MECHANICS

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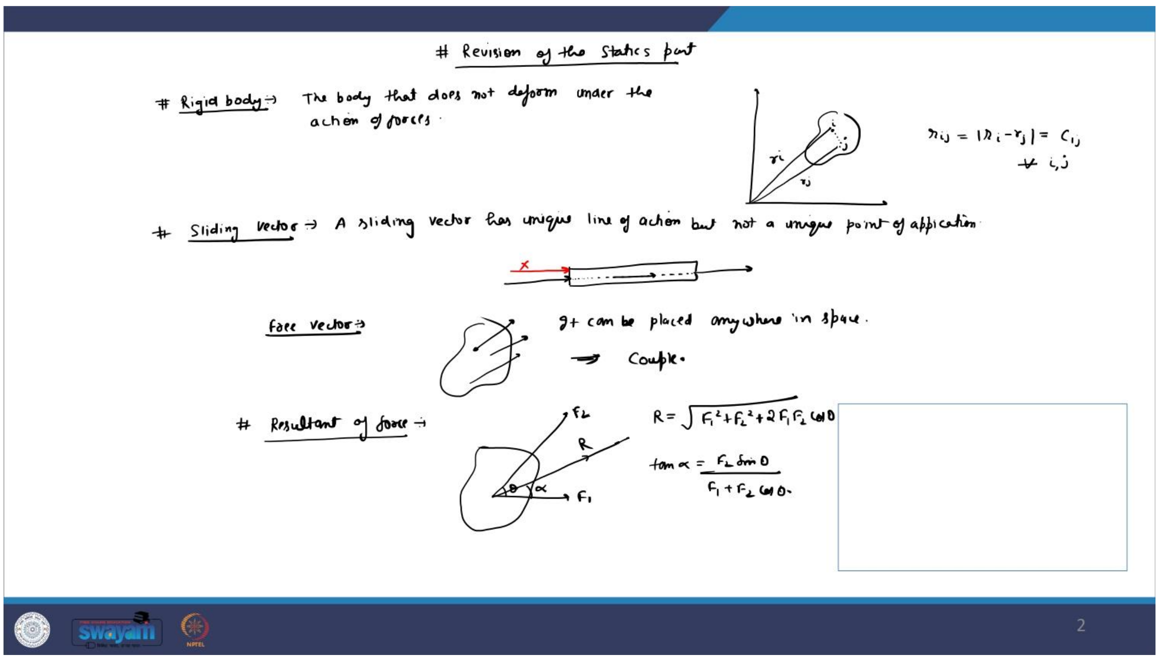
Lecture 29

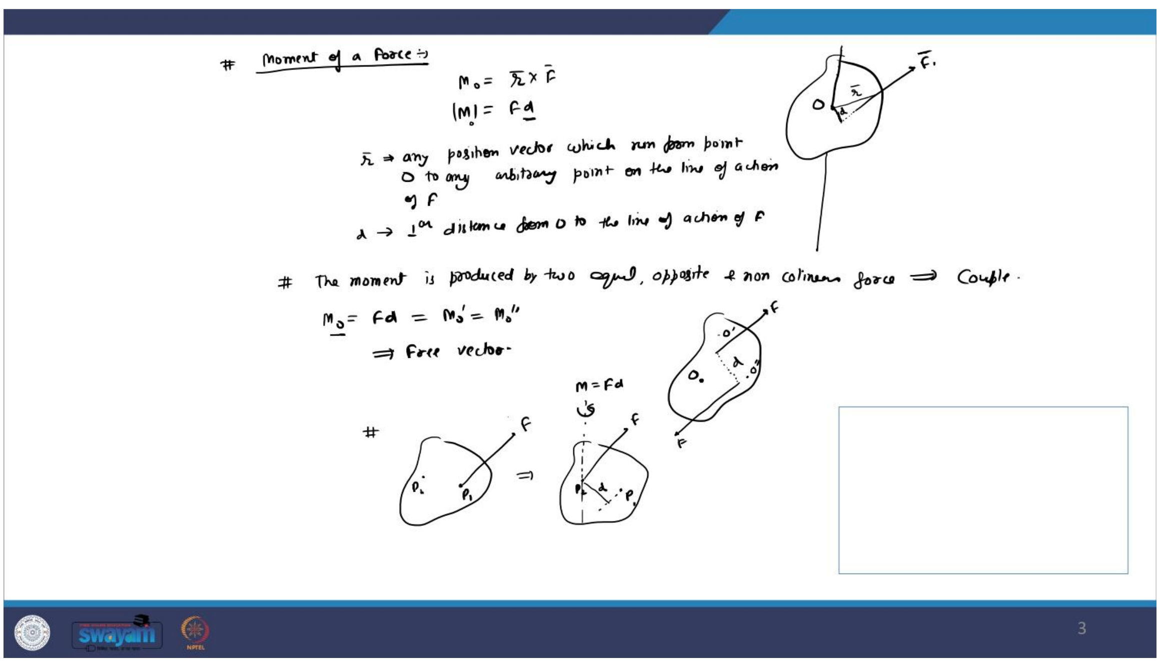
Revision: static

Hello everyone, welcome to the lecture again. In the last 28 lectures, we have covered the static part which was half of the syllabus of this course. Therefore, it is right time now that we revise the concept that we have learned so far. So, this lecture is about the revision of the static part and we started with the concept of the rigid body.

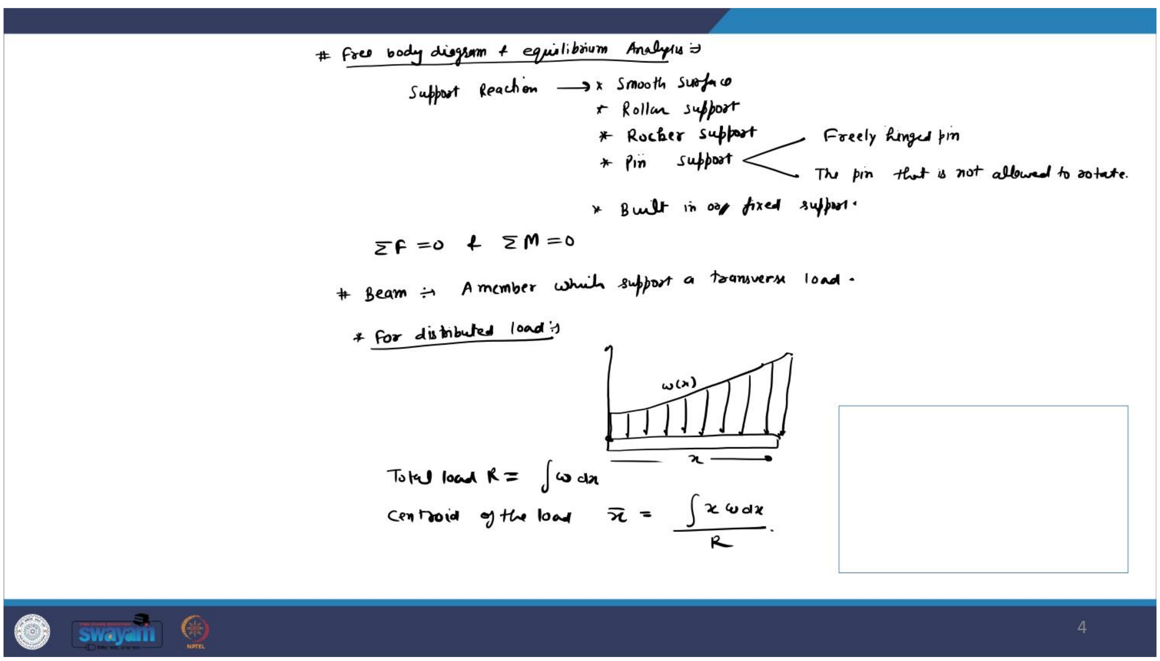


A rigid body does not deform under the action of the forces. So, suppose I have this rigid body and let's say I have the i^{th} particle and I have the j^{th} particle and let's say from the origin the position vector is r_i and r_j In that case, r_{ij}, which is the spacing between the particles, so this is $r_{ij} = |r_i - r_j|$, it has to be constant under the action of force, okay.

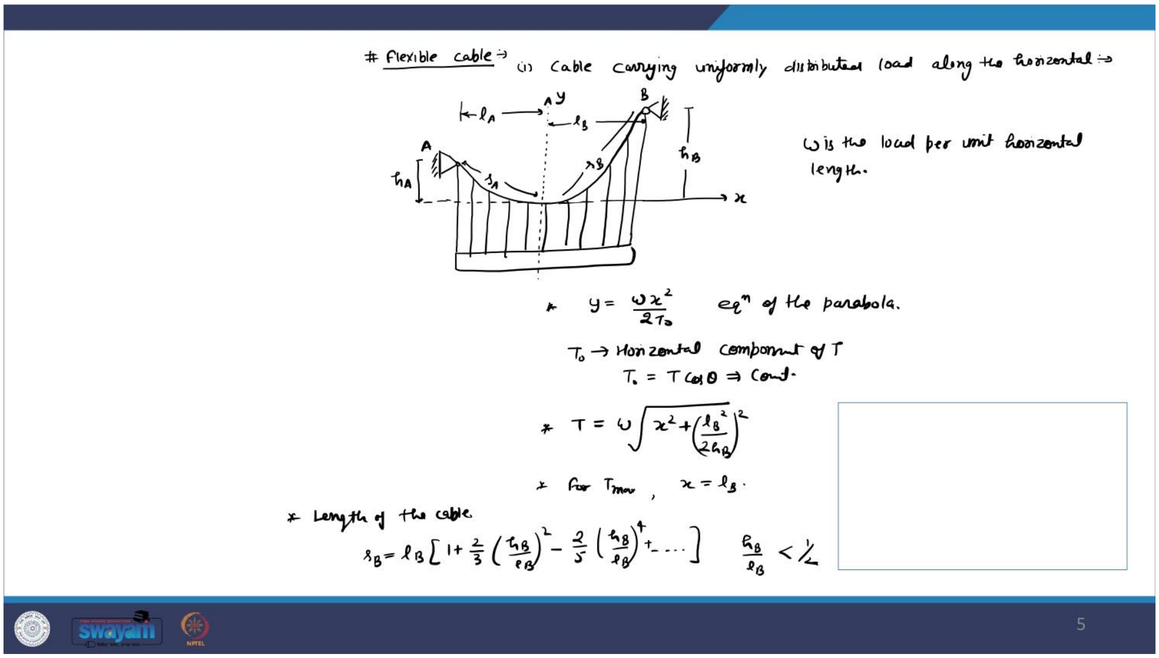
So, $r_{ij} = |r_i - r_j| = C_{ij}$, C is constant and this is true for all i and j, okay. So, the body that does not deform under the action of forces is called the rigid body. Now, we also discuss what is sliding vector and what is free vector. So, sliding vector. So, let's say I have a body and on this body you apply a force. Then, in that case, the action of this force does not change if you apply the same force on the line of action of it. So, you can apply the force here or the same force you can apply here or the same force you can apply there. So, in that case, the force becomes a sliding vector. So, a sliding vector has unique line of action, but not a unique point of application. So, for example, you cannot shift this force to some other location because then it will also introduce the torque. So, this is not possible, but along the line of action you can slide it. So, therefore, force is a sliding vector. free vector. So, free vector is something that can be positioned anywhere in the space. So, for example, let's say I have this rigid body and there is a free vector. So, let's say this is a free vector in that case this vector you can place anywhere in the rigid body. So, it can be placed anywhere in space and the example is the couple. So, couple is a free vector. Now, we also saw how we can find out the resultant of the force. So, again let's say I have a rigid body and on this rigid body force F_1 and F_2 is acting and let's say the angle between them is θ, then the resultant $R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2cos\theta}$ and the resultant, so this is the resultant R and the angle that it makes from F_1, let's say it is α, then $tan\alpha = F_2sin\theta/(F_1 + F_2cos\theta)$.



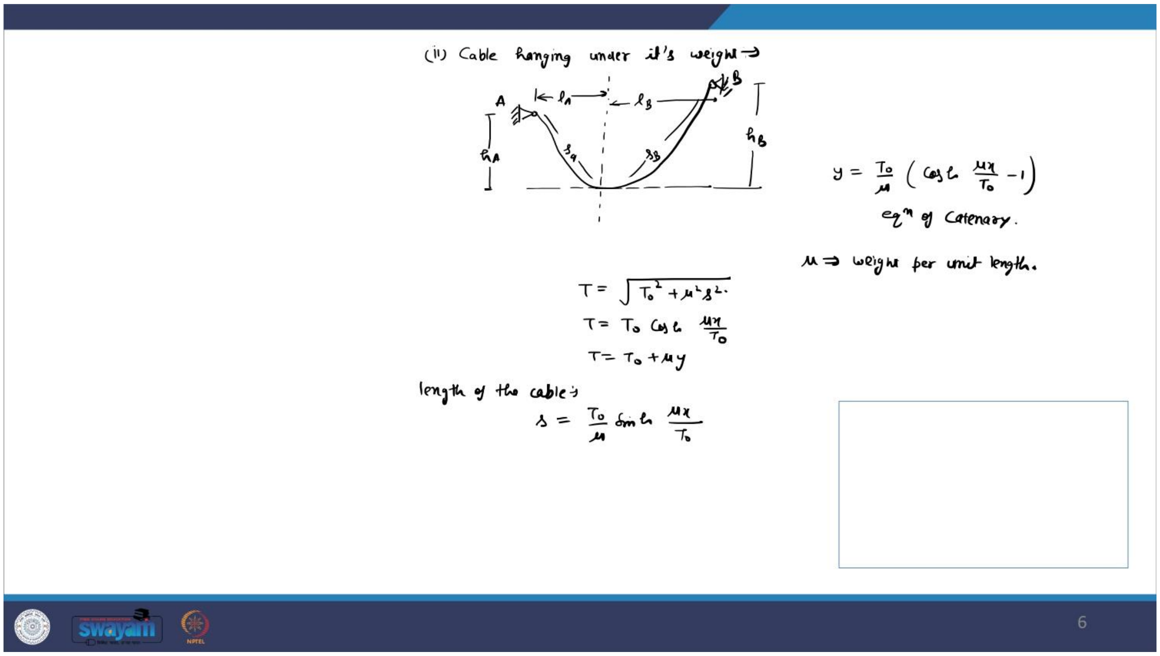
We also discuss the moment of a force. So, let's say I have a rigid body and on this rigid body, a force F is acting. So, let's say this is the point about which I want to determine the moment. So, let's say this is O, then from O you draw any position vector r on the line of action of F. So, $M_O = r \times F$ and $|M| = Fd$, where this d is the perpendicular distance of this force from O. So, r is any position vector which run from point O to any arbitrary point on the line of action of F and d here is the perpendicular distance from O to the line of action of F. Now, if the moment is produced by two equal opposite and non-collinear then it is called couple. So, here the situation is following. You have this rigid body and let's say a force F is acting like that and another force F is acting like this. Then we saw that the moment of this force about any point O is equal to the force multiplied by the perpendicular distance between them and this moment about O is independent of point O. So, you can calculate the moment about let's say O' or O''. So, in that case, $M_O = Fd = M_O' = M_O''$. So, therefore, this becomes a free vector. We also discussed that suppose there is a rigid body and on this rigid body a force F is acting at point P_1 and you want to shift this force at some other point P_2 keeping the same external effect. So, this can be done by having this force at point P_2 and then you have to account for the moment and the moment will be force into perpendicular distance d. So, this will be the d.



Next, we discuss the free body diagram and equilibrium analysis. So, for the free body diagram particularly we look at the support reactions. Support reactions of smooth surface, support reaction from roller support, rocker support, pin support and herein we discuss two case freely hinged pin and the pin that is not allowed to rotate. So, if the pin is not allowed to rotate then it also support the torque and the the built-in or fixed support. For the equilibrium analysis, we use $\sum F = 0$ and $\sum M = 0$. Then, we discuss the beam and beam is a member which supports a transverse load and if the load is distributed. So, suppose there is a beam and on this beam, we have a load that is some function of x. Then this distributed load, it can be replaced by the concentrated load and the total load $R = \int wdx$, where w is the loading intensity and the centroid of the load, which is $\bar{x} = \frac{\int xwdx}{R}$.

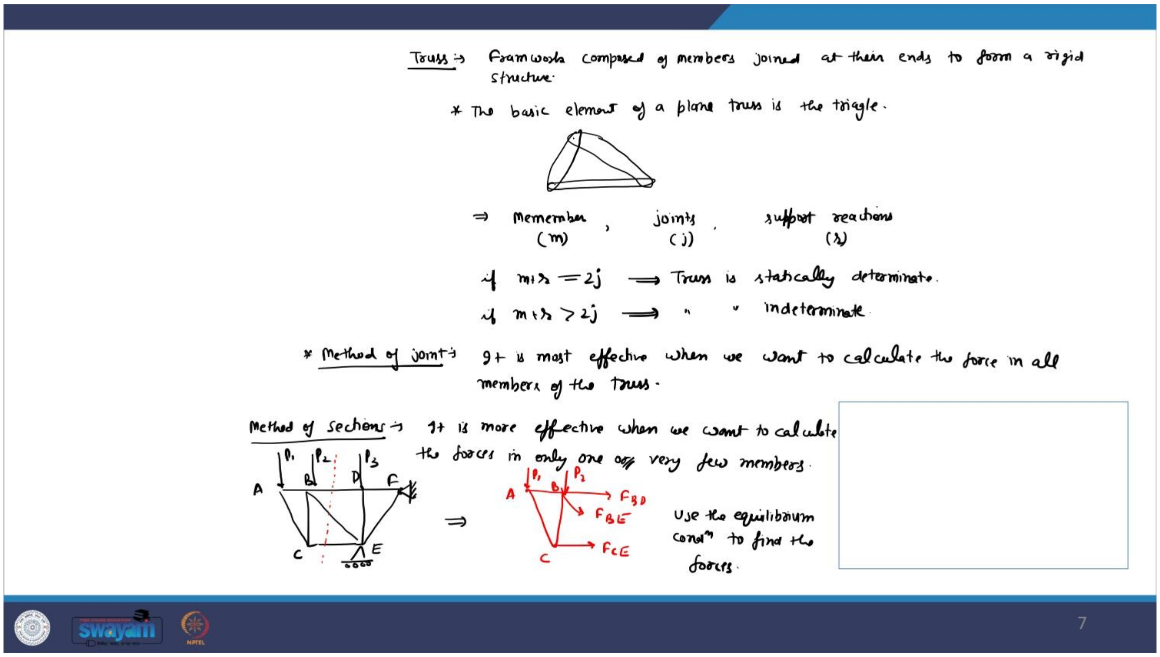


We then discuss the flexible cable and it has two cases. In the first case, we discuss the cable carrying uniformly distributed load along the horizontal. So, we have this cable. Let's say this point is A, this point is B, then we fix our axis at the minimum point of this cable. So, this is the x-axis, this one is the y-axis and let's say this height is h_A, this height is h_B, this length is s_A, this length is s_B and this length is l_A and this length is l_B and w is the load per unit horizontal length, then the equation of the cable $y = \frac{wx^2}{2T_0}$, which is the equation of the parabola. Here, this T_0 was the horizontal component of T or horizontal component of the tension and it was $T = T_0 cos\theta$ and that was constant. The tension in the cable can be find out using $T = w\sqrt{x^2 + \left(\frac{l_B^2}{2h_B}\right)^2}$. And for maximum T, so for T_{max}, my x has to be maximum. $x = l_B$. Now, the length of the cable is can we find out using $s_B = l_B(1 + \frac{2}{3}\left(\frac{h_B}{l_B}\right)^2 - \frac{2}{5}\left(\frac{h_B}{l_B}\right)^4 + \dots)$. Similarly, you can find out s_A, then the total length will be s_A plus s_B. Herein, you have to note that your $\frac{h_B}{l_B} < \frac{1}{2}$.



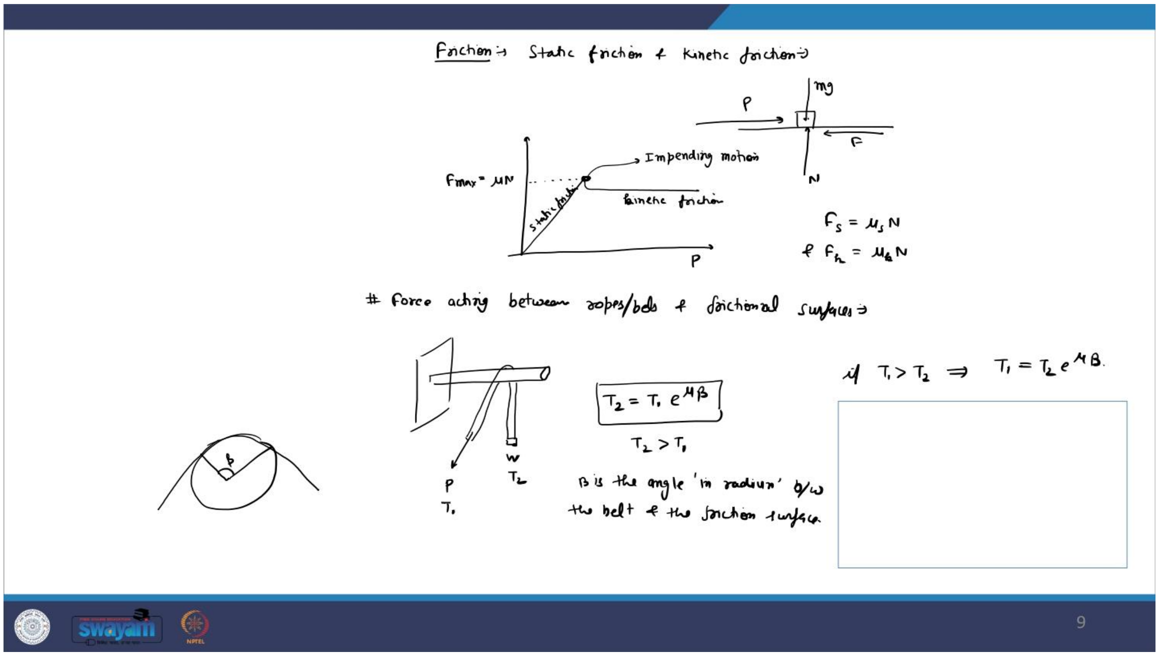
In the second case, the loading intensity was uniformly distributed along the length of the cable. So, this was the case wherein you can think of the cable which is hanging under its own weight. So, again let's say there is a cable and let's say this is the minimum point of the cable. So, we fix our axis here. This point is A, this point is B. So, therefore, this height is h_A, let's say. This height is h_B. This length is s_A. This length is s_B and this is l_A and this one is l_B. So, the equation of the cable in this case comes out to be $y = \frac{T_0}{\mu}\left(\cosh\left(\frac{\mu x}{T_0}\right) - 1\right)$ and this is the equation of catenary. Here, this μ was the weight per unit length of the cable and the tension T in the cable was $T = \sqrt{T_0^2 + \mu^2 s^2}$ or T can also be calculated using $T = T_0 \cosh\left(\frac{\mu x}{T_0}\right)$ or $T = T_0 + \mu y$. Now, the length of the cable, let's say s_A or s_B, so $s = \frac{T_0}{\mu} \sinh\left(\frac{\mu x}{T_0}\right)$. So, you calculate this for s_A and for s_B and then you sum them up to find out the total length.

We next discuss truss. Truss is a framework which is composed of members which are joined at their ends to form a rigid structure and the basic element of a plane truss is the triangle. So, if you want to make a truss, you have to use three members which are joined together and then you can extend this to make bigger truss. Now, if the members in the truss are m, the joints are j and the support reaction is r, then if $m + r = 2j$, in that case the truss is statically determinate. That means we can find out all the forces that are acting on the truss. And if $m + r > 2j$, then the truss is indeterminate. Now, there are two methods to find out the forces in the truss. So, one is the method of joint and this method is more effective when you want to find out the force in all the members. When we want to calculate the force in all members of the truss. The second method was

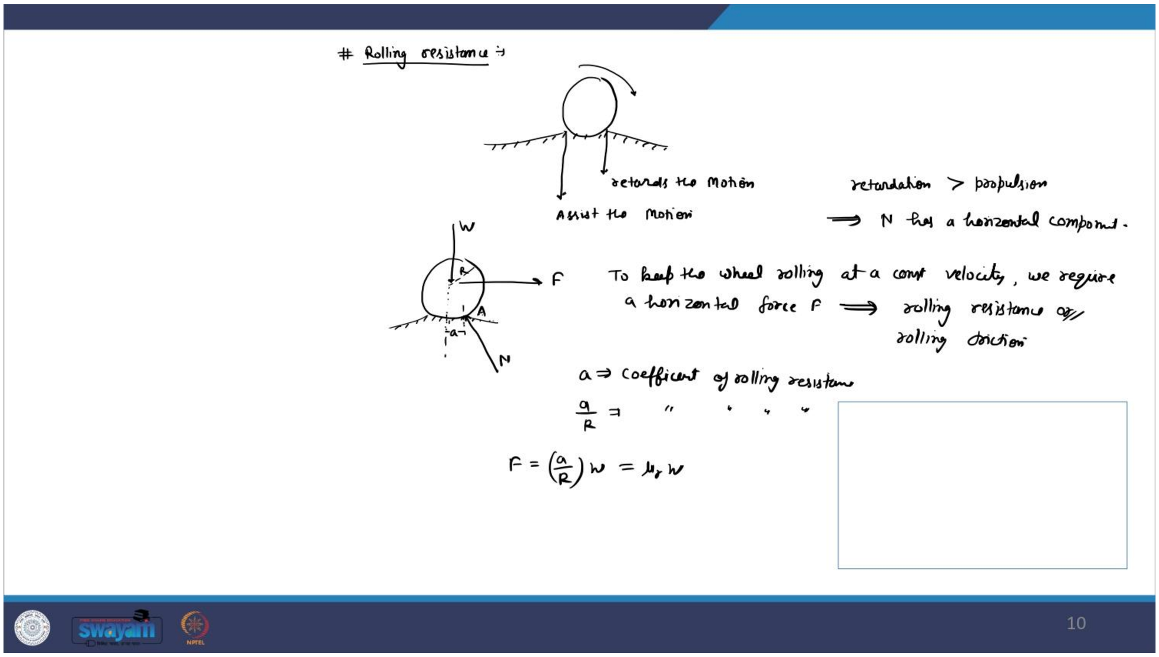


method of sections and this method was more effective when we want to calculate the forces in only one or very few members of the truss. Let's say we have the following truss. So, this point is A, B, C, D, E and F and at E, there is a roller support and let's say there are external forces also P_1, P_2 and P_3 then in method of section you divide this truss into two complete part and we divide it such that it does not cut more than three members because we have three independent equations. So, therefore, when we cut this, we have the following free body diagram of the truss on the left hand side and then we use the equilibrium condition to find out the forces in the truss.

We next studied the principle of virtual work and the statement of the principle of virtual work is following. If a rigid body is in equilibrium, then the total virtual work of the external forces acting on the rigid body is 0 for any virtual displacement of the body. So, that means $\delta U = 0$. Let's say the external forces P, Q and a moment M is acting on the rigid body, then if δx is the virtual displacement corresponding to force P, δy is the virtual displacement corresponding to force Q and $\delta\theta$ is the virtual displacement corresponding to the moment M, then $P\delta x + Q\delta y + M\delta\theta = 0$. Now, the next step is to represent this δx, δy and $\delta\theta$ in terms of generalized coordinate. So, your x, y and θ, they should be represented in generalized coordinate and what is generalized coordinate? Well, the generalized coordinate are the least possible coordinate which completely define the position of the system. For example, for a simple pendulum, the generalized coordinate is θ and the degree of freedom is 1. We then studied the stable and unstable equilibrium So, let's say we have a particle on the following curve. Let me denote the minimum and maximum point on this curve. Then A and C represents the point of stable equilibrium and B and D they denotes the point of unstable equilibrium. So, suppose you displace the particle from point A, then it has a tendency to come back. Therefore, at point A, the equilibrium is stable, while at B, if you displace it slightly, it is not going to come back. So, therefore, it becomes an unstable equilibrium and to find out the stable and unstable equilibrium, you find $\frac{dV}{dq} = 0$. And for stable equilibrium, your $\frac{d^2V}{dq^2} > 0$ and for unstable equilibrium, $\frac{d^2V}{dq^2} < 0$. Note that here q is the generalized coordinate.



We next discuss the friction, particularly the static friction and kinetic friction. Let's say I have a rigid body on a rigid surface and the mass of the body is mg. In that case, the normal force N is going to act upward. Let's say you apply the external force P, then the frictional force is going to develop. And if you plot the frictional force as a function of external force P, then initially the F is going to increase. It attains a maximum value. So, $F_{max} = \mu N$ and then the body start to move. So, in this case, you have static friction and herein you have kinetic friction. And this is the point wherein the motion just begins. So, this is the point of impending motion. The value of the static friction is $\mu_s N$ and similarly, the kinetic friction is equal to $\mu_k N$. We also studied the force acting between ropes or belt and frictional surfaces. So, for example, suppose I have a peg. and on this peg, I have a rope. Let's say you have a weight W which is suspended like this and it is held by a force P. So, let's say this is T_1 and this is T_2, then we got $T_2 = T_1 e^{\mu\beta}$ where $T_2 > T_1$ and β is the angle in radian between the belt and the frictional surface. So, for example, you have this circular surface and you have a belt like this, then β is the angle that the belt is making with the surface. Now, in this case, if my $T_1 > T_2$, now we have $T_1 = T_2 e^{\mu\beta}$.



So, finally, we discuss about the rolling resistance and herein, we assume that the deformation is there in the body as well as in the surface. So, suppose I have a body and this body is rolling. So, the surface and the body, they deform. The deformation in the front, it retards the motion and the deformation in the back, it assists the motion. However, the retardation is always larger than the propulsion and because of that, the normal force N has a horizontal component and because of that you have to apply a force to keep the wheel rolling at a constant velocity. So, the situation is following, you have this wheel and we have the normal force N which has a component around the horizontal. its weight is W and you have to apply a force F so that this wheel is keep rolling at constant velocity. So, to keep the wheel rolling at a constant velocity, we require a horizontal force F and this force F is called the rolling resistance or the rolling friction, okay. Now, let's say A is the point of application of this force N and a represents the horizontal distance between point A and the vertical line of center of the wheel. So, let's say this is a, then a is called the coefficient of rolling resistance or $\frac{a}{R}$ is also called the coefficient of rolling resistance. And the value of $F = \left(\frac{a}{R}\right) W = \mu_r W$, where μ_r is the rolling resistance. With this, let me stop here. See you in the next class, wherein we will start the discussion about the dynamics part. Thank you.