MECHANICS

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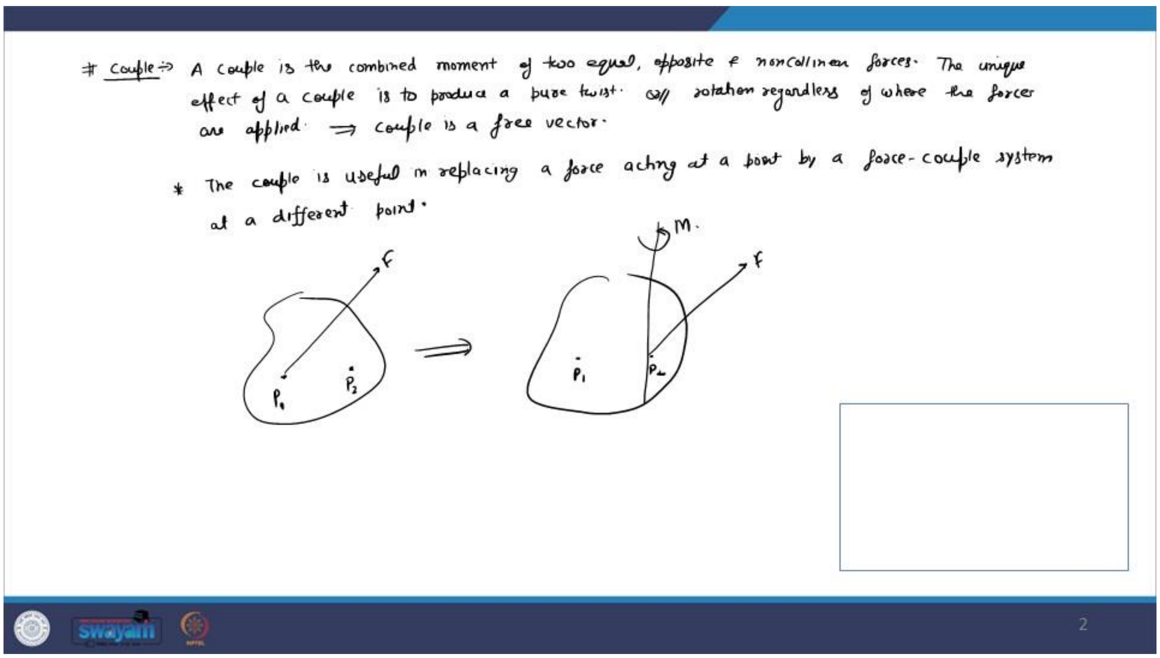
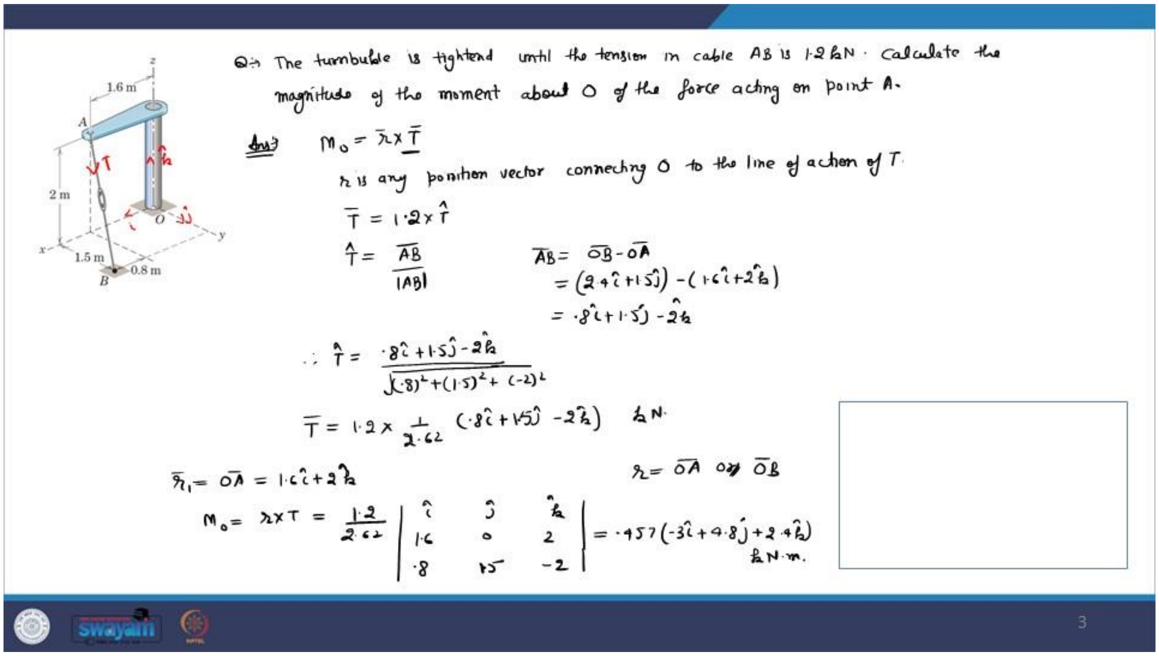
Lecture 05
Examples: moment in three dimensions

Hello everyone. Welcome to the lecture again. In the last lecture, we look about the basic concept of moment and couple. Today, what we are going to do is we are going to look at two examples wherein we will find out what is the moment in three dimension. So, before that, let me once again define the couple.



A couple is the combined moment of two equal, opposite and non-collinear forces. The unique effect of a couple is to produce a pure twist or rotation regardless of where the forces are applied and therefore, couple is a free vector. We have also seen that the couple is useful in replacing a force which is acting at a point by a force couple system at a different point.

So, again what do we mean by that? Suppose we have a rigid body and a force is acting at point P_1 and we want to replace this force at point P_2, then we can replace this force at point P_2 by having the same force at P_1 and then a couple about the point P_2. So, with this, now let us look at two examples.



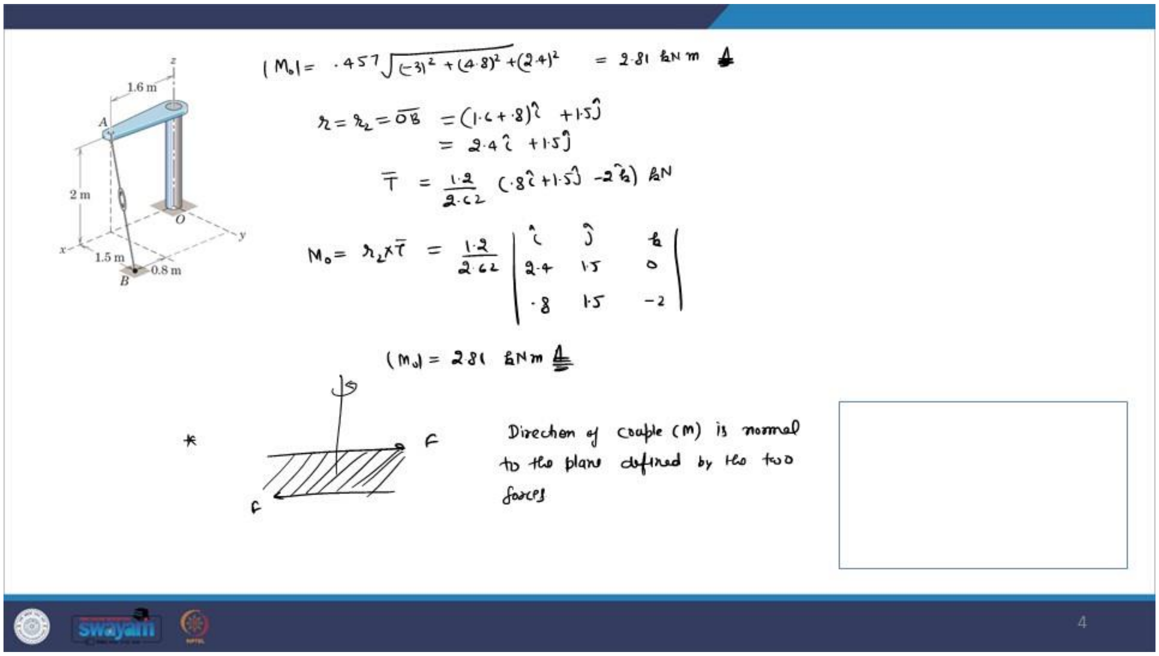
So, the problem statement of the question is following. The turnbuckle is tightened until the tension in cable AB is 1.2 kN. Calculate the magnitude of the moment about O of the force acting on point A. So, in this question, this turnbuckle is tightened. So, the tension will be in this direction and let us say that i j and k are the unit vector along the x, y and z direction. So, $M_O = r \times T$ where this r is any position vector connecting O to the line of action of T. So, T the magnitude is given it is 1.2 and its direction will be in the direction $\hat{T}$ which we have to find. So, this $\hat{T} = \frac{\overrightarrow{AB}}{|AB|}$ because it is acting in the direction AB.

Now, let us see what is AB. $AB = OB - OA = (2.4\hat{\imath} + 1.5\hat{\jmath}) - (1.6\hat{\imath} + 2\hat{k})$ and this comes out to be $0.8\hat{\imath} + 1.5\hat{\jmath} - 2\hat{k}$. Therefore, $\hat{T} = \frac{0.8\hat{\imath}+1.5\hat{\jmath}-2\hat{k}}{\sqrt{0.8^2 + 1.5^2 + (-2)^2}}$. So, it is. Therefore, $\vec{T} = 1.2 \times \frac{1}{2.62} \times 0.8\hat{\imath} + 1.5\hat{\jmath} - 2\hat{k}\ kN$.

So, now we have T over here. We have to find out what is r. Now, r is in our hand and we can take any point as I said which connects O to the line of action of T. Let us take some convenient r. So, r for example, you can take either OA or OB. Let us first take $r = OA$.

So, let me call it r_1 and this is OA. So, $OA = 1.6\hat{\imath} + 2\hat{k}$.

Therefore, the moment $M_O = r \times T = \frac{1.2}{2.62}\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1.6 & 0 & 2 \\ 0.8 & 1.5 & -2 \end{vmatrix} = 0.457(3\hat{\imath} + 4.8\hat{\jmath} + 2.4\hat{k})\ kNm$.

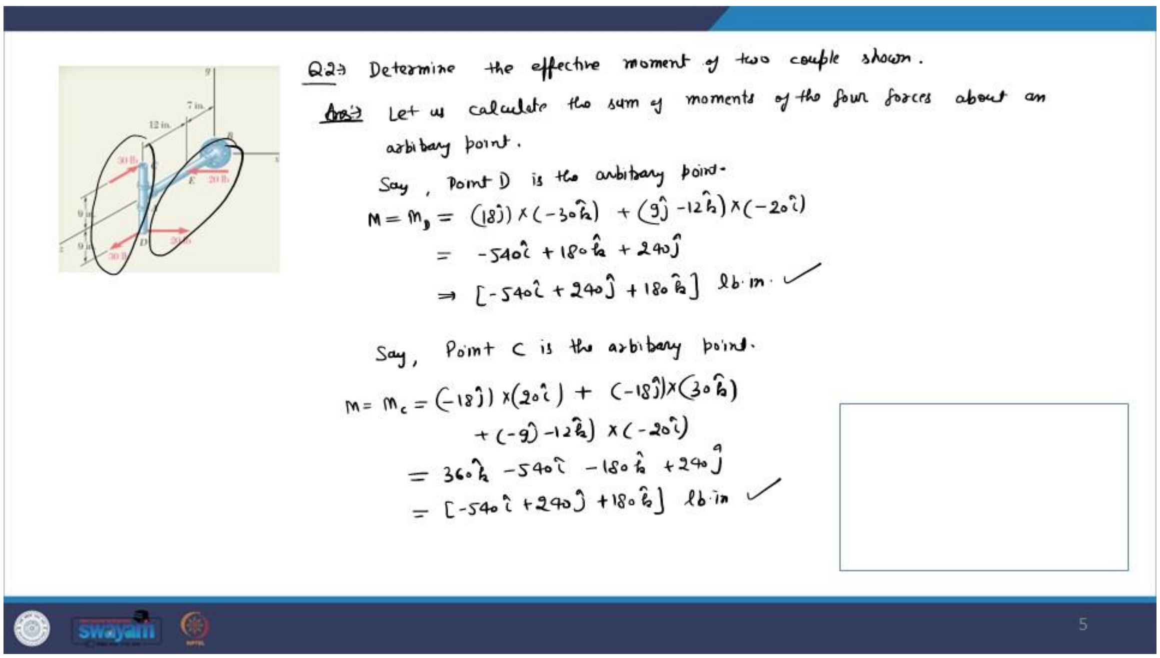


Now, $M_O = 0.457\sqrt{(-3)^2 + (4.8)^2 + (2.4)^2} = 2.81\ kNm$.

Now, as I said, we can also take our position vector as OB. So, let us call this $r_2 = OB$ and I will show you that if you use this r, even then you will get the same moment, okay. So, $OB = (1.6 + 0.8)\hat{\imath} + 1.5\hat{\jmath}$ or this is $2.4\hat{\imath} + 1.5\hat{\jmath}$ and we already have what is T. So, $\vec{T} = \frac{1.2}{2.62} \times 0.8\hat{\imath} + 1.5\hat{\jmath} - 2\hat{k}\ kN$.

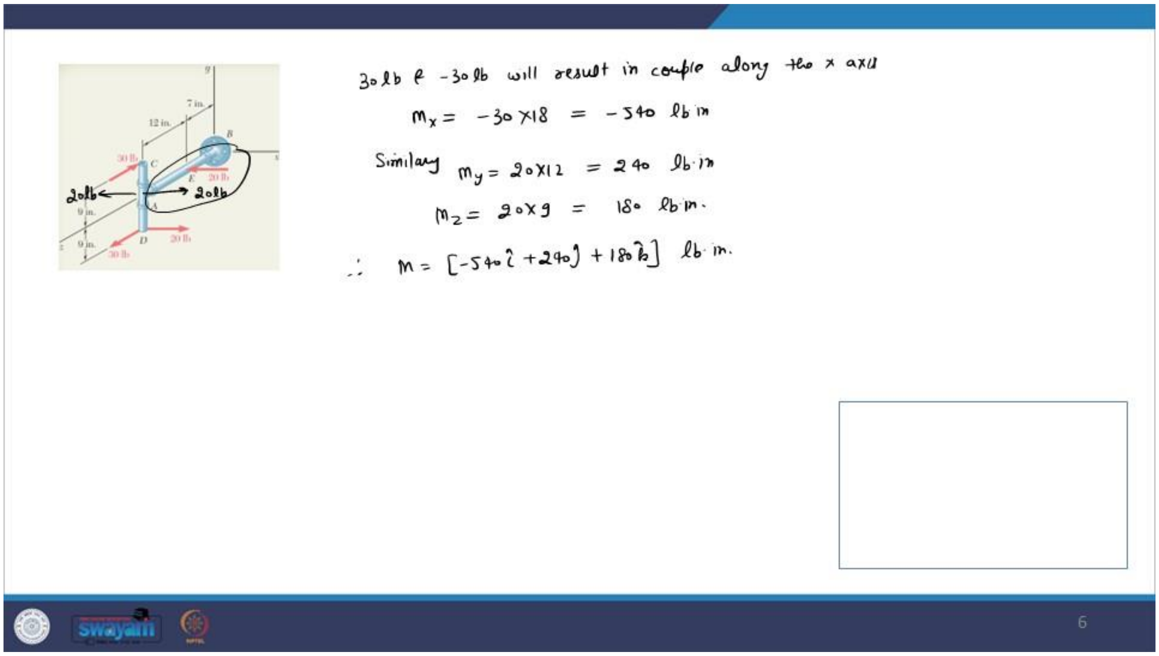
So, therefore, the moment $M_O = r_2 \times T = \frac{1.2}{2.62}\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2.4 & 1.5 & 0 \\ 0.8 & 1.5 & -2 \end{vmatrix}$ and again it comes out to be $2.81\ kNm$. So, you see that it is independent of r, this is what we are keep saying. Now, we have seen that if we have two equal and opposite forces, then they form a couple and the direction of couple, let us call it M, is always normal to the plane which is defined by the two forces. So, therefore, this is the plane and the couple will be normal to the plane.

Now, let us look at the second question and here the problem statement is determine the effective moment of two couple which is shown here. So, to find out the moment, let us choose some convenient point. So, let us calculate the sum of moments of the four forces about an arbitrary point and let's say we choose point D as our arbitrary point. The moment about D, let us represent it by M_D will be, so first of all, these forces will not contribute because they are passing through D So, we have to calculate the moment of this force and the moment of that force. So, for 30 pound force, the moment will be $18\hat{\jmath} \times (-30)\hat{k}$ and for 20 pound force, the moment will be $(9\hat{\jmath} - 12\hat{k}) \times (-20\hat{\imath})$ and



this will be $-540\hat{\imath} + 180\hat{k} + 240\hat{\jmath}$ or we can write it as $-540\hat{\imath} + 240\hat{\jmath} + 180\hat{k}\ lb.in.$

Now, let us choose C as the arbitrary point this time. So, let us say point C is the arbitrary point, then the M_C. So, in this case, this force will not contribute because it is passing through C. So, therefore, the 20 pound force, this force, let us first calculate the moment of this. So, it will be $(-18\hat{\jmath} \times 20\hat{\imath})$ plus the 30 pound force. So, it is again $(-18\hat{\jmath}) \times 30\hat{k}$ and then we have this 20 pound force. So, its $(-9\hat{\jmath} - 12\hat{k}) \times (-20\hat{\imath})$. So, again it is $360\hat{k} - 540\hat{\imath} - 180\hat{k} + 240\hat{\jmath}$ and this is $-540\hat{\imath} + 240\hat{\jmath} + 180\hat{k}\ lb.in.$ So, you can see here that the moment is independent of whether we calculate it about D or we calculate it about C. Now, why this happens? This happens because in this question, you can see that the couples are acting. So, for example, this and this, they makes a couple, and this and this, they makes a couple. So, there are two couples which are acting. So, therefore, you know, the moment is independent upon where you are calculating. We can also calculate the moment of this by adding a zero force at point A. So, let us add a 20 pound force at point A and also a minus 20 pound force.



So, basically we have added two force at point A, 20 and -20. Now, this makes a couple. This makes another couple and these two forces also make a couple. So, there are three couples and let us find out their moments. So, as I said, 30 lb and -30 lb will result in a couple along the x axis. Why is that? Because we have these two forces and let us connect them, then this is the plane and the moment will be perpendicular to this plane, okay. So, therefore, it will act in the x direction. So, the moment is one of the forces multiplied by the distance. So, $M_x = -30 \times 18 = -540\ lb.in$. Similarly, we have this 20 pound force which will give you the moment along the y direction. So, $M_y = 20 \times 12 = 240\ lb.in$ and $M_z = 20 \times 9 = 180\ lb.in$. So, we can write down the $M = (-540\hat{\imath} + 240\hat{\jmath} + 180\hat{k})lb.in$. So, again we got the same answer. So, by three different methods, we have find out the moment and you know as expected it comes out to be the same. Now, let me stop here and let us meet in the next class. Thank you.