MECHANICS

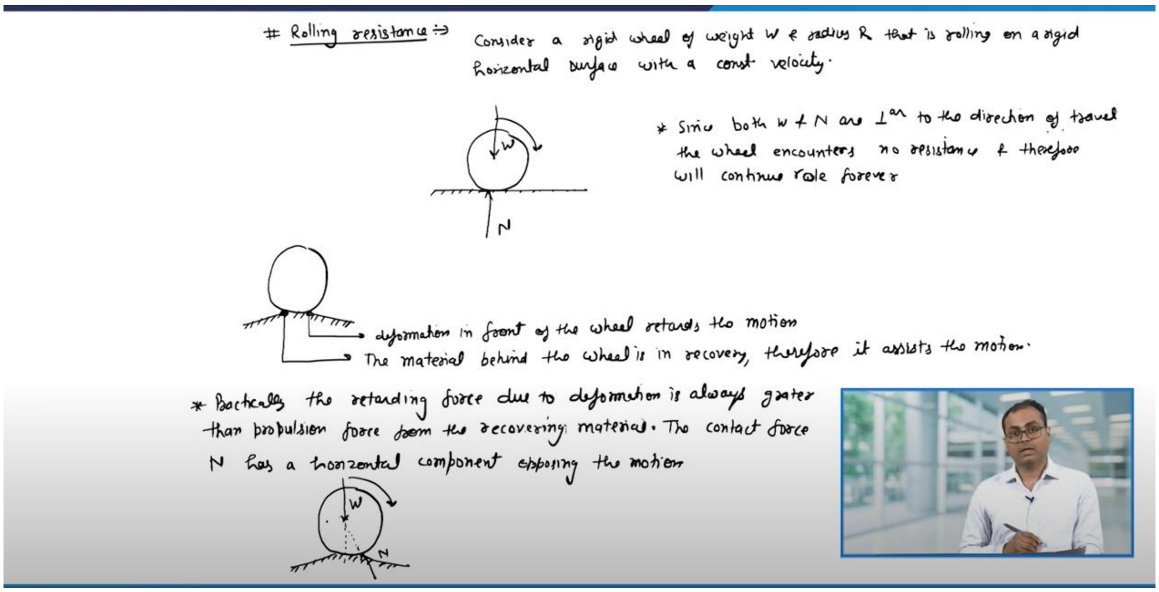
Prof. Anjani Kumar Tiwari

Department of Physics

Indian Institute of Technology, Roorkee

Lecture: 28
Rolling resistance

Hello everyone, welcome to the lecture again.



Today we are going to study the final topic on friction that is rolling resistance. Please note that up to now we have discussed the friction between the rigid bodies that is the deformation was assumed to be negligible. So, let us understand rolling resistance by an example.

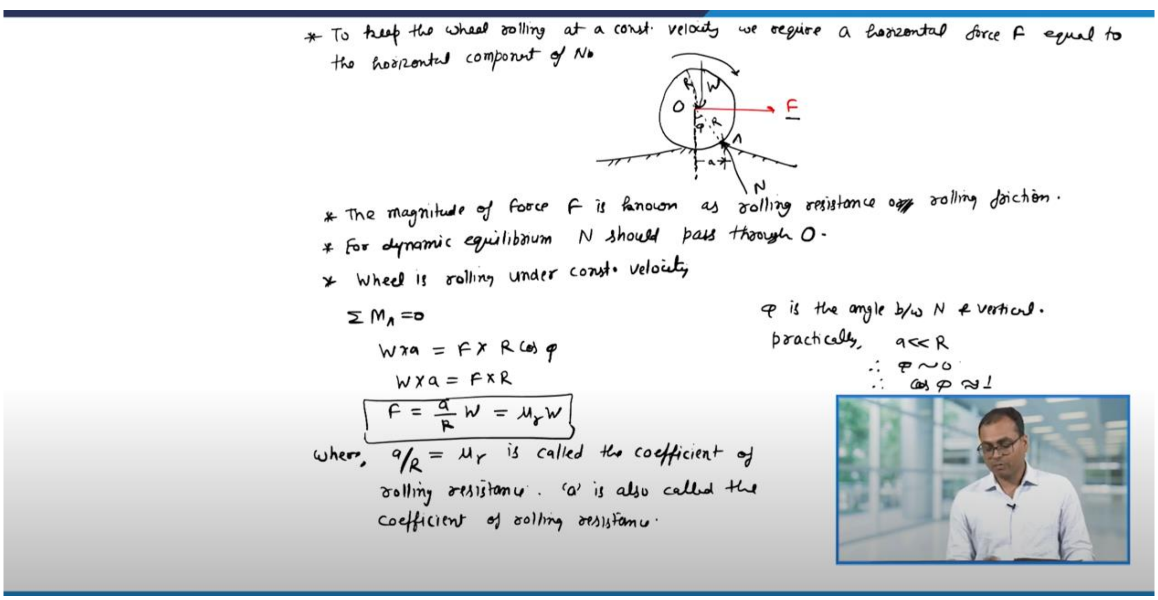
Let us consider a rigid wheel of weight W and radius R that is rolling on a rigid horizontal surface with a constant velocity. Okay. So, the situation is following. We have a rigid wheel and this wheel is rolling on a rigid surface. Okay.

So, let us say its weight is W and in that case, there will be a normal force from the surface on the wheel and let us say this normal force is N. Now, since both W and N, they are perpendicular to the direction of travel, the wheel will encounters no resistance and therefore, it will continue to roll forever. But we know that in the real world, both the wheel and the surface, they deform in the region of the contact. So, therefore, the practical situation is following.

We have this wheel and then we have the surface. Both the surface and the wheel, they deform. The deformation in the front part, it tries to stop the motion. So, the deformation in front of the wheel, it retards the motion and the material which is behind the wheel that is in recovery.

So, therefore, it assists the motion. And practically, we know that the deformation in the front of the wheel is larger than the recovery. Therefore, eventually the wheel is going to stop. So, practically the retarding force due to deformation is always greater than the propulsion force from the recovering material.

And because of that the contact force N will have a horizontal component which will oppose. So, practically the situation is following. We have this wheel which is rolling on the surface and let us say its weight is W, then N is going to act such that it will oppose the motion and this reaction force N has to pass through the center of the wheel.



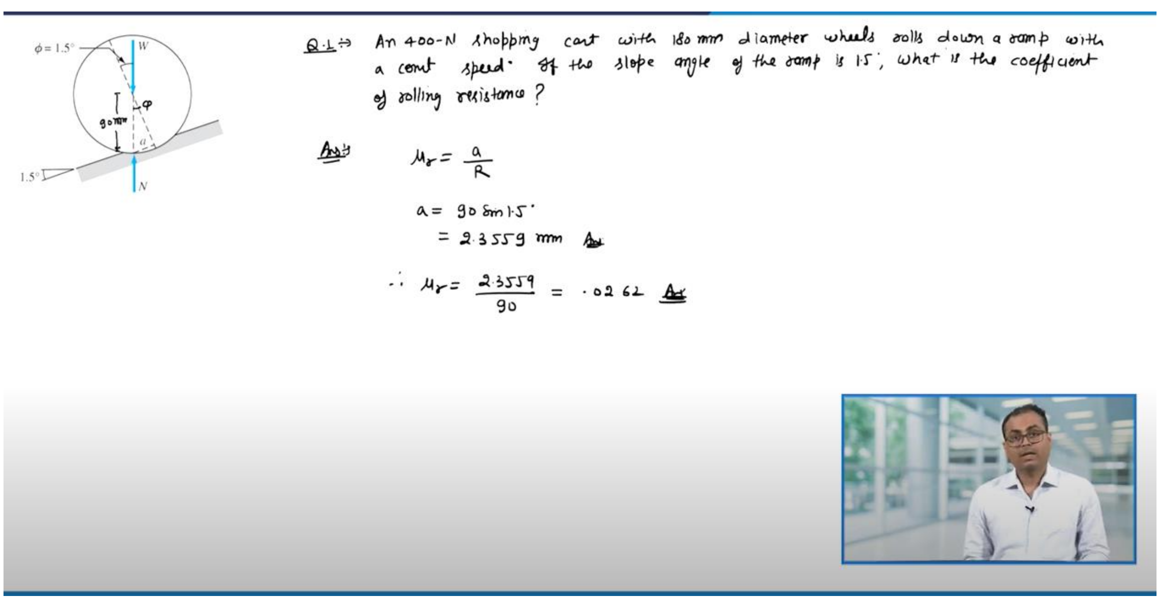
Now, note that to keep the wheel rolling, at a constant velocity, we require a horizontal force, let us say F, which will be equal to the horizontal component of N. So, again we have this wheel. This is on a surface and this wheel is rolling. Let us say its center is O, weight is W, then the reaction force will act like that.

Now, this will pass through the center of the wheel. Now, we have to apply a force F so that this wheel will keep rolling with a constant velocity. The magnitude of force F is known as rolling resistance or rolling friction.

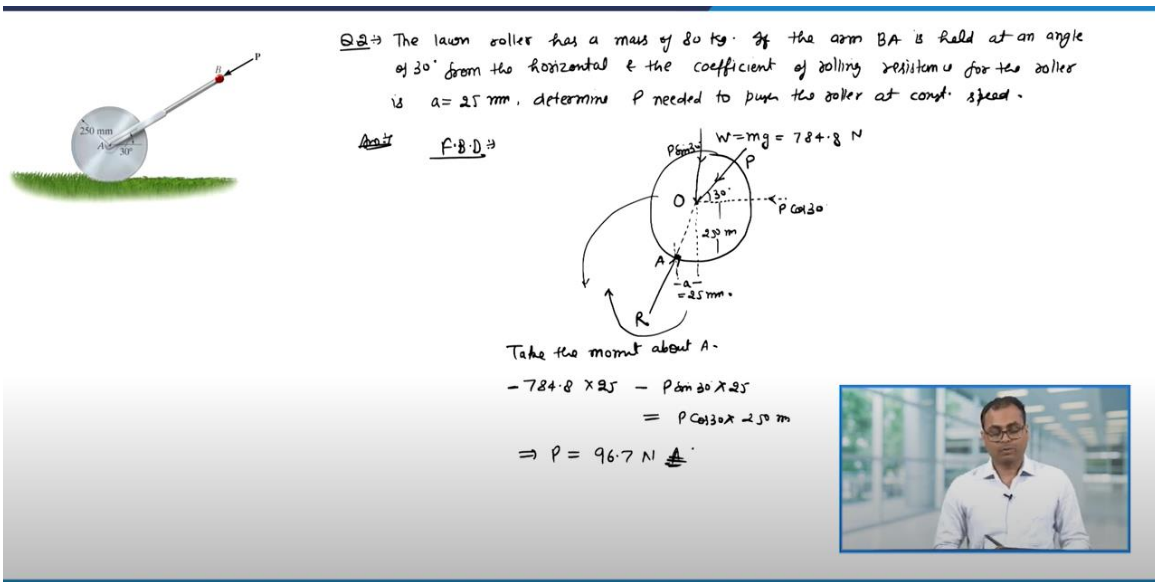
Now, let me mention this again that under the influence of the force F, since the wheel is rolling at a constant velocity, therefore, the wheel is in dynamic equilibrium and therefore, the N should pass through O. So, for dynamic equilibrium, N should pass through O. Now, let us say A is the point of application of N and small a represents the horizontal distance between point A and the vertical line of center of the wheel. Okay? And since this wheel is rolling under constant velocity, therefore, the moment about A will be 0, okay?

So, let us say the radius of this wheel is R, then W into the perpendicular distance which is a equal to the force F into its vertical distance. So, which is, so let us say this angle is ϕ and since this is R, then the vertical distance will be $Rcos\phi$. Now, note that ϕ is the angle between N and vertical, okay? And practically, this a is much, much smaller than R. Therefore, ϕ can be assumed to be 0^0.

Therefore, $cos\phi$ is almost equal to 1. So, we get $Wa = FR$. Or $F = \frac{a}{R}W$ and this can be written as $\mu_r W$ where this $\frac{a}{R}$ or μ_r is called the coefficient of rolling resistance. Sometimes you will see that a is also called the coefficient of rolling resistance. And to identify whether a is called or $\frac{a}{R}$ is called the coefficient of rolling resistance, you have to look at the dimensions.



Now, with this very basic concept, let us look at some of the examples. The first problem statement is following. A 400 Newton swapping cart with one 80 mm diameter wheels rolls down a ramp with a constant speed. If the slope angle off the ramp is 1.5^0, what is the coefficient of rolling resistance? Now, in this question, it is given that the diameter of the wheel is $180\ mm$. Therefore, the radius will be 90 mm and we know that the coefficient of rolling resistance μ_r is $\frac{a}{R}$. Now, from the geometry, you can see that a is $90sin1.5^0$. So, you can take this as phi and then a will be $90sin1.5^0$, which comes out to be $2.3559\ mm$. So, either you can call this the coefficient of rolling resistance in millimeter or you can calculate μ_r. So, that will be $\frac{2.3559}{R}$. So, divide by 90 and that comes out to be 0.0262.

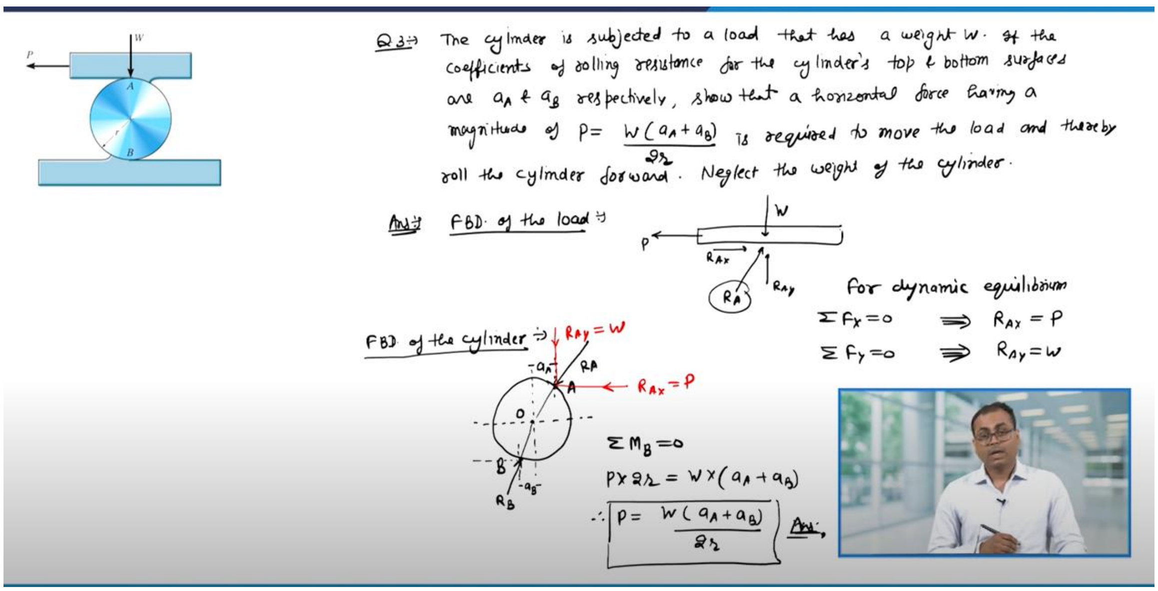


Now, let us look at another problem statement. The lawn roller has a mass of 80 kg. If the arm BA is held at an angle of 30^0 from the horizontal and the coefficient of rolling resistance for the roller is $a = 25\ mm$, determine P needed to push the roller at constant speed. So, let us look at the free body diagram of the roller that we have. So, we have this roller and its weight is $W = mg$.

So, you can multiply 80 by 9.81. So, you get $784.9\ N$. Then we have a force which is acting at an angle of 30^0 from the horizontal. So, this is P, this angle is 30^0.

So, I can, you know, decompose this force into the normal and the horizontal force. So, this will be $Pcos30^0$ and that will be $Psin30^0$. Now, for the dynamic equilibrium, a reaction force R is going to act on this, okay? And let us say, like this.

So, note that this R has to pass through the center of the roller and it is given that a, which is the distance between the point of application of R and the normal distance from point O. So, this is a and it is given that a is 25 mm. So, to find out P, let us take the moment about A. So, in that case, the R will go away. So, we have 784.8 Newton force. Its distance from point A is 25 mm and it has a tendency to rotate in the clockwise direction. So, therefore, it will be negative minus $-Psin30 \times 25$. So, again this force P also has a tendency to rotate in the clockwise direction. So, that will be equal to $Pcos30$ into the radius of the roller which is $250\ mm$ and this will be positive because that has a tendency to rotate in the anticlockwise direction and this gives you $P = 96.7\ N$.



Let us look at another problem statement. The cylinder is subjected to a load that has a weight W, if the coefficients of rolling resistance for the cylinders top and bottom are a_A and a_B, respectively show that a horizontal force having A magnitude of $P = W(a_A + a_B)/2r$ is required to move the load and thereby roll the cylinder forward and it is given that neglect the weight of the cylinder okay. So, first let us look at the free body diagram of the load that we have.

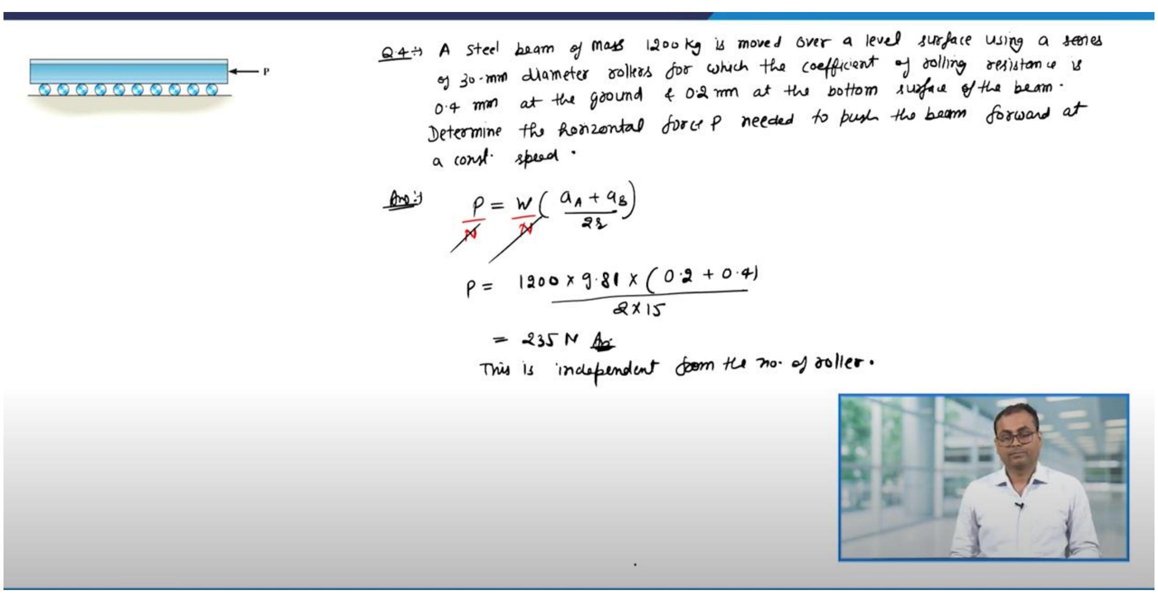
So, we have this load and on this load a force P is applied and its weight is W. Now, you have to note that everything is in a dynamic equilibrium. So, therefore, a reaction force from the cylinder is going to act on it. So, let us say the reaction force is like that. Let us say it is R_A.

So, this R_A will have both the horizontal and the vertical component. So, let us say the horizontal component is R_{Ax} and its vertical component is R_{Ay}. , then because everything is in dynamic equilibrium, we can use $\sum F_x = 0$. So, we have $\sum F_x = 0$ and this gives me $R_{Ax} = P$. Similarly $\sum F_y = 0$ is going to give me $R_{Ay} = W$. Now, let us look at the free body diagram of the cylinder.

So, we have this cylinder. Now equal and opposite R_A is going to act on it and this R_A has to pass through the center of the cylinder. Let us say it is acting at a distance of a_A and similarly from the bottom the reaction force is going to act. And again, it will pass through the center of the cylinder. So, let us say this is R_B and it will act at a distance of a_B.

Let us say this point is A and this point is B. Now, since everything is in dynamic equilibrium, therefore, the moment about B should be 0. So, we can take $\sum M_B = 0$. And before that, let me break this force R_A into the horizontal and vertical components. So, this is R_{Ax} and just now we have found that this $R_{Ax} = P$ and $R_{Ay} = W$.

So, for dynamic equilibrium $\sum M_B = 0$. So, we have this force P and its vertical distance from point B is $2r = W$ into the distance from point B, the normal distance and that is equal to $a_A + a_B$. Therefore $P = W(a_A + a_B)/2r$.

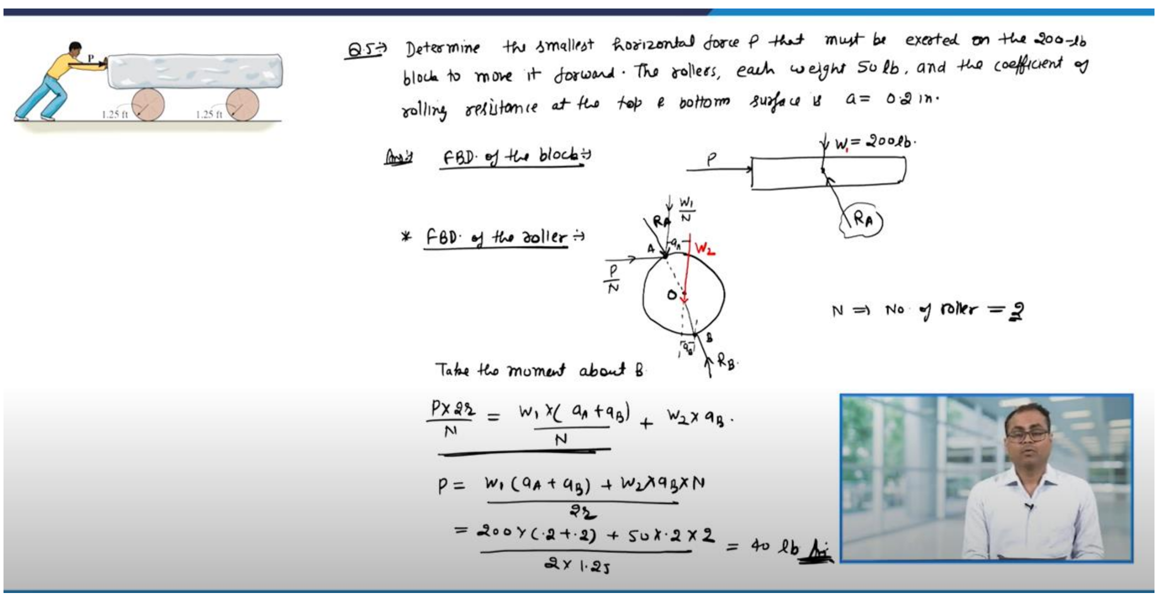


Now, based on the same concept, let us look at one more problem. So, the problem statement is following. A steel beam of mass 1200 kg is moved over a level surface using a series of 30 mm diameter rulers for which the coefficient of rolling resistance is 0.4 mm at the ground and 0.2 mm at the bottom surface of the beam. Determine the horizontal force P needed to push the beam forward at a constant speed.

So, we can use the formula or the result that we have derived in the previous question. We have $P = W(a_A + a_B)/2r$. Now, note that this force P is going to be divided over N number of rollers. So, let us say there are N rollers. Similarly, the weight will also get divided over N rollers.

So, eventually this N will get cancelled and we have the formula that we derived earlier. So, P will be equal to W. $W = 1200 \times 9.81 a_A$. a_A Is given. It is 0.2 and a_B is $\frac{0.4}{2} r$. r is 15 mm because the diameter is 30 mm.

So, that comes out to be $235\ N$. So, please note that in this case when the rollers were massless, we found that the force that is required to move this beam with a constant speed is independent of the numbers of rollers that we have. So, this result is independent from the number of roller.



Now, let us look at an example where the roller has the mass. So, the problem statement is following. Determine the smallest horizontal force P that must be exerted on the 200 pound block to move it forward. Each weight 50 pound and the coefficient of rolling resistance at the top and bottom surface is $a = 0.2\ inch$.

So, let us first look at the free body diagram of the block that we have. So, we have this block, force P is applied on it and its weight is $W = 200\ Pound$. So, since everything is in dynamic equilibrium, therefore the reaction force from the roller is going to act on it and it will act like this. Now, let us look at the free body diagram of the roller.

So, we have this roller, its center is let us say O. Now, since you have this reaction force A, therefore, the reaction force is going to act like that and this time, this roller also has a weight. Let us say this is W_2 and this one is W_1. Now, from the free body diagram, the component of R_A along the horizontal direction will be P/N, where N is the number of roller that we have.

Similarly, the component of R_A along the vertical direction will be W_1/N. Similarly, you have the reaction force from the bottom surface and let us say this point is B, this point is A, this distance is a_A and this distance is a_B. So, everything is in dynamic equilibrium. Therefore, we can take the moment about B. So, we have $P2r$ because the distance is $r + r$ equal to $2r/N$ equal to $\frac{W_1(a_A+a_B)}{N} + W_2 a_B$.

So, note that here we have used the previous result that we have derived. So, from here, we get $P = W_1(a_A + a_B) + W_2 a_A a_B N$. So, let me mention here that N is the number of ruler which is equal to $2/2r$. So, let us put the value. We have $200 \times 0.2 + 0.2 + 50 \times 0.2N = \frac{2}{2r}$. r Is 1.25 and this is equal to 40 $Pound$. With this, let me stop here. See you in the next class. Thank you.