MECHANICS

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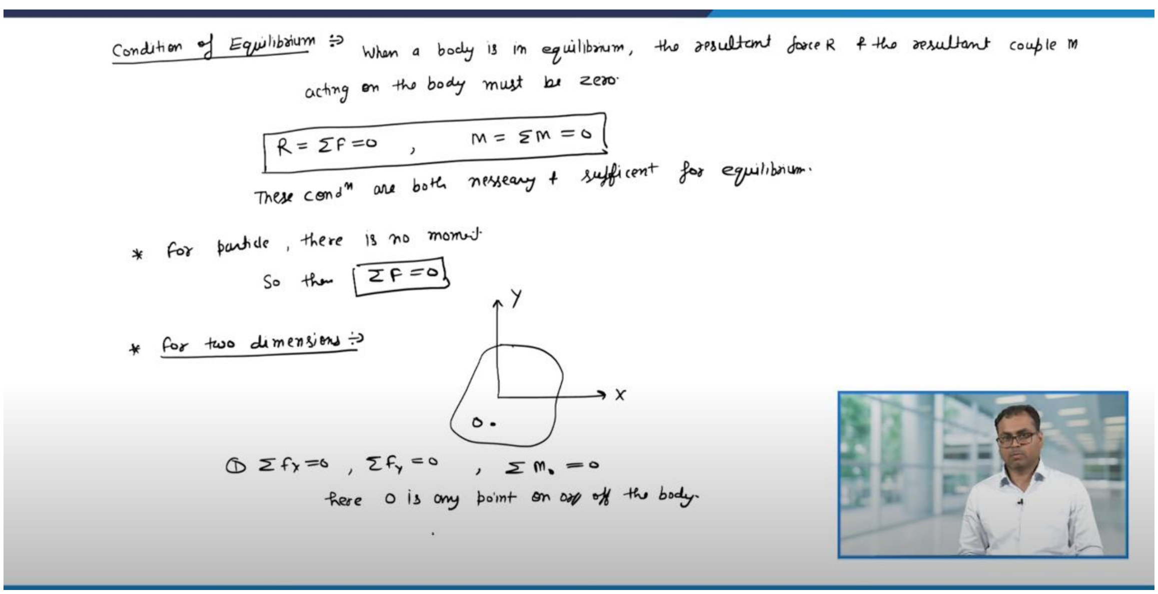
Lecture 07

Equilibrium of rigid bodies in two and three dimensions

Hello everyone, welcome to the lecture again. In the last lecture, we looked at the concept of free body diagram, how to make a free body diagram. We also learned about the reactions exerted by various supports.



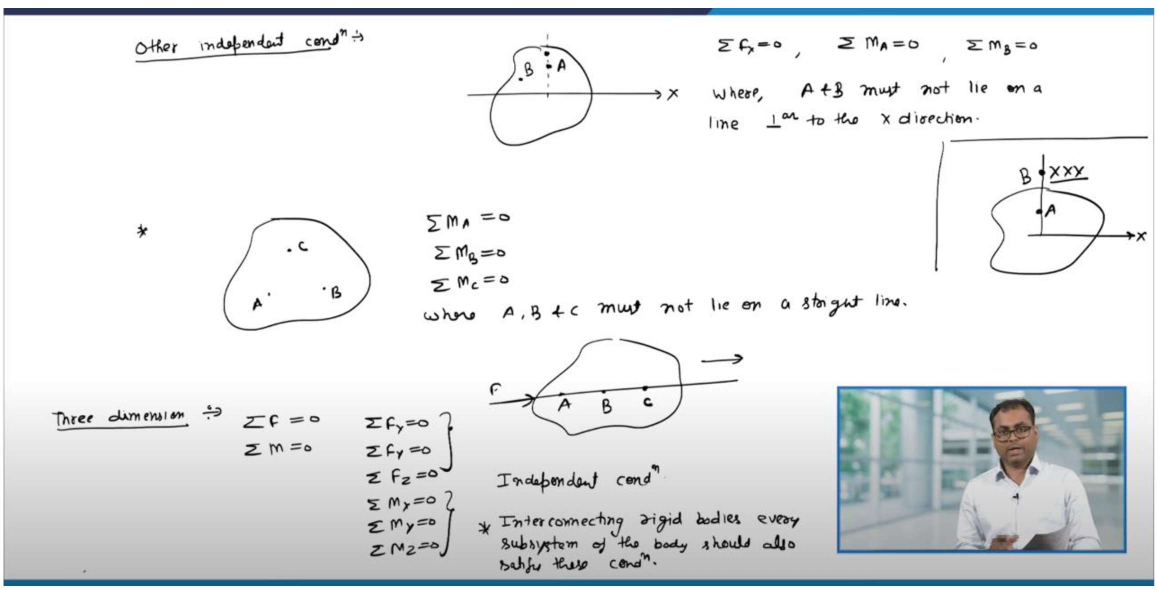
Today, what we are going to do is, we are going to analyze the equilibrium of rigid bodies in two and three dimensions. So, for the equilibrium, the body should neither move nor it should rotate about any axis.

That means the condition of equilibrium is when a body is in equilibrium, the resultant force R and the resultant couple M which is acting on the body must be 0. That means R total force which is sum of the force this should be 0 and M which is sum of all the moments should be 0. These conditions are both necessary and sufficient for equilibrium.

Now, suppose I have a particle. So, for particle there is no concept of rotation. So, therefore, the condition that M equal to 0 is no longer required. So, for a particle there is no moment.

Therefore, the condition is $\sum F = 0$. Now, this equation are vector equation. So, actually $F = 0$ means $F_x = 0, F_y = 0$ and $F_z = 0$. Now, suppose the body is in 2D. So, for two dimensions.

So, let us say I have a body and this body is in two dimension. So, these are x and y. In this case, there are various conditions that are you know necessary and sufficient. So, for example, the condition can be $\sum F_x = 0$, $\sum F_y = 0$ and the moment about any arbitrary point which may be or may not be on the body should be 0. So, for example, let us take this point. So, the moment about this point should be 0. That means the moment about point O should be 0, okay. Here, O is any point on or of the body.



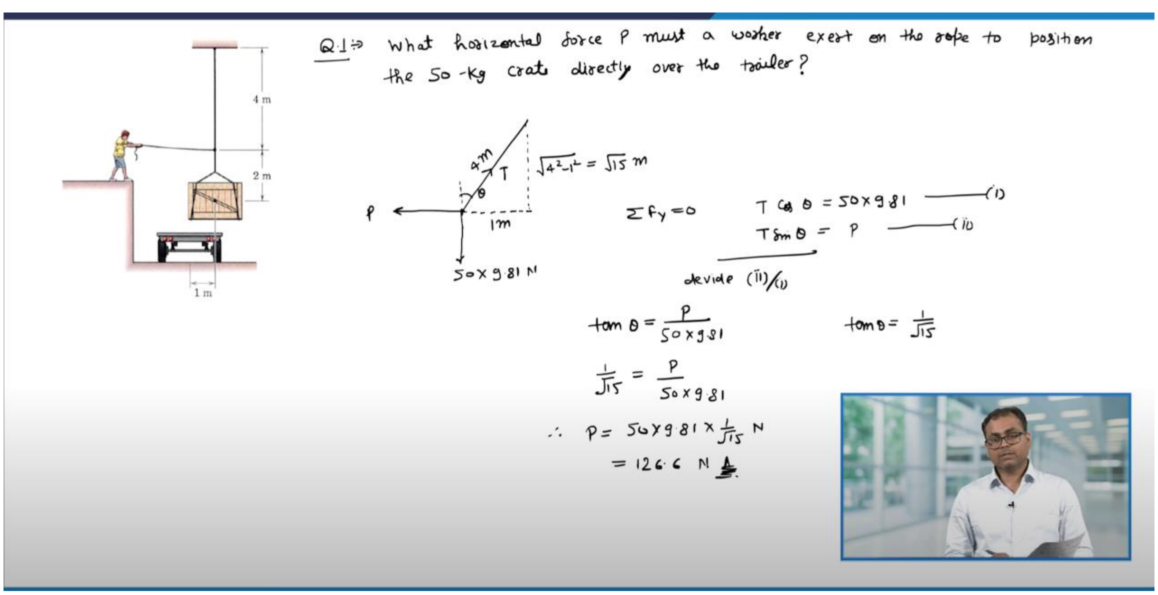
Now, let us look at other independent condition for two-dimensional object. So, let us say again I have a rigid body. Let us say this is some x-axis. In that case, summation F x should be equal to 0. Moment about a point A should be equal to 0 and also the moment about some other point B must be 0, but this point B should not lie to a single line which is perpendicular to x-axis. So, for example, this point B should not lie on this x, it should be somewhere else. So, let us say here. So, where A and B must not lie on a line perpendicular to the x direction.

Just to repeat this point again, we have this rigid body, I have this x-axis, this is point A. Then B cannot lie here. It should be somewhere else, then this is also necessary and sufficient condition. Now, let us look at other conditions. Again, suppose I have a rigid body and in this rigid body, you take three points A, B and C. So, if the moment about A is 0, moment about B is 0 and moment about C is 0, then also the body will neither move nor rotate.

But here, these points A, B and C, they must not lie on a straight line. Why so? Because if A and B, and C, they lie on this line. Now, suppose you apply the force, then the moment about A is 0, moment about B is 0, and moment about C is also 0. However, under the influence of this force, the body is going to move.

Therefore, to protect this, point A, B, and C must not lie in a single line. Now, for three dimensions, the condition is that is $\sum F$ should be 0 is $\sum M$ should be 0, okay. So, this implies that is $\sum F_x$ should be 0, is $\sum F_y$ should be 0, is $\sum F_z$ should be 0, is $\sum M_x$ should be 0, $\sum M_y$ should be 0, and $\sum M_z$ should. So, these are six independent equations. Note that for the equilibrium, when you test, you have to test all these conditions.

Only then we can say that the body is in equilibrium. So, it might happen that some of the conditions are satisfied and some of the conditions are not satisfied. If that is the case, then the body is not in equilibrium. So, these are the independent conditions and all of them should be tested. Now, suppose we have some interconnecting rigid bodies. Then in this case, every subsystem of the body should also satisfy these conditions.



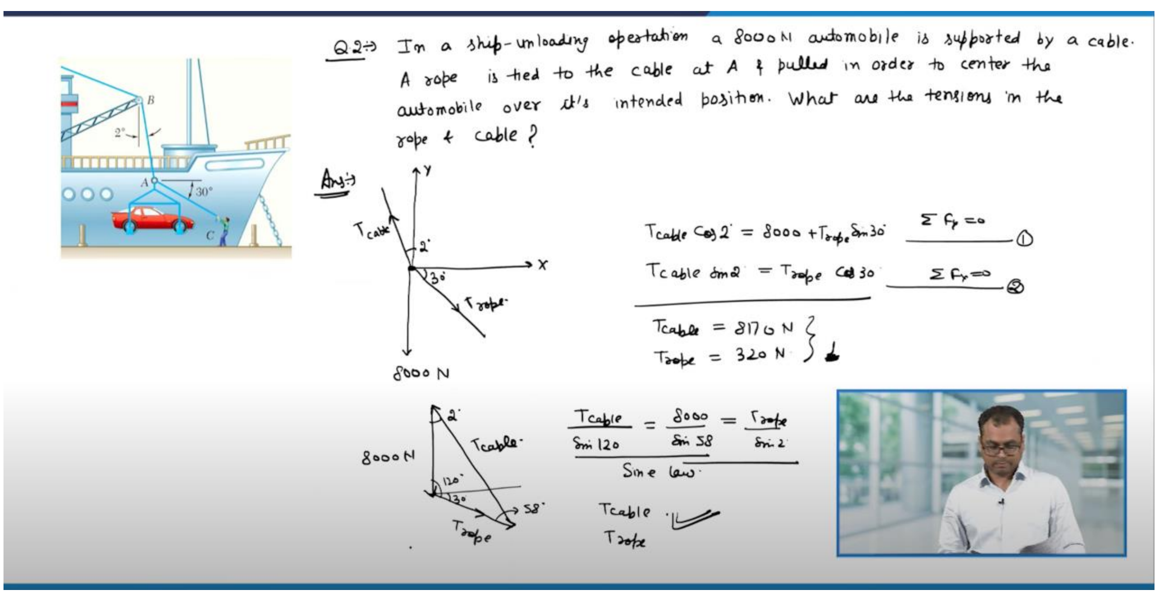
Now, with this very basic introduction about the equilibrium, now let us look at few examples which are based on these. Now, let us look at the first example, what horizontal force P must a worker exert on the rope to position the 50 kg crate directly over the trill.

Now, let us look at the free body diagram of this question. So, we have a force P which is acting in this direction. Under this force, the rope is going to be like that and we have the mass of the thread which is 50 kg. So, 50 into $9.81N$ and this arm is $4m$. This arm is $1m$ and let us say it is making an angle theta from the vertical.

So, therefore, this arm will be $4^2 - 1^2 = \sqrt{15}m$. The tension force on the rope, it is going to act in the opposite direction of the weight. So, therefore, this will be the direction of T and this is the free body diagram. Now, let us balance the vertical and horizontal forces. So, first let us balance the vertical forces.

So, $\sum F_y$ should be equal to 0. That means the forces in the y direction should be balanced. So, $Tcos\theta$ should be 50×9.81 and $Tsin\theta$ should be P. Now, if we divide 2 by 1, then we get $tan\theta = \frac{P}{50 \times 9.81}$ and $tan\ \theta = 1/\sqrt{15}$.

So, therefore, $\frac{1}{\sqrt{15}} = \frac{P}{50 \times 9.81}$. Therefore, P will be $50 \times 9.81 \times \frac{1}{\sqrt{15}}$ and this comes out to be $126.6N$.



Let us look at another example. And here the problem statement is following. In a ship unloading operation a $8000N$ automobile is supported by a cable. A rope is tied to the cable at A and pulled in order to center the automobile over its intended position. Question is what are the tension in the rope and cable?

So, the first step is to make the free body diagram. So, let us make the free body diagram. Let us say this one is the x-axis and this is y-axis. In the cable, let us say the tension is T_{cable} and in the rope, Let us say the tension is T_{rope} .

The weight of the automobile is working downwards and it is $8000N$. This angle is 2^o and this angle is 30^o. Now, note that both the tension T_{cable} and T_{rope}, they are acting away from the object. With this, we complete the free body diagram. Now, let us balance the forces.

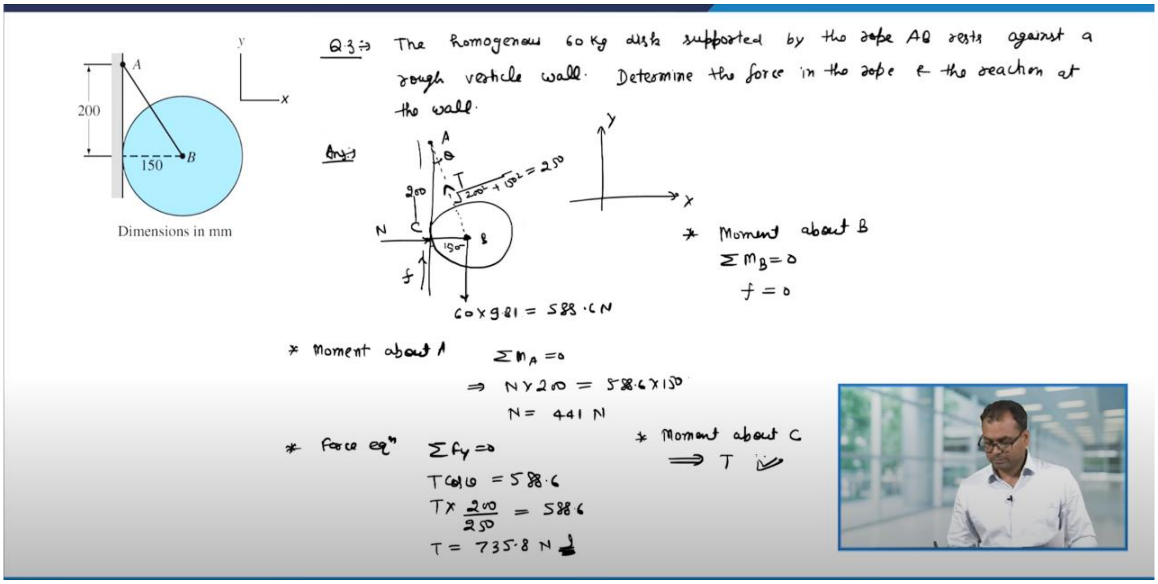
So, T cable. So, let us first balance $\sum F_y = 0$. So $T_{cable}\ cos2^o = 8000 + T_{rope} sin30^o$. Let us call it equation number 1 and now let us balance the horizontal forces.

So, $T_{cable}\ sin2^o = T_{rope}\ cos30^o$. This is$\sum F_x = 0$. So, here there are two variables and there are two equations. So, we can find out what is T_{cable} and T_{rope}. T_{cable} Comes out to be $8170N$ and T_{rope} comes out to be $320N$.

Now, we can also solve this problem using the sine law. So, let us look at the forces. So, $8000N$ force is acting downwards. T_{rope} Is also acting like this and these forces are balanced with T_{cable}. Now, this angle is 2^o.

This angle is 30^o. Therefore, this angle will be 58^o. Now, we can use the sine law. So$\frac{T_{cable}}{sin120^o} = \frac{8000}{sin58^o} = T_{rope}/sin2^o$ because this angle is120^o.

So, we have used the sine law. And again, we have two equations, this and that. From here, we can find out what is T_{cable}? And what is T_{rho}? So, any of the method we can use.



Let us look at one more example. So, the problem statement here is following. The homogeneous 60 kg disc supported by the rope AB. It rests against a rough vertical wall.

Determine the force in the rope and the reaction at the wall. So, first let us make the free body diagram. We have this wall, we have this disc and its center is B from here. It is kept in the equilibrium by using a rope. So, therefore, the tension in the rope is going to act in the upward direction and this point is C, this point is A. Now, its weight is going to act downward.

It is 60 kg. So, therefore, 60×9.8, it is 588.6 kg. Newton. This wall has friction. So, therefore, the friction force is going to act.

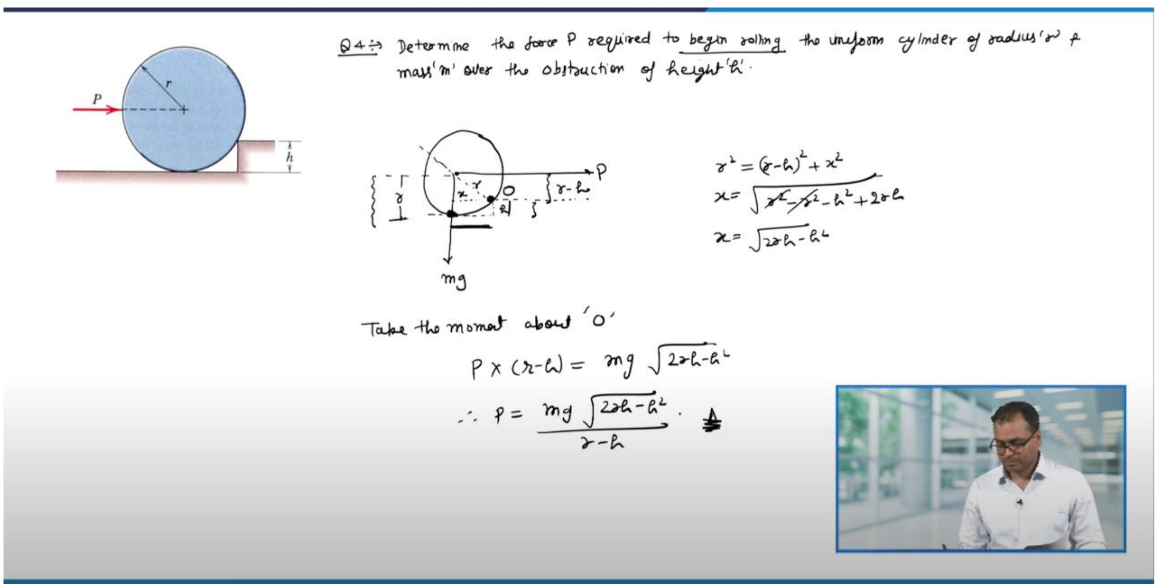
Let us say the direction of the friction force is upward. From the wall, there will be a normal force and let us also put the axis, x-axis and y-axis. So, this completes the free body diagram of this problem. So, the friction force can be found out if we take the moment about B because if we take the moment about B, then the moment of T will be 0, N will be 0 and the weight is also, its line of action is also passing through B. So, therefore, that will also be 0. So, therefore, we have only one unknown.

So, let us take the moment about B and $\sum M_B$ is 0. So, this tells us that F will be 0. So, even though this is a frictional wall, the frictional forces will be 0. Now, to determine M, let us take the moment about A.

So, that means $\sum M_A$ should be 0 because the body is in equilibrium. So therefore, $N \times 200$ because this is 200, this is 150. So, $200 = 588.6 \times 150$ and this gives you $N = 441N$. Now, to determine T, let us use the force equation.

$\sum F_y = 0$. So, it will be $Tcos\theta$ where θ is at this angle equal to 588.6. So, $Tcos\theta$ will be 200, divide by 250 because this arm will be square root 200 square plus 150 square which is 250. So, this should be equal to 588.6.

Therefore, T comes out to be $735.8N$ now, we can also find T just by taking the moment about C. So, this you can take as a homework. Moment about C can also give you T.



Now, let us look at one more example. And here, the problem statement is following. Determine the force P required to begin rolling the uniform cylinder of radius r and mass m over the obstruction of height h. So, first, let us look at the free-body diagram. We have this uniform cylinder and here, the force P is acting. So, since it is a sliding vector, let me put the P here, and its mass is m g, and this is the support that we have, and this height is h. This is

Now, since the cylinder is just begin rolling, so therefore, it will detach from this point. Therefore, no reaction will act over here. From here, we will have some kind of reactive forces and since it is unknown, so therefore, let us take the moment about this point O.

Now, to write down the moment, we should know the distance from O to the line of action of m g. So, let us say this one is x. First, let us determine x. Now, since this distance is r, this is h. Therefore, this will be $r - h$ and this is r so we can find out x because now $r^2 = (r-h)^2 + x^2$ or x will be $\sqrt{r^2 - r^2 - h^2 + 2rh}$, this will get cancelled, and x will be $\sqrt{2rh - h^2}$. So, now we know this distance. Now, let us take the moment about O. So, we have force P, its vertical distance is $r - h$ equal to the mg force multiplied by the distance, which we have just calculated. It is $2rh - h^2$. Therefore, P will be $\frac{mg\sqrt{2rh-h^2}}{r-h}$. With this, let me stop here and see you in the next class with more examples. Thank you.