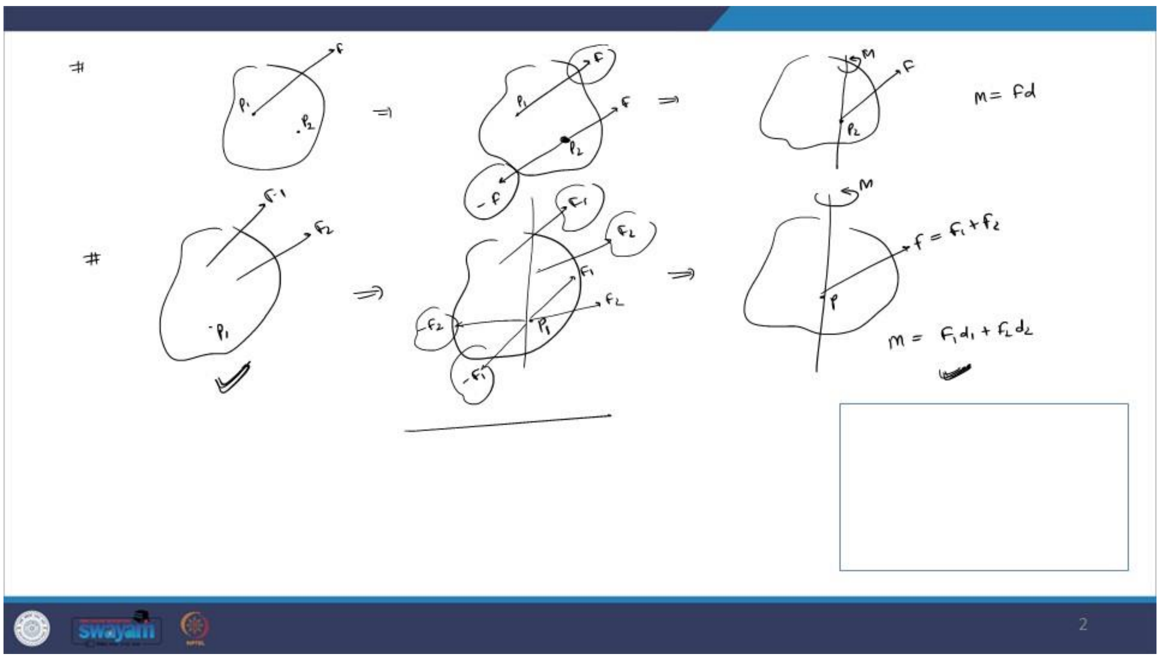
MECHANICS
Prof. Anjani Kumar Tiwari
Department of Physics
Indian Institute of Technology, Roorkee

Lecture 04
Examples of couple moment

In the last lecture, we look at the concept of couple and we see how we can write down a force into a force and a couple system. Today, we are going to do couple of examples based on the same concept. So, let me just quickly revise what we have done in the last lecture.

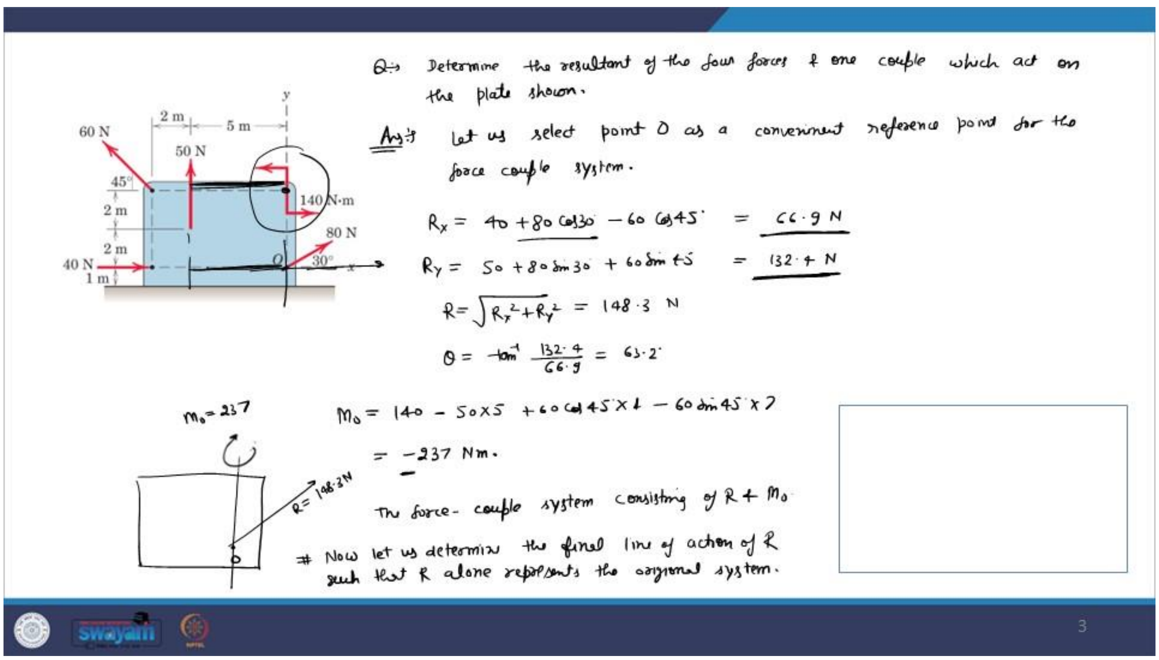


Suppose I have a rigid body and on this rigid body, a force F is acting at point P_1 and I want to translate this force at point P_2. So, what I do is at point P_2, I add two equal and opposite forces. Then this force and this force, they make a couple, and this couple is a free vector, so I can place it at point P_2. So, at point P_2, I put a couple, let us denote it by M and what we have is only this force F and the value of the couple M is force times the distance between the point P_1 and point P_2.

Now, let us say I have two forces now, which are F_1 and F_2. They are acting on this body and we want to see their effect at point P_1. So, what we can do is at point P_1, so I had this force F_1, I had this force F_2. So, at point P_1, I can add equal F_1 and minus F_1.

So, this force and this force is going to make a couple. Similarly, this force F_2, I can add equal and opposite amount. So, this force and this force is again going to make a moment. So therefore, I am left with F which is equal to F_1 plus F_2 and a moment M which is you know M_1 plus M_2.

So, moment $M = M_1 + M_2 = F_1 d_1 + F_2 d_2$. Again, I am not taking care of the sign, but it will be you know $F_1 d_1 + F_2 d_2$. So, now we do not need these intermediate steps, what we need is the initial and the final result wherein we take these forces, we translate them to the desired point and then calculate the moment about that point and then you know you also calculate what is the resultant force at that point. So, let us make use of these concepts and let us look at this problem statement.



So, here the problem statement is determine the resultant of the four forces and one couple which act on the plate shown. So, here in this example, you can see that there are four forces which are acting on this plate and also there is a couple, and we have to find out what is the resultant of these forces and the couple. So, in this let us choose some convenient point about which we will calculate you know all the forces and moment etcetera. So, for that let us choose that O is our convenient point. So, let us select point O as a convenient reference point for this force couple system.

So, first let us calculate all the forces that are, you know, acting on it and let us calculate their x and y component.

So, $R_x = 40 + 80\ cos30° - 60\ cos45°$.

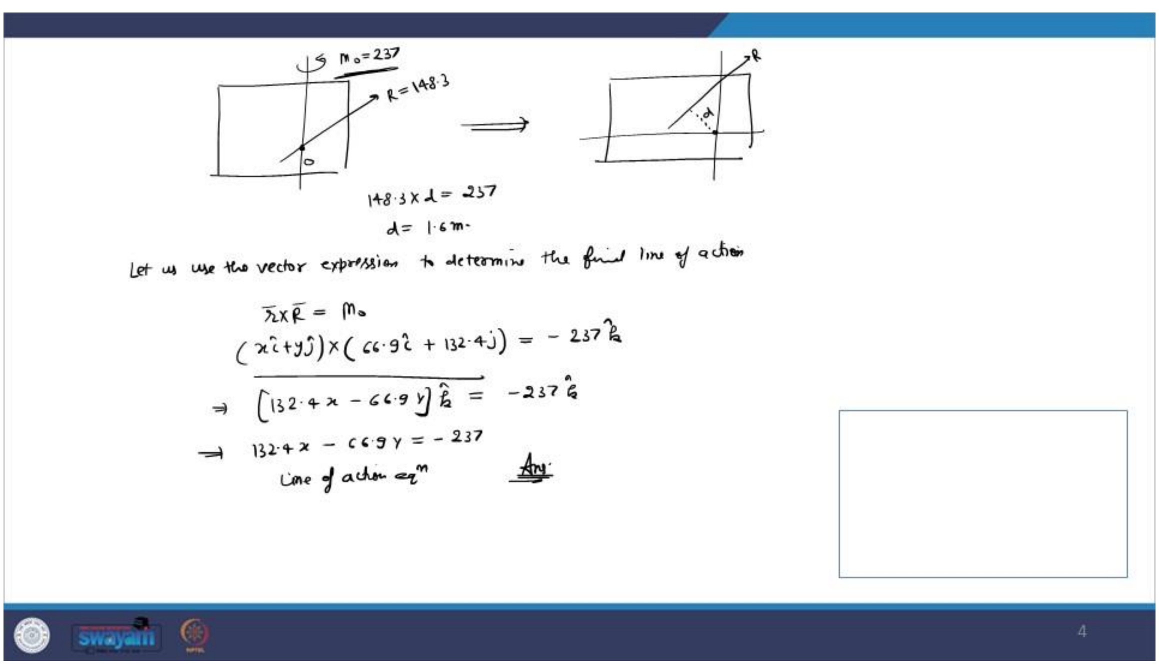
Because this is acting in the x direction, then we have this 80 N force, its component along the x-axis will be 80 $cos30°$. Similarly, we have 60 N force, its component along the x-axis will be $-60\ cos45°$.

So, if we calculate that because we know the values of cos 30 and cos 45, this comes out to be 66.9 N. Similarly, we can calculate the y component of the force and it will be $50 + 80\ sin30° + 60\ sin45°$ and this comes out to be 132.4 N. Now, R will be $\sqrt{R_x^2 + R_y^2}$ and this comes out to be 148.3 N and similarly, the angle that the resultant make from the x-axis, it will be $\tan^{-1}\frac{R_y}{R_x}$. So, it will be $\tan^{-1}\frac{132.4}{66.9}$, this will be 63.2°. Now, M_O will be 140 because it is a couple, it is a free vector. So, either I can place it here or I can as well take it at point O. So, it will be 140 minus, now let us calculate the moment of these forces about O. So, it will be 50 multiplied by 5 because this distance is 5. So, therefore, about O, this distance will be $5 + 60 \times cos45° \times 4 - 60 \times sin45° \times 7$.

$$\Rightarrow M_O = 140 - 50 \times 5 + 60 \times cos45° \times 4 - 60 \times sin45° \times 7.$$

And this you can find out it will be minus $-237\ Nm$. So, so far what I have done is we have taken this plate and about point O we find out the moment and the moment comes out to be 237 and since it is negative, so that means it is in the clockwise direction. So let me just change this direction. So, it will be 237 about O and then we have a resultant at 63.2° and its value is 148.3 N. So, this is the force couple system consisting of R and M_O.

So, what we have done so far is we have replaced all these four forces and a couple into a moment and a resultant force R. Now, let us determine the final line of action of R such

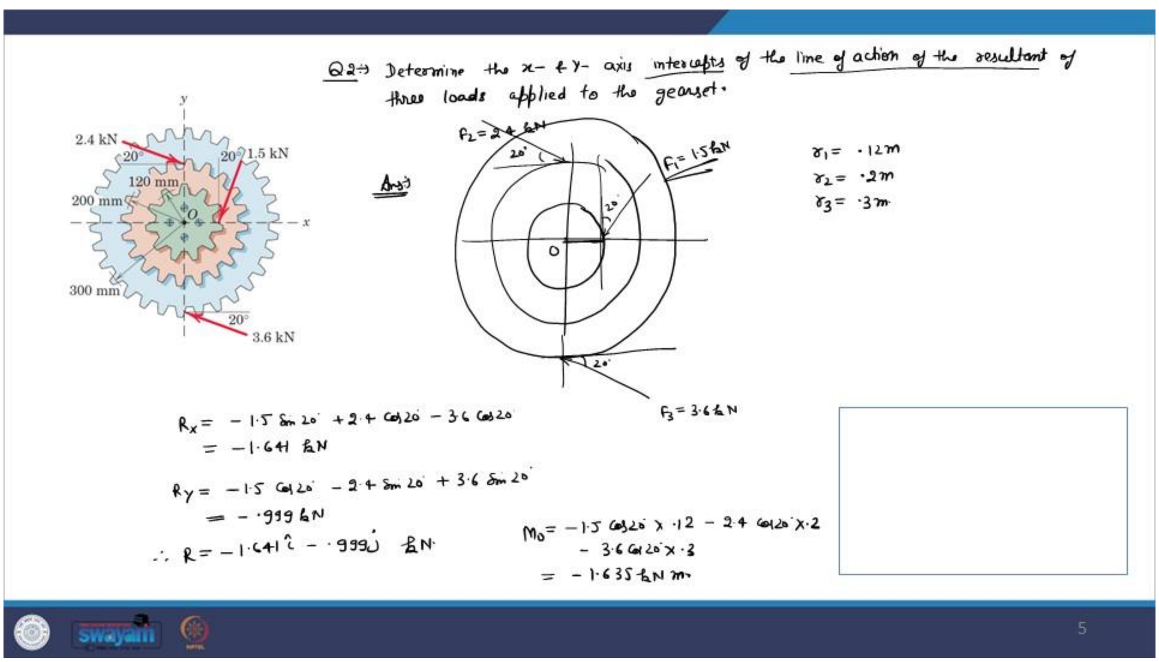


that R alone represents the original system. So, what do I mean by that? So, up to now we have this point O and the moment was 237 and the resultant was 148.3.

Now this we want to replace by a single force R and this we are going to do by moving this force R by a distance d. So, let us see if we can account for this moment M by translating this force R. So, for this we have the force multiplied by distance should give me the moment. So, from here, I can find out how much I have to translate this force so that I can account for the moment. So, d comes out to be $1.6\ m$. So, that means I have to translate this force by $1.6\ m$. So, therefore, I can account for the moment. Now, let us use the vector expression to determine the final line of action.

So, let us say I have a point $x\ y$ on this resultant and I use $r \times R = M_O$. So, this distance multiplied by the force should give me the moment and let us say $r = x\hat{\imath} + y\hat{\jmath}$. Now, R, I have already calculated. So, $R = r_x\hat{\imath} + r_y\hat{\jmath}$. So, therefore, it will be $66.9\hat{\imath} + 132.4\hat{\jmath}$ and this should give me $-237\hat{k}$ because the moment is -237.

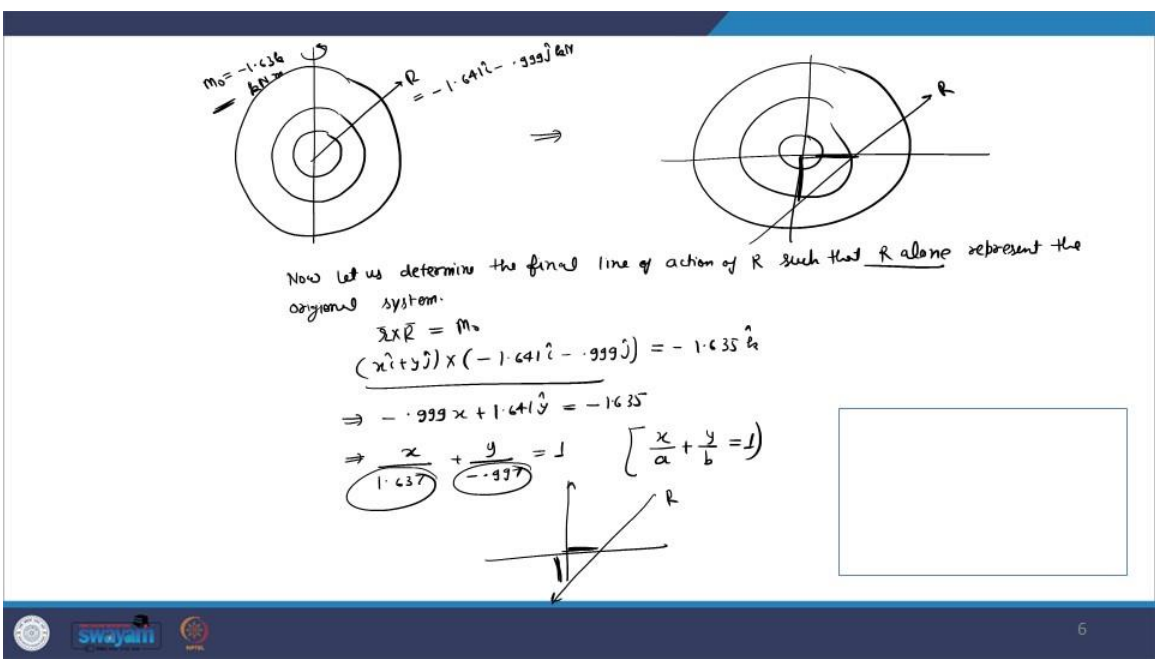
Now, this is a vector product and it can be find out So, it will be $132.4x - 66.9y\ \hat{k}$ and this would be equal to $-237\hat{k}$. Therefore, $132.4x - 66.9y = -237$ and this is the final line of action equation of the force.



Now, let us look at the second question on the same concept and the problem statement is following. So, this is question number 2.

Determine the x and y axis intercepts of the line of action of the resultant of three loads applied to the gear set. So, let us say we have this gear set and there are three forces of amplitude 3.6, 1.5 and 2.4 kN which are applied on this gear set and we have to find out what is the resultant force and what is its line of action.

So, we can draw the schematic. So, O is at the center and we have the first gear, then we have the second gear and we have the third gear and the first force is acting on the first gear from vertical it is making a 20° let us say it is F_1 and the value is of course 1.5 kN then F_2 is making 20° from the horizontal. So, let us say this is F_2 and its value is given, it is 2.4 kN and again F_3 is acting on third gear, its value is 3.6 kN and it is also making an angle 20° from the horizontal. Now, the radius of the first gear is 0.12 m, radius of the second gear is 0.2 m and radius of the third gear is 0.3 m. Now, let's say O is the convenient point about which we are going to find out the line of action and the resultant. So, $R_x = -1.5\, sin20° + 2.4\, cos20° - 3.6\, cos20°$ and you can find its value, it comes out to be $-1.641\, kN$. Similarly, we can find out $R_y = -1.5\, cos20° - 2.4\, sin20° + 3.6\, sin20°$ and it comes out to be $-0.999\, kN$. Therefore, $R = 1.641\hat{\imath} - 0.999\hat{\jmath}\, kN$. Now the $M_O = -1.5\, cos20°$, so, this is the vertical component of this force. Then I have to multiply it by the perpendicular distance. So, multiplied by the radius which is 0.12. Similarly, for the second force, $-2.4\, cos20° \times\ 0.2 - 3.6\, cos20° \times 0.3$ and this gives me $-0.1635\, kN$.



So, what we have done so far is, we have replaced this forces which were acting on the gear by a resultant R and the moment about O and the moment $M_O = -1.635\ kNm$ and the $R = -1.641\hat{\imath} - 0.99\hat{\jmath}\ kN$. But in the question statement, it was asked to find out the line of action of the resultant and what are its intercept on the x and y axis. So, that means it has asked you to replace it by a single resultant R and then find out what is the x intercept and what is the y intercept. So, now we have to account for this moment by replacing this R. So, let us do that.

So, now let us determine the final line of action of R such that R alone represents the original system. For that, let us use our master equation $r \times\ R = M_O$ and let us take $x\hat{\imath} + y\hat{\jmath} = r$ multiplied by the resultant $R = -\ 1.641\hat{\imath} - 0.999\hat{\jmath}$ and this is equal to $-1.635\ \hat{k}$. Now, this is a vector cross product can do that. It comes out to be $-0.999x + 1.641y = -1.635$. Now, we can rewrite this equation as $\frac{x}{1.637} + \frac{y}{0.997} = \ 1$ and this is in the form of $\frac{x}{a} + \frac{y}{b} = \ 1$. Therefore, these are the intercepts on the x and the y-axis. So, the resultant is like this, and this is your x-intercept, and this is your y-intercept.

So, with this, let me stop here. In the next class, we will look at the similar examples, but they are in three dimensions. Thank you.