Acoustic Materials and Metamaterials
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Lecture – 07
Standing Waves and Modes

Hello, and welcome to the lecture 7 in our series on Acoustic Materials and Metamaterials. I am Dr. Sneha Singh an Assistant Prof. at the Department of Mechanical Industrial Engineering at IIT Roorkee and today's topic is on Standing Waves and MKodes.

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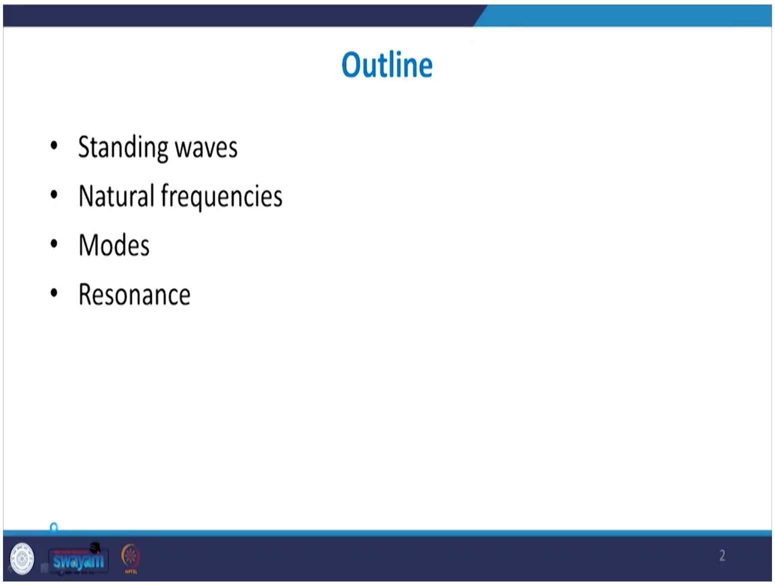

So, before that we have studied about sound wave propagation through a homogeneous medium and then sound wave propagation through a when the sound wave encounters the boundary of a second medium. Now, we will and both of these waves were travelling waves. So, in the very beginning I told you that mechanical waves or in general can be either traveling wave or standing wave and all the derivations and all the study we did till now as for forward propagating or a backward propagating wave so, it was a wave which also varied with respect to space sinusoidally.

Todays, lecture we will study about the standing waves and how are they created. So the outline is as follows, we will first study about standing waves and then what do you what is meant by natural frequency of a system, what are modes and then we will have a brief discussion on the phenomena of resonance.

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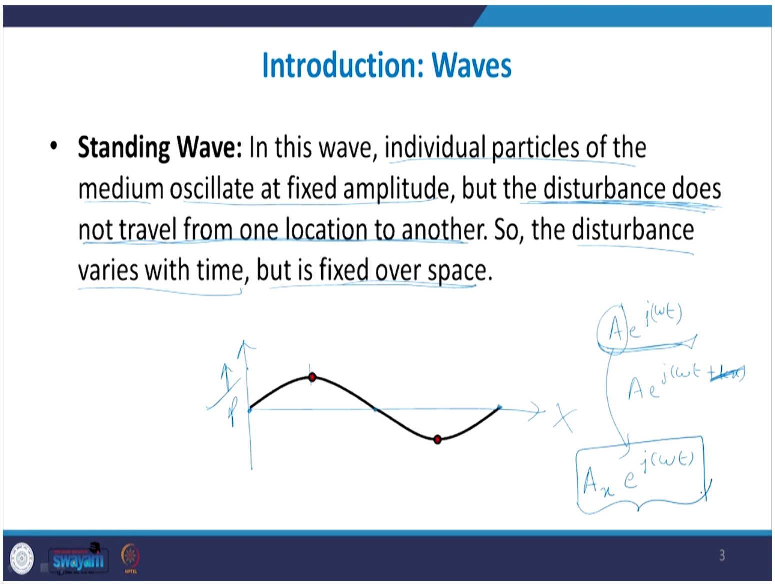

So, as described earlier as well standing wave is a wave where the individual particles of the medium they oxalate at a fixed amplitude, but the disturbance does not travel from one location to another. So, here whatever disturbance is created and one location, it is not propagating over space. So, it the disturbance varies with time, but not over space.

And, this animation shows to you a typical standing waves, you can see here suppose this is the X axis, and this is the pressure then you can see that individual particles at different X locations they are may so, every particle here is doing a sinusoidal motion. So, every particle is oscillating with and it is doing a sinusoidal motion with respect to time. So, every particle is doing something with respect to time, but it Is not doing anything with respect to space; it Is not varying with respect to space. So, this term is not there.

And, at every location it is doing some sort of motion and the amplitude of the motion, is dependent on the location. So, for example, in this location you have maximum amplitude, here you have 0 amplitudes and so on. So, the amplitude itself is a function of space. So, A is some function of space, e to the power j omega t can be a it can be a standard equation or a general equation for a standing wave.

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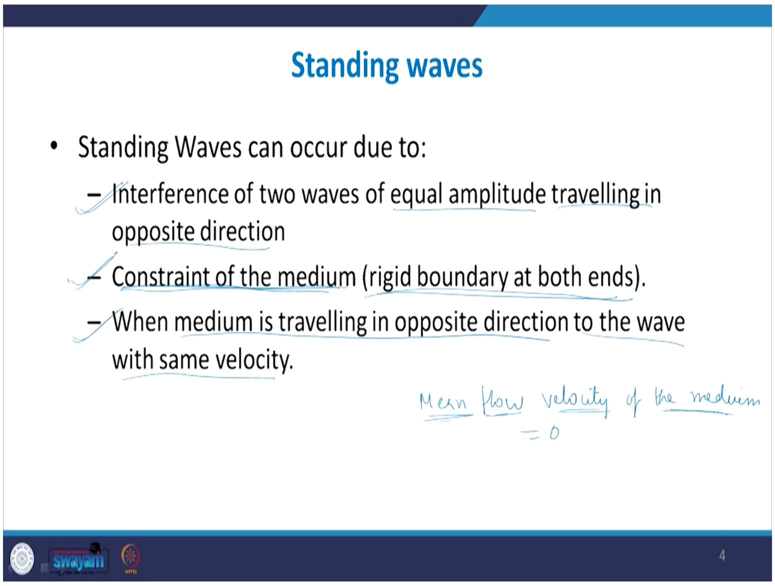

Now, how are such waves created? There are many ways when we where there are many such phenomena where standing waves occur. For example, let us say when two waves of equal amplitude, traveling in opposite direction they interfere with each other in a medium. The second one is when we have the constraint of the medium itself; so, let us say we have waves are generated in a confined medium.

So, a medium that is surrounded by rigid boundaries. So, there is a fixed boundary condition for a medium. So, in such a constraint medium there also we encounter standing waves, and the third common occurrence is when the medium itself is traveling in opposite direction to the wave with same velocity.

And, now, in this particular course right at the very beginning we assumed that the assumption we made when, we were doing sound wave propagation was that we assume that the mean flow velocity of the medium is 0. So, the medium is not moving and then suddenly a disturbance is created and the particle starts oscillating. So, that was the mean as the assumption we made at the very beginning; so, third case we will not be studying here we will study about the first two cases. So, let us see what happens.

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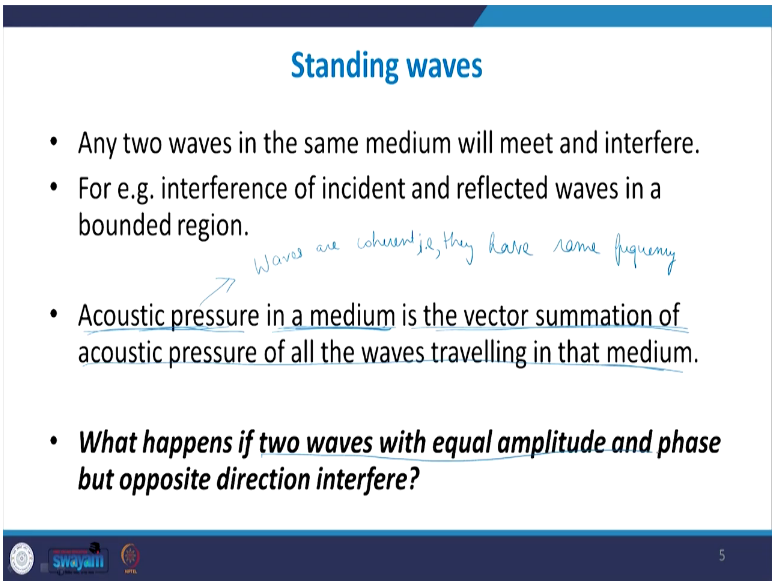

So, the first case: now, whenever there are more than one wave in a medium, I told you in the beginning let us say we have 2 or 3 sinusoidal waves they meet and they interfere. So, if these waves they are they have the same frequency then the total pressure can simply be found by the principle of superposition, so, it will be a summation of their individual pressures. So, it is going to be so, the acoustic pressure in such case; when two or more waves with the same frequency they are interfering, it will be the vector summation of the acoustic pressure of all the waves traveling in the medium.

Then, you do remember here that the condition here is that; the individual the waves are coherent which means, they have same frequency. So, two or more waves with same frequency they are interfering with each other then we can directly obtain the total pressure at a point as a vector summation of the individual pressure of these waves. Now, what happens if two waves with equal amplitude and phase, but opposite direction they are interfering?

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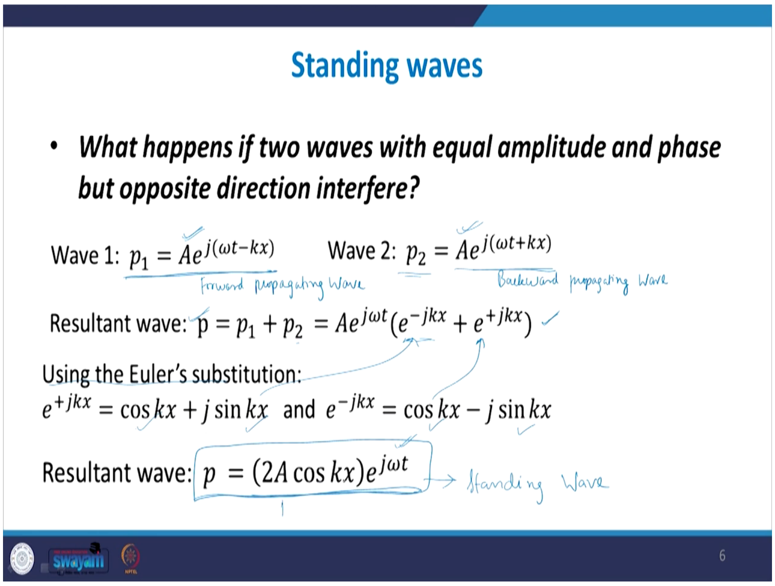

So, let us study this case. So, let us say we have wave 1 and the pressure of that wave is given as Ae to the power j omega t minus k x and the wave 2 is given as Ae to the power j omega t plus k x. So, as you can see here both of them have the same amplitude, there is no phase difference and 1 is propagating forward so, this is the forward propagating wave and, this is the backward propagating wave. So, they both are traveling in the opposite direction with equal amplitude and phase.

So, the resultant wave will be a sum of these two waves, the resultant pressure; so, you can sum the two up so, what you get is Ae to the power j omega t which is the constant and then a summation of this quantity e to the power minus j k x plus e to the power plus j k x.

Now, using the definition of e to the power minus j k x, the Euler substitution; this is my definition in this particular function is cos k x plus j sin k x and this is cos k x minus j sin k x. So, when you sum them together, you are only left with the cos terms the sin terms they cancel each other out. So, if you put these substitutions here so, the end result you get is two times of cos k x into A e to the power j omega t. So, this is the form of wave you are getting. So, this is the resultant wave and as you can see this is a standing wave. Where its only varying sinusoidally with time and the amplitude varies with the amplitude is fixed over a particular spatial location.

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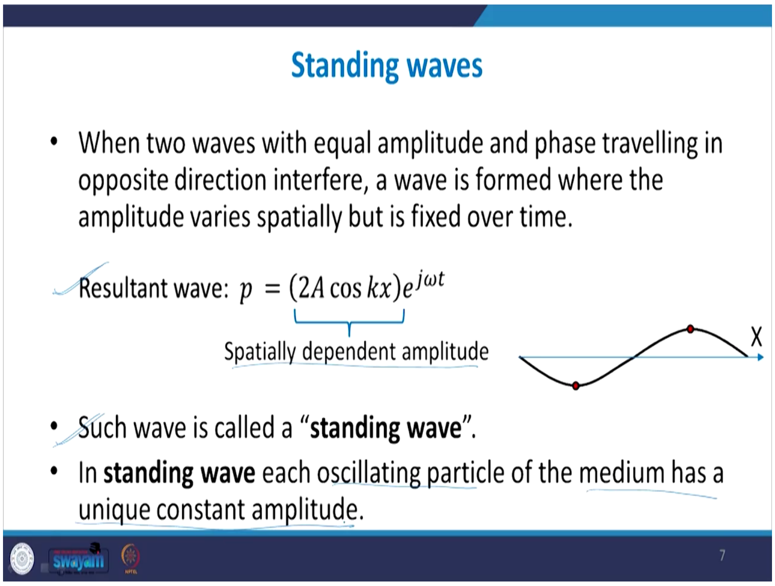

So, this is a animation again and this is our resultant wave, that we have found. So, here the amplitude it is spatially dependent. So, for difference for different X locations the amplitude is different, but it is fixed and there is a sinusoidal motion with respect to time, and such wave is called as a standing wave. So, here each oscillating particle has a unique constant amplitude depending upon whatever is its special location.

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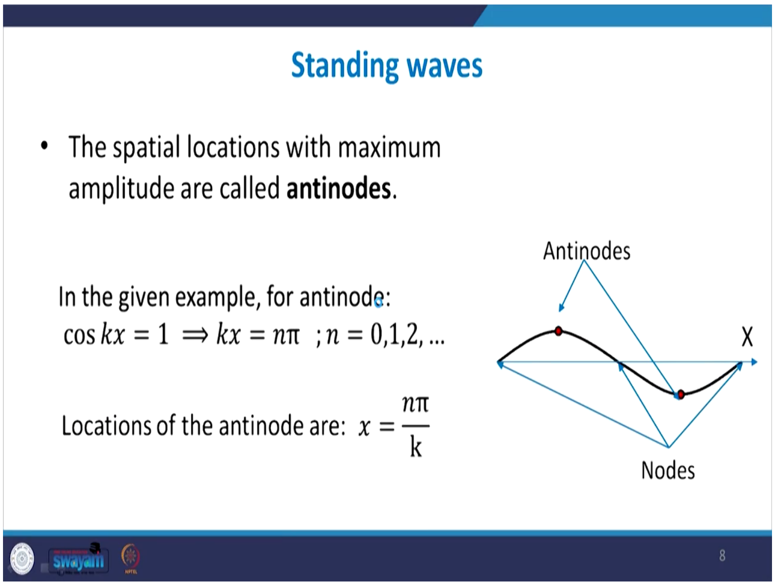

So, let us find out, where are the different terminologies here? So, in a standing wave you see that there are certain points where the amplitude reaches a maximum value. So, in this particular animation, these red dots these red dots are the places where the amplitude is the maximum and these are called as the antinodes, the red dot places.

So, how can you find the location of such antinodes in this particular example? So, in this particular example, this is the pressure equation this is the equation for the amplitude, right. So, when the amplitude will be maximum? When this quantity becomes maximum? So, when 2A cos k x becomes maximum. Which means, cos k x has to become maximum for the maximum amplitude and as we know the cosine function has the maximum value as 1. So, we can simply put cos k x equals to one and solve it.

So, when you put it here; so, which means that this particular angle has to be an integral multiple of pi, because cos at all the integral multiples of pi let us say, cos 0 which is equal to cos pi, which is equal to cos 2 pi, which is equal to cos 3 pi and so on. They are all 0 so, cos becomes 0 at integer multiples of pi. So, this is the value we substitute so, the location of the antinode then comes out to be x is equal to n pi by k.

So, if we know, what is the frequency of the wave? Which are interfering? So, if we know the frequency then we can find out the wave number as omega by c and then we can find out what are the locations, where we will get antinode? In the same way, we can find these spatial locations for nodes. So, nodes are those locations where the amplitude is minimum. So, we have minimum amplitude. So, as you can see these are the 3 fixed nodes here in this figure; these are the 3 fixed nodes. So, obviously, add the nodes the amplitude has to be the minimum so, which means, cos k x will become minimum which is 0. The mod value of this so, mod of cos k x becomes 0 which means; so, when is a cosine function 0? It is at the in odd multiples of pi by 2.

So, which means, k x has to be an odd multiple of pi by 2. So, this is a general form n can be 0 1 2. So, if you put n equals to 0 it becomes pi by 2, if you put n equals to 1 it becomes 3 pi by 2 and so on.

So, the location of node can then be found as k x is equal to 2 n plus 1 pi by 2 so, x becomes 2 n plus 1 pi by 2 k. So, this will become the location of the nodes.

So, we studied the first case where two waves with equal amplitude and phase traveling in opposite direction they interfere and what we got was the equation for a wave which is a standing wave, and it has certain places where we have nodes and certain places where we have antinodes and how we derive the locations for these nodes and antinodes.

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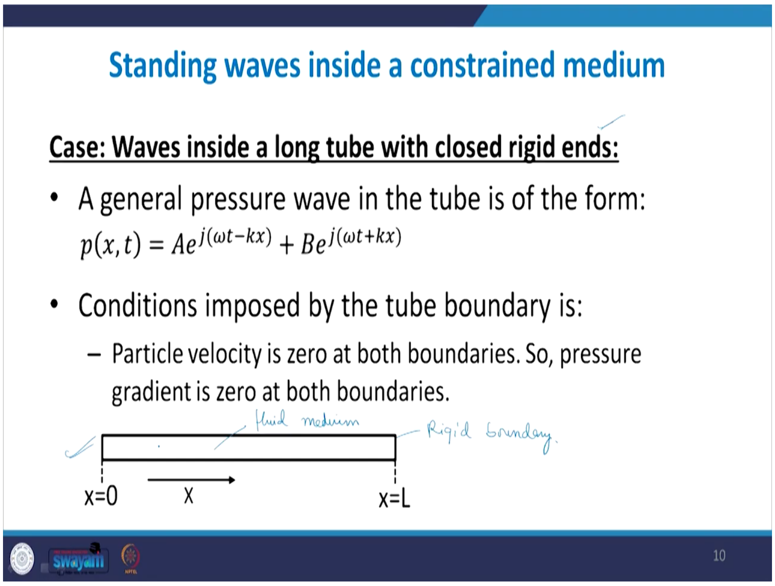

Let us study the second case, which is when the waves are generated inside the long tube with closed rigid end. So, now, we are taking the case of a constraint medium. So, here there is a constraint which means, that the medium has got fixed boundaries. So, the waves are being generated in a long tube with a closed rigid end. So, let us see here; so, waves this represents fluid medium, in this particular figure and all this is the rigid boundary. So, this is a constrained medium.

Now, in a constraint medium if you have a very hard surface then the acoustic particles they are accelerating, but as soon as they inquire encounter a rigid boundary they cannot which means, the boundary has got a very high impedance its rigid and hence it will not allow any further passage of sound waves. Which means, that so, the acoustical the acoustic particles they are accelerating so, we have a certain particle velocity, but once it reaches a rigid end then the particle velocity has to become 0, it cannot impinge further beyond the boundary.

So, the condition that a rigid boundary imposes is that; particle velocity becomes 0. So, this is the condition. So, whenever we have rigid boundaries, the particle velocity becomes 0.

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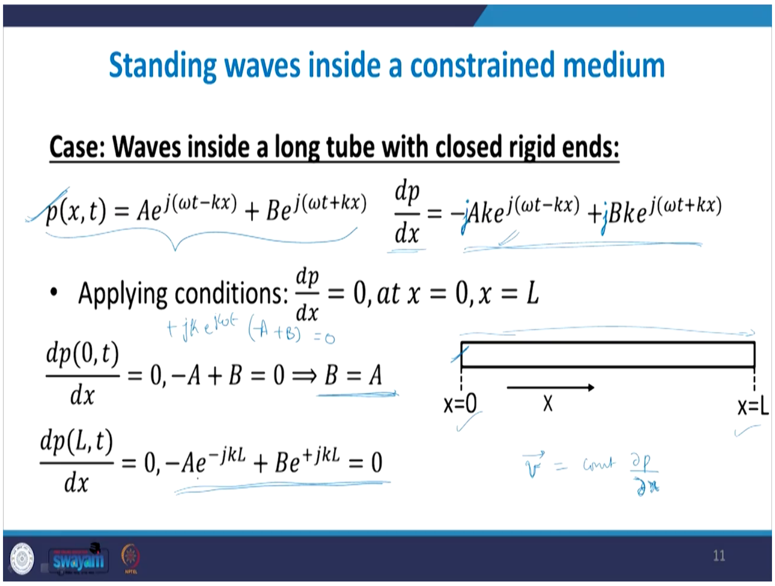

So, let us use this condition. So, the pressure let us assume a general pressure equation for the wave inside this long tube. Now, inside this long tube because, here the lateral dimension we have assumed to be very small the tube is very long so, the length is very long. So, in that case the waves generated are only harmonic waves.

So, we take the general harmonic wave equation. So, we take 2 different waves because, we did not know what kind of wave it could be? It could be any kind of wave. So, we take either it is; we take a combined solution a wave that is forward propagating and a wave that is backward propagating. So, we take this common general solution for general expression for the wave inside this tube and because, the velocity is 0 at both the ends at x equals to 0 and L.

And, we know that the velocity function itself is some proportion some constant into del p by del x. So, in the very beginning in our lecture 2, when we were deriving lecture 2 and 3; when we were deriving the equations for pressure and velocity so, we found that the velocity using the Euler's relation comes out to be some function of del p by del x. So, when velocity is 0 which means, pressure gradient has to be 0.

So, we find d p by d x here and d p by d x we have differentiated with respect to x. So, minus j k comes out, minus j k comes out so, here the j term is missing here. So, j k has come out and this is the term we get. So, we apply these conditions now, d p by d x equals to 0 at x 0 and L. So, let us say putting x equals to 0 what we get is, if we put x equals to 0 in this expression then all the common terms this is going to be 0 and this is non 0 non 0 and non 0 and similarly, this common term is also eliminated so, the only terms we are left with and this becomes 0 and 0 so, the only term we are left with is minus a plus b is equal to 0. The common terms they cancel out.

So, minus j k e to the power j omega t times of A minus A plus B is equal to 0, this is effectively what you will get; when you put x equals to 0 in this expression. So, overall minus A plus B equals to 0 which means, A is equal to B this is the first equality we are getting using a by applying the first boundary condition now, let us apply the second condition which is d p by d x becomes 0 at x equals to L. So, at both places the gradient of pressure is 0.

So, when you apply this what you get is; again Ae to the power minus j k L last time, we had put x equals to 0 so, this term cancelled out. So, minus Ae to the power j k L plus Be to the power plus j k L will become 0. If you put this x equals to L in this equation and B is equal to A so, overall we can write it as we take this A constant so, this becomes this minus this quantity.

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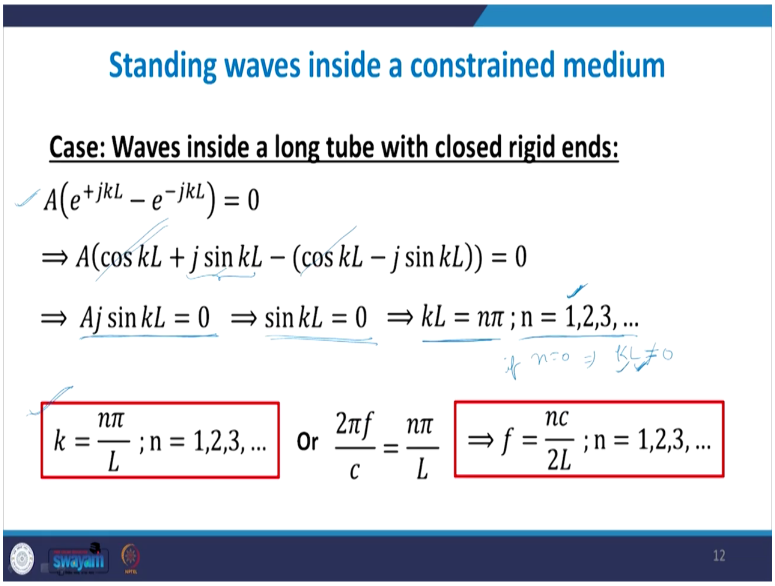

So, this minus e to the power plus j k L minus e to the power minus j k L is equal to 0. Both of them A and A, we take it as. So, this is the expression we get. So, using the Euler's relationship we get cos k L plus j sin k L minus cos k L minus j sin k L in brackets is equal to 0.

So, when you subtract them the thing that you are left with is, j sin k L this is what you are left with. So, A j sin k L actually, you are left with 2 j sin k L. So, overall this quantity has to be 0, cos terms cancels out. So, which means, sin k L has to be 0. Again sinusoidal function will be 0 whenever, it is at 0 pi 2 pi and so on. So, a sin function becomes 0 when the angles they are integral multiples of pi. So, this is the condition here so, we put k L is equal to n times of pi n is equal to 1 2 3 and so on.

So, here we have not started the value from 0, we started from 1. Why? Because, if n was 0 if n is equal to 0 this means, k L is also 0 and this cannot be true because, we have a tube L is non 0 and some wave is propagating k L equals to 0 would effectively mean there is no wave in the tube. So, we are not taking that condition. So, this is a non 0 quantity, this is a non 0 quantity so, this will be non 0. So, that is why we start this solution from n equals to 1. So, this becomes our overall solution k becomes n pi by L n equals to 1 2 3 and so on. and we can replace this k as 2 pi f by c, which is omega by c then f can be found as if you equate this then you get is n c by 2 l where n is from 1 2 3.

So, when you solve this what you are getting is that we started with a general wave so, we have a long tube and we started with let us say it has any general harmonic wave and when we put the boundary conditions, that we have rigid boundary at x equals to 0 and L then we arrived at a solution that k is some n pi by L, where n is 1 2 3 and so on and, f is n c by 2 L where n is 1 2 3 and so on. So, both k and f now, become fixed. Which means, that under normal conditions or under steady state conditions, any pressure in the wave will only have some fixed frequency values; it cannot have any random frequency value.

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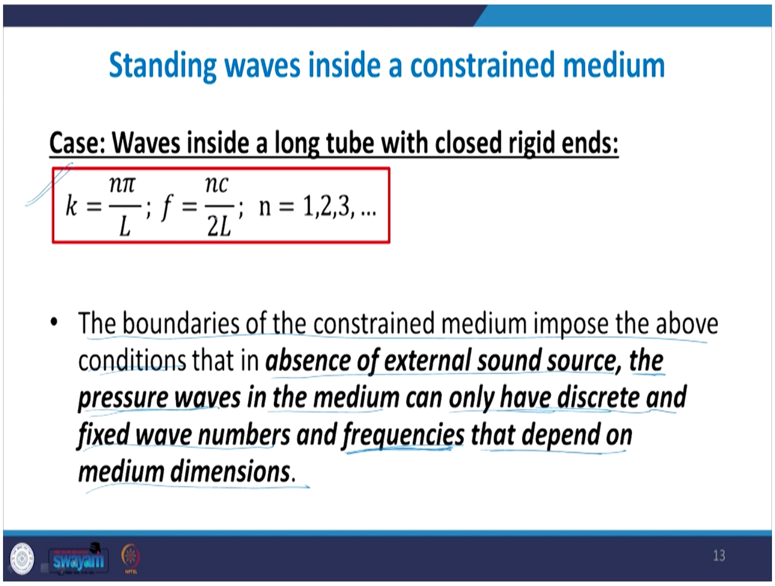

So, this is what we have come to terms with, it is that the boundaries of this constrained medium they impose the above conditions; that in the absence of any external sound source. Whatever pressure waves that exist in the natural state in this medium they can only have discrete and fixed wave numbers and they can have discrete and fixed frequencies and these depend upon the medium dimensions L.

So, in other terms you can say that the frequency or the wave number they are getting quantized. So, in a constrained medium under steady state condition it can only allow there or it can only allow certain frequencies. So, now, any random frequency wave will not exist, only certain allowable frequencies will exist and these allowable discrete frequencies in this constrained medium is called as its natural frequencies.

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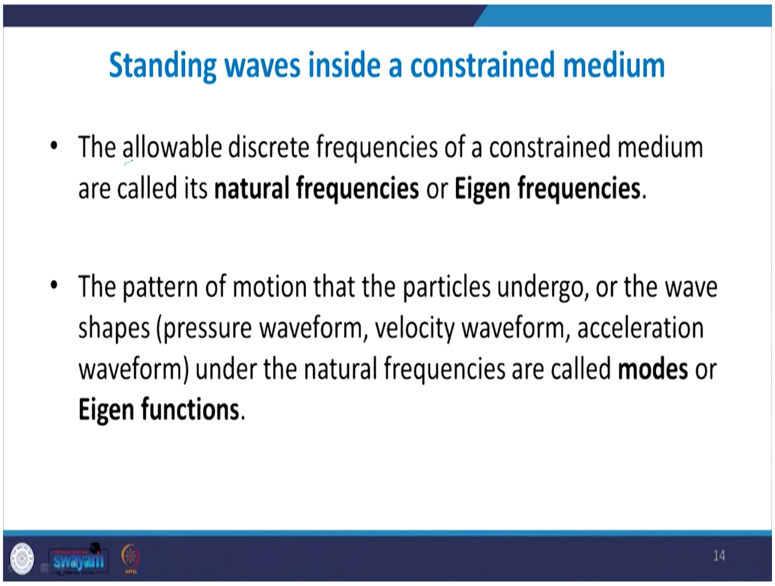

So, these frequencies are because, of the conditions imposed by its boundary and the dimensions of the medium. So, these allowable frequencies which we got here this is the frequency. So, only these frequencies can exist they are called as natural frequencies or the Eigen frequencies of the medium and the pattern of motion that the particle undergoes or the wave shapes such as the pressure waveform, velocity waveform, acceleration waveform under these frequencies are then called as the modes or the Eigen functions.

So, the allowable frequencies these discrete allowable frequencies are called as the Eigen frequencies or natural frequencies and, when you put these values of frequencies then the kind of function you get for pressure, velocity or acceleration. So, the kind of the kind of wave form you are getting at these frequencies these are called as the modes or the Eigen functions

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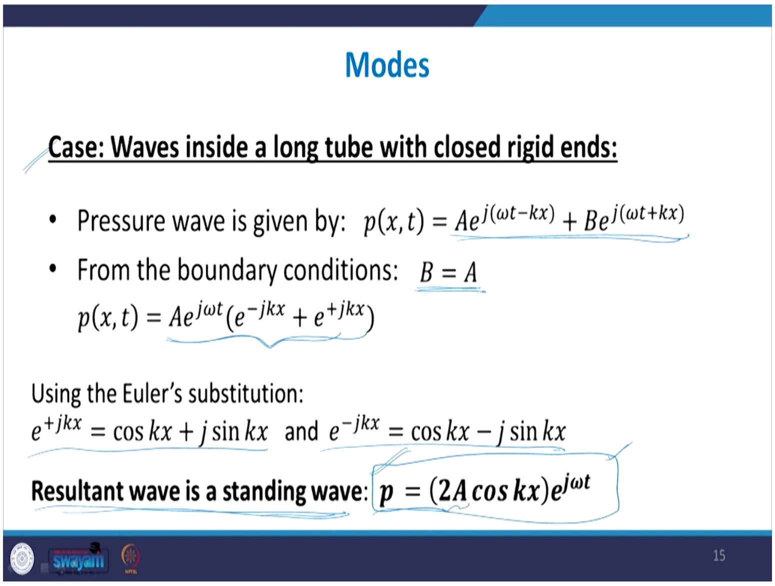

So, lets see, what are the modes of this particular case? So, in this particle case; the pressure wave was given by this particular equation B is equal to A, which we found out from the first condition. So, putting B equals to A this is the overall wave we are getting. Ae to the power j omega t e to the power minus j k x plus e to the power plus j k x, again using this Euler substitution here, what you get here is that the resultant wave you get is a standing wave. So, here also when you put the boundary conditions the resultant wave comes out to be a standing wave in this form.

So, this is the wave equation which we are getting.

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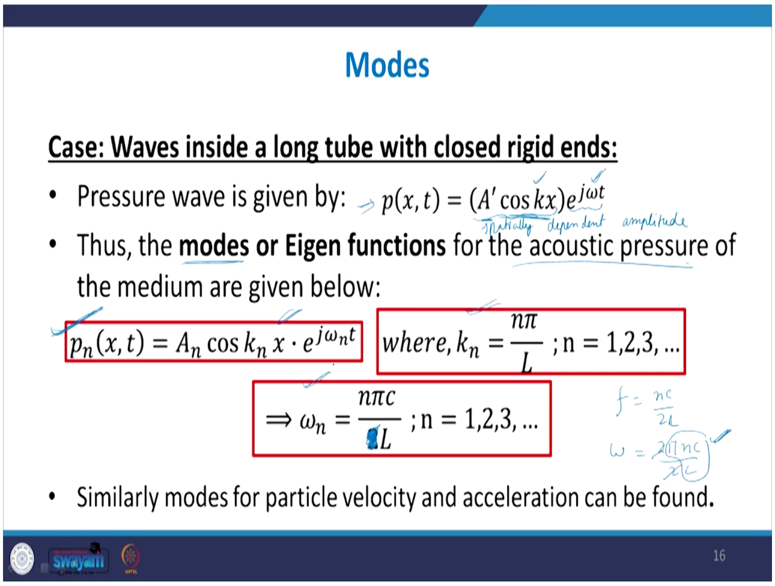

So, this is the pressure wave here I have just used, I have replaced this twice a with another constant a dash so, it is some constant a dash times cos k x; this is the spatially dependent amplitude and this is the time varying function. So, we are getting a pressure wave solution.

Now, how do you find the moods or the Eigen functions for this acoustic pressure? Let us find what are the moods of the acoustic pressure? So, for in that case, because we have n such frequencies, we have frequency is the frequency we can have n such frequencies f n is equal to n c by 2 L, where n is equal to 1 2 3 and so on. So, the mood will simply be some An cos k n x dot e to the power j omega n t.

So, we have use this same function here, but now, we know that this k is fixed and this omega is also fixed and the k for any particular mood is given by n pi by L and omega becomes because, we omega becomes we know that frequency was n c by 2 L. So, omega will become correspondingly 2 pi; so, n was given by n c by 2 L. So, omega is 2 pi f. So, it becomes 2 pi times of n c by 2 L so, you get is pi c by l. So, this is a correction here this 2 will not be here, we will get is n pi c by L, ok. So, this is what you get.

So, the overall solution that you get is that; this is the expression for a mode, where k n is given by this and the omega 1 is given by c. So, we are getting fixed pressures for fixed frequencies.

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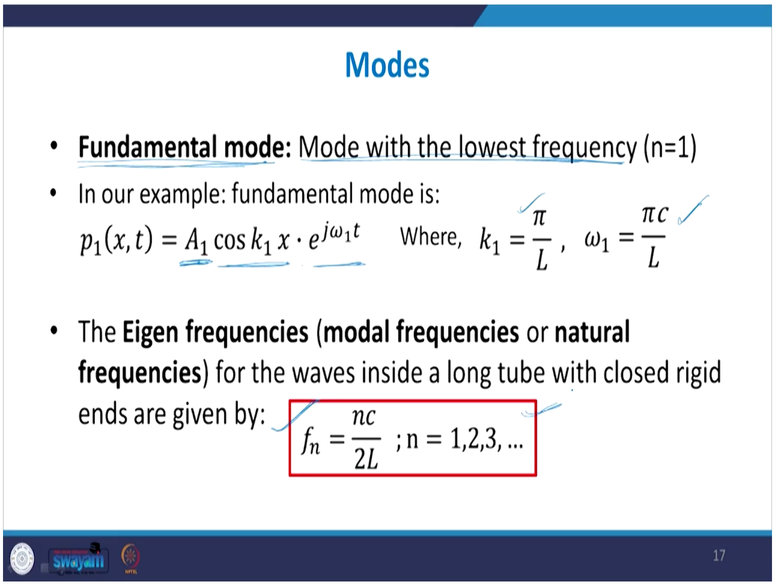

Similarly, we can find out the mood for acceleration and velocity. Now, what do you mean by so, do you know what is modes? Modes are simply when you put the values of these are natural frequencies then whatever function you are getting that is a mood. So, what do you mean by a fundamental mood? The fundament we know, that these modes can have n such different values depending upon n equals to 1 2 3 and so on.

So, the minimum value when n is equal to one. So, the lowest order value of the mode becomes the fundamental mode. So, this is the mode with the lowest frequency value. So, in this case the lowest frequency is obtained at n equals to 1. So, we can get the first mode as p 1 x comma t is some amplitude A one times cos k L x into e to the power j omega 1 t, k L becomes pi by L and this becomes this was n pi c by L so, this becomes pi c by L.

Now, Eigen frequencies or modal frequencies or natural frequencies; this is the definition for the Eigen frequencies. So, f n is equal to n c by 2 L. So, just like we have fundamental mode, we have also a term called fundamental frequency which means, the Eigen frequency with the lowest value.

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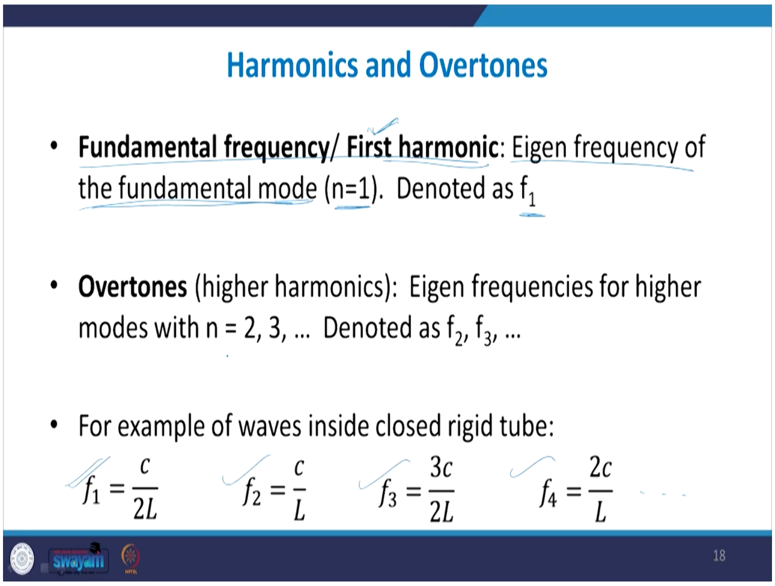

So, in this case this fundamental frequency or the first harmonic is the Eigen frequency with the first of the fundamental mode or the frequency with the lowest value.

So, we denote it as f 1 here. So, f 1 by this formula becomes c by 2 L similarly, we have the second mode, third mode, fourth mode and so on. The second mode will be 2 c by 2 L which is c by L. Third mode will be three c by 2 L. So, we can obtain first, second, third, fourth mode and so on and so, forth.

So, the first mode is also called as the first harmonic and all the higher modes are then called as the overtones or the higher harmonics, ok.

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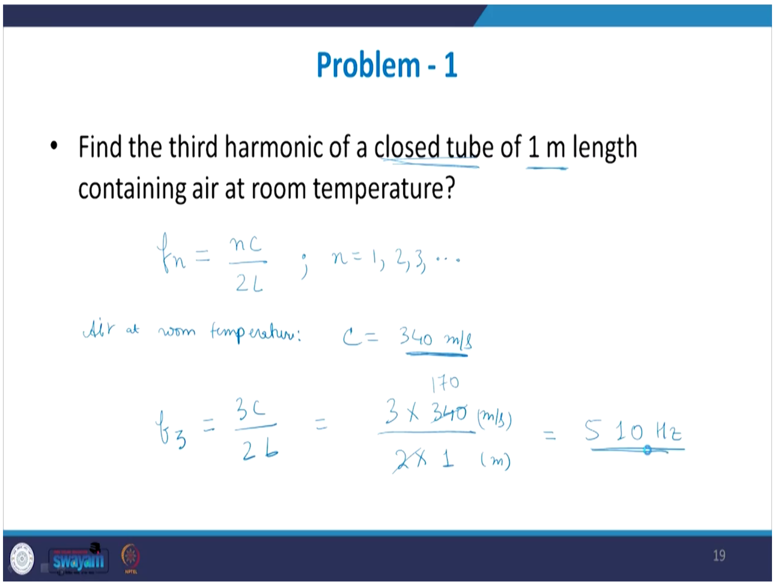

So, let us solve a problem. So, here we have find the third harmonic of a closed tube of one meters length, containing air at room temperature. So, it is a closed tube which means, that the frequencies they will follow this relationship where n is 1 2 3 and so on. And, it contains air at room temperature so, air at room temperature means, the speed is going to be 340 meters per second so, this is the value of the speed of sound I have taken for air at room temperature. You these values are fixed you can either memorize them or when you are when I give you questions then you can rho c values can be known to you or c values will be provided for any medium.

So, when a medium is given at a fixed temperature the speed of sound is always fixed, and it depends upon rho and c values so, it depends upon the B and the rho values so, the speed of sound is fixed; so, here we have taken this c as 340 meters per second for air at room temperature and what we have to find is third harmonic which means, we have to find f 3; and f 3 will be 3 c by 2 times of L. Which is 3 into 340 divided by what is the length of the tube? It is 1 meters, 1 meters. So, what you get is, it should be close to about 510 hertz. So, this is the value you are getting for f 3.

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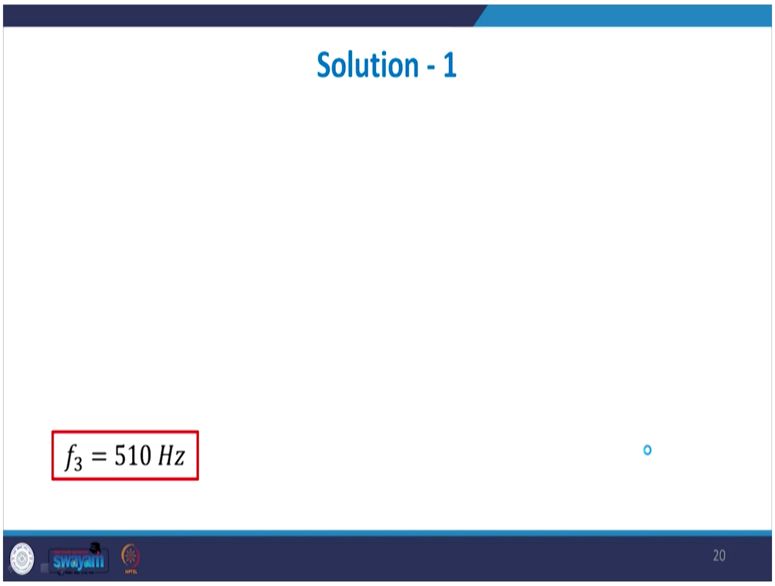

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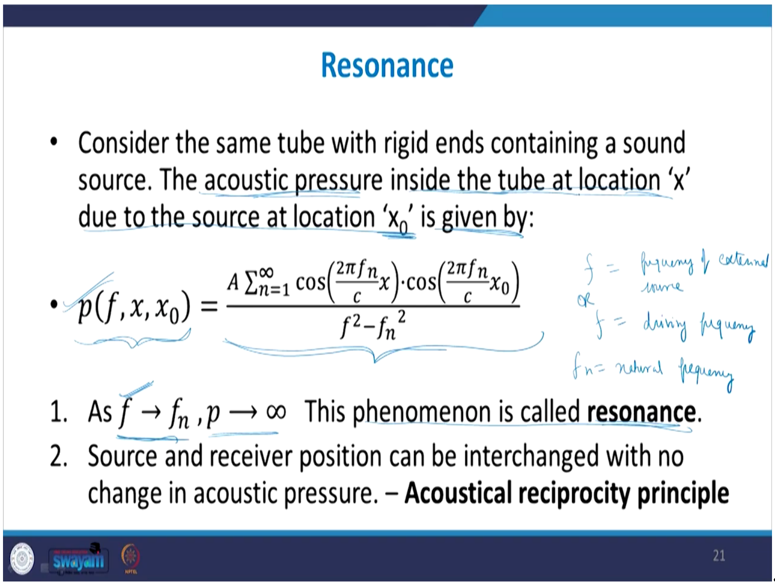

Now, we will quickly summarize and tell you about a concept called resonance. So, now, we know that in a constrained medium when there is no external source so, under normal condition. So, let us say we had some excitation. So, we got a constrained medium we gave some excitation and sound waves got generated and, then we stopped giving the external source. Then after a certain point of time, it will reach a steady state condition and under that steady state condition only normal moods; only its natural frequencies will exist. So, it will only exist in these fixed modes, but what if we have a continuous external source given to such medium?

So, it has been found that the acoustic pressure inside a tube at location x due to a source at location x naught is given by this particular expression. So, when you see that so, here if you look at this denominator this is the f is the frequency of the external source or we call this as driving frequency and f n is the natural frequency, of the medium.

So, whenever this f becomes equals to f n. So, whenever the external frequency which is applied tends to f n or the natural frequency, then the pressure amplitude reaches almost infinity. So, we have very loud pressures or very loud sounds at or near its natural frequency and this phenomena is called as resonance. And, again you can see that here the source location is taken as x naught and the location of measurement is x if we even interchange it.

So, you can look at the nature of this function it is interchangeable. So, if you interchange the 2, then also you will get the same solution this is called as the principle of reciprocity. So, the pressure wave that is obtained is it will remain the same if you interchange the location of source and receiver.

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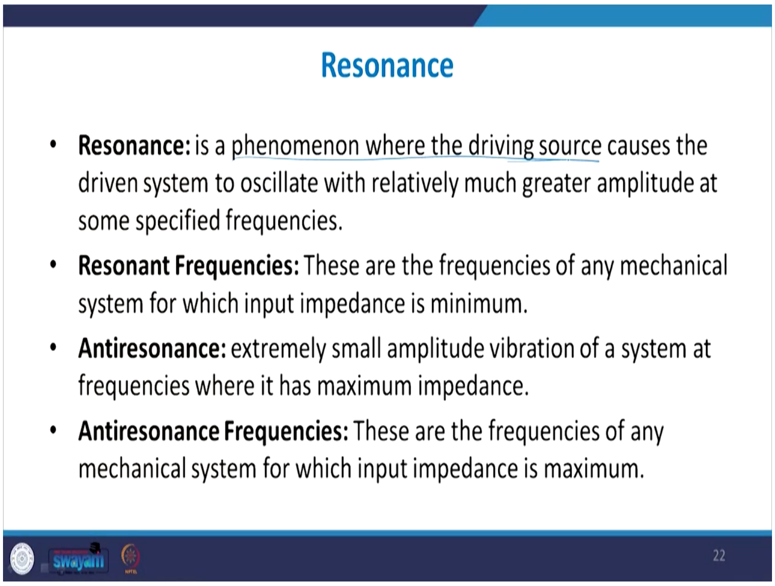

So, with this resonance is simply defined as a phenomena where so, it is a phenomena where the driving source causes the driven system to oscillate with very much larger. So, relatively much greater amplitude at some specified frequencies. And, as we saw in this particular case resonance is obtained wherever the driving frequency approaches or becomes equal to natural frequency, but in general resonance can be obtained at any frequency it is simply a phenomenon where when some excitation is given suddenly at certain frequencies the system starts to vibrate or oscillate with very large amplitude. So, that is the phenomena of resonance and the frequencies at which it occurs is called as the resonance frequencies, then we also have a phenomena called anti resonance; which is no matter how much excitation you give to a system, there is very low response the response is very low or the oscillation is very low that is called as anti resonance and the frequencies at which it occurs is called as the anti resonance frequency.

So, as you see for a for many most of the constrained mediums, that response is highly frequency dependent. So, with this I would like to close the discussion on standing waves modes and resonance and see you for the next lecture.

Thank you.