MECHANICS

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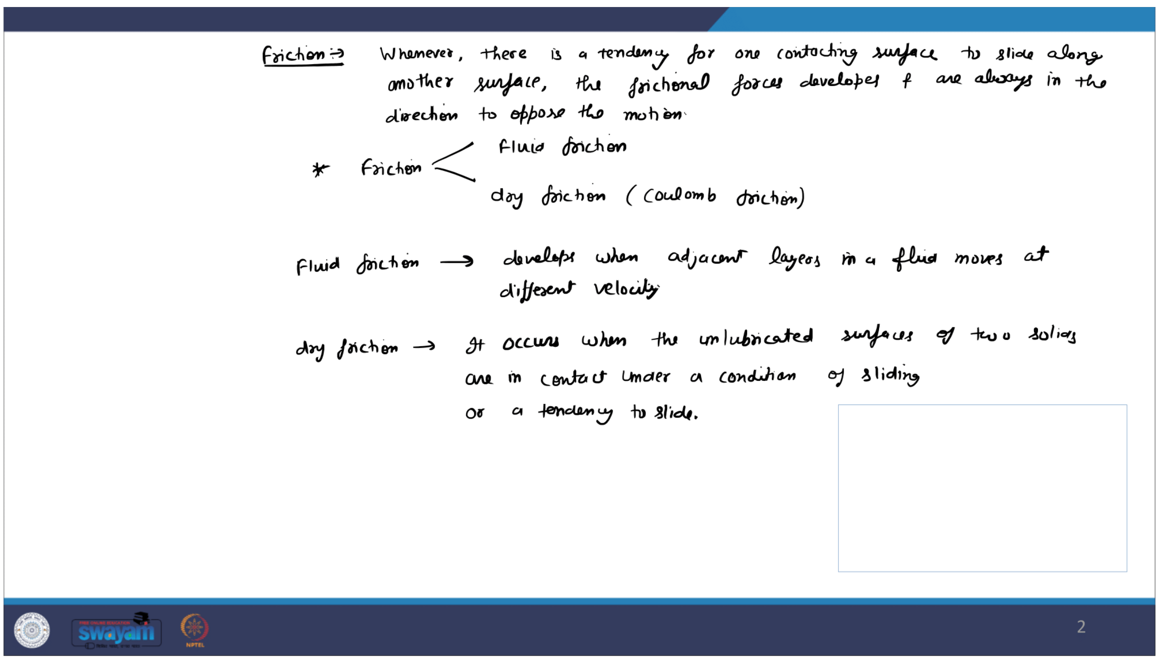
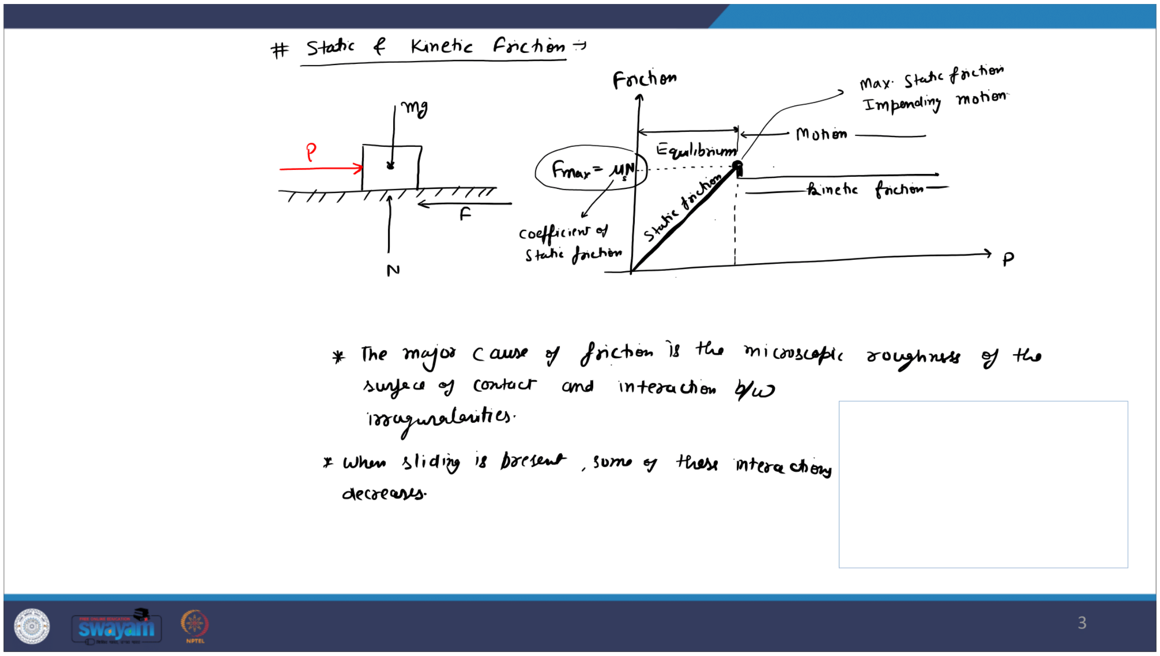
Lecture 24
Friction

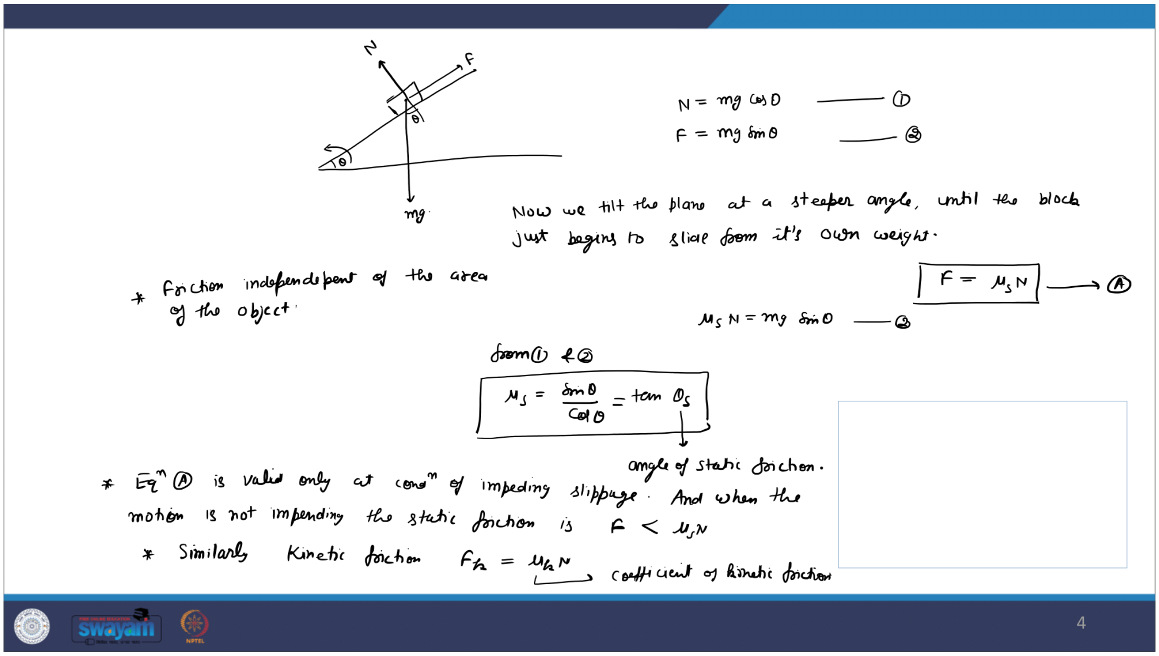
Hello everyone, welcome to the lecture again. Today, we are going to discuss friction. The equilibrium problem that we have analyzed so far were frictionless. Therefore, the reaction forces were always normal to the contact surface. However, we know that all the real surface, they have friction.



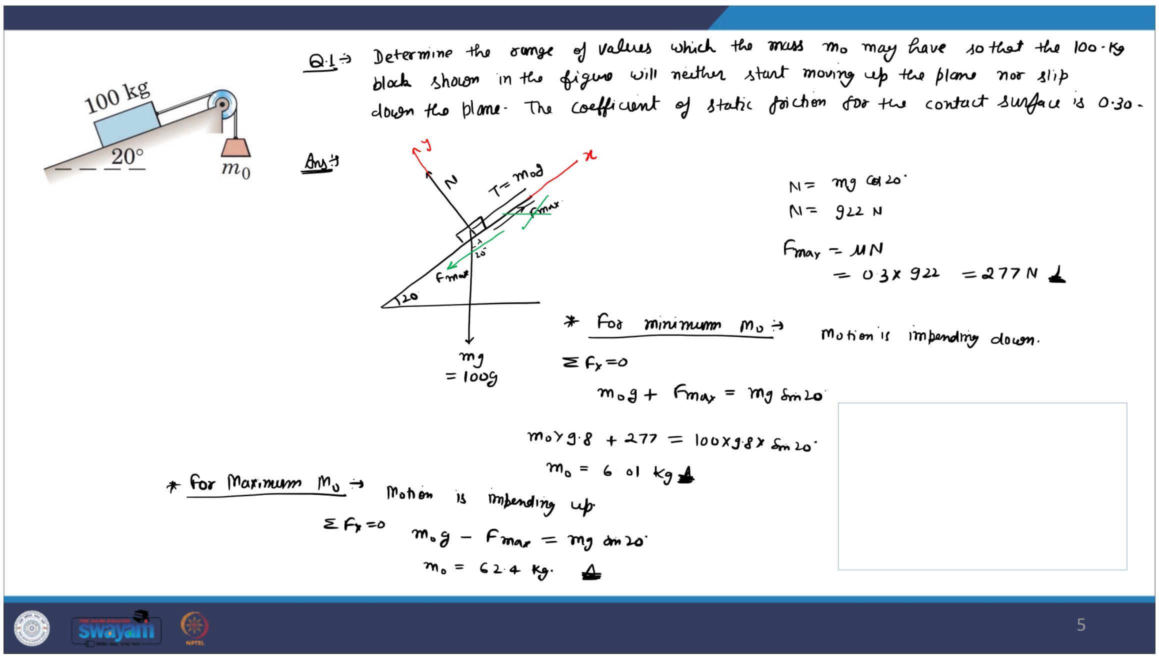
Therefore, they provide a force component that is tangent to the surface and these are called the frictional forces. So, let us see what is friction. Whenever there is a tendency for one contacting surface to slide along another surface, the frictional forces develops and these forces are always in the direction to oppose the motion. Very basically, we can divide these forces into let us say two categories. One is the fluid friction and another one is dry friction or sometimes it is also called the coulomb friction. Let me briefly state what is a fluidic friction. So, these fluidic frictions develops when adjacent layer in a fluid moves at different velocities. Okay and the dry friction it occurs when the unlubricated surface or surfaces of two solids are in contact under a condition of sliding or a tendency to slide.



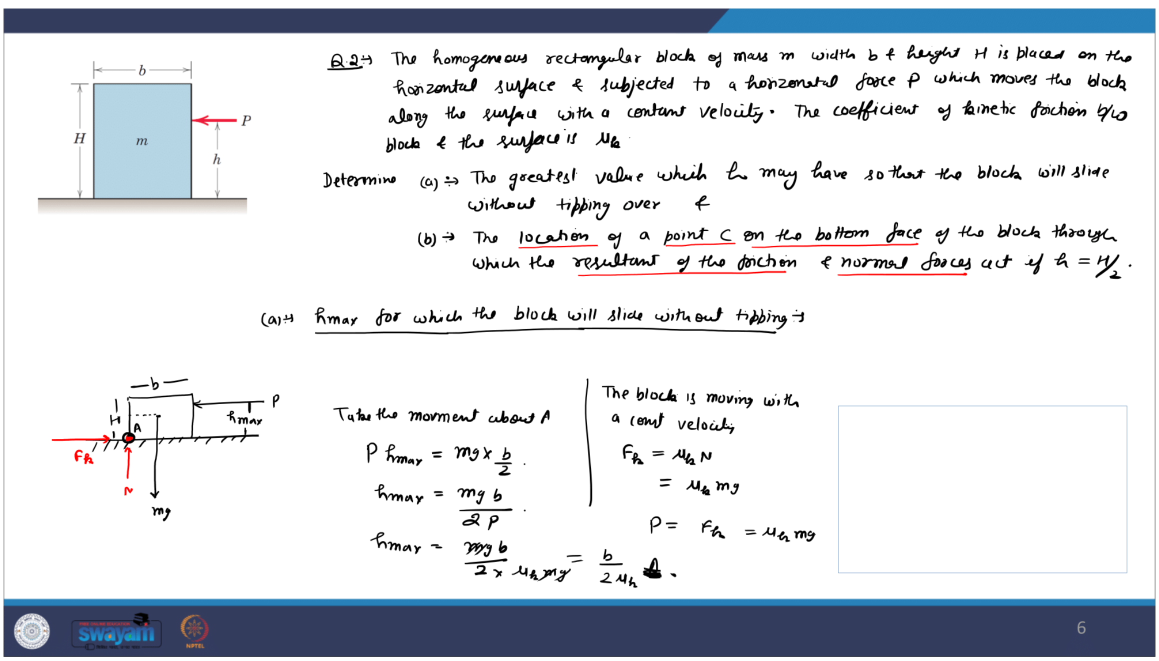
So, if you have two surface which are in contact and they have a tendency to slide in that case we will have dry friction. Now, let us discuss the static and kinetic friction. To understand this, let's say we have a rough surface and on this surface, we have a block of mass m therefore its weight will be mg which will act downward and let us say we apply a force P along the horizontal therefore to oppose the motion there will be a frictional force F in the opposite direction and of course the normal force will be perpendicular to the surface. Now let us see what happens when we increase P. So, as you increase P, the frictional forces will increase to oppose the motion and after a certain value of P, the motion will start. So, up to here, let's say the motion has not begin. So, therefore, the body is in equilibrium. And of course, this is the friction. So, the maximum value of the friction $F_{max} = \mu N$, where this μ is the coefficient of static friction. And after a certain value of P, as I said, the motion will start and the friction will decrease slightly and then it will be constant. So, this is the region where the motion starts. So, up to here, we have equilibrium and then we have motion. This is the point wherein the friction is maxima. So, here we have maximum static friction and this is the point where the motion has just begin. So, therefore, this is also the point where we have impending motion. Therefore, this region, the friction is static or static friction and in this region we have kinetic friction because the motion has started, okay. Now, let me also mention that the major cause of friction is the microscopic roughness of the surface of contact and interaction between irregularities. When sliding is present that means when the motion starts then some of these interactions they decreases. Therefore, you can see that it has decreased slightly and then it is independent of the applied force P.



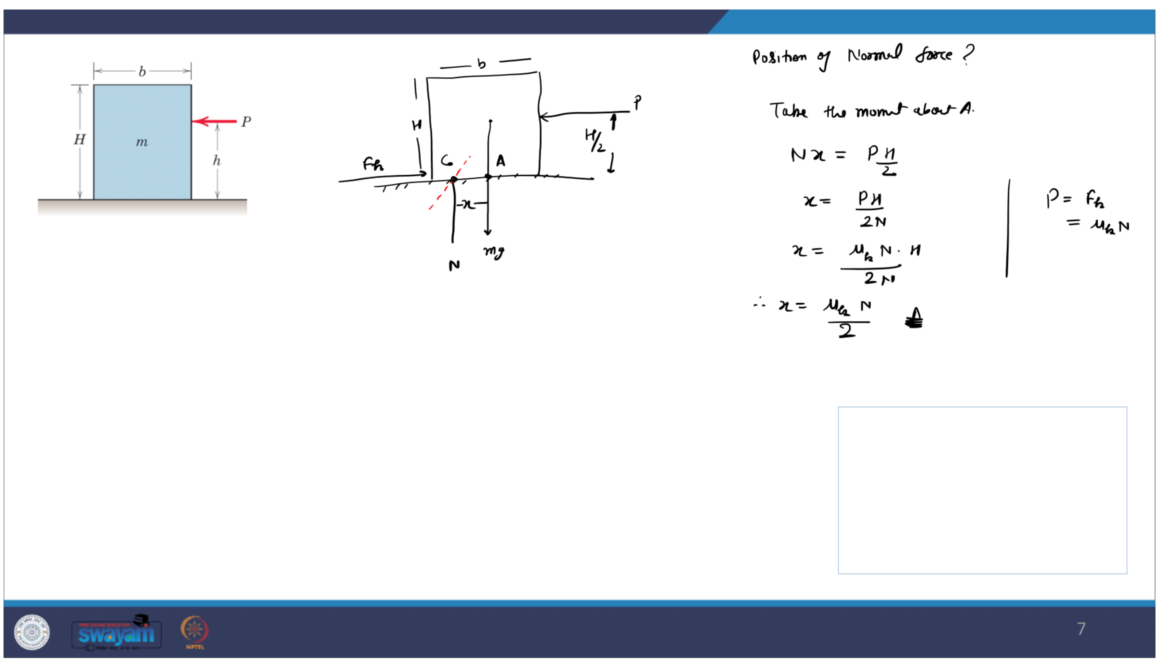
Now, let us analyze the coefficient of friction. For that, let's say we have an inclined plane and it is such that its inclination angle we can change. Let's say on this we have a block of mass m. So, therefore, its weight will be mg and the normal force act perpendicular to the inclination. So, therefore, this angle will also be θ and the frictional forces F will develop to oppose the motion. So, therefore, we can use the condition for equilibrium. So, we have $N = mgcos\theta$ and the friction force $F = mgsin\theta$. Let us say that this is equation number 1 and equation number 2. Now, what you do is you start to tilt the angle so that the block just begins to move. So, what we are doing is now we tilt the plane at a steeper angle until the block just begin to slide from its own weight. So, we also know that the maximum value of the friction $F = \mu_s N$. This is the maximum friction that is possible. Let us call this equation number A. Then from A and equation number 2, we have $\mu_s N = mgsin\theta$. This is the equation number 2. We have just rewrote it. So, from equation number 1, and 2, you can see that $\mu_s = \frac{sin\theta}{cos\theta} = tan\theta_s$. Here, this θ_s is called the angle of static friction. Now, you have to note that equation number A is valid only at condition of impending motion or impending slippage. So, when the motion just begin and when the motion is not impending, then the static friction $F < \mu_s N$. So, here the frictional forces are always smaller than $\mu_s N$. So, this is what we wrote. When the impending motion is not there, in that case, the frictional forces will always be smaller than $\mu_s N$. And once the motion starts, then we have kinetic friction. So, similarly, you have the definition of kinetic friction. Let us denote it by F_k, k for kinetic and this becomes $\mu_k N$ where this μ_k is the coefficient of kinetic friction. One more point I would like to mention here that this F or the frictional forces you can see that they are independent of the area of the object. So, in all this formalism the area of the object does not come into picture. So, let me mention here that the friction is independent of the area of the object.



With this very brief introduction, let us now look at some of the problem based on this concept. So, this is the first problem statement. Determine the range of values which the mass m_0 may have so that the 100 kg block shown in the figure will neither start moving up the plane nor slip down the plane. The coefficient of static friction for the contact surface is 0.3. Now, to analyze this, let us first make the free body diagram of the problem. So, we have an inclined plane at 20°. We have mass of 100 kg. So, therefore, $mg = 100g$ that will act downwards. We have a normal force which will be perpendicular to the inclination. So, let us say it is N and there is a rope which hangs a mass of m_0. So, therefore, the tension in the rope will be $T = m_0 g$ and to find out the value of T, let us take our axis along the inclination. So, let's say this is x-axis and this is y-axis and let us balance the force along the y direction. So, we have $N = mgcos20°$. Therefore, $N = 922\ N$. And once we know N, we can find out the frictional forces, its maximum value. So, $F_{max} = \mu_s N = 0.3 \times 922 = 277\ N$. Now, in the question, it has asked to determine the range of values of m_0 so that this block neither moves up nor it goes down. So, let us first calculate the minimum value of m_0. For minimum value of m_0, we can think that the impending motion of the $100mg$ block will be downwards. So, therefore, you can say that the motion is impending down. Therefore, the frictional forces will be upward. So, the frictional forces that will develop it will be upward. Now, let us balance the force along the x-axis. So, we have $m_0 g + F_{max} = mg\ sin20°$. Now, let us put the values. So, we have $m_0 \times 9.8 + 277 = 100 \times 9.8 \times sin20°$. From here, we can find out what is m_0 and $m_0 = 6.01\ kg$. Now, let us calculate the maximum value of m_0. So, for maximum m_0, when the m_0 is maximum, then the motion of the $100\ kg$, you can think of it is going up. So, in this case, the motion is impending up. Therefore, in this case, the frictional forces are going to act downward. So, this will not be there. Now, let us write down the force balance along the x-axis. So, let us put $\sum F_x = 0$. So, we have $m_0 g - F_{max} = mgsin20°$ and again the value of F_{max} and mg is known. So, therefore, from here, you can calculate m_0. It will be $62.4\ kg$. So, this implies that if the value of m_0 is less than $6.01\ kg$, then the $100\ kg$ block will move downward and when its value is more than $62.4\ kg$, then the $100\ kg$ block will move upward. Therefore, for the equilibrium, the value of m_0 should be between 6.01 to $62.4\ kg$.



Now, let us look at another problem statement and here the problem statement is following. The homogeneous rectangular block of mass m width b and height H is placed on the horizontal surface and subjected to a horizontal force P which moves the block along the surface with a constant velocity. It is given that the coefficient of kinetic friction between the block and the surface is μ_k and we have asked to determine (a) the greatest value which h may have so that the block will slide without tipping over and (b) the location of a point C on the bottom face of the block through which the resultant of the friction and normal forces, they act if $h = \frac{H}{2}$. Now, let us look at part (a). Herein, we have asked to find out the h_{max} for which the block will slide without tipping. So, let me make the free body diagram. So, you have this block of mass m, therefore mg, its height is given H, width is b and we are applying a force P to a height h_{max} so that the tipping does not happen. When the tipping will happen, it will happen at this point. Therefore, the normal force will act over here and the frictional force will oppose the direction of the motion. So, therefore, the kinetic friction will be in the opposite direction of P. So, it is given that the block is moving with a constant velocity. Therefore, the friction force, kinetic friction $F_k = \mu_k N$ and $N = mg$. Therefore, it will be $P = \mu_k \times mg$. Now, to find out the value of h_{max}, let us take the moment about this point. Let us call this point A and take the moment about A. So, in that case, the contribution of N and F_k will go away because they are passing through A. So, we have force $P \times h_{max} = mg \times \frac{b}{2}$. Therefore, $h_{max} = \frac{mgb}{2P}$. Now, we can further simplify it because from the free body diagram, you can also see that $P = F_k = \mu_k mg$. Therefore, $h_{max} = \frac{mgb}{2\mu_k mg}$ and mg will get cancelled. It will be , $h_{max} = \frac{b}{2\mu_k}$.



Now, in the second part, it has asked you to find out the location of the point on the bottom face from which the resultant of the frictional force and normal forces act if $h = H/2$. So, let me formulate this problem. So, again you have the surface and we have this block, its mass is going to act downward. The friction force will act in the opposite direction of P. This height is given. It is $H/2$. And we have to find out from where the line of F_k and N normal force will pass. So, let's say our normal force N act at a distance of x. Then the resultant of N and F_k will also pass through this point. Therefore, if we know where the N is acting, we know from where the resultant is passing. So, let us say this point is A, this point is C and we have to find out what is the position of normal force. For that, what we can do is we can take the moment about A. So, if I do that, then I have $Nx = P \times \frac{H}{2}$, the contribution from F_k will not be there and mg will also be not there because both of them are passing through point A. Therefore, we get $x = \frac{PH}{2N}$ and again we know that $P = F_k$ because we have to balance the force along the x-axis and this will be equal to $\mu_k N$. Therefore, $x = \frac{\mu_k NH}{2N}$. Therefore, $x = \frac{\mu_k H}{2}$. So, therefore, the N has to be passed at a distance of $\frac{\mu_k H}{2}$ and the resultant will also pass through this point. With this let me stop here. See you in the next class. Thank you.