Acoustic Materials and Metamaterials

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Lecture – 09
Sound Signal Analysis - II

Welcome to the lecture 9 on this series of Acoustic Materials and Metamaterials. In the previous class we were discussing about various ways of representing sound and measuring sound. So, we studied about spectrum analysis and how the sound is represented as a distribution of acoustic intensities over a frequency scale and that frequency scale is then divided into small bands for analysis just to increase the computation time. We rather than having a continuous scale we divide the frequency scale into small bands and we analyze the intensity over these individual bands to get the full spectrum. So, in this class also we will continue a discussion on Sound Signal Analysis.

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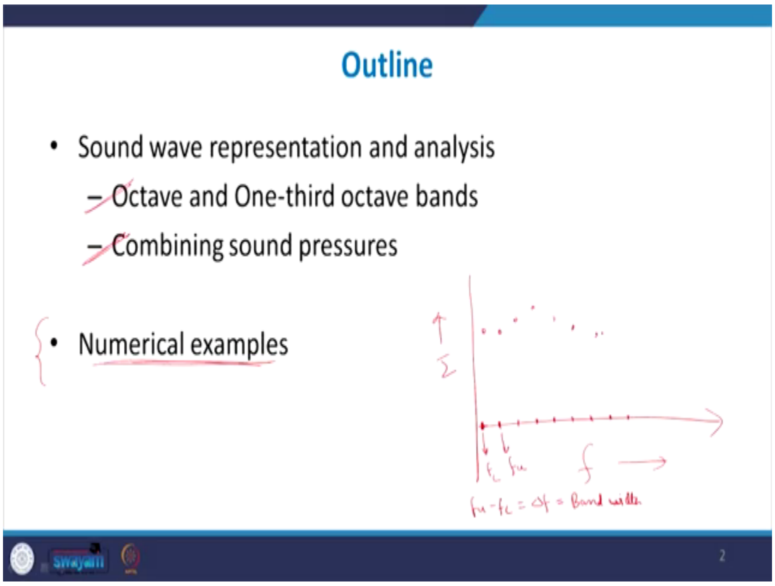

So, here the outline for the course is we will study about octave and one third octave bands and then we will study how do we combined the sound pressure levels for different sound waves. And then after that we will have a tutorial session where will do some numerical problems for a better understanding.

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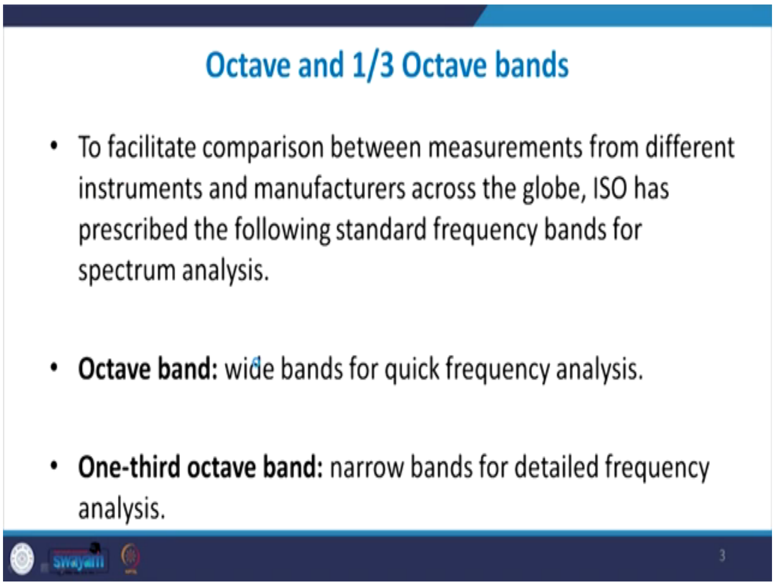

So, as I said before this spectrum analysis is done in noise control because human hearing is highly frequency dependent and the knowledge of how the intensity is distributed over frequency is very important.

And the usually a spectrum is calculated by dividing the frequency scales into certain bands. So, if you have a frequency scale like this a continuous frequency scale it is divided into bands. So, it is cut into such bands and then so, each band is characterized by some upper frequency and some lower frequency. And the difference between this upper and lower frequency gives you the net width of the band which is called as the bandwidth.

So, different bands are obtained and then the intensity distribution can be studied over these fixed bands. So, it is sort of like a histogram kind of a graph. So, this increases the computation time. So, we study it by digitizing the frequency scale. So, two common types of bands exist. This is done to facilitate comparison between the manufacturers from all across the globe and from the measurements from different instruments. So, ISO has specified that all across the globe we have some standard bands so that we can compare the data of one researcher with another researcher at any part of the world.

So, the two common bands which are used are octave band and one third octave band. So, octave band here the bandwidth is wider. So, this is a more wider kind of a band something like this and this is a more finer band one third octave band. So, if we get suppose four bands in a particular frequency range, one third will be every band is again divided into three separate bands. So, we get a more finer detail here. We will see that in the subsequent slides also. So, this is a wider bands for quick analysis. This is a narrow band computation time can be slightly higher, but it will give you a more detailed frequency analysis.

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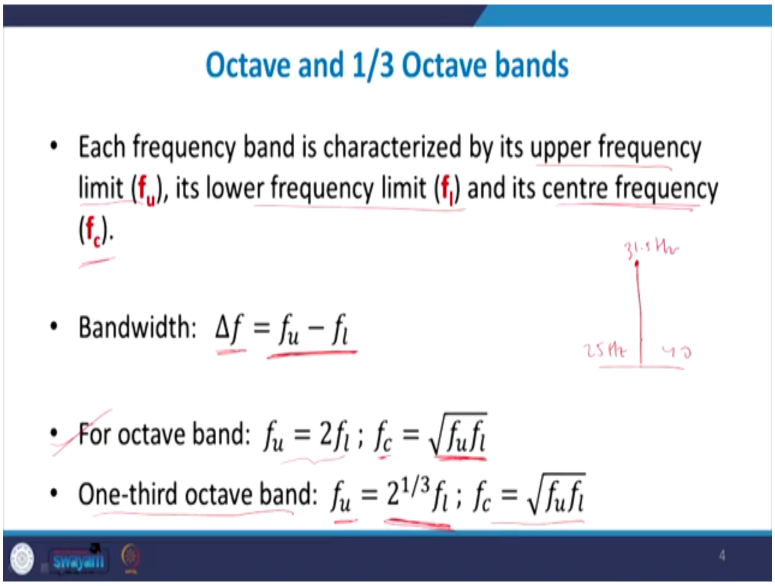

And each of these band it is characterized by some upper frequency limit and lower frequency limit and the center frequency f c. The bandwidth is simply the difference between the upper and the lower frequencies. And for octave band the way the calculation is done is that we start from for both bands we start from somewhere around 25 hertz or so, and the lowest frequency and then for a octave band the upper frequency is just the double of the lower frequency. And this center frequency then is the geometric mean of upper and lower.

But for one third octave band the upper frequency is 2 to the power 1 by 3 times of lower frequency. So, as you can see here this is twice of this is 2 to the power 1 by 3 of this. So, the width of this particular band will be more because here upper frequency is more apart than lower frequency.

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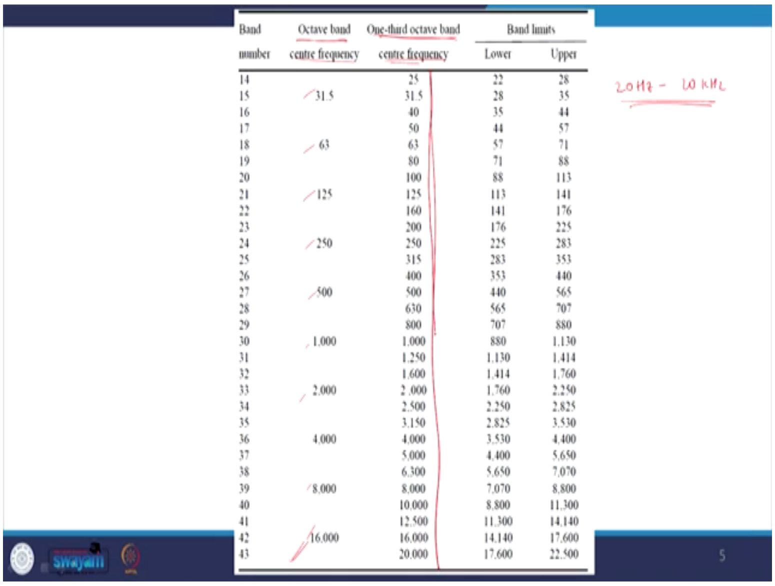

Band number	Octave band centre frequency	One-third octave band centre frequency	Band limits Lower	Band limits Upper
14		25	22	28
15	31.5	31.5	28	35
16		40	35	44
17		50	44	57
18	63	63	57	71
19		80	71	88
20		100	88	113
21	125	125	113	141
22		160	141	176
23		200	176	225
24	250	250	225	283
25		315	283	353
26		400	353	440
27	500	500	440	565
28		630	565	707
29		800	707	880
30	1,000	1,000	880	1,130
31		1,250	1,130	1,414
32		1,600	1,414	1,760
33	2,000	2,000	1,760	2,250
34		2,500	2,250	2,825
35		3,150	2,825	3,530
36	4,000	4,000	3,530	4,400
37		5,000	4,400	5,650
38		6,300	5,650	7,070
39	8,000	8,000	7,070	8,800
40		10,000	8,800	11,300
41		12,500	11,300	14,140
42	16,000	16,000	14,140	17,600
43		20,000	17,600	22,500

So, this table lists the bands. So, this is the this is just giving you the center frequency for a particular band. So, we have a band from let us say 25 to 40 hertz the geometric mean of occurs somewhere around 31.5 hertz. So, the center frequency, so, every band has got two frequencies which are the limits, but the center of that is taken as the center frequency and to for easier notation we simply donate a denote a band with its center frequency.

So, this is a this is how the audible range is divided in an octave band. So, these are the center frequencies of the octave band. And then one third octave band the same frequency range 0 to 16 kilo hertz can be divided. Sorry, the audible frequency range is 20 hertz to 20 kilo hertz. So, 20 hertz to 20 kilo hertz is divided into these bands for octave scale and one third octave band it is so many. So, as you can see here the number of bands are limited they are more wide. So, number of bands obtained are limited.

So, the observation point is also limited. So, we get a quick analysis, but it is not very detailed, this is a more final band and gives you more detailed analysis.

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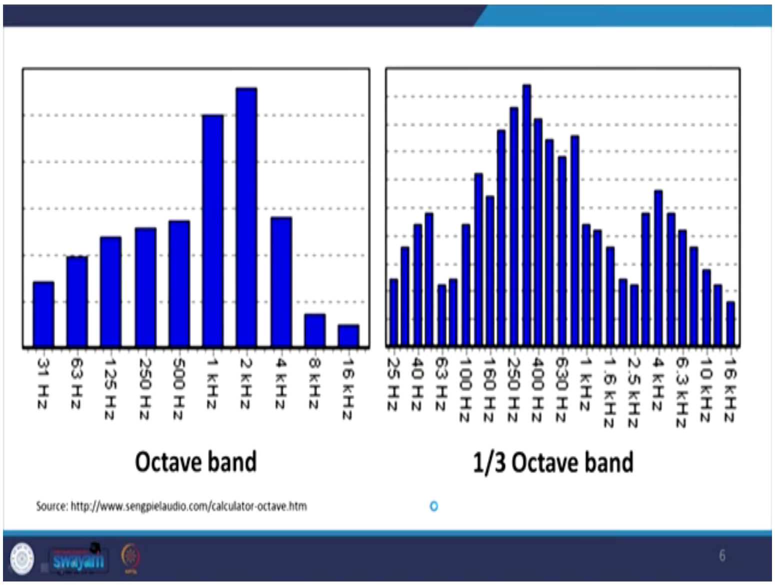

So, to compare the 2 let say we have once we have the same signal this is how the signal comes out in octave band and this is how the signal looks like in one third octave band. So, when you observe this is your octave band you will see that this is how the signal is varying. The intensity is lower here then there is a peak around here 1 and 2 kilo hertz and then it again decreases like this. So, this is what observation you get from octave band, but when you do a more finer analysis then you get certain dips and peaks which you missed out in the octave band analysis.

So, when you do a more finer detail then you see that ok, no although roughly this is the distribution pattern, but if you go for a more detail analysis what you see is that even at this point it is not continuously increasing, but there is a dip here and then also again it is not continuously decreasing, but there is a peak here. So, you had missed a dip and a peak which you now observe when you do a more detail analysis. So, now, let us study that let us say we have 2 3 machines in a single room and every machine is emitting some sound level.

So, one is emitting let say 60 dBs other is emitting let say 40 dBs 50 dBs and so on. Then what will be the combined effect of these machines when they are working together. So, this we need to know. So, for that we are going to see how the SPLs they combine together. So, if we have individual SPL sources SPL 1 plus SPL 2 plus SPL 3, so, their combined effect will be what?

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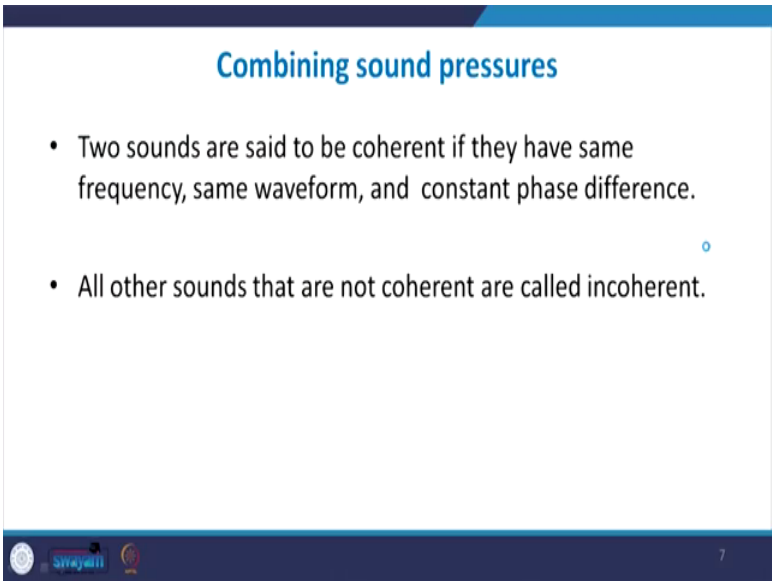

So, for that we need to know that there are 2 types of sounds, when 2 sounds they have the same frequency same waveform and constant phase difference. So, usually then they are called as coherent sound. So, coherent sounds are usually generated from the same source because frequency is highly dependent on the source. So, suppose the same source is generating 2 signals at some phase difference then they are coherent.

So, if two sounds are coherent then the way sound pressure levels are combined will be different and all the other sounds which do not have the same frequency and fixed phase difference and the same waveform they become incoherent. So, most of the common sounds encountered are incoherent in nature because they are signals coming from everywhere; some coming from a particular machinery some coming from vibrating surface some coming from a general human talking somewhere outside.

So, in general any particular area is a combination of many sound waves and these sound waves are in general incoherent in nature. It is very rarely that we find coherent sounds that are coming together. So, the addition and subtraction depends upon nature in which these sounds they interact.

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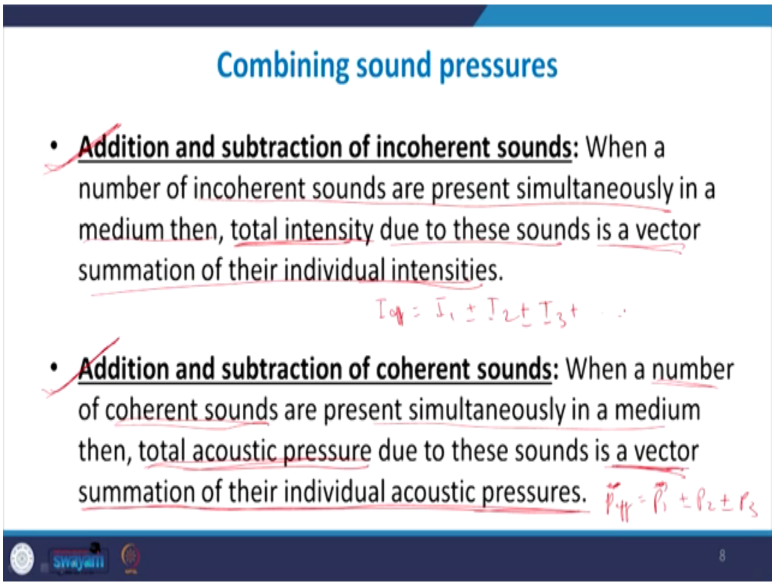

So, we had already studied before when we were doing the standing waves. So, what we studied was that suppose we have two different waves then the resultant wave could be a standing wave. The resultant wave is simply obtained by summing or subtracting in whatever case. So, it is simply a vector summation of their acoustic pressures and that is how we got a standing wave, when we had two waves travelling in opposite direction, the equivalent wave become a sum of the acoustic pressure of the two waves. So, in that case both the waves they had the same frequency component, they had the same omega and k.

So, they were coherent in nature. So, that kind of addition and subtraction is only applicable to coherent sounds. So, what happens when the coherent sounds or the sounds with the same frequency and waveform they are simultaneously present in a medium then the total acoustic pressure is a vector sum of the individual acoustic pressures. And that is what we have done in our previous class when we were doing standing waves, when we are solving the equations of standing waves.

So, we are simply adding up the pressures together and when we also studied about the sound propagation through medium boundaries. So, whenever we are deriving the conditions like the equation of continuity, so, we used to simply add the pressure of incident and the reflected wave to get the total pressure at the left hand side of the boundary, again in that case also both these waves they had the same frequency. So, they were coherent and that is why the summation of pressure was valid, but if the sound waves they have different frequencies they are incoherent in nature then in that case when such incoherent sounds are present simultaneously in a medium then the pressure summation is not valid. What happens is that the total intensity due to these sounds is then a vector summation of their individual intensities.

So, here the p effective is simply p 1 not p 1 plus minus p 2 plus minus p 3 and so on and here the intensity is getting summed and so on. So, not the pressure, but the intensities. So, let us study this case one by one, let us take the incoherent case first.

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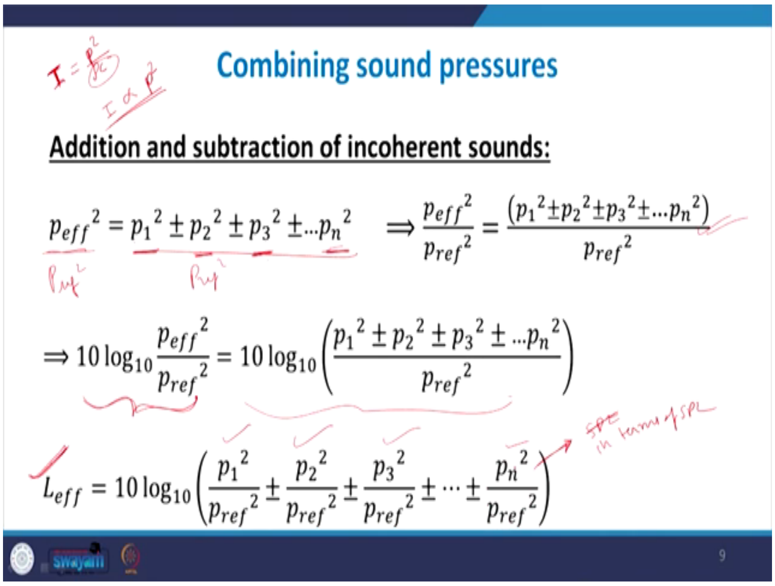

So, suppose in a medium we have n different sounds and its and the individual pressure of each sound is p 1, p 2, p 3 and so on till p n. So, we have n such different sounds. And they are present together in the medium and we have to find how the totals what will be the equivalent sound pressure level in terms of their individual sound pressure levels.

So, we know that the intensity is a summation of their individual intensity. So, what we do is intensity again because all of them are together present in the same medium and p is equal to p square by rho c; so rho c is same. So, I is directly proportional to p square. So, we can represent the summation of intensity as the summation of their square of RMS pressures. So, p effective the net effective RMS pressure square will be p 1 square plus minus p 2 square plus minus p 3 square plus minus p n square and so on.

So, here I have used plus and minus both because so, that this equation is valid both for the summation of sound as well as subtraction. So, you can use it for both cases to add the sounds or to subtract the sounds, so, usually you square it the squares they follow this rule.

Now, if we divide both the sides by p reference square dividing them by p reference square this is what you get and if you do 10 log 10 of this on both the sides, so, this is the value you get. Now this is what by definition this becomes the sound pressure level of the effective wave front. So, this is the effective sound pressure level. When all these individual sound waves they are present together then the total sound pressure level is given by this one.

So, this is the total sound pressure level which is given by this expression 10 log 10 we have separated this. So, we have got 10 log 10 this plus this plus this plus this and so on. So, now, let us derive let us see how these expressions calculate these expressions in terms of their respective SPLs.

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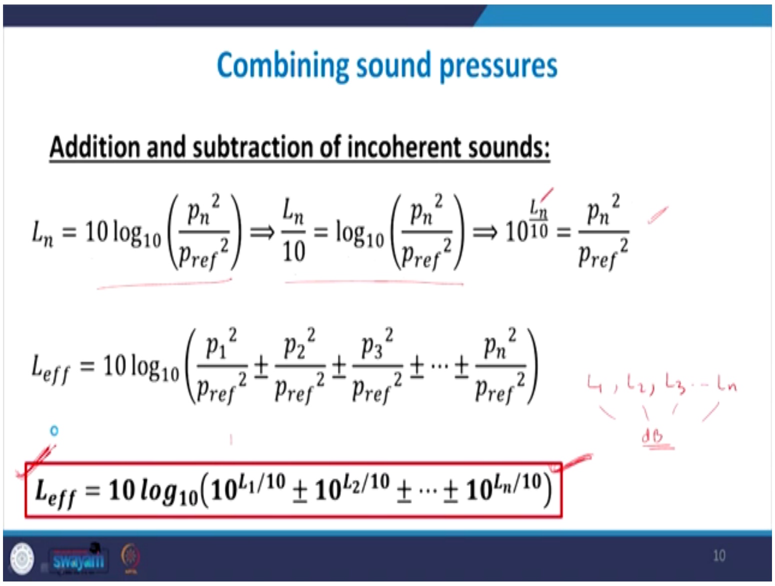

If you calculate this expression in terms of the SPLs so, let us say the SPL of the nth sound the formula for that will be 10 log 10 p n square by p reference square. So, you if you solve this equation then L n by 10 becomes become log 10 of p n by p reference square. And then you do a 10 to the power on both ends. So, this log cancels out and what you get is 10 to the power L n by 10 is equal to p n square by p reference square.

So, you are able to obtain the value of this expression in terms of the level of the sound. So, again putting these respective values what you get is the total effective SPL for suppose we have sounds that are emitting SPL 1, 2. So, suppose we have n sources with L 1, L 2, L 3 and so on such individual way, and sources having individual levels in decibel. So, these are all in decibels.

Then the total decibel the total decibel level when all the sounds are present together will be the total decibels when all the sounds are present together will be given by this expression. So, this is how the SPL combines. So, it is a complicated expression, but you need to remember it. So, this is when the sounds are incoherent in nature.

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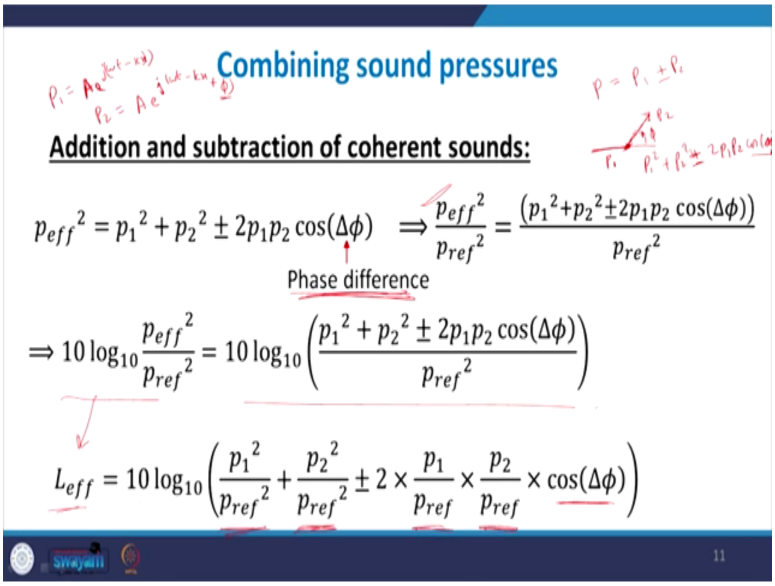

But what if they are coherent they are coming from the same source then we know that p is simply a vector summation of p 1 plus minus p 2 whatever. So, suppose say we have one pressure is given as this particular vector p 1 and the other pressure has some phase difference phi and this p 2. So, we can simply use the vector summation equation, we know that the vector summation for 2 different p for 2 different vectors is going to be mod of 1 vector square plus 2 second vector square plus 2 p 1 p 2 times cos whatever is the angle between the 2 del phi and similarly if it is in subtraction then a negative will come only in this term. So, we are using this vector summation equation here where this is the phase difference. So, what is phase difference? Suppose 1 p 1 is A e to the power j omega t minus k x, it is a forward propagation wave the other p 2 is some A e to the power. So, this is e to the power j omega t minus k x plus some angle. So, some it is having the same waveform only that the starting point is now different.

So, there some phase difference. So, this is what we have taken into account, we divide both the sides by p RMS square p reference square sorry. So, p reference square we divided by. So, this is the expression we get taking 10 log 10 this is the expression we get. So, SPL effective this 10 log 10 is SPL effective it becomes 10 log 10 of p 1 by p reference whole square plus p 2 by p reference whole square plus minus twice of p 1 by p reference into p 2 by p reference into this. So, let us find this individual terms in terms of their corresponding SPL.

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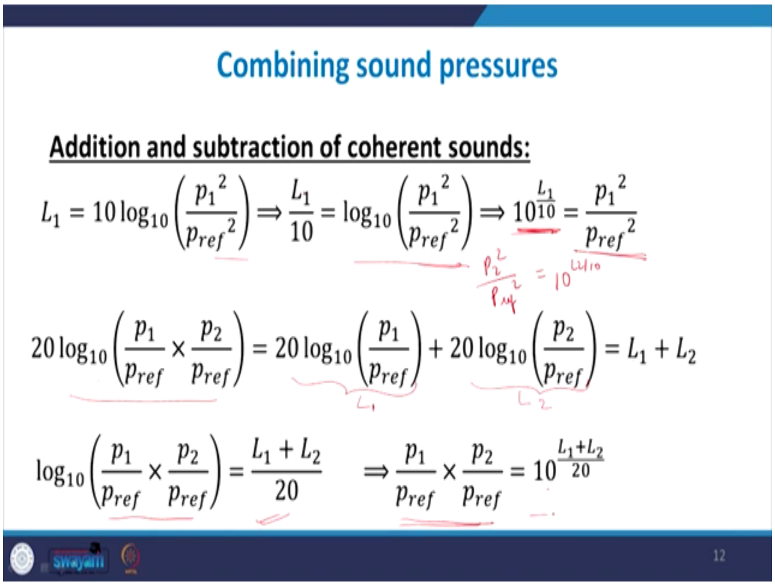

So just like we did it in the previous case, the L 1 of the sound will be this. So, L 1 by 10 will becomes this quantity if you take 10 to the power on both the ends.

So, this comes out to be 10 to the power L 1 by 10. Similarly p 2 square by p reference square will be 10 to the power L 2 by 10. So, we have found these values in terms of their levels. Let us find the value for this. Now if we take 20 log 10 of this particular expression then this can be written as by the property of log it can be written as 20 log 10 of p 1 by p reference plus 20 log 10 of p 2 by p reference. And this is what? This is level of 1 this is the level of sound 2. So, L 1 plus L 2.

So, this is coming out to be L 1 plus L 2 in so, which means that this log thing this thing will be L 1 plus L 2 by 20 solving this equation. So, once this into this is log 20 you take 10 to the power on both the sides. So, you get this expression as 10 to the power L 1 plus L 2 by 20.

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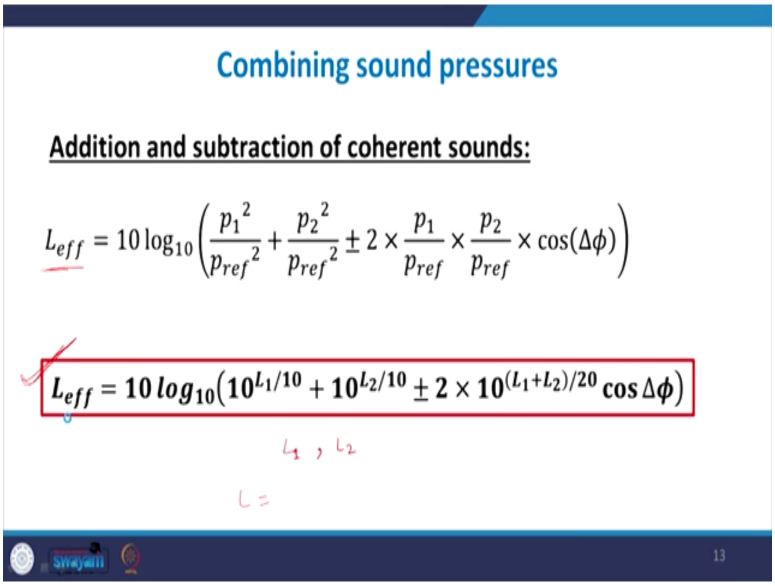

So, this was the expression for the total SPL when 2 different coherent sounds are present together when we put these values in terms of the respective levels. So, this is what we get. So, here suppose we have 2 different sound sources which have level L 1 and L 2 some decibels. Let say one has 40 one has 50 decibels. Then the total then when they are present together the total decibel is given by this formula you put this formula here and you get the total decibels for coherent sounds.

So, you have got these two formula here this one and this one. This is the most commonly used one because most of the sounds there incoherent in nature. Now let us start with solving some problems.

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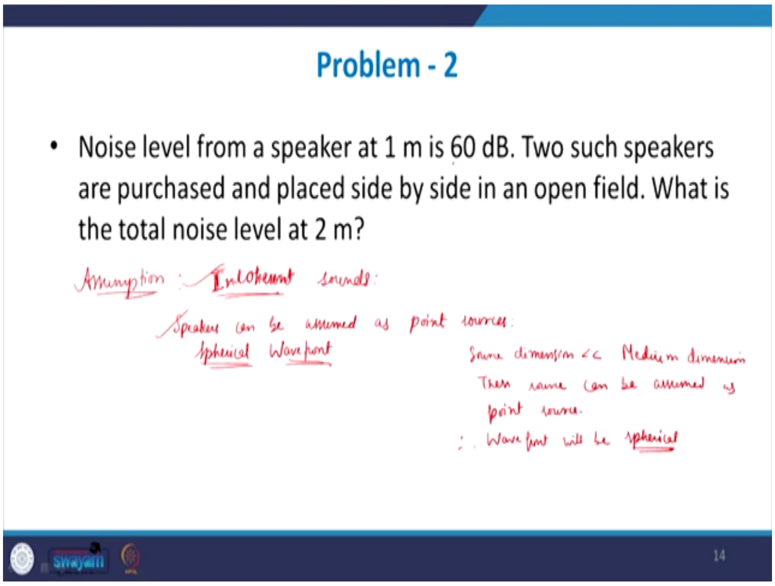

So, the first problem here is the sound level from one speaker is 60 dB at 1 meter and two such speakers are purchased and placed side by side in an open field, what is the total sound level at 2 meters? So, if the nature of the waves is not given its not given whether it is coherent or incoherent then we see that these are two different speakers placed together.

So, we assume it is incoherent in nature; assumption that incoherent sounds. Now let us see. Now we know that also when the type of wave front is not given to you and it is only said that there are two speakers. So, in general in real life whenever there is no constraint in the medium we have open field or an on large space we have two small two sources which are small which whose dimensions are relatively small compared to the dimension of the space. Then we assume that they can be assumed as point sources.

So, the speakers can be assumed as point sources. So, these point sources will have spherical wave front. So, whenever we have whenever the dimension of the source is much smaller compared to the dimension of the medium. So, when source dimension much smaller than the dimension of the medium, so, if a small speaker is present in a large room then it can be assumed then source can be assumed as point source.

So, wave front will be therefore, wave front will be spherical. So, this is the assumptions which we have to take whenever solving such problems. So, now, we have these two assumptions, let us solve the problem ok. So, these values are given to us.

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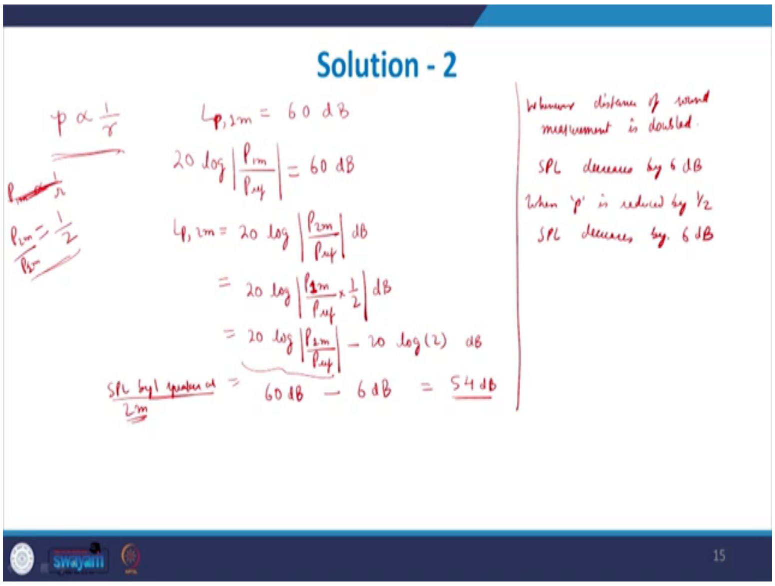

So, because it is a spherical wave front, so which means that the pressure will be inversely proportional to the distance of measurement. So, pressure is inversely proportional to the distance of measurement and it is given that the pressure coming from the first micron the pressure coming from 1 microphone at 1 meters distance is 60 dB. This is given to us. Then this means that 20, so this means that 20 log of the pressure at 1 meter by p reference this comes out to be 60 dB. Then the pressure at 2 meters will be what? The pressure at 2 meters will be 20 log of p at 2 meters by p reference this will be the total pressure at 2 meters and we know that this is the pressure wave front.

So, pressure at 1 meters is inversely proportional to 1 by r. So, pressure at 2 meters by pressure at 1 meter will be what? It will be 1 by 2 it is inversely proportional to r. So, now, the distance has been doubled from 1 meter to 2 meter. So, the pressure is going to reduce by half. So, this comes out to be 20 log p of 2 meters by p reference. So, p of 2 meters is now converted into p of 1 meters into 1 by 2 using this relationship.

So, what we get is this becomes 20 log p at 1 meters by p reference minus 20 log of 2 decibels. And this value we already know what is the pressure value? What is the SPL at 1 meters? It is 60 dB. So, this is 60 dB minus 20 log 2 which is going to be 6 dB. So, this comes out to be 54 dB. So, this is the SPL by one speaker at 2 meters is coming out to be 54 dB.

So, here an important observation is that whenever distance of sound measurement is doubled then the SPL decreases by 6 dB. So, if the distance is doubled that does not mean then the pressure will reduce by half. So, the pressure is reduced or you can say whenever pressure is when p is reduced by half SPL decreases by 6 dB. So, this is the more correct way of saying this.

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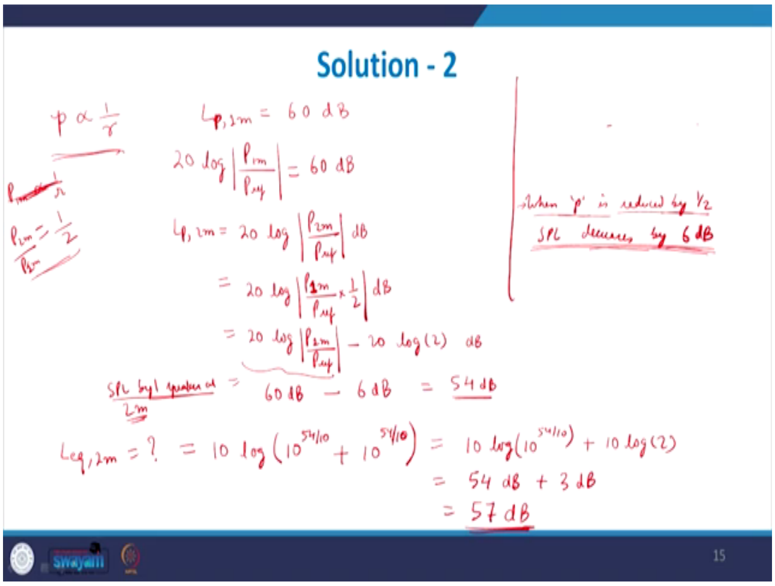

So, this is first observation. So, whenever the acoustic pressure reduces by half then the SPL over here it reduces by it decreases by 6 dB. SPL does not produce by half it reduces by 6 dB. So, we have obtained this. Then the equivalent pressure at 2 meters is what we have to find and this will be using the formula 10 log 10 of 10 to the power the equivalent, the level at 2 meters for 1 microphone which is 54 by 10 plus 10 to the power. The second loudspeaker is also making the same noise. So, it is 54 by 10; so, two sounds.

So, now, we have two speakers place side to side there may then that total SPL at 2 meters will be 10 log 10 of SPL due to 1 divided by 10 plus 10 to the power. So, we have used this formula of combination combining the incoherent sound sources we have used this formula. So, this formula we have used.

So, what you get is it is simply 10 log of 10 to the power 54 by 10 multiply by 2 which we take out which becomes 10 log of 2. So, this 10 to the power log this cancels out. So, what we get is this is 54 decibels plus 10 log 2 is going to be 3 decibels. So, the total decibel we are getting is 54 decibels and that is going to be an answer.

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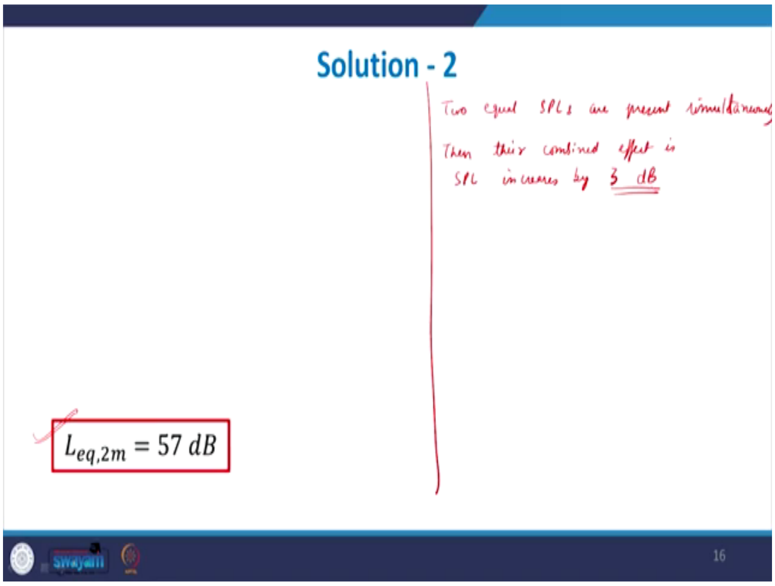

So, the answer is the total sound is 54 decibels. Another observation that we get is that if two equal SPLs are present simultaneously. So, two sound sources when they have the same SPL, then their combined effect is SPL increases by 3 dB. So, as you can see, so, you can see that the decibels can it compresses all this difference. So, here we have two let say we have two sound sources, one is emitting 60 dB sound, the other is emitting 60 dB sound. Then the combined effect is not going to be 120 dB, it is going to be 63 dB . So, it increases by 3 dBs.

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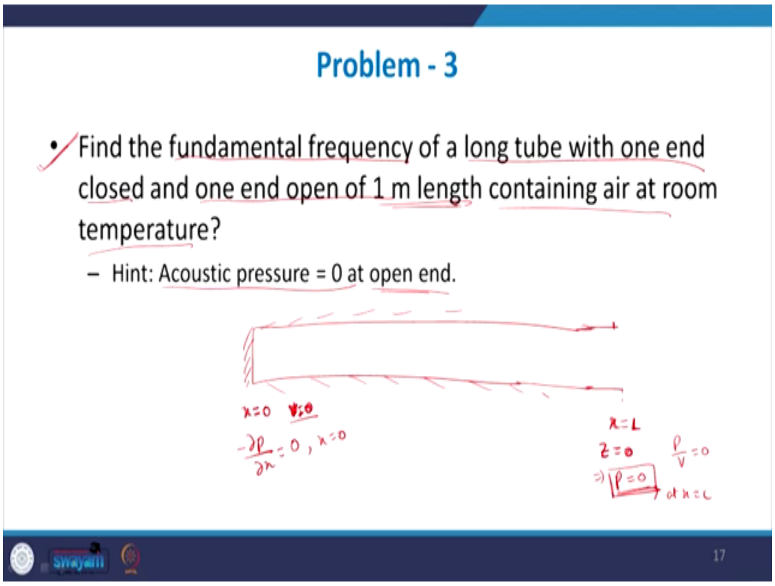

Let us solve one last problem for this class. So, this is a recall problem from what we studied in standing waves lecture. So, here we have to find the fundamental frequency of a long tube with one end closed one end open and 1 meter length containing air at room temperature. So, we already know how to find fundamental frequency of a long tube which is closed at both the ends.

Now, here we have a long tube which is closed at one end open at one end. So, here the trick here is that the acoustic pressure is 0 at open end. So, what we mean here is that suppose this is a long tube, one end is rigid this is also rigid and this end is open. So, what happens at this end is that whatever velocity is entering here is just going out. So, this end is just like an open window. So, here it is offering no resistance to the flow of sound waves. So, which means that the p the impedance is going to be 0 at this end.

So, let say at x equals to 0 we have a closed end and this is at x equals to L. So, this is at x equals to L at the other end we have a open end. So, at the open end it offers no resistance to the flow of sound. So, the z becomes equals to 0. The impedance is 0 there is no resistance which means p by v is 0 which means p is equal to 0. So, the acoustic pressure here becomes 0. So, this is the condition we use for x equals to L p 0 at x equals to L and minus del p by del x is going to be 0 at x equals to 0, why because v is 0.

So, at the rigid end it offers no it offers full resistance to the flow of air. So, no particle can go beyond that. So, velocity particle velocity become 0 at rigid end and the pressure becomes 0 at the open end. So, these are the two conditions.

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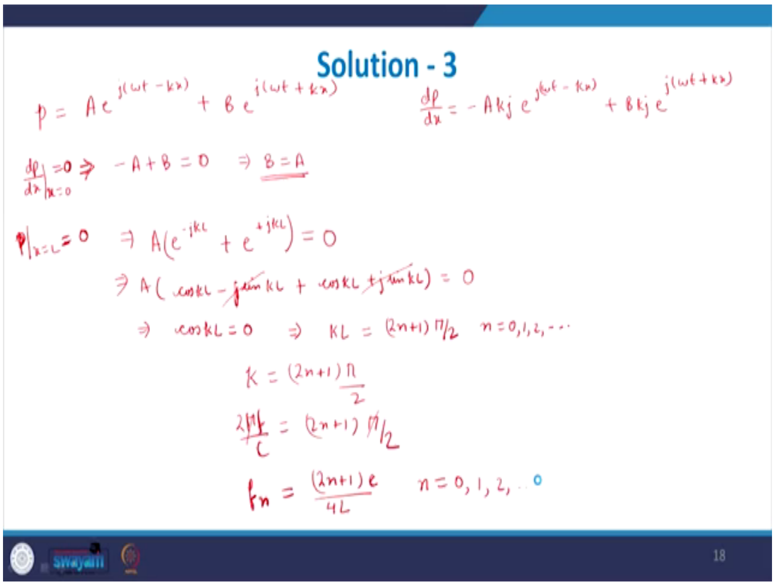

So, let us solve using this condition. So, we assume a general form of pressure A e to the power j omega t minus k x. So, it is a composition of a forward and the backward propagating wave. So, applying the first condition d p by d x at x equals to 0. So, this will be. So, let us calculate what is d p by d x, it is going to be minus A k j times e to the power j omega t minus k x plus B k j e to the power j omega t plus k x. So, putting this at x equals to 0, what will be the value? This cancels out this is the common term, overall what we get is minus A plus B is equal to 0 when you solve it.

So, B is equal to A is what you get putting the second condition that is p is equal to p at x equal to L is equal to 0. So, this was the first condition. So, from the first condition we got this value, the second condition is the acoustic pressure becomes 0 at the open end. So, putting that here what you get is now B is equal to A.

So, which means A e to the power minus j k L plus e to the power plus j k L, here e to the power j omega t is the common term which we have removed. This becomes equal to 0. So, when you put this and using the Euler's relation what we get is cos k L minus j sin k L plus cos k L plus j sin k L this entire thing will be 0.

So, this cancels out. So, what overall we get is that cos k L has to be 0 because A is non-zero. This implies that when will be the cos function 0, at the odd multiples of pi by 2. So, this is going to be the condition, n starts from 0 1 2 3 and so on. So, at n equals to 0 we have pi by 2 as the first solution and then we continue. So, this is what we get then we have to find out the frequency I think, we have to find the fundamental frequency. So, let us derive what is the equation of frequency.

So, this means that k is equal to 2 n plus 1 times pi by 2 2 pi f by c is equal to this quantity. So, f is what? The natural frequency is given by 2 n plus 1 times 4 L into c 2 n plus 1 c by 4 L where n is 0 1 2 so on and we have to find the fundamental frequency.

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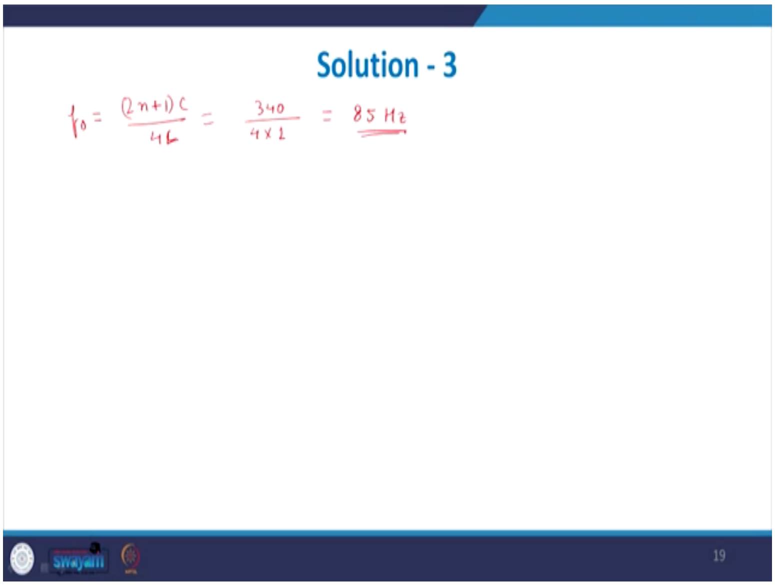

So, f naught or the lowest frequency will be when you put n equals to 0 in this equation what you will get is it will be c which we take as 340 for here at room temperature into 4 into 1 meter which is the length of tube. So, it comes out to be approximately 85 hertz.

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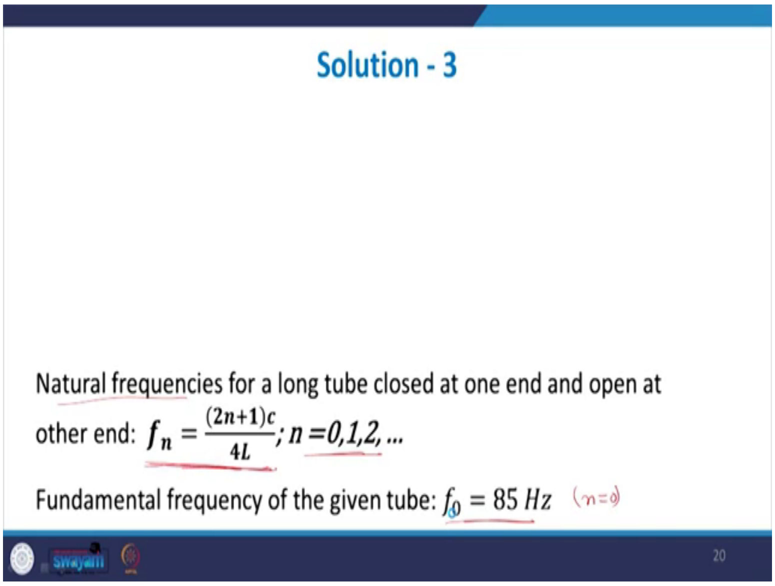

So, this is the solution, here I have noted it down. So, the natural frequencies for open closed tube is given by this and the first mode is when we put n equals to 0. So, this is the first mode. So, anyways.

Thank you for listening to this lecture.