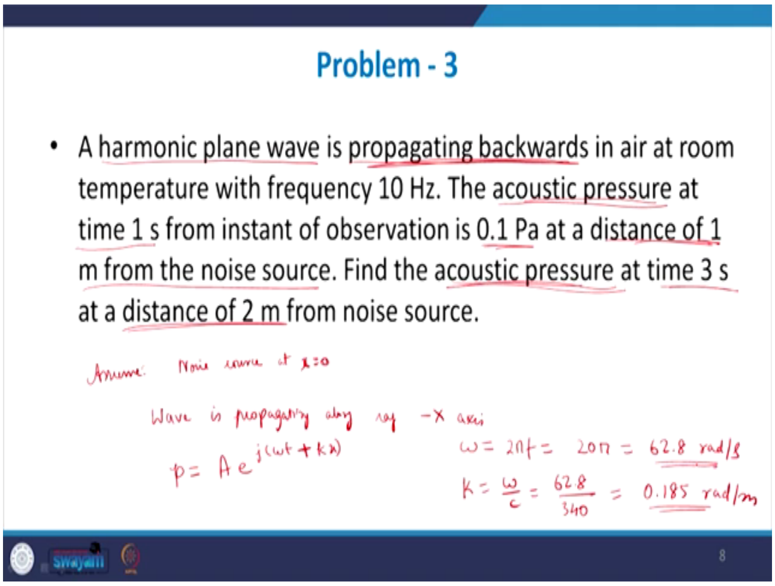

Now, let us solve another problem, this is the last problem for this lecture and using the same values. So, we have the same wave, the same harmonic wave with the frequency 10 Hertz that is propagating at propagating in the air at room temperature. Now, we have two points now see here we have acoustic pressure is given to be, the acoustic pressure at time 1 seconds is 0.1 Pascal's at a distance of 1 meter from the noise source.

So, let us assume first we assume noise source because noise source is what is generating the sound. So, the noise source we assume as the point where the sound originates with x equals to 0 for the noise source. Then, in that case the time; so, for the first the first acoustic pressure is found at t equals to 1 seconds and x equals to 1 meter, then we have to find what is the acoustic pressure at t equals to 3 seconds and at a distance of x equals to 2 meters.

So, I am also assuming that the direction of propagation is not given let us say it is ok, one thing is given to you here it is at the harmonic plane wave is propagating backwards. So, in that case we will have to assume whether it is negative x direction, negative y or negative z. So, the second assumption is that wave is propagating along let us say negative X axis. So, the only thing is given is it is backward propagating. So, the form of equation will be Ae to the power j omega t plus k x; so, the plus sign will be there because it is a backward propagating wave.

So, let us find the value of omega and t it is the same value for the as the last problem f is given to be 10 Hertz. So, this comes out to be 20 pi which is 62.8 radians per second and k comes out to be omega by c 62.8 and for air at room temperature. This is 340 meters per second; so, we get these values. Conditions are same so, we get the same omega and k as in the last problem.

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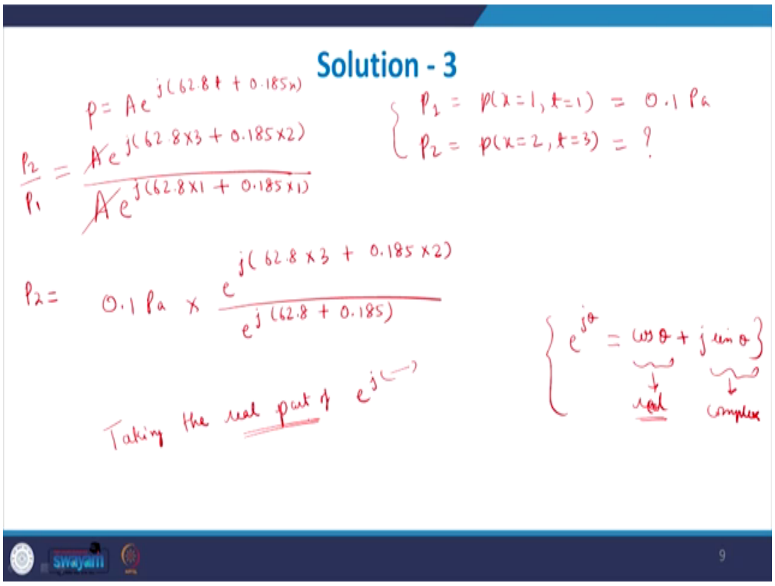

So, now we can write this equation as Ae to the power j 62. 8 t plus 0.185 times x. And p 2 by p 1, here p 1 is simply p at x equals 2. So, what is the first pressure it is at x equals to 1 meters and time t equals to 1 seconds, and the second is at 3 seconds and 2 meters and p 2 is simply pressure at x equals to 2 and t equals to 3 seconds.

So, this is given to us as 0.1 Pascal's and this is the value you have to find. So, if you divide p 2 by p 1 Ae to the power j 62.8 times 3 plus 0.185 times of 2 divided by Ae to the power j 62.8 times of 1 plus 0.185 times of 1, this cancels out. So, p 2 will simply be 0.1 Pascal's times e to the power j 62.8 times 3 plus 0.185 times of 2 divided by e to the power j 62.8 plus 0.185, this will be the value of the pressure p 2.

So, if you see now just take the real part taking the real part because we only use the complex form for the sake of calculations, the actual value is always obtained from the real part. So, taking the real part of e to the power j this thing, where e to the power j theta is simply cos of theta plus j times sin of theta. So, this is the real part and this sin the sin is the complex part.

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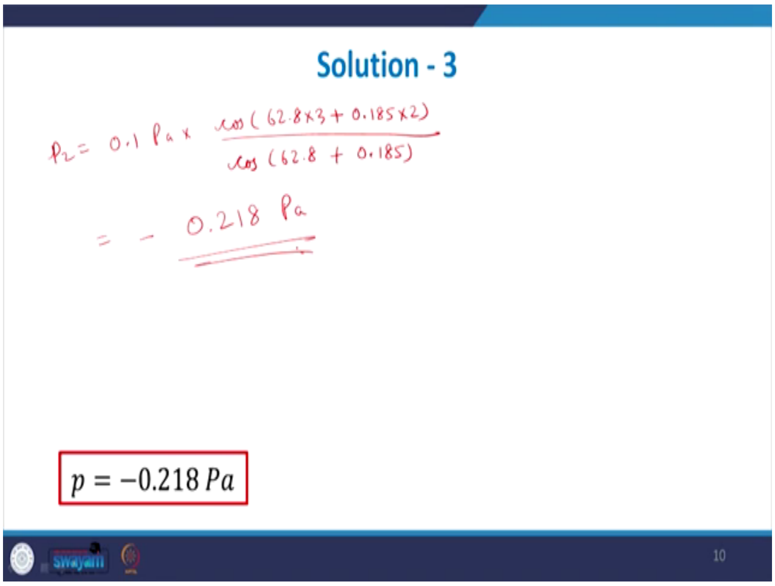

So, for the solution; now, final solution; now we are going to take the real part of this. So, what you are going to get is p 2 will be 0.1 Pascal's multiplied by cos of 62.8 times 3 plus 0.185 times of 2 divided by cos of 62.8 plus 0.185, if you solve this thing the answer that you will get is 0.218 Pascal's, ok.

So, now we have seen three solutions; so, 3 different types of problems which can be solved. So, let us now begin to learn about some new topics. So, the first quantity we are going to study is called as a sound power. So, this is a quantity associated with sound wave, now we know that a sound wave what does it do it travels through space. Suppose, we have a source which is generating the sound, let it could be a vibrating surface or it could be a vibrating sphere. So, whatever be the source it is generating some sound, and then so, it is generating the sound in the form of energy sound is also a form of energy just like heat, electricity.

So, it is generating the energy and that is radiating throughout the space and this energy is being transported from the source to the listeners here. So, it is; so, basically what does the soundwave do it transmits the energy generated from the source throughout the space. So, the sound wave in general does some work. So, the sound power is defined as the rate at which the sound energy it is emitted from the source. So, this is the definition of sound power.

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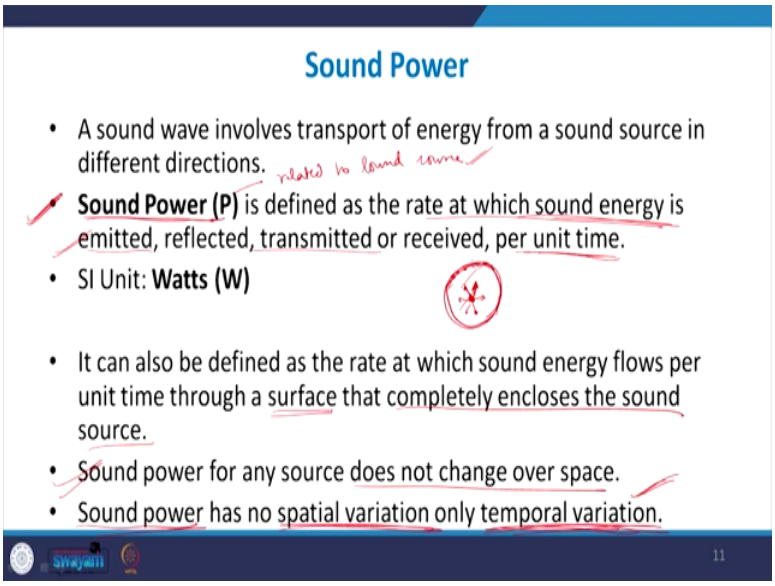

So, whenever there is a sound then there will be some source. So, sound power is something which is related to the sound source. So, what is the rate at which this sound source is generating sound energy? So, it is the rate at which sound energy is being emitted or being transmitted per unit time. So, if we have a fixed source then the sound power will be fixed and it will not vary with respect to space. So, if we have suppose one sound source here, then it starts generating the energy which starts spreading over space.

So, whether we measure the sound power at a distance of 1 meters or we measure it at a distance of 2 meters and so on. The power will remain constant because its independent of space the because it is the total energy that is being generated per unit time and because we have the same fixed source the power is going to be the same it would not vary with space.

So, as you see here sound power does not change over space it is totally dependent on the sound source, but it can vary over time. Let us say we have some tuning fork can be an example of a sound source. So, our tuning fork has been struck and it starts vibrating and sound is generated, but it would not keep in an ideal situation when it is set to motion it will keep vibrating infinitely, but that is not the ideal situation that we live in.

So, we have many things such as for example, we have air friction, due to the friction due to of the air particles damping will takes place. So, slowly and slowly the amplitude of vibrations will start to decrease because of the air resistance and slowly-slowly after a certain point of time the fork will come to rest.

So, usually in ideal situations any sound source, it is not infinite it does not it does not emit energy infinitely. So, usually over time it decreases. So, the sound power does not does not vary with space, but it does vary with time. So, the sound power depends on the sound source and with time if the sound source is getting diminished then the power will diminish, but with space it will remain the same.

Now, we have defined this as the rate at which the sound energy is being emitted by a source this can also be defined in another way. Let us say we have a. point source and we enclose it throughout with some spherical shell or some surface, then the amount of energy this sound source is emitting in space per unit time is the sound power.

So, we can also define it as a total amount of energy that is incident on a surface enclosing this source per unit time because whatever energy this source is emitting will ultimately be incident on this surface which is completely enclosing the source. So, the surface will take in all the power. So, if we have a surface which completely encloses the sound source then the energy incident on that surface per unit time will be the same as the energy that is being emitted which will be the sound power. So, this is another definition for sound power.

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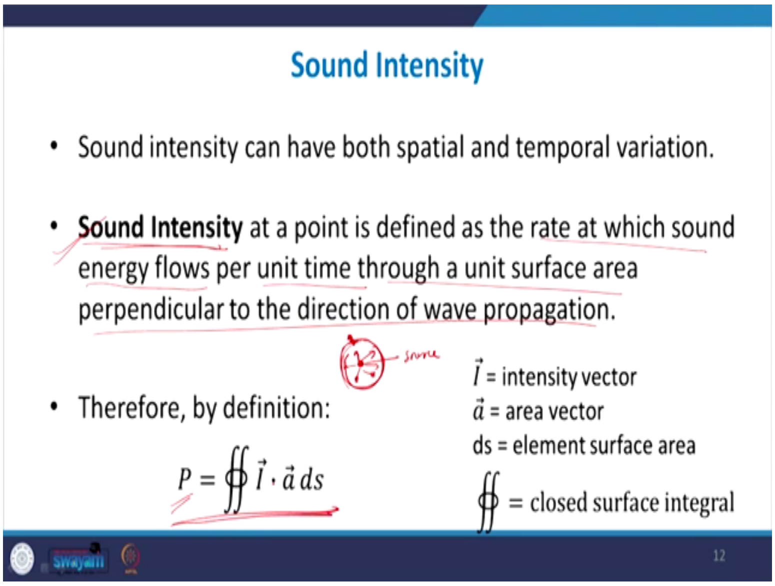

Now, we have a second quantity called as sound intensity. So, sound power was energy generated per unit time, this is the sound energy per unit time per unit area. So, by definition you know that if it is per unit time per unit area. So, which means that intensity multiplied by area should give you the sound power. So, this is the formal definition of sound intensity, the rate at which sound energy flows per unit time through a unit surface area perpendicular to the direction of wave propagation.

So, with the same explanation let us say, we have a we have some source which is emitting some sound energy per unit time and we have some surface outside the source which is completely enclosing it. So, the total energy incident on this surface will be the power and intensity is whatever is the energy incident per unit area. So, if we have I and we have the area vector we integrate it throughout the surface, we get sound power; so, this is by definition.

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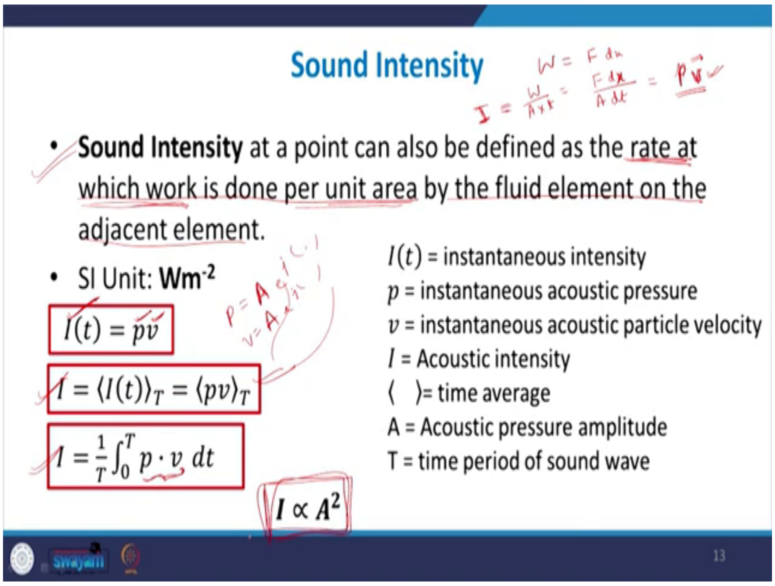

Now, another way to look at sound intensity is we know that when the sound energy is being transmitted then it is the oscillations of the particle which are doing work. So, one particle starts oscillating, it passes on some energy to the next layer of particles which start oscillating and then they pass on the energy and this is how the energy gets transported.

So, usually; so, as we define the sound power we said that it was the sound energy being generated per unit time which is the same as the sound energy being transmitted per unit time provided there is no loss of power in between in the air particles. So, sound intensity can be defined as the rate at which this fluid element is doing work per unit time per unit area. So, sound intensity becomes the rate at which work is being done per unit area by the fluid element to the adjacent element.

So, it is the rate of the rate at which this fluid element does work per unit area. So, the work done is simply force times the distance through which the work is being done. So, work done per unit time per unit area is force distance per unit time per unit area; so, f by a becomes the pressure and dx by dt becomes the velocity. So, the rate at which the work is being done per unit time per unit area can be written as the pressure being applied multiplied by the velocity through which the velocity at which the pressure is being applied.

So, the at any instant the intensity will be simply a product of the acoustic pressure times the particle velocity using this relationship, because I is simply work done by the fluid element per unit area per unit time which we got as pressure into the velocity. Now, usually this p and v together they are multiplied and I t is the instantaneous intensity, but whenever we refer to a sound wave then we usually talk in terms of what is its intensity then we usually say what is its intensity averaged over a particular time period.

So, we take the averaged value. So, this is the intensity that we talk about we simply take the average value over a particular time period.

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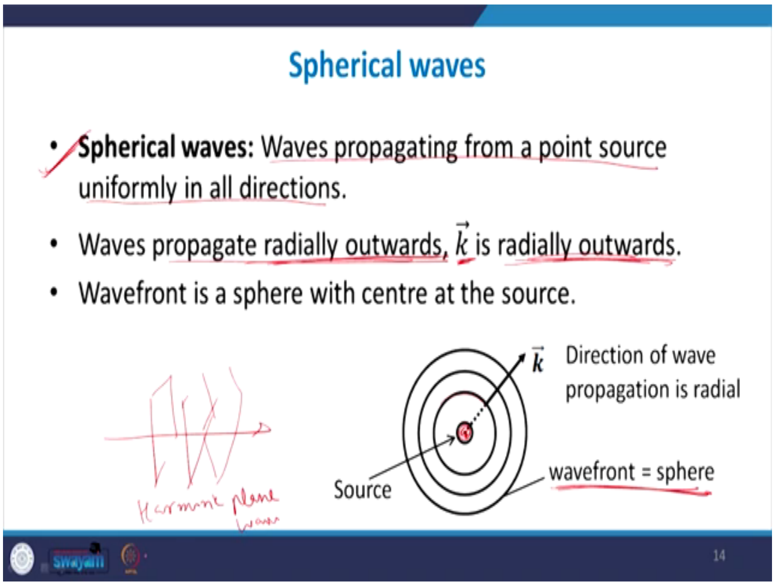

So, with these two definitions let us quickly revise what is called as spherical waves, now in the previous lectures I already showed you a wave front where we had a pulsating sphere and then the sound wave is being generated.

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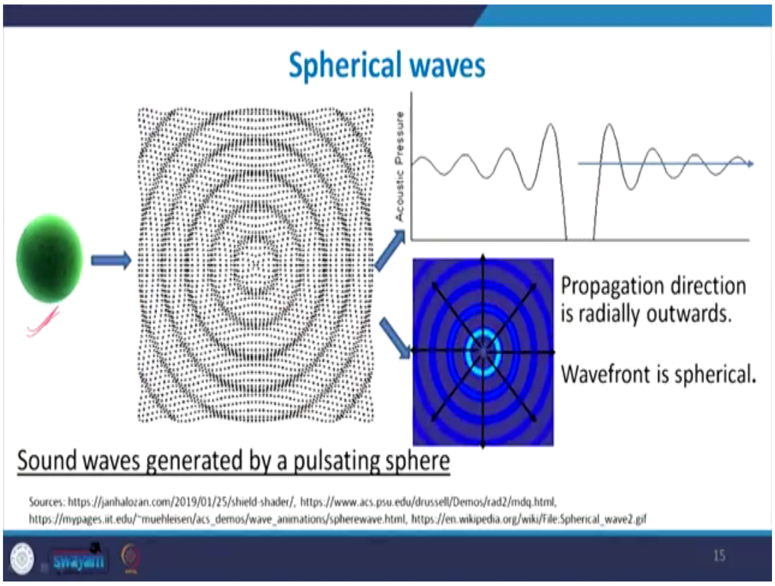

As you can see here we had a pulsating sphere and this is the form of motion of the fluid particles around this pair. So, this was the wave front here it was spherical in nature. So, as you can see the sound, the direction of sound wave propagation is always radially outwards. So, at any point you take the wave is going radially outwards. So, if you assume one particular sphere let us say, we assumed one, this one particular sphere then throughout that sphere the phase is going to be constant. So, as you can see here it is constant phase throughout. So, it has a spherical wave front.

So, this is a typical example of a spherical wave and the formal definition is that spherical waves are waves propagating from point source uniformly in all the direction. So, when a point source it starts generating waves and it is radiated uniformly in all directions then the waves created are spherical waves. So, here the direction of propagation is always radially outwards from the point source.

So, the k vector; k vector corresponds to the direction of wave propagation. So, this will also be radially outwards and the wave front is simply any sphere. So, this is if this is the source then the wave; so, if you take any sphere that is concentric with this particular sound source then this will become a wave front. So, all these sphericals shells here are the wave front because at any point there the phase will be constant, this is the definition of a spherical wave.

So, the wave front is a sphere here in case of harmonic plane waves. The wave front was planar to the direction of wave propagation this was for harmonic waves; harmonic plane waves and for harmonic spherical waves we have spherical wave fronts.

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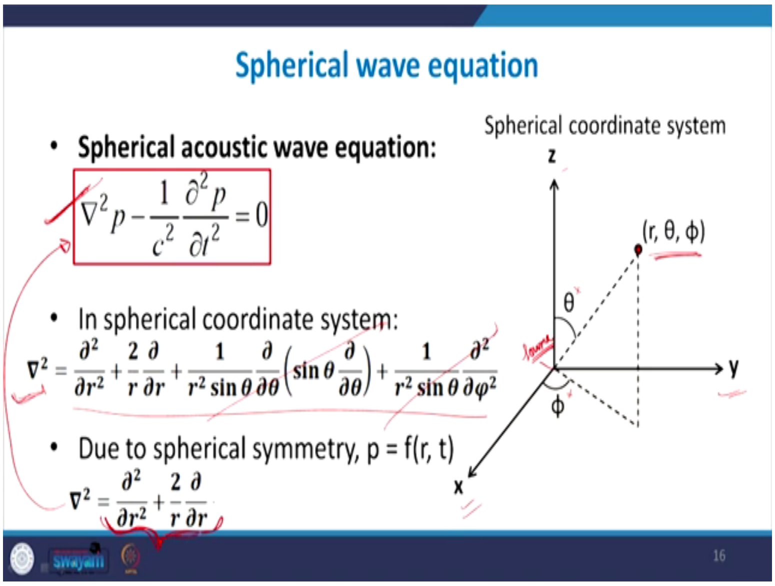

So, let us derive the equation for a spherical wave, we use the same equation for any 3D wave. Now, here as you see the spherical wave is a 3D, wave harmonic wave was only propagating along a single direction. So, we took only one component of nabla, this operator (Refer Time: 28:09) operator, but here we are taking this value.

So, this is the general wave equation and we apply it here. So, as you see the kind of symmetry it has at any point r suppose this is the location of source, this is the noise source. So, at any pointr in this spherical coordinate system you will have the wave. So, we have defined it because it has a spherical symmetry. So, instead of taking a Cartesian system we are taking the spherical coordinate system, the nabla square operator for a spherical coordinate system is given by this expression.

Now, we already know that due to the spherical symmetry this is a spherical source, it is generating the sound uniformly. So, by the definition the wave spherical waves means that the sound is being generated uniformly in all directions. So, the so, if we have a we take a point here or we take a point here or we take a point of here the pressure should be the same because it is uniformly being distributed in all directions.

So, the angle it should not depend upon these angles. So, because of this spherical symmetry they does not depend upon angles. So, we simply remove these terms. So, we can find out the equation for a spherical wave front either by using this equation and for this operator we replace it with this particular expression.

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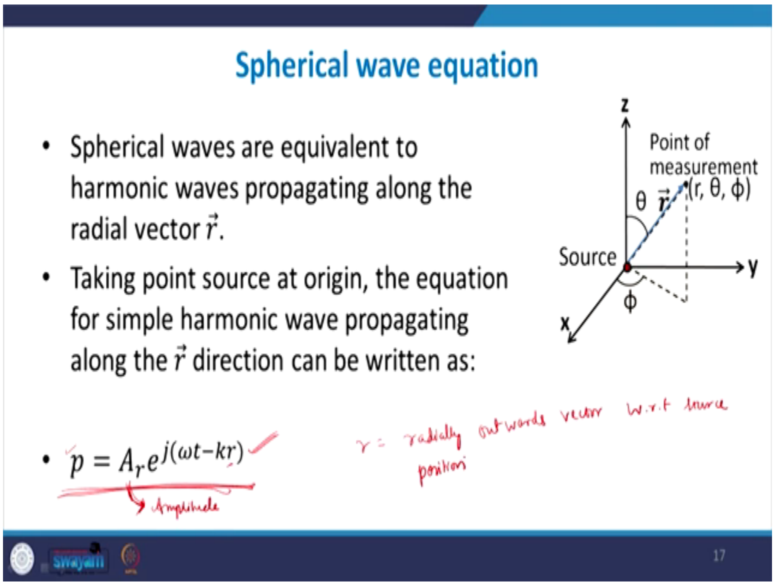

There is another simpler way of deriving spherical wave equation and I am going to discuss that what is the simple way of deriving. So, now, we already know the expression for a wave that is propagating along a particular direction. So, a spherical wave can be assumed to be let us say a wave that is propagating along this radial direction.

So, this is the general equation. So, here we have the pressure p this means it is the amplitude, now this amplitude could be a function of r could be independent of r we do not know yet. So, this is just the amplitude at r and e to the power j omega t minus kr. So, this is the general expression for any wave that is propagating along the direction r and so, we use this thing. So, here r is the radially outwards vector. So, r is the radially outwards vector with respect to source position.

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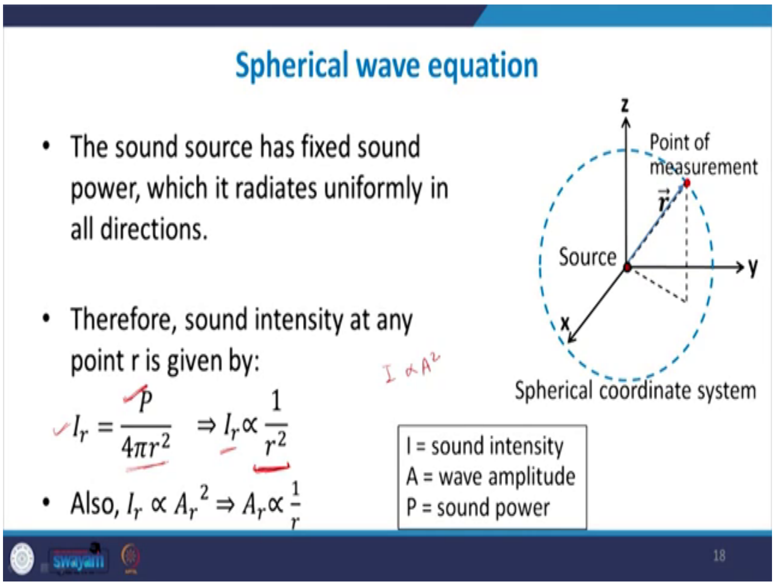

Now, this source is radiating the energy uniformly. So, whatever energy is being incident. So, if we have let us see we are finding what is the acoustic pressure at a at a point r then whatever energy is being radiated we assume a spherical shell around it, then whatever intensity energy is being radiated is going to be incident on this shell.

So, the power is constant p remains constant. So, the intensity at a point p, at the intensity at a distance r can be given by the constant sound power because sound power will not change with space. So, this p is fixed sound power divided by the area over which the power is being distributed. So, here the area of this entire spherical shell is 4 pi r square. So, what we get is that at any point of time this I r will be inversely proportional to r square because this is constant.

So, intensity at a point r is going to be proportional to r square and we also know that intensity is proportional to amplitude square. So, when we were discussing about intensity as you said intensity was a product of p and v, and p is so, intensity was a product of p and v and p is something some amplitude e to the power j omega t minus k x, v is also some amplitude e to the power j and some function. So, they both share the same amplitude. So, p into v will be some function multiplied by A square.

So, intensity was directly proportional to A square. So, since intensity is proportional to a square and the intensity is inversely proportional to r square. So, which means the amplitude is going to be inversely proportional to r. So, for a spherical wave at any point r the amplitude is inversely proportional to r.

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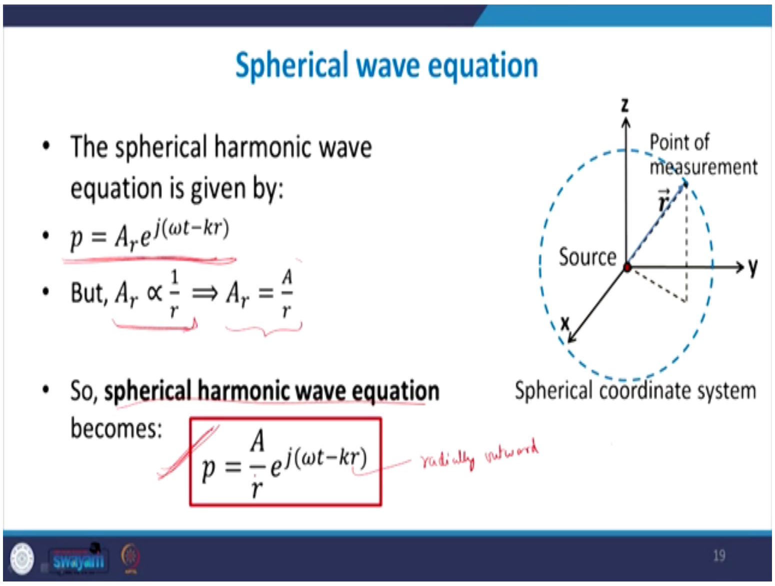

So, if we take this equation then A r is inversely proportional to r. So, we can replace it with some proportionality constant A so, it becomes some constant A by r. So, the form of equation we get for a spherical harmonic wave is going to be A by r e to the power j omega t minus k r. So, here where r is the vector that is radially outwards, the radially outward pointing vector. So, r is the radial outwards direction, it is pointing outwards and the amplitude it varies inversely with r. So, this is the form of the equation. So, we will discuss further on this in our next lecture.

Thank you.