MECHANICS

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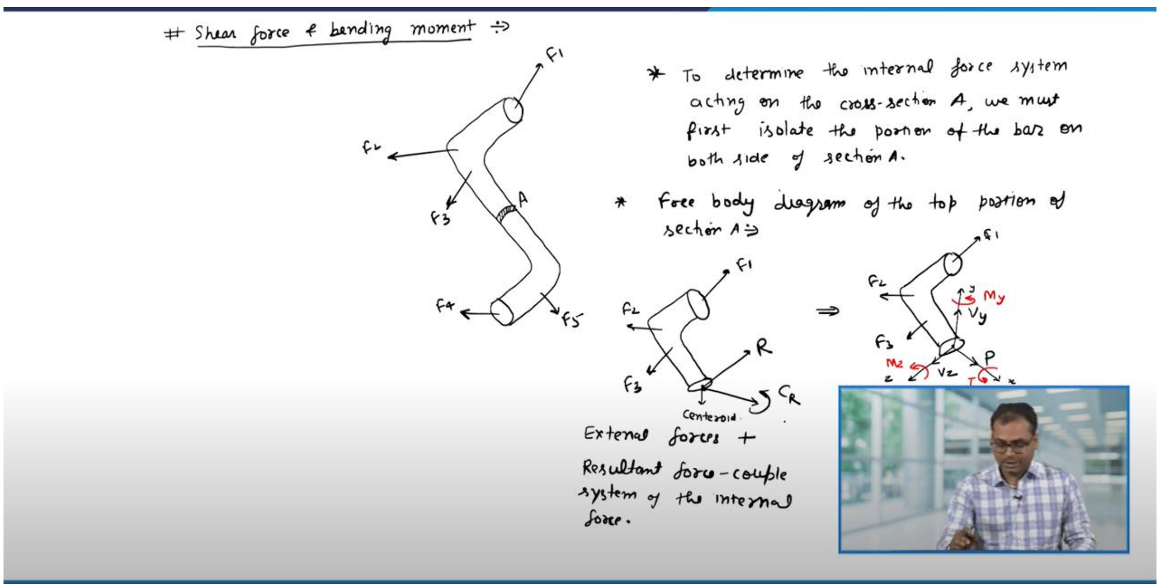
Lecture: 18

Shear force and bending moment

Hello everyone, welcome to the lecture again. In this course so far, we have seen that if the external forces are given, then we can find out the reaction forces and of course, if it is a cable, then the tension along the cable.



Today, we are going to learn that if the external forces in a member is given, then we can also calculate the internal forces by the analysis of equilibrium. Now, these forces are particularly the shear force and the bending moments. So, let us understand what is shear force and the bending moment.

So, let us understand it by example. So, as I said, suppose the external forces are given. So, let us say I have a bar and let us say the bar is like this. And on this bar, let us say various external forces are acting.

So, let us say this is F_1, F_2, F_3, F_4, and F_5. So, these are the external forces and let us say we want to determine what are the forces that are acting on this at a particular area of this bar. So, let us say this is the cross section. Let us say this cross section has an area of A, then to determine the internal forces or the internal force system which are acting on the cross section A. So, first what we should do is we should isolate this part.

So, we must first isolate the portion of the bar on both side of section A, okay. So, here you can see that there are two parts. So, let us isolate them and let us look at the free body diagram of the top section first, okay. So, let us look at the free body diagram of the top portion of section A. So, let me isolate it.

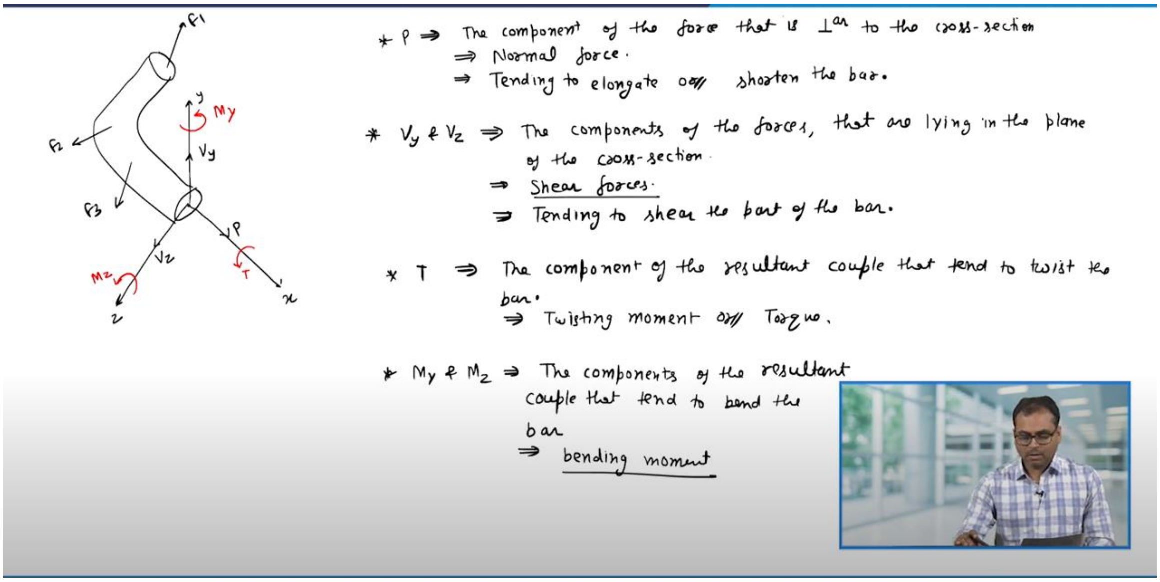
So, we have the force, external force, which is F_1, F_2, and F_3, and then because the whole body is in equilibrium, therefore, on this part, there will be a resultant, let us say R and then along arbitrary line, let us say there is a couple C_R. So, we have in this free body diagram, we have external forces plus the resultant force couple system of the internal force. So, note that this resultant R is acting at the centroid.

Or at the center of this cross-section and this axis is for the couple that is also passing through the centroid. Now, it is convenient to write down this resultant R in terms of x, y, and z coordinates. So, let us rewrite this. So, we have this cross-section and then we have the centroid. And, of course, we have force F_1, F_2, and F_3.

Now, this resultant R, i can write down in terms of the x, y, and z coordinates. So, let us say this is the x-axis, this one is the y-axis, and let us say this one is the z-axis. So, along the x-axis, let us say the component of R is P along the y-axis, it is V_y, and along the z-axis, the component of R is V_z, okay? Now, we will also have the couple.

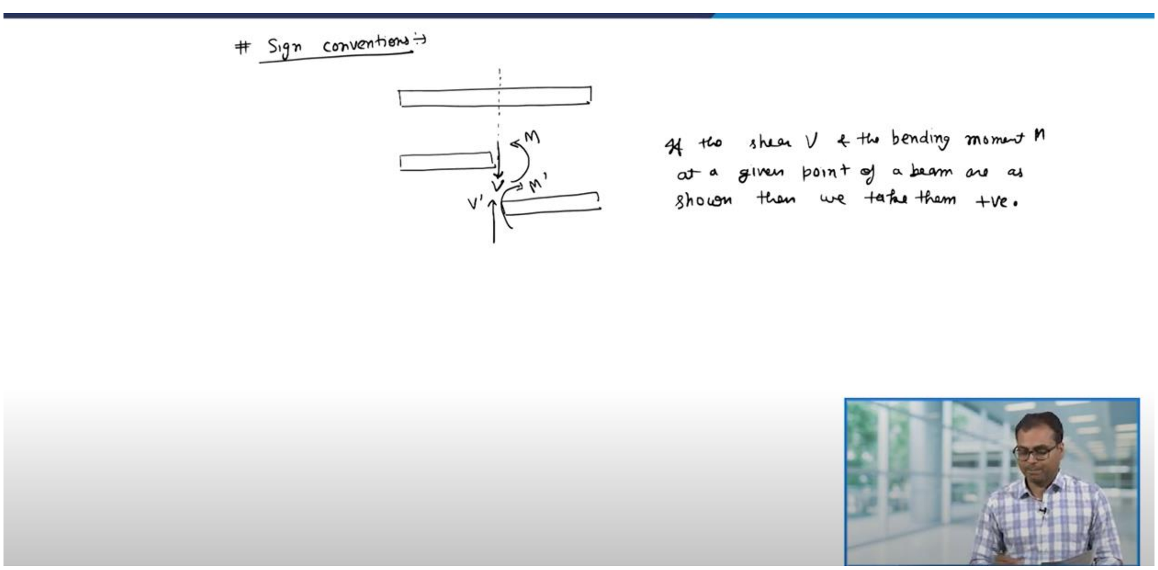
So, along the x-axis, let us say the couple is P. Along the y-axis, the couple is M_y, and along the z-axis, the couple is M_z. Now, let us look at the physical interpretation of this P, P,V_y M_y, and V_z, M_z.

So, let us first look at what is P. P is the component of the force that is perpendicular to the cross section. Now, this force is also called the normal force because it is perpendicular to the cross section.



It has a tendency to elongate or shorten the bar. Now, let us look at what is V_y and V_z. So, this is the component of the forces that are lying in the plane of the cross-section. And these forces are called the shear forces. And they have a tendency to shear the part of the bar. Now, we have couple T. This T is the component of the resultant couple that tend to twist the bar.

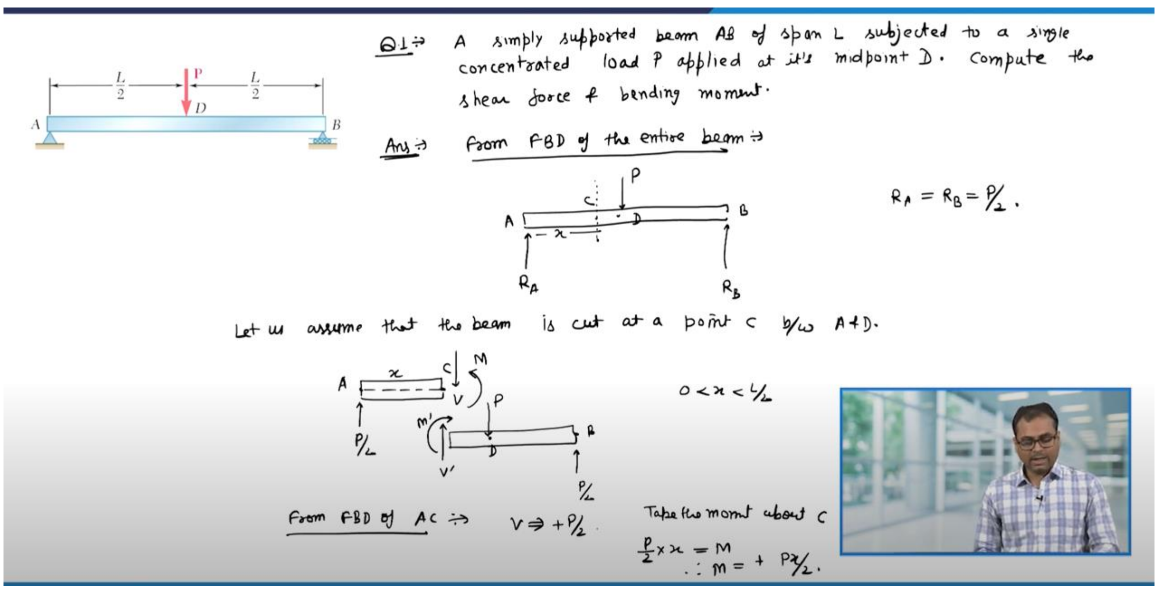
So, therefore, this is also called the twisting moment or torque. Then we have M_y and M_z. These are the components of the resultant couple that tend to bend the bar and they are known as the bending moment. So, we are interested in the shear force and the bending moment.



Now, to calculate this, we also have to follow some sign convention. So, let us look at the sign convention. So, suppose I have a bar and on this bar, you want to calculate the bending moment and the shear force. So, what we do is we think of as if we cut this bar into two parts. So, let us say I cut this bar at this point.

Now, on the left hand side of the bar, let us think that the shear force is acting downwards. So, this V is like this and the moment is in this direction. In this case, it will be positive and on the right hand side of the bar, let us think that the bending force is V' and it is upward and let us say the moment is in this direction, then we can take this as positive sign.

So, if the shear force or shear V and the bending moment M at a given point of a beam are as shown then we take them positive.



Now, let us understand these concepts by one example. So, let us say the question statement is following. So, suppose you have a simply supported beam AB and it has a length of L.

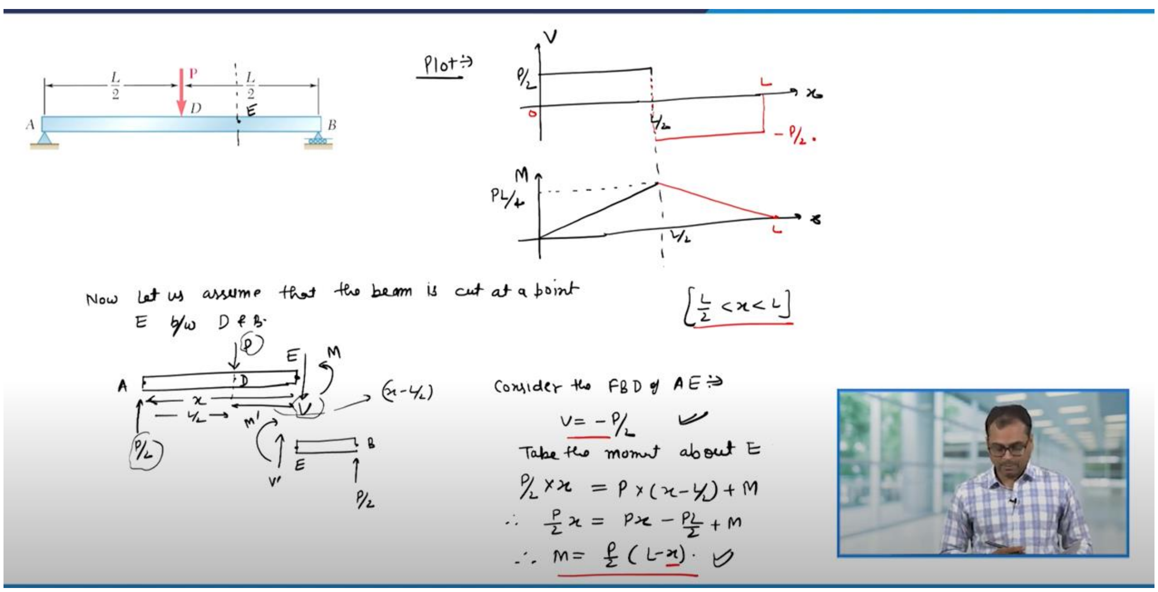
So, of span L which is subjected to a single concentrated load P applied at its midpoint D, and we want to calculate what is the shear force and what is the bending moment on this. So, we want to compute the shear force and bending moment. So, first let us look at the free body diagram of the entire beam so that we can find out the reaction forces at A and B. So, let us look at the free body diagram of the entire beam.

So, let me put all the forces that are acting. So, at point A, we have a force in the y direction. So, let us call it R_A. At B, you have a force again in the y direction R_B, and then at the midpoint, which is D, we have external force of P. Now, from here, you can clearly see that R_A equal to $R_B = P/2$.

So, now, we got the reaction forces, which are acting at A and B. Now, to find out the shear force and bending moment, let us assume that the beam is cut at a point C, which is between A and D, okay? So, let me cut the beam at this point. So, let us call this point C and let us look at the free body diagram of both the part.

So, on the left hand side, we have point A, point C. At point A, we have a reaction force of $P/2$ and then at C, we will have force V and the moment M. Note that I have taken the positive sign. So, therefore, V will be downwards and M will be upward. And if my answers are negative, in that case, I will change the direction of V and M. Now, on the right-hand side, we have point B. Here, the reaction force is $P/2$ again. And at the midpoint,

So, this point is D, I have external loading of P and over here I will have the vertical force V prime and the bending moment M prime. Now, this free body diagram is for the reason x equal to 0 to $L/2$. So, you can see from free body diagram of AC that V is nothing but plus P by 2 and you can take the moment about point C and you can see that $\frac{P}{2}x$. So, let me take this length as x, this length as x. So, $\frac{P}{2}x = m$. So, therefore, m is also plus $P_x/2$.



So, therefore, both the shear and the bending moments are positive. Now, let me plot this. So, let me first plot the shear force. So, this is x and just now we have calculated that the value of the shear force is $P/2$. So, it is independent of x and we have calculated this in the region between 0 and $L/2$.

So, let me put here that this is my $L/2$. So, its value is constant. So, we are plotting B and the value was $P/2$ and also let us look at the shear force. So, the shear force was $P_x/2$. Now, note that this is a function of x and it will be a straight line which is passing through the origin.

So, let me say that this is my M and this is $L/2$. So, it will just rise and its maximum value will be at $x = L/2$. So, let me put $x = L/2$ over here. So, we get $P_L/4$.

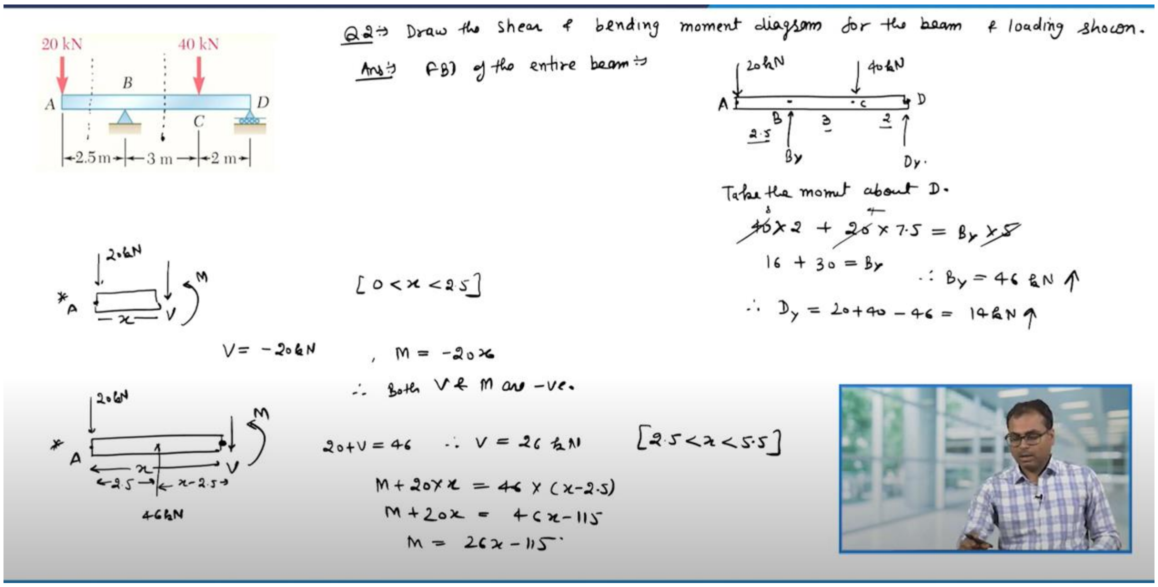
So, that is the maximum value. This is $P_L/4$ and of course, it will be like this. Now, to calculate the bending moment and shear force in the region between $L/2$ and L, So, let us say x is between $L/2$ and L. Let us take a point E over here and let us assume that the beam is cut at this point.

So, now let us assume that the beam is cut at a point E which lies between D and B. So, now let us look at the free body diagram of the left-hand side and right-hand side. So, we have point A, the forces that are acting at A is $P/2$ and then at the midpoint, so this is $L/2$, we have a point D and at this point, the external force P is acting Now, at this end, let us take the positive convention. So, we have bending moment M and shear force V and on the right hand side, we have point B, external force is $P/2$ and this point is of course, E, this point is also E, we have V prime force and the bending moment So, this distance is let us say L_x this we also know that this is $L/2$.

So, therefore, this distance will be $x - L/2$ ok. Now, consider the free body diagram of AE. So, you can clearly see that V should be $-P/2$ because we have force V here and then we have force P and this force is $P/2$. So, therefore, V will be $-P/2$ from $\sum F_y = 0$.

Now, to find out M, let us take the moment about E. So, we have force $\frac{P}{2}x$ equal to force P into its perpendicular distance is $x - \frac{L}{2} + M$. So, we got $\frac{P}{2}x = Px - \frac{PL}{2} + M$, and this gives you $M = \frac{P}{2}(L - x)$. So, let us look at the plot of these. So, you can clearly see here that V is $-P/2$ in the region $L/2$ to L. So, therefore, we have this equal to $-P/2$. Now, let us plot the bending moment.

So, this bending moment because of minus sign, it will come down. And if you put the value of $x = L$, in that case, M should go to 0. So, this is the plot for the shear force and bending moment. Now, let us look at one more example.

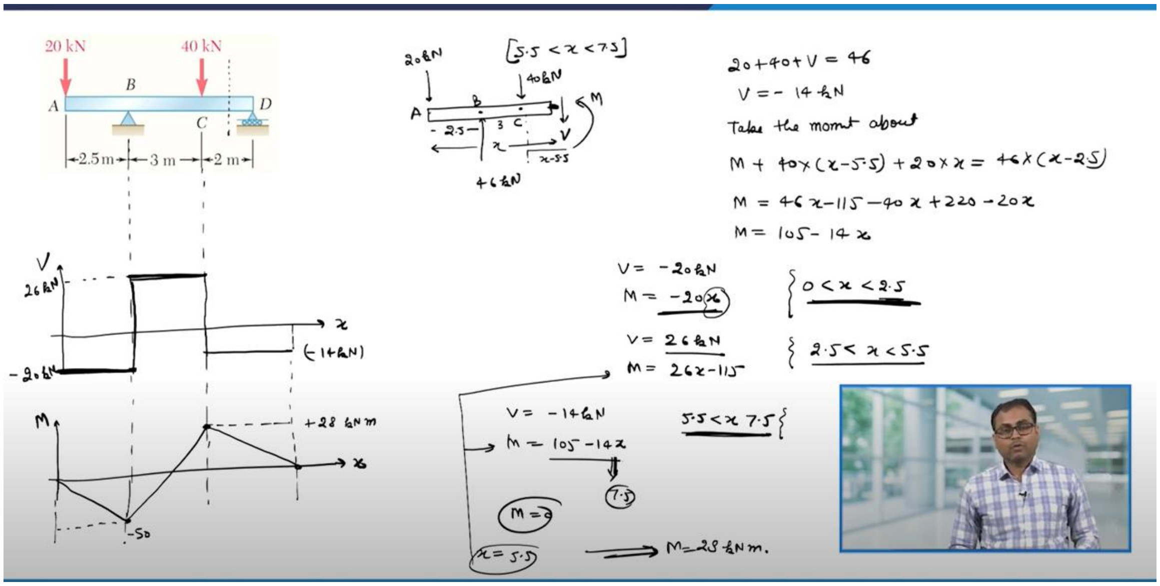


Here, the problem statement is following. Draw the shear and bending moment diagram for the beam and loading shown okay. So, as we did first, let us find out the reaction forces and for that, let us consider the free body diagram of the entire beam.

So, free body diagram of the entire beam. So, we have this beam at point A, you have an external loading of 20 kN. At point B, We will have a vertical reaction, let us say it is B_y. At point C, you have an external loading of 40 kN and at D, you have a roller support.

So, let us say the reaction force is D_y and this is $2.5m$, this is $3m$ and this one is $2m$. To find out what is B_y, let us take the moment about D. So, we have 40 into the perpendicular distance is $2m$ plus 20 into the perpendicular distance is $3 + 2 + \ 2.5$. So, which is $7.5m$ equal to D_y into the perpendicular distance is $3 + 2 = 5$. So, we get $16 + 30 = B_y$.

Therefore, B_y is 46 kN and its direction is upward. Therefore, we can calculate from the force balance equation $D_y = 20 + 40 - 46 = 14\ kN$ and this is also acting upward.



To find out the shear forces and bending moment, let us first cut the beam between A and B. So, we are interested first in the region wherein x is between 0 and 2.5.

And let us look at the free body diagram of the left hand side of this beam. So, we have point A here and at A, I have an external force of $20\ kN$. Let us say this distance is A and over here, there will be a shear force V and the bending moment M. From this free body diagram, you can see that V comes out to be$-20\ kN$. So, that means I have to change the direction, and also, M is$-20x$. So, therefore, both V and M are negative.

Now, let us cut the beam between B and C and let us look at the free body diagram again of the left hand side. So, this time we have this beam, we have this point A and at A there is $20\ kN$ of force, at B we have just now calculated that there will be a reaction force and its value is $46\ kN$. Now, here again, let us say V is acting like this and M is like that.

So, you can clearly see from the force balance equation, $20 + V = 46$. Therefore, V will be $26\ kN$. Now, to find out the M, let us take the moment about this point. So, we have $M + 20x$. So, again, this is $x = 46 + x - 2.5$ because this length is 2.5. Therefore, this will be $x - 2.5$.

And from here we get $M + 20x = 46x - 115$ or $M = 26x - 115$, okay. So, this is in the region wherein my x is between 2.5 and $5.5m$, okay. Now, to find out the shear force between C and D, let us cut the beam like this and here my x is between $5.5m$ and $7.5m$. So, what is the free body diagram of the left hand side? So, we have a 20 kN force and then at $2.5m$, I have 46 kN of force and then, so this is point A, this is point B, then at point C, I have an external loading of 40 kN and then let us say the shear force is acting like this and the bending moment is like that. This length is $3m$ and this is let us say x. So, therefore, this length will be $x - 5.5$. So, from here, you can see from the force balance equation that V is $-14\ kN$. Because we have $20 + 40 + V = 46$. So, therefore, V will be $-14\ kN$.

Now, to find out the moment, let us take the moment about this point. So, we have $M + 40x - 5.5 + 20x = 46x - 2.5m$. So, we got $m = 46x - 115 - 40x + 220 - 20x$. So, this gives you $m = 105 - 14\ x$. So, now let me summarize these results. So, in the region between 0 to 2.5, we got V equal to minus 20 kilo Newton and m equal to minus 20 x. Then in the region between 2.5 to 5.5, we got V equal to 26 kilo Newton and M was 26 x minus 115 and then in the region between 5.5 and 7.5, we got V equal to minus 14 kilo Newton and M equal to 105 minus 14 x. Let us understand it by the plot. So, let us say this is x-axis and we are plotting V. So, first we have V equal to minus 20 kilo Newton. And then it is 26 kilo Newton and then its value is minus 14 kilonewton.

So, this is minus 14 kilo Newton, this one is minus 20 kilo Newton. This is the plot of the shear force. Now, let us look at the bending moment. So, this is the plot of the M. So, let us look here between 0 to 2.5, it is $-20x$. So, therefore, it will be like this and what is the maximum value of this M? Well, its maximum value will be at $x = 2.5m$.

So, let me put $x = 2.5$. So, if you put that, you get $M = -50$. So, therefore, this is -50. And then in this region, its value is $26x - 115$. So, it will go like this.

And then in this region between $5.5m$ and 7.5 its value is $105 - 14x$. So, it is going to decrease because of $-x$ factor ok. Let us see here at $x = 7.5$. So, let us say $x = 7.5$. You can see that M comes out to be 0.

So, therefore, it has to pass through this. And let us look at the value over here. So, here my x is $3 + 2.5$, so x is basically 5.5. Let us put $x = 5.5$ either in this equation or in that equation.

You can see that if you put $x = 5.5$, then you get $M = 28\ kNm$. So, therefore, this value will be plus 28 kNm. So with this, let me stop here. See you in the next class. Thank you.