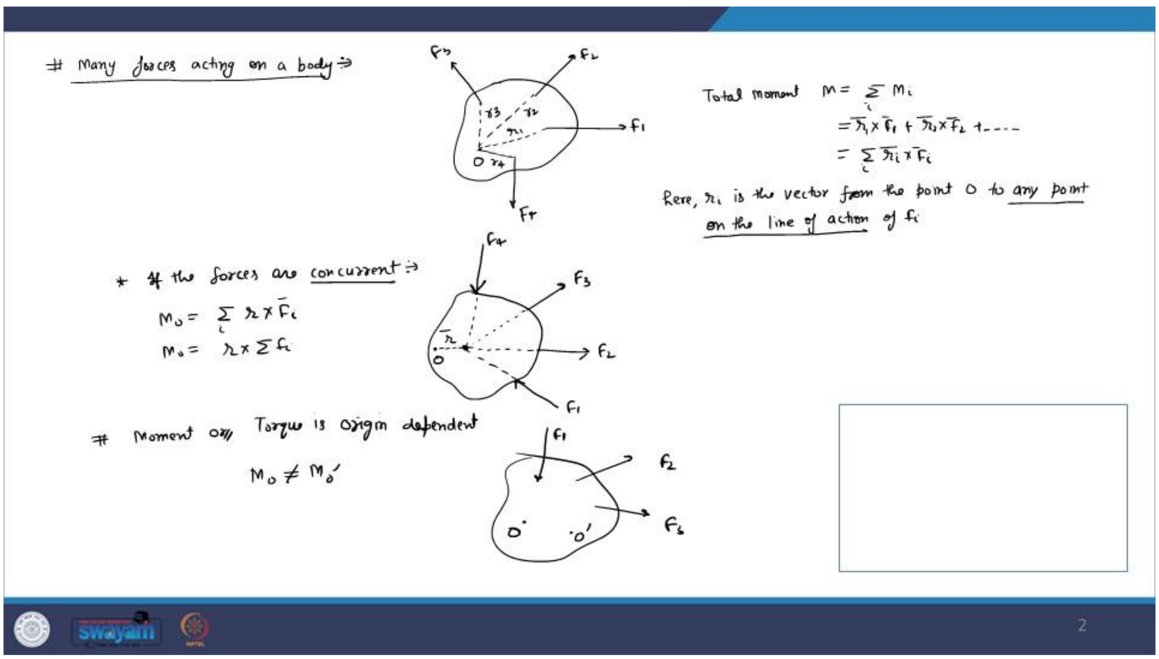
MECHANICS

Prof. Anjani Kumar Tiwari
Department of Physics
Indian Institute of Technology, Roorkee

Lecture - 3

Couple moment and reduction of a force system to a force and a couple

Hello everyone. Welcome to the lecture again. In the last lecture, we looked at how to calculate the moment of a force about a given point O. Today, what we are going to do is we are going to learn how we find out the couple moment and reduce a force system to a force and a couple. So, let us start.



Let's say we have many forces which are acting on a body. So, suppose I have a rigid body and on this rigid body there are many forces, let's say F_1, F_2, F_3 and F_4 and let us say we want to calculate the moment of these forces about a point O. So, how do we calculate the moment?

From O, we draw a vector r_1, r_2, r_3 and r_4. Then,

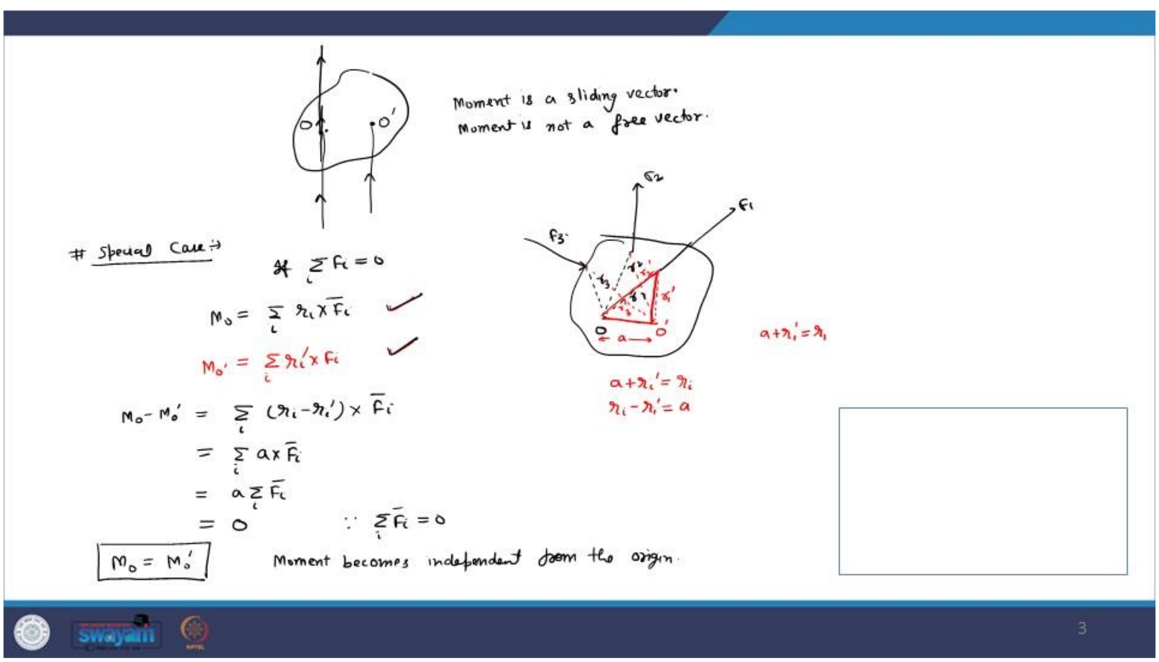
total moment, $M \ = \ \sum_i M$.

So, here it will be $\bar{r}_1 \times \bar{F}_1 + \bar{r}_2 \times \bar{F}_2 +$.. so on and this can be generalized as $\sum_i \bar{r}_i \times \bar{F}_i$. Now, note that here your r_i is the vector from the point O to any point on the line of action of F_i. So, it should be noted that it is any point on the line of action.

Now, let us look at a situation where the forces are concurrent. Now, we already know the definition of the concurrent force. That means that all the forces, they pass through a common point. So again, let us say I have a rigid body and all the forces that are there, they pass through a common point here in, let's say, this point.

And suppose we want to calculate the moment of this body about a point O. So, from O, I draw a vector. Let's say this vector is r. Then the moment about O will be $r \times F_i$. Why there is common r? This is common because it connects point O to all the forces.

So, let's say this is F_1, F_2, F_3 and F_4 then r, I can take outside and the sum of the forces. Now, We also know that the moment, so from here it is very clear that the moment or the torque is origin dependent. What do I mean by that? So, the comment is that moment or torque is origin dependent. So, that means that if you have a rigid body and let us say various forces are acting on it, so let us say F_1, F_2 and F_3 and you calculate the moment about O or you calculate the moment about O', then the moment about $O \neq O'$.



Now, from here, it is also clear that suppose I have a rigid body and you want to calculate the moment about this point O. So, you draw a line which is passing through O and the moment about this line or the moment about this line or the moment about this line will be the same because they are same line which is passing through O. So, this tells us that the moment is a sliding vector. But note that the moment about this point will be different than the moment about this point. So, therefore, moment is not a free vector. So, that means it is a sliding vector, but it is not a free vector.

Now, let us look at the special case where all the forces acting on the body is such that their sum is 0. That means that if the sum of the forces which are acting on the body is 0. So, let us say I have a rigid body and I want to calculate the moment about O' and the forces are such that their sum is 0.

So, let us say this is force 1, this one is force 2 and this is force 3. So, what we do? From O, you draw a vector on these forces. So, let us say this is r_1, this is r_2 and this is r_3. Then, $M_O = \sum_i r_i \times F_i$.

Now, let's say we want to calculate the moment about another point O' which is at a distance a from O. So, if you want to calculate the moment about O', then it will be you have first you have to draw the position vectors from point O to the force. So, therefore, this will be $r_1{}'$ this will be $r_2{}'$ and it will be $r_3{}'$. So, $M_{O'} = \sum_i r_i{}' \times F_i$.

Now, you can take any triangle. So, for example, let us take this triangle. So, here you can say that $a + r_i' = r_i$.

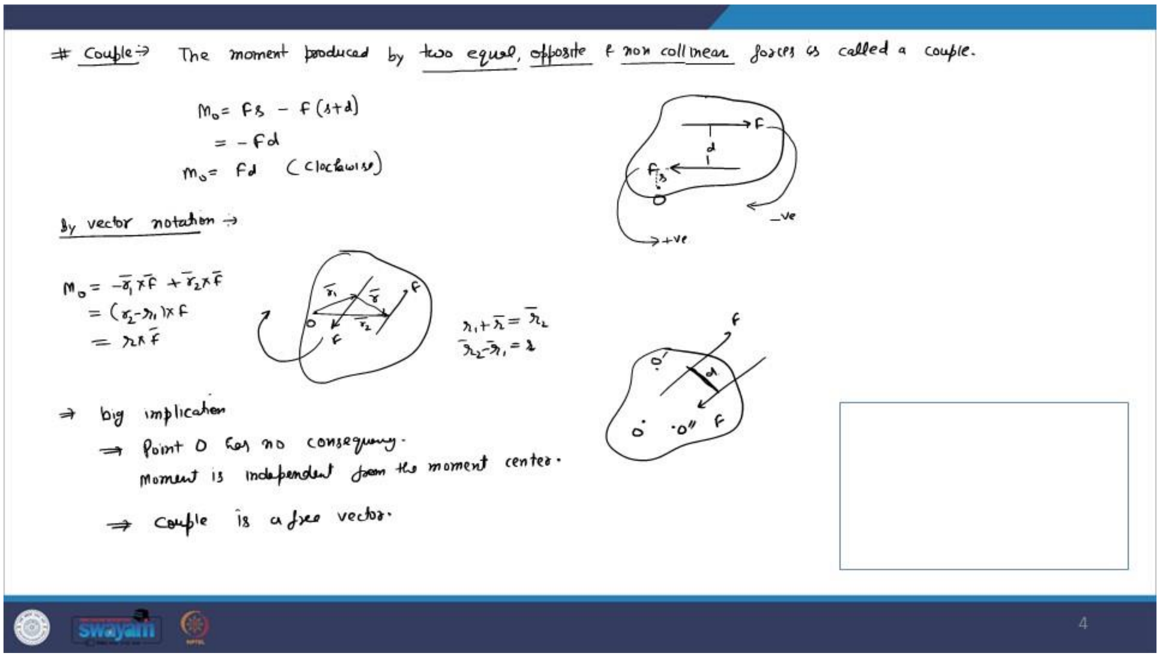
So, we can clearly see that $a + r_1' = r_1$ and we have generalized it for any r_i. So, therefore, $r_i - r_i' = a$. So, now we have the moment about O and we have the moment about O'. Now, let us look at the difference between these moments. So, $M_O - M_O' = \sum_i (r_i - r_i{}') \times F_i$.

But just now we have seen that $r_i - r_i' = a$, which is the spacing between O and O'. So, therefore, this will be equal to $\sum_i a \times F_i$. But since a is constant, we can take outside the summation. And we have assumed that the sum of the forces are such that they are equal to 0. So, therefore, this will be 0 since $\sum_i F_i = 0$. What is this?

This means that under the case where the sum of the forces are 0, in that case, $M_O = M_O'$. So, this implies that the moment become independent from the origin. So, therefore, the moment becomes independent from the origin.

Now, let us see what is couple. So, first let me define couple.

The moment produced by two equal, opposite and non-collinear forces is called a couple. Let us look at the example. Again, suppose I have a rigid body and there are two equal forces. So, let us say one force is this and the other force is this. So, these are two equal forces.



They are opposite and they are non-collinear. That means they are not in the same line. Then the moment which will be produced by this kind of force, we call it couple. So, to formulate the problem, let us say the spacing between them is d and let us say you want to calculate the moment about a point O. So, what do you do? From O, you draw a perpendicular distance and let us say this is s and you calculate the moment about O. So, this force has a tendency to rotate about O in the clockwise direction.

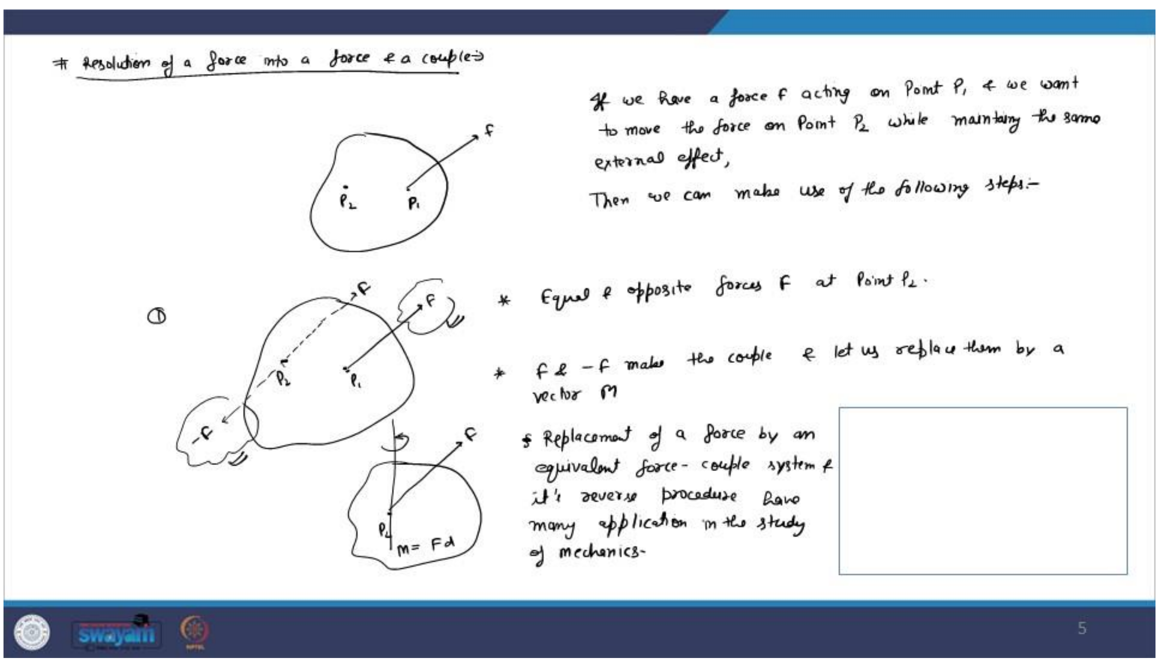
So therefore, it will be negative and the tendency of this force about O has a tendency to rotate in the anti-clockwise. So, therefore, this will be positive. Let us calculate the moment of these two forces about O. So, the moment of these forces about O will be the first force multiplied by the distance. This is positive minus the second force and the perpendicular distance from O and it will be $s + d$. So, this is the moment about O.

And you can see that it comes out to be $-Fd$. Or we can write down that the moment about O is just Fd and it is in the clockwise direction. We can also, you know, get the same result if we, you know, take a position vector r and connect it to the line of action of the forces. Let us do it again by the vector notation. So, again, let's say I have this rigid body and I have two forces F and F which are equal and opposite. And let's say I want to calculate the moment about O. So, I draw a vector r_1 to the line of action of this F.

And again, I draw another vector r_2 on the line of action of s. And let us say this $r = r_2 - r_1$. So, we can see that $r_1 + r = r_2$ or $r = r_2 - r_1$. So, let us calculate the moment about O. So, moment about O is I have this force F which has the tendency to rotate in the clockwise direction. So, therefore, it will be $-r_1 \times F + r_2 \times F$ and this can be written as $(r_2 - r_1) \times F$ or $r \times F$. So, from here we can see that the point O has no consequence.

So, basically this has a very big implication. We can see that point O has no consequence and the moment is independent from the moment center. Why is that? So, basically you have this rigid body, we have a couple So, this is F, this is F.

You calculate the moment about this point or you calculate the moment about that point or you calculate the moment about this point. In all the cases, the moment will be same because moment is independent of their position. It only depends upon the spacing between them. So, from here, it is also clear that couple is a free vector because you can place it any point inside the rigid body.

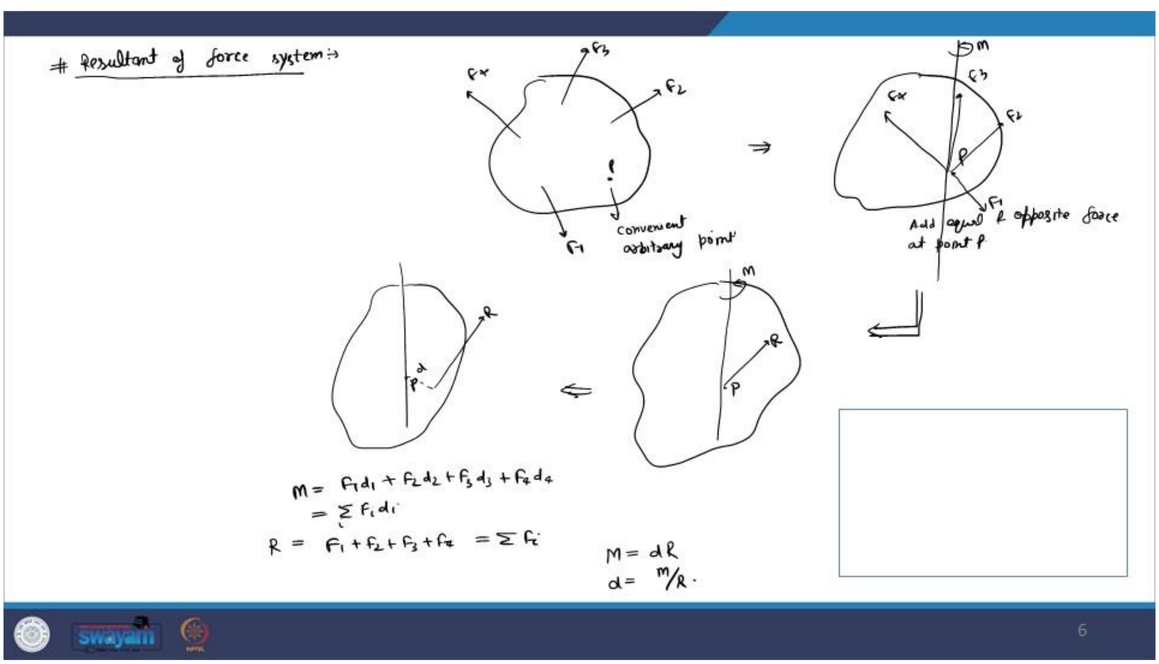


Now, let us look at the resolution of a force into a force and a couple system. Now, we are going to see the resolution of a force into a force and a couple. Let's say I have a rigid body and on this rigid body, there are two points, let us say P_2 and P_1 and on P_1, there is a force F acting. Suppose we want to move this force from P_1 to point P_2 and while maintaining the same external effect. So, let me write it down.

If we have a force F acting on point P_{21} and we want to move this force on point P_2 while maintaining the same external effect, then we can make use of the following steps. So, please follow it. So, we have a rigid body and on this rigid body, we have force which is acting at point P_1.

So, what we can do is at point P_2, so this was the original system, at point P_2, we can add two equal and opposite forces of magnitude F. So, this is like adding a zero force at point P_2. So, what we have done is we have add equal and opposite forces F at point P_2. Now, you also know that this F And this F, they are equal, they are opposite and they are non-collinear force.

Therefore, they are going to make a couple. So, F and $-F$, they make the couple and let us replace them by a vector M, where M is the couple. So, this F and this F, I am replacing it by a couple. Now, couple is a free vector. I can place it anywhere. So, let me place it at point P_2. So, now we are left with this force F and a couple. its value is M and this is equal to one of the forces multiplied by the distance between them. So, let me mention here that



this replacement of a force by an equivalent force couple system and also its reverse procedure, it have many application in the study of mechanics and we will use it later.

Now finally, let us look at the resultant of the force system. let's say we have a rigid body and again many forces are acting on it. So, let's say F_1, F_2, F_3 and F_4 and let's say we want to calculate the moment of this about a point P. This is some point which is convenient to us and of course, it is arbitrary, but it is convenient. So, this is convenient arbitrary point.

So, again the process is similar. You have this rigid body and as I said I want to calculate the moment at point P. So, at P we add equal and opposite forces. So, a this is F_1, this is F_2, this is F_3, this is F_4 and you know we also have the couple because equal and opposite forces has make couple. So, at this point we also have couple M and then at P, we can find out the resultant force. So, let's say the resultant force is R. This is nothing but $F_1 + F_2 + F_3 + F_4$ in this case and we have the moment M.

Then, we can also account for this M by translating this resultant R from point P. So, let us say we translate it by a distance d And by this, we can also account for the moment M. So, let me just write down these quantities. So, M, for example, here, it will be $F_1 d_1 + F_2 d_2 + F_3 d_3 + F_4 d_4$. I am not taking care of the sign here. So, it will be just $\sum_i F_i d_i$.

And what is the resultant R? Resultant R will be $F_1 + F_2 + F_3 + F_4$, which is nothing but $\sum_i F_i$. Now, how much I have to translate? So, I know that M or the moment is dR. So, therefore, $d = M/R$.

So, let me stop here. Let us discuss couple of examples in the next class. Thank you.