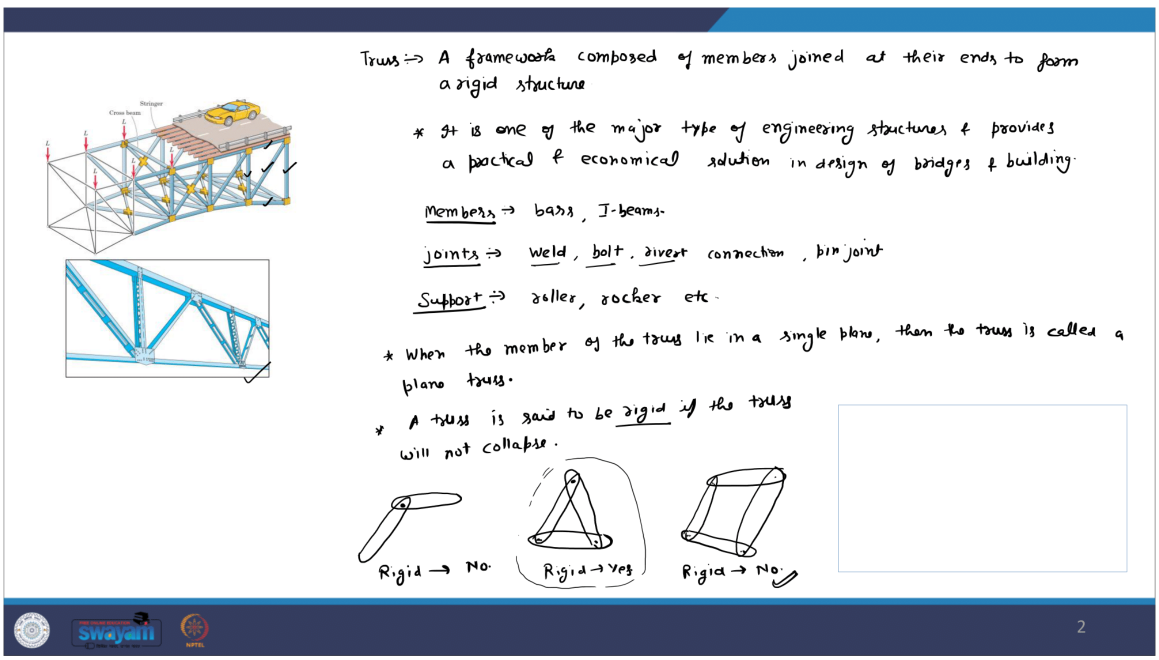
MECHANICS

Prof. Anjani Kumar Tiwari
Department of Physics
Indian Institute of Technology, Roorkee

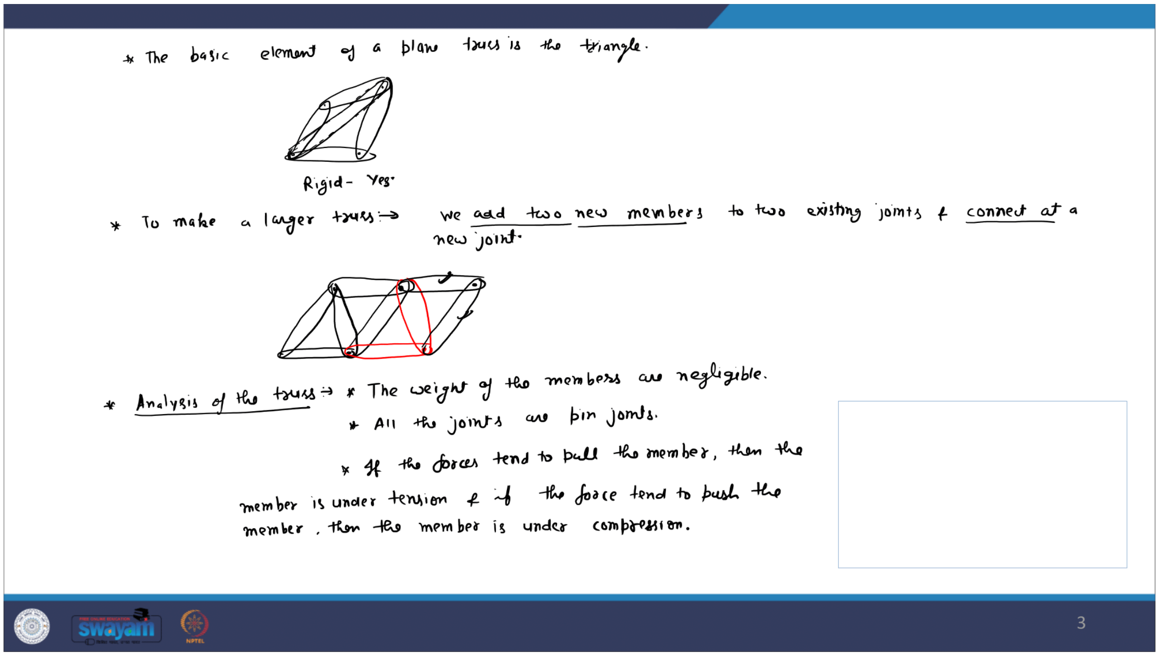
Lecture 15
Structures: Plane Trusses

Hello everyone, welcome to the lecture again. Today, we are going to start a new topic called structure and particularly we are going to look at plane trusses. So, let me first define what is a truss. So, you can see the picture over here. So, for example, this is a truss, and this is the example of a planar truss because the members are in a plane.



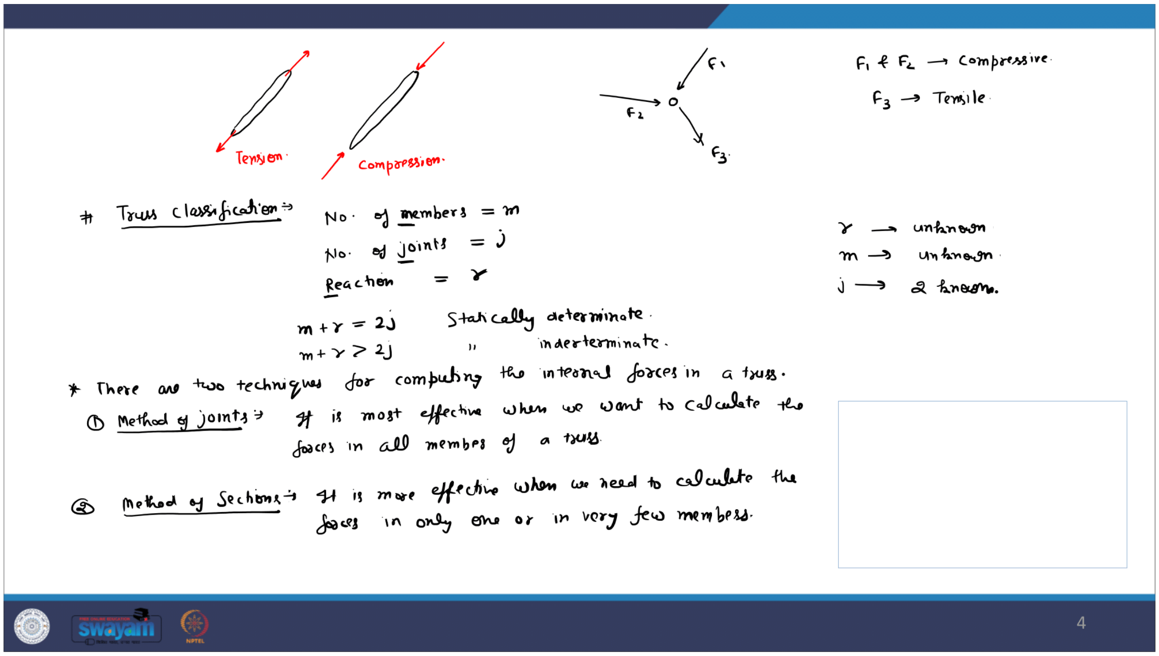
So, by definition, a truss is nothing but a framework which is composed of members which are joined at their ends to form a rigid structure. So, for example, you can see here, these are the members, and they are joined at their end. So, these are members, and these are the end, and they are joined together to form a rigid structure. What do I mean by rigid? Rigid means that when we have this structure, this structure should not fell down. This truss is one of the major type of engineering structure and it provides a practical and economical solution in design of bridges and buildings. Why it is economical? Because you can see here the entire structure is not filled with the material. So, therefore, this need less material and therefore, it is more economical. Now, as you can see here, in the truss, there are many, you know, components. So, the first one is the member. So, these members are nothing but these beams that you see here. So, for example, this one, this one, this one, this one and this one, they are the member. So, basically, they are bars or they can be I-beams. Now, we can als see in the truss that these members or the bars, they are joined at their end. So, there are joints. These joints can be welding or by weld. They can be bolt or by bolting. They can be revert connection or more commonly it can be pin joint. Now, when I say pin joint, So, if you have pin joint then we already know that the pin joint cannot support the moment. So, basically about this pin joint there is a freedom that the beams or the bars can rotate slightly. Now, if your truss is big, then we also, need to support it, okay. So, you will see in many problem that you have support which will help this truss, okay. So, these are for large truss and the supports can be roller support or rocker support, etc. Now, this is general definition of the truss. Now, the second figure that you see here, this is a plane truss. So, this is a general example of the truss. Now, let us see what is a plane truss. When the member of the truss, So, when I say member, I mean bars and I-beams, they lie in a single plane then the truss is called a plane truss. And as I already said, what is a rigid truss? A truss is said to be rigid if the truss will not collapse. Now, let us look at some example. Let's say I have a bar and I connect another bar using a pin. Now, this pin, so, from the pin, the bar can rotate. So, therefore, you can visualize that this truss is not rigid because from this end, the bar are going to rotate. So, this cannot be sturdy. So, therefore, if the question is, is it a rigid truss, then the answer is no. Now, let us look at another structure. So, let's say I have this bar. Now, I connect another bar and then I put one more bar. Now, even if these bars are, you know, they are free to rotate about these points, still this structure will not fell down, okay. So, therefore, if I ask that is this a rigid structure, then the answer is yes. Now, let us look at this structure. So, suppose this time I have four bars. Now, if you give this structure the freedom to rotate about these points, then you can see by yourself that this structure will not be sturdy and it will fell down, okay. So, therefore, again this structure is not a rigid structure, okay.

So, you can see that this triangle structure is basically the building block to make a rigid structure. So, the basic element of a plane truss is the triangle. So, let's say in this example wherein we have you know four members. So, let's say I have four members. So, as I said that this structure or this truss is not a rigid, but since the basic element of a truss is triangle, So, therefore, if you connect one more, you know, member like this, then this structure will become a rigid structure. So, now, this is a rigid structure. So, how do I



make larger truss? So, to make a larger truss, what you have to do is you have to add two new members to existing joint and connect a new joint. So, basically you start with a triangle structure. So, let me start with a triangle structure because this is our basic building block. Now I have to make a larger structure. So, what I have to do is I have to add two new members. So, let me add two new members. So, let's say one member is like this and another member is like that. So, I have had two new member at two existing joint. So, this is one joint, this is the second joint. So, I have had two members and then you connect it with a new joint. So, let us put a new joint here. So, therefore, now the structure or the truss has become big. Now, again if I have to extend it, let us again connect two new member. So, let me connect this member and then let me connect one more member. and then again I put a new joint here, okay. And if I have to extend it further, So, again I can do the same thing. Well, I can put one member, then I can put the second member and add another joint. Now, let us see how do we analyze this truss. So, analysis of the truss. So, by the way, what is the meaning of analysis? Meaning of analysis is to find out how much tension is, you know, acting between different members, what is the tension at the pin, etc. So, to analyze the truss, let us first make some assumption or some idealization. And the first assumption will be the weight of the members are negligible. So, we will not consider the weight of these members because they will support, you know, much, much larger weight than their own weight. And we will also, consider that all the joints are pin joints.

So, as I said previously that the joints can be welding, it can be bolt, it can be revert connection or it can be pin joint. For the analysis, we will consider that the joints are pin joints. Now, in these trusses, you will see that there are member and sometimes these members are under tension or sometimes they are under compression.



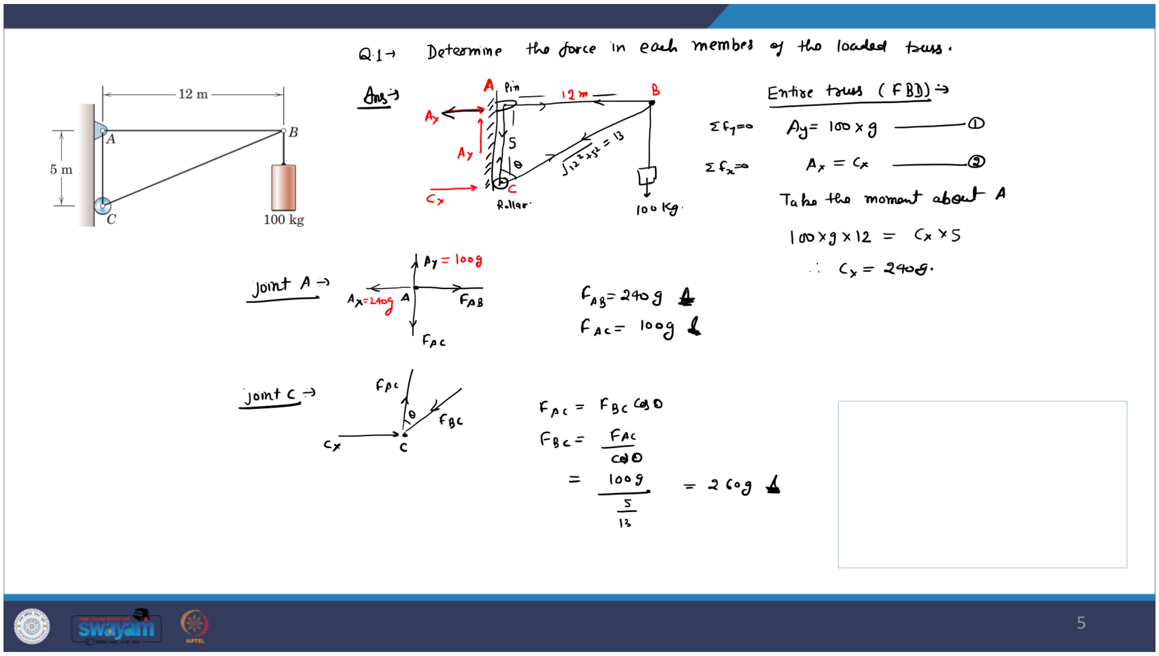
So, let us see in which case they are, you know, under tension and in which case they are under compression and what is the meaning of that. So, if the forces tend to pull the member, then the member is under tension and if the force tend to push the member, then the member is under compression. So, for example, let's say this is one of the member and the forces that are acting on this member is, let's say, like that. In this case, this member is under tension. Similarly, let's say, again, I have this member and in this case, the forces that are acting on the member are like that. In this case, this member is under compression. Now, similarly, you can also, think of the pin that is there.

So, let's say I have this pin and on this pin, let's say various forces are acting. So, let's say this is F_1. This is F_2 and then I have F_3. Then you can see that force F_1 and F_2, they are pushing the pin. So, therefore, it will be a compressive force. And F_3 is the tensile force. Tensile or tension force, whatever you want to call it. Now, let us look at the classification of the truss. So, truss classification. So, as I discussed earlier, in the truss, there are, you know, many members.

Let's say the number of the members are m. So, here this member, I write down m and let's say the number of joints are j and suppose this truss is kept on some surface. So, therefore, you will have the reaction force. So, let's say there are some reactions r. Now, if $m + r = 2j$. In this case, the truss is called statistically determinate truss. So, this is called statistically determinate. What is the meaning of that? Well, statistically determinate means you can calculate all the forces that are acting on the truss. And if your $m + r > 2j$. In this case, the truss is statistically indeterminate. Okay. And how we get this condition? Well, r is the reaction forces and usually we do not know these reaction forces. So, if there are r reactions, then these r's are unknown. Okay. And if there are m members, So, there will be m forces which will act on it. So, therefore, m is also, unknown. Now, for every joint, there are, you know, j joint. For every joint, we have, you know, two scalar forces, F_x and F_y, because it is a planar structure. So, therefore, there is no force acting in the z direction. So, we have F_x and F_y. So, there are two known. And note that moment equation we cannot use because this structure does not support the moment. So, there are $2\,j$ known and $r + m$ are unknown. So, therefore, if $m + r = 2j$, only then we have sufficient equation and we can determine all the forces that are acting on the truss. Now, let us see how do we analyze, you know, or how do we compute all the forces. So, there are two methods.

The first one is called the method of joint. So, just to tell you that there are two techniques for computing the internal forces in a truss. The first one is let's say method of joint. So, this technique of method of joint, it is most effective when we want to calculate the forces in all member of a truss. So, we will look at, you know, these in detail, but when you have to calculate all the forces or all the internal forces that are acting on the truss, in that case, we will use method of joint. And if, So, the second one is method of section, and this is most effective or more effective when we need to calculate the forces in only one or in very few members. So, now with this very basic introduction, now let us look at couple of examples which are based on method of joint.

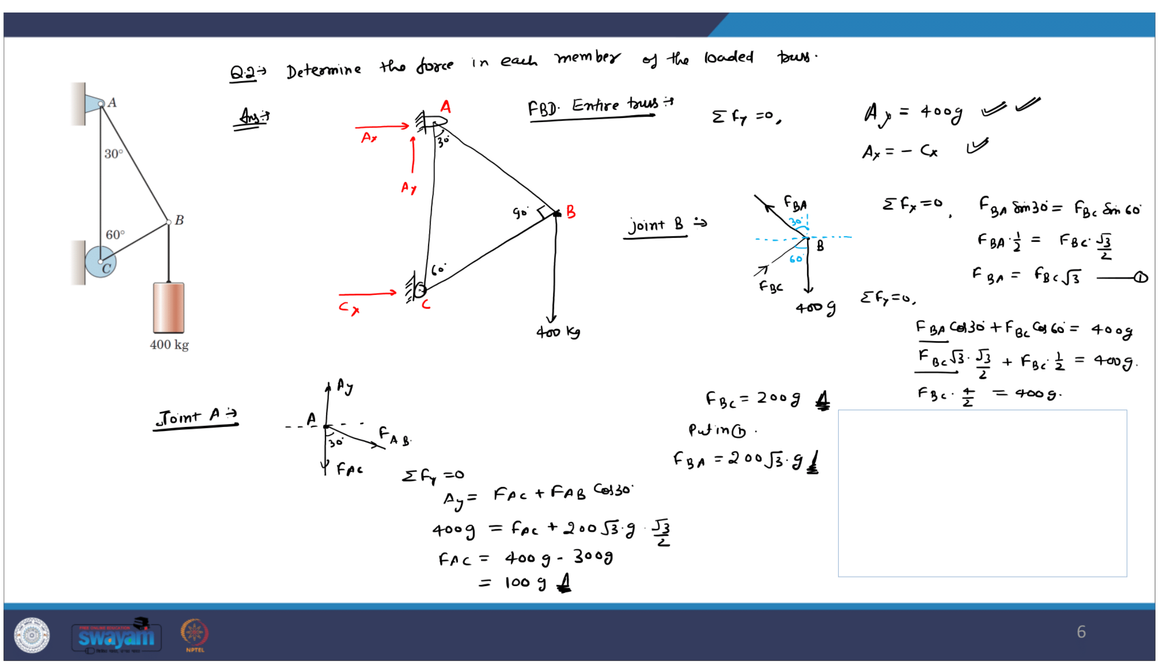
So, this is of course, very simple truss and the question statement is determine the force in each member of the loaded truss. Loaded because we have external load which are acting at point B. So, to analyze this, the first thing is to make the free body diagram of the entire structure or the entire truss. So, let us make the free body diagram. So, we have, So, just for simplicity, I am just making this, but it is not required. So, you have a pin here and you have a roller support at this point. So, this is a pin support, this is a roller and then we have all these members like this and then at B it is connected by a pin, then



you have a 100 kg load. Now, let me draw various forces. So, because it is a pin joint, there will be two force, two reaction force. So, let's say because this point is A, let's say the horizontal force is A_x and the vertical force is A_y. At C, since it is a roller support, So, therefore, it cannot support the reaction in the y direction. So, it will have only horizontal force. So, this point is C. So, let us call this force C_x. So, this makes the free body diagram this is 12 m and as I said to analyze this let us first consider the entire truss.

So, entire truss and we are looking at free body diagram. So, let us apply the force equation $\sum F_y = 0$. So, clearly you can see if I use $\sum F_y = 0$, you get $A_y = 100 \times g$. Let us call it equation number 1. And if I use, So, this is $\sum F_y = 0$ and if I balance the forces in the x direction, then you can see that $A_x = C_x$. Actually, there should be a minus sign because both are in the same direction. So, therefore, the direction for either A_x or C_x should be opposite. So, let me make this A_x in this direction. So, now it is correct $A_x = C_x$. Let us call it equation number 2. Now, to find out what is the value of C_x or A_x, let us take the moment about A. So, if I take the moment about A, then the contribution of A_x and A_y will go away. So, we have 100 kg force. So, $100 \times g$ is the weight multiplied by the perpendicular distance, which is 12 m equal to C_x into the perpendicular distance is 5 m . $100 \times g \times 12 = C_x \times 5$ So, this immediately gives me that $C_x = 240g$ and therefore, I also, know what is A_x. So, this reaction forces and you know other values they are calculated by considering the free body diagram of the entire truss.

Now, let us do it joint by joint. So, let us consider joint A. So, let us make the free body diagram of joint A. So, this is point A and here I have a force. So, this is let's say A. So, at A, you will have a force which will act from A to B and I also, have a force A_y which is acting at point A and I will have a force acting from A to C. So, this is F_{AC} and then there is a force A_x which is acting at point A. Now, some of the force we have already find out. So, let us put those values. So, for example, A_y I have calculated. This is 100 g and A_x also, we know because the value of $A_x = C_x$ and $C_x = 240g$. So, therefore, this is 240 g. So, this makes the free body diagram of joint A and now we can balance the forces. So, you can see that $F_{AB} = 240g$ and $F_{AC} = 100g$. So, we have find out how much forces are acting in AB and AC. Now, to further calculate the other forces, let us look at, you know, other joint. So, let me consider joint C. So, now let us see how many forces are acting at C. So, at C, you will have a force C_x, then you will have a force which will act from C to B and we will have a force which will act from C to A. So, let us call it F_{AC} or F_{CA} whatever and from this direction to balance it, I know that the force should act downward. So, this force direction is not very important. So, as I said you know earlier that if the force comes out to be negative in that case you change the direction of the force. But here it is very obvious because I put the force F_{AC} upward. So, therefore, to balance the vertical force I have to put F_{BC} downward. Now, let us find out the forces. let's say this angle is θ. So, I said this angle is θ. So, I can find out the values of $tan\theta$, $sin\theta$ and $cos\theta$ from the geometry. So, let us balance the force $F_{AC} = F_{BC}cos\theta$ or $F_{BC} = \frac{F_{AC}}{cos\theta}$ and F_{AC} just now I have calculated it is $\frac{100g}{\frac{5}{13}}$ because this will be $\sqrt{12^2 + 5^2} = 13$. So, therefore, this comes out to be $F_{BC} = 260g$. So, therefore, we have calculated all the forces that are acting in different part of the truss.



Now, let us look at another example, question number 2. So, here again the problem statement is determine the force in each member of the loaded truss. So, again let us make the free body diagram of this. So, we have a pin joint here and we have a roller support here. And then this angle is given as 90° because this is 60°, this is 30°. So, therefore, this has to be 90° and from here you have a weight of 400 kg. Now, let us put the forces, the reaction forces. So, since this is a pin joint, So, we will have a force in the x direction and the force in the y direction. Here, it is a roller support. So, therefore, there will be only force in the x direction. So, this point was A, this point was C, this point was B. So, therefore, let me put a force in the x direction. So, this makes the free body diagram. Now, let us balance the vertical forces. So, you can see from the vertical forces, we get $A_y = 400g$. And also, you can see that $A_x = -C_x$. So, this we get by considering the free body diagram of the entire truss. Now, let us look at it by joint by joint. So, let me consider the free body diagram of joint B. So, at B, you can see that there is a force of 400 g which is acting downward and then there will be a force along AB. So, let me put the direction like that and there will be a force along BC, okay. So, let me put this force like that because I have to balance the horizontal forces of F_{AB}. So, therefore, I am going to put it like this and let us call it F_{BC}. Now, from the geometry, you can see that this angle is 30° and this angle is 60°. So, now let us balance the forces in the x direction. So, let me use $\sum F_x = 0$. This gives you $F_{BA} sin30° = F_{BC} sin60°$ and $sin30° = \frac{1}{2}$. So, therefore, $\frac{F_{BA}}{2} = \frac{F_{BC}\sqrt{3}}{2}$. So, I get $F_{BA} = F_{BC}\sqrt{3}$. Let me call it equation number 1. Now, let us balance the force in the y direction. So, $\sum F_y = 0$. So, I get $F_{BA} cos30° + F_{BC} cos60° = 400g$ and let us put the value of $cos30$ and $cos60$. So, I get F_{BA} from equation number 1 is $F_{BC}\sqrt{3}$ and $F_{BC}\sqrt{3}.\frac{\sqrt{3}}{2} + F_{BC}.\frac{1}{2} = 400g$. So, I get $F_{BC}.\frac{4}{2} = 400g$ or $F_{BC} = 200g$. And if I put it in equation number 1, then I get $F_{BA} = 200\sqrt{3}g$. So, I calculated the force along BC and along BA. Now, to calculate other forces like F_{AC}, let us consider joint A. So, let us look at the free body diagram of joint A. So, at A, I have a force which is acting in the y direction, A_y force and then there will be a force along AC and there is a force along AB and the angle between F_{AC} and F_{AB} is given, it is 30°. Now, let us use the force balance along the y direction. So, $\sum F_y = 0$. So, we get $A_y = F_{AC} + F_{AB} cos30°$. A_y, we have calculated, $A_y = 400g$. So, you can look at this equation and $F_{AC} + F_{AB}$. $F_{AB} = 200\sqrt{3}g$ and $cos30 = \frac{\sqrt{3}}{2}$. So, therefore, I get $F_{AC} = 400g - 300g$ or $F_{AC} = 100g$. So, with this, we have calculated all the forces that are acting in this member. So, with this, let me stop here. Thank you.