MECHANICS
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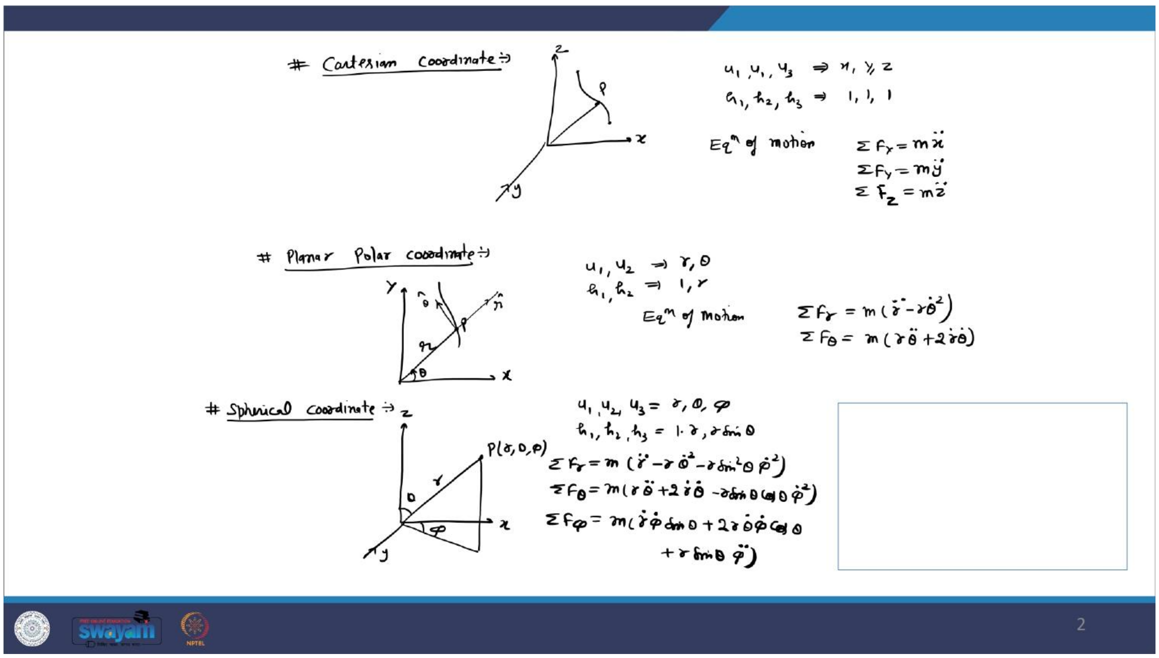
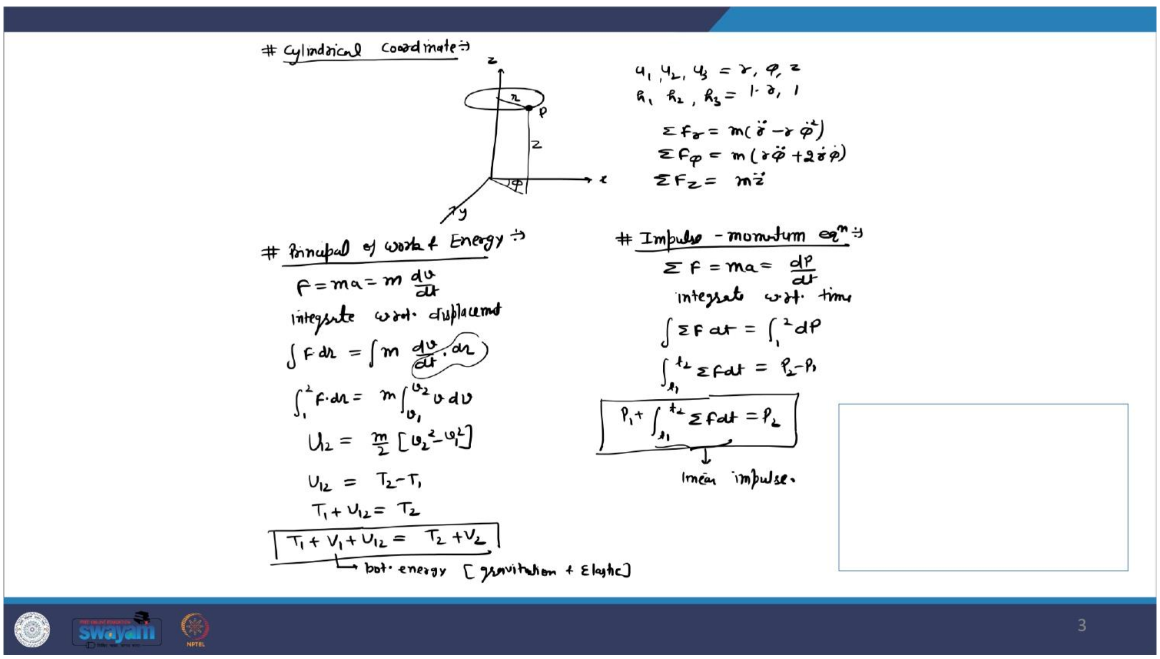
Lecture 60
Revision: Dynamics

Hello everyone, welcome to the lecture again. In this last lecture, we are going to revise the key concept that we have learned in the dynamics part that was the second half of this course.



We started the discussion by looking at the Newton equation of motion in different coordinate system namely the Cartesian coordinate, the planar polar coordinate, spherical coordinate and cylindrical coordinate system. So, in Cartesian coordinate, we have the x, y and z-axis and here this u_1, u_2 and u_3 are the x, y and z and the scale parameters $h_!$, h_2 and h_3, they were 1, 1 and 1 and the equation of motion of the particle. So, let us say the particle is at point P and this particle is moving. So, in this case, we saw that the unit vector does not change, it does not change the direction as the particle move from one point to other point and the equation of motion was $\sum F_x = m\ddot{x}$. $\sum F_y = m\ddot{y}$ and $\sum F_z = m\ddot{z}$. In planar polar coordinate system, we have the r and θ coordinate. So, the particle moves. So, its position vector is r and this r, the angle that it makes is θ. So, the unit vector along r is $\hat{r}$ and in the perpendicular direction, we have $\hat{\theta}$. So, here our u_1 and u_2 or the axis, they are r and θ and the scale factors h_1 and h_2 they were 1 and r and the equation of motion was $\sum F_r = m(\ddot{r} - r\dot{\theta}^2)$ and $\sum F_\theta = m(r\ddot{\theta} + 2\dot{r}\dot{\theta})$. Now, note that in planar polar coordinate system when the particle moves from one point to other point in that case the unit vector also changes the direction this was not the case in the Cartesian coordinate. Let us now look at the spherical coordinate. So, in spherical coordinate system we have the r θ and ϕ. So, let us say the position of the particle is P. So, that is defined by r, θ and ϕ. r is this r. θ is the angle that it makes from the z-axis and phi is ϕ and u_3, they are r, θ and ϕ and h_1, h_2 and h_3 are 1, r, $rsin\theta$. Therefore, the equation of motion along the r direction that is $\sum F_r = m\left(\ddot{r} - r\dot{\theta}^2 - rsin^2\theta\dot{\phi}^2\right)$, $\sum F_\theta = m(r\ddot{\theta} + 2\dot{r}\dot{\theta} - rsin\theta cos\theta\dot{\phi}^2)$ and $\sum F_\phi = m(\dot{r}\dot{\phi}sin\theta + 2r\dot{\theta}\dot{\phi}cos\theta + rsin\theta\ddot{\phi})$.



In cylindrical coordinate system, we have r,ϕ and z. So, let us say this is x-axis, y-axis and z-axis, then the position of the particle, let us say P is defined by r, ϕ and z. So, u_1, u_2 and u_3, they are r, ϕ and z, where r is the radius of the cylinder, z is the height of the cylinder and ϕ is the angle that this r makes from the x-axis. And, we saw that h_1, h_2 and h_3, the scale factors were 1, r and 1 and the equation of motion in the r direction, so $\sum F_r = m\left(\ddot{r} - r\dot{\phi}^2\right)$, $\sum F_\phi = m\left(r\ddot{\phi} + 2\dot{r}\dot{\phi}\right)$ and $\sum F_z = m\ddot{z}$. Then, we looked at the principle of work and energy and the equation we got by integrating the Newton's law with respect to displacement. So, principle of work and energy. So, we started with $F = ma = m\frac{dv}{dt}$. We integrated this with respect to displacement. So, we have

$$\int F.dr = \int m\frac{dv}{dt}.dr$$

$$\int_{1}^{2} F.dr = m\int_{v_1}^{v_2} v\,dv$$

$$U_{12} = \frac{m}{2}(v_2^2 - v_1^2)$$

$$U_{12} = T_2 - T_1$$

$$T_1 + U_{12} = T_2$$

Here T_1 and T_2 are the kinetic energy, the initial and the final kinetic energy and U_{12} is the work done by the external forces. And if there are elastic members are also present, then it can be written as $T_1 + V_1 + U_{12} = T_2 + V_2$. So, here V_1 is the potential energy which include the gravitational and elastic potential energy and U_{12} is the work done by the other forces. We also look at the impulse momentum equation and the impulse momentum equation we got again from the Newton's law by integrating with respect to time. So, again we have

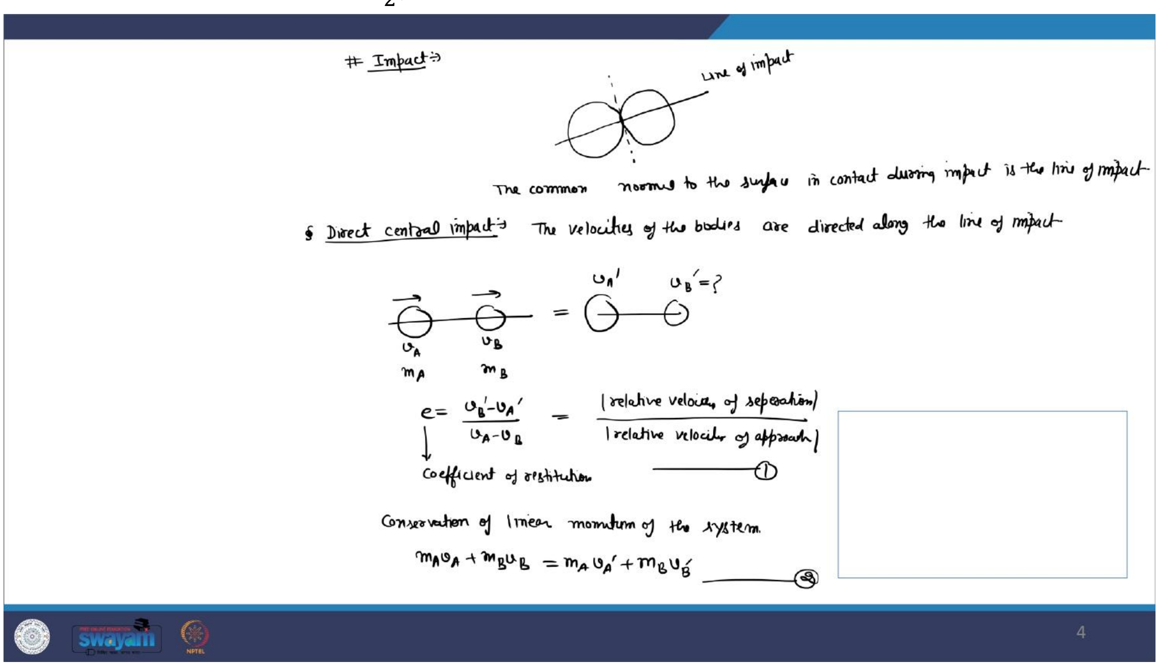
$$\sum F = ma = \frac{dP}{dt}$$

$$\int \sum F\,dt = \int_{1}^{2} dP$$

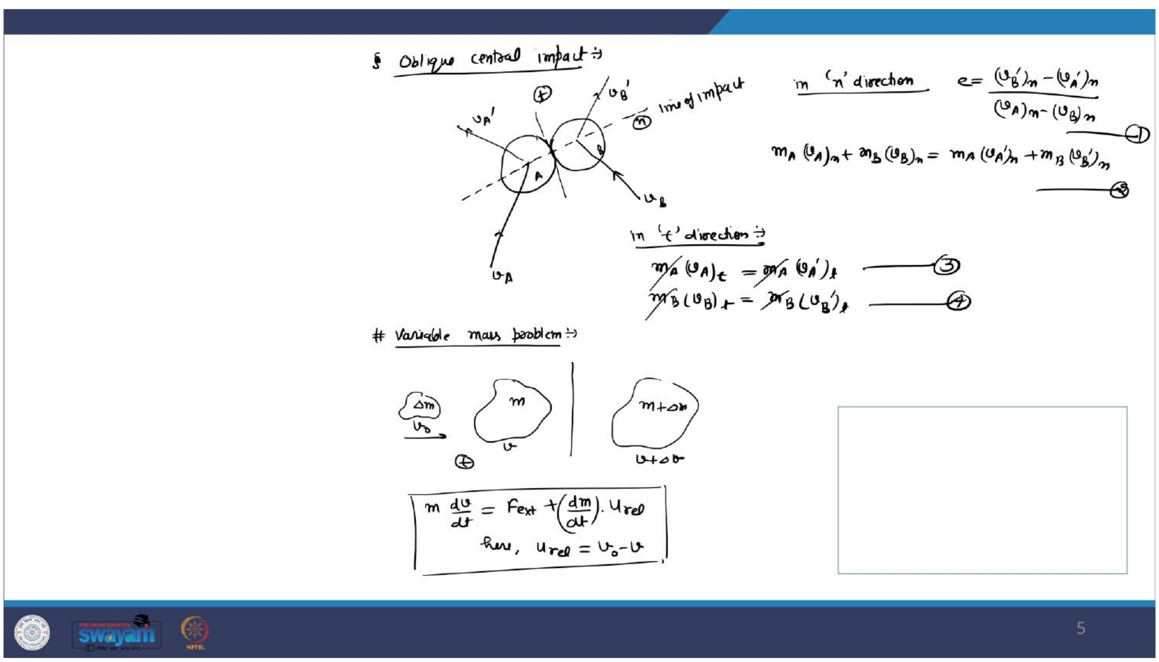
$$\int_{t_1}^{t_2} \sum F\,dt = P_2 - P_1$$

$$P_1 + \int_{t_1}^{t_2} \sum F\,dt = P_2$$

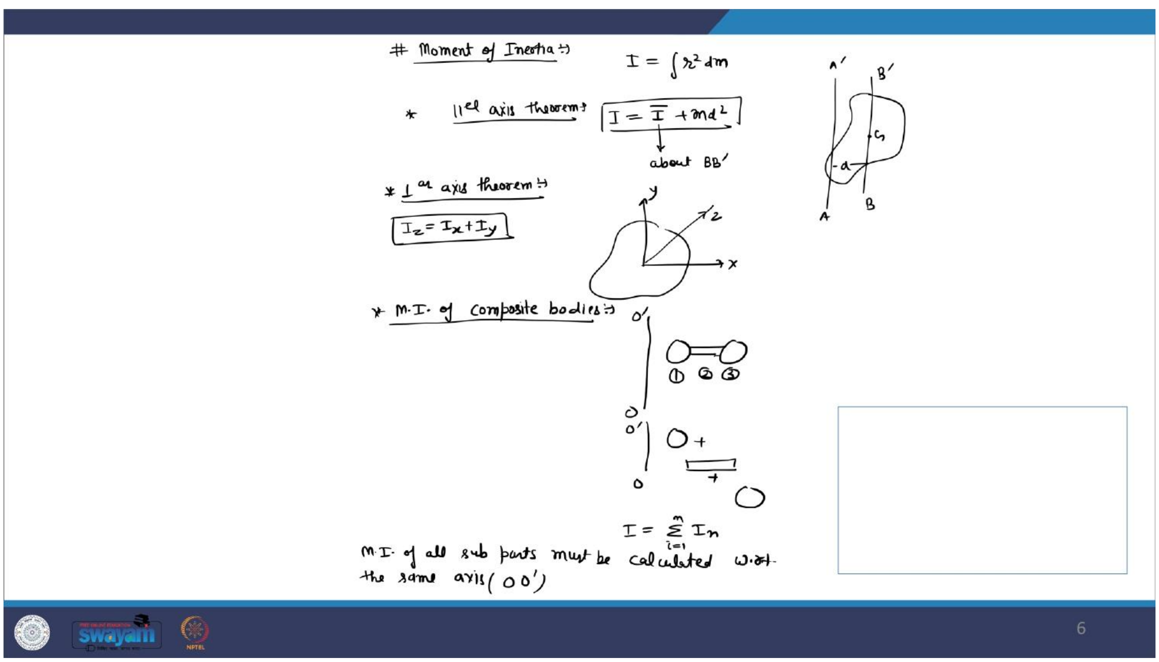
So, this equation tells that if the initial momentum is P_1 and the linear impulse is this, then the final momentum will be P_2.



Then we studied the impact. Let us say I have two rigid bodies. Then the tangent to the contacting surface is this and the common normal to the surface in contact during impact is the line of impact. So, here in that common normal is this. So, therefore, this is the line of impact. We discussed two cases. One was the direct central impact. So, here in the velocity of the bodies are directed along the line of impact. So, that means if this is the line of impact, then the velocities are along it. Let us say the velocity of this body is v_A, the mass is m_A, and the velocity of this body is v_B, mass is m_B. Then for the impact, v_A has to be larger than v_B and after the impact, the velocity of body A is let us say $v_A{}'$ and the velocity of body B is $v_B{}'$. Then the question was what are these velocities? And for that, because there are two variables, we need two equations and one equation comes from the definition of coefficient of restitution. So, we got $e = \frac{v_B' - v_A'}{v_A - v_B}$. So, this e was the coefficient of restitution. And $v_B' - v_A{}'$ is the relative velocity of separation of the bodies and $v_A - v_B$ is the relative velocity of approach. and the conservation of linear momentum for the whole systems gives us. So, this comes from the second equation comes from the conservation of linear momentum of the system. So, we have $m_A v_A + m_B v_B = m_A v_A' + m_B v_B{}'$. So, we have two equations and two unknowns. So, therefore, the problem was solved. Then we look at the oblique central impact.

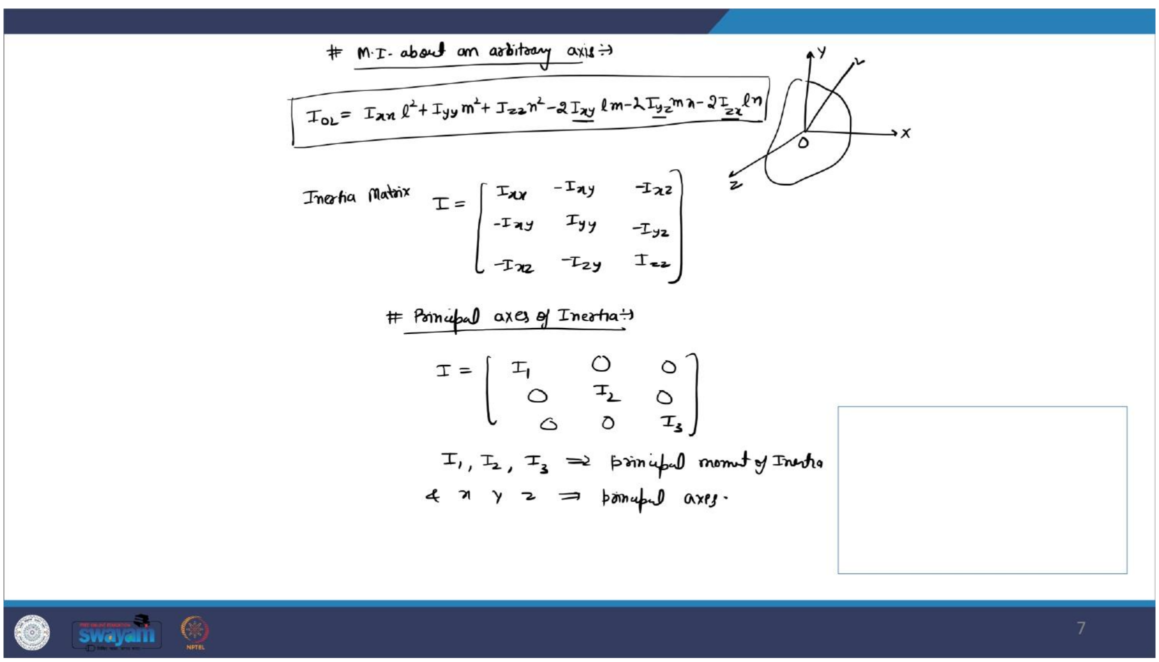


So, here in we have let us say two bodies and the line of impact is this. Let us say the velocity of body A is v_A and the velocity of body B is v_B. Then the question was after the impact, what is the velocity of the bodies? So, let us say this is $v_B{}'$, this one is $v_A{}'$ and the directions are also involved. So, there were four parameters, two for the velocity and two for the direction. Let us say the direction of the tangent to the contacting surface is t and the direction of the line of impact is n. In that case, we have in the n direction, we have $e = \frac{(v_B')_n - (v_A')_n}{(v_A)_n - (v_B)_n}$ and we have the conservation of linear momentum for the whole system. So, this is the same case as before $m_A(v_A)_n + m_B(v_B)_n = m_A(v_A{}')_n + m_B(v_B{}')_n$. And then two equation comes in the t direction. So, we have in t direction. Since there is no impulse, so therefore, the momentum of each particle remains conserved. So, we have $m_A(v_A)_t = m_A(v_A{}')_t$ and $m_B(v_B)_t = m_B(v_B{}')_t$. So, we have four equations and four unknown. So, again the problem is solved. Then, we discuss the variable mass problem. So, herein we consider a system which is either gaining the mass or losing the mass. To formulate the problem, we said well, let us say there is a small mass dm which is moving with a velocity v_o and we have a bigger mass, let us say it is denoted by m moving with a velocity v, then at time t, this is the initial case and then after they combine together, then we have the mass $m + dm$ and the velocity is $v + dv$. Then we use the impulse momentum relation to find out the equation for the variable mass problem and it was $m\frac{dv}{dt} = F_{ext} + \left(\frac{dm}{dt}\right).u_{rel}$ where this u_{rel} was $v_o - v$ or the velocity of the smaller mass minus the velocity of the bigger mass.



Then we look at the moment of inertia. The moment of inertia of a body is defined as $I = \int r^2 dm$ and then we look at parallel axis theorem. So, suppose I have a rigid body. And we have an axis which is passing through the centre of mass, let us say G and let us say this is BB' and we want to know what is the moment of inertia of this body about AA' axis which is parallel to BB' and at a distance of d. Then the moment of inertia I about AA' is

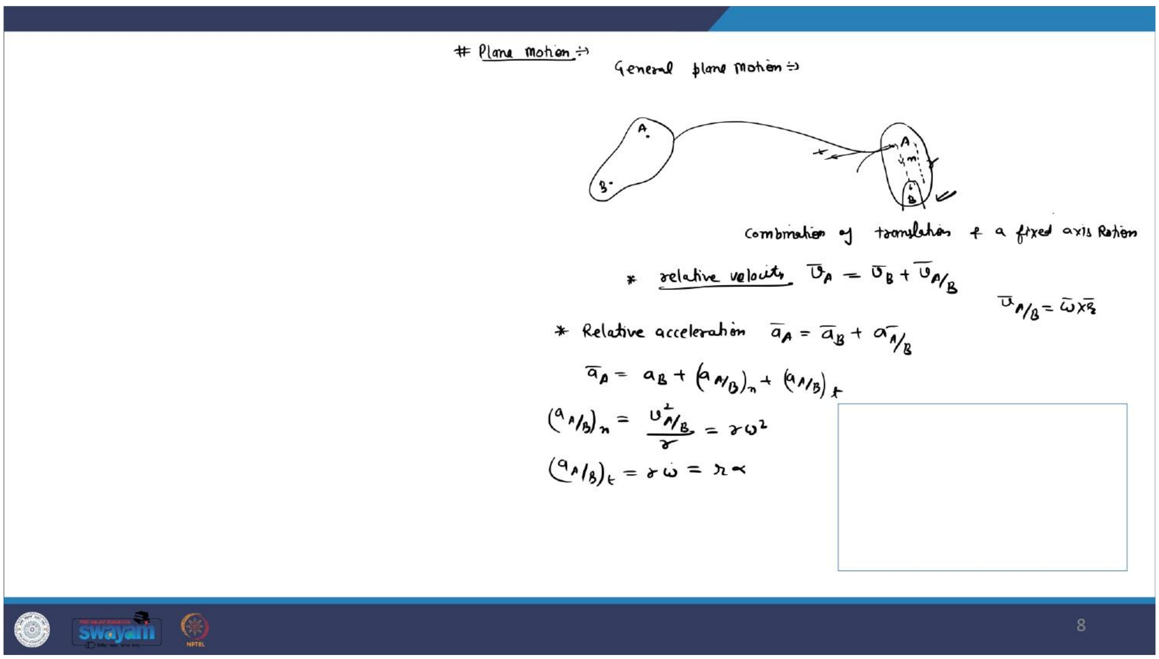
$\bar{I} + md^2$. Here $\bar{I}$ is the moment of inertia about the centre of mass axis. So, this is about BB' and I is the moment of inertia about AA' which is an axis which is parallel to BB' and at a distance d. Then we discuss the perpendicular axis theorem. Let us say there is a body which is in two dimension, let us say in the xy plane and then we want to calculate the moment of inertia of this body about the z direction. Then the moment of inertia about z is $I_z = I_x + I_y$. Then we discuss the moment of inertia of composite bodies. Let us say I want to calculate the moment of inertia of this object about OO' then the moment of inertia of this object will be let us say this is object 1, 2 and 3 then the moment of inertia about OO' will be the moment of inertia of the first object plus the moment of inertia of the second object plus the moment of inertia of the third object. Mathematically I can write down as I the moment of inertia of the composite body equal to the moment of inertia of all subparts and then sum them together. Herein, we have to note that the moment of inertia of all subpart, it must be calculated with respect to the same axis here OO'.



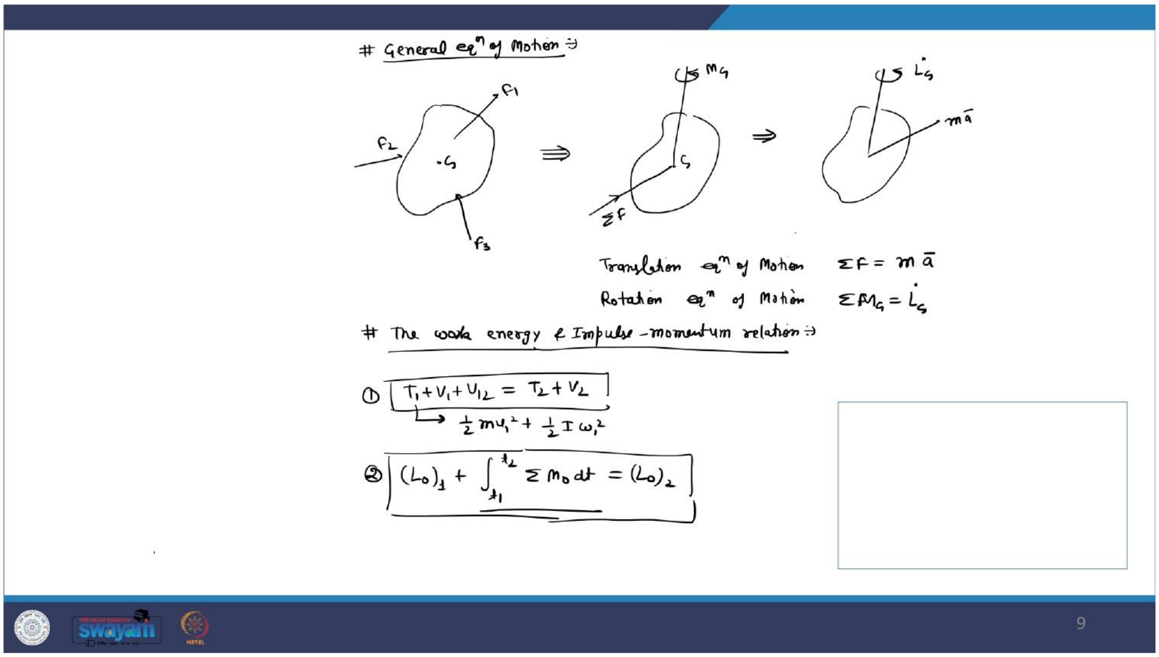
Then we look at the moment of inertia of the rigid body about an arbitrary-axis. So, let us say I have a rigid body and we have the x, y and the z-axis and I want to calculate the moment of inertia about let us say OL, then we saw that $I_{OL} = I_{xx}l^2 + I_{yy}m^2 + I_{zz}n^2 - 2I_{xy}lm - 2I_{yz}mn - 2I_{zx}ln$. So, here I_{xx} is the moment of inertia about the x-axis. Here we encounter I_{xy}, I_{yz} and I_{zx}. These are the product of inertias and the inertia matrix I is

$$I = \begin{bmatrix} I_{xx} & -I_{xy} & -I_{xz} \\ -I_{xy} & I_{yy} & -I_{Yz} \\ -I_{xz} & -I_{zy} & I_{zz} \end{bmatrix}$$

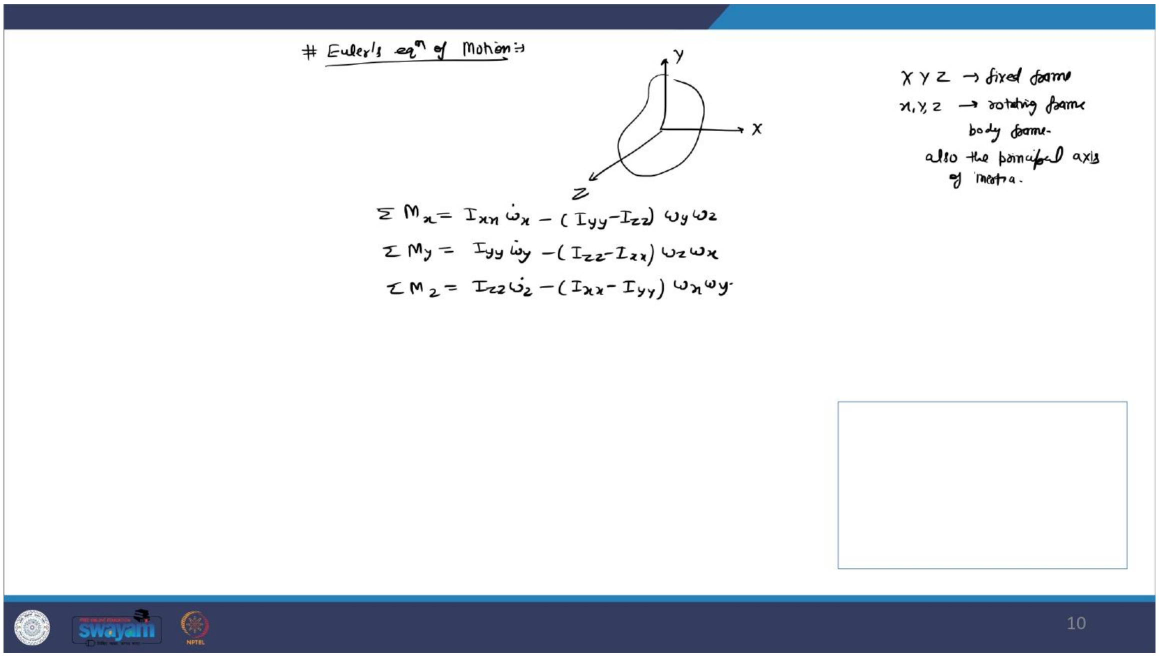
Then we discuss the principal axis of inertia. We saw that if we examine the rigid body and its moment and product of inertia for all possible orientations, then there is always an orientation for which the product of inertia vanishes and in that case, $I = \begin{bmatrix} I_1 & 0 & 0 \\ 0 & I_2 & 0 \\ 0 & 0 & I_3 \end{bmatrix}$ and all the product of inertia are 0. So, here this I_1, I_2, and I_3, these are called the principal moment of inertia. And this x, y and z-axis for which, you know, all the product of inertia vanishes, they called the principal axis.



We then discuss the plane motion and we saw that a rigid body execute a plane motion when all parts of the rigid body they move in a parallel plane. Particularly, we discussed the general plane motion. And this general plane motion is the combination of a translation and a fixed axis rotation. So, let us say I have a rigid body and let us say there are two points on the rigid body B and A and this body performs a general plane motion. So, the general plane motion can be analyzed by considering it as a combination of the translation of let us say the point B and a fixed axis rotation of point A with respect to B. The relative velocity of point A will be the velocity of B plus the velocity of point A with respect to B. $\bar{v}_A = \bar{v}_B + \bar{v}_{A/B}$. Now, since point A is making a fixed axis rotation about point B, therefore $\bar{v}_{A/B} = \bar{\omega} \times \bar{r}$ and the relative acceleration of point A will be the acceleration of point B plus the acceleration of point A with respect to B. $\bar{a}_A = \bar{a}_B + \bar{a}_{A/B}$. Now, $\bar{a}_{A/B}$ has two components, the normal component and the tangential component because this is making a fixed axis rotation. So, we have the n component and the t component. So, therefore, . $\bar{a}_A = \bar{a}_B + \left(\bar{a}_{A/B}\right)_n + \left(\bar{a}_{A/B}\right)_t$ and the value of the normal acceleration in fixed axis rotation is $\left(\bar{a}_{A/B}\right)_n = \frac{v_{A/B}^2}{r} = r\omega^2$ where r is this distance. $\left(\bar{a}_{A/B}\right)_t = r\dot{\omega} = r\alpha$.



We then discuss the general equation of motion of the rigid body. Let us say I have a rigid body and on this rigid body various forces are acting and G is the center of mass, then this situation can be analyzed by replacing all the resultant forces at some chosen point. So, let us say the chosen point is G. So, as if at G, all the forces are acting and then there is a corresponding couple or the moment about the mass center G and then we can write down the equation of motion. So, we have translation equation of motion which is $\sum F = m\bar{a}$ where $\bar{a}$ is the acceleration of the center of mass and then we have a rotation equation of motion which is $\sum M_G = \dot{L}_G$, where L is the angular momentum. So, because of this, we have $m\bar{a}$ and because of m, there is $\dot{L}_G$. We then looked at the work energy and impulse momentum relation for the rigid body and we saw that their form remains the same as they were for the particle. Herein, we have the work energy equation as $T_1 + V_1 + U_{12} = T_2 + V_2$ and the kinetic energy $T_1 = \frac{1}{2}mv_1^2 + \frac{1}{2}I\omega_1^2$ because the body has also the rotational energy. And the impulse momentum equation was $(L_0)_1 + \int_{t_1}^{t_2} \sum M_0 dt = (L_0)_2$. Here, $(L_0)_1$ is the initial angular momentum and $\int M\, dt$ was the external angular impulse and $(L_0)_2$ is the final angular momentum.



Finally, we discussed the Euler's equation of motion. So, we said that suppose I have a rigid body and X, Y, Z are the fixed frame and x, y, z are the rotating frame which are fixed with the body. So, basically they are the body frame and if this is x, y, z are also the principal axis of inertia, then we have the equation of motion of the rigid bodies as

$$\sum M_x = I_{xx}\dot{\omega}_x - \left(I_{yy} - I_{zz}\right)\omega_y\omega_z$$

$$\sum M_y = I_{yy}\dot{\omega}_y - (I_{zz} - I_{xx})\omega_z\omega_x$$

$$\sum M_z = I_{zz}\dot{\omega}_z - \left(I_{xx} - I_{yy}\right)\omega_x\omega_y$$

Well friends, with this, we come to the end of this particular course on mechanics. I hope that this course was useful to you and you were able to learn and understand various concepts related to Newtonian mechanics. I understand that there is always a scope for improvement in teaching and therefore, I look forward to get valuable feedback from you so that we can make it better with time. Taking this course under the NPTEL platform was also a learning experience for me and I am very grateful that I got this opportunity. I thank the TA Nikita Chaudhary and Diksha Sharma for helping me throughout this course. I hope to see you next time probably with a different course on physics.

Thansk you.