NOISE CONTROL IN MECHANICAL SYSTEMS

Prof Sneha Singh

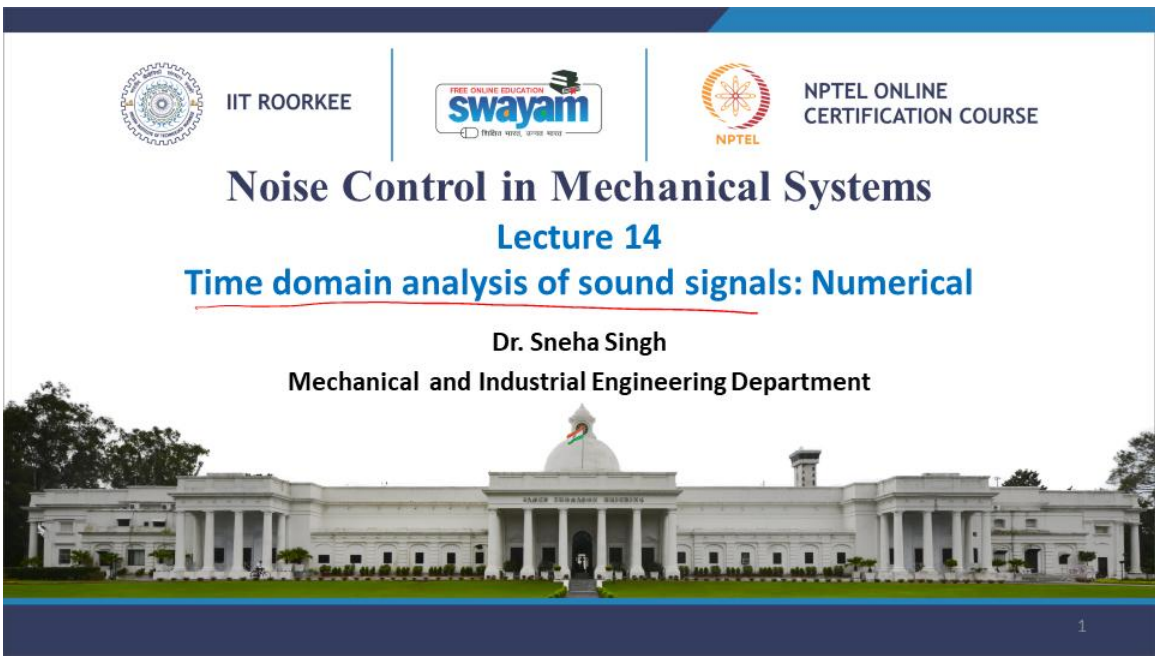
Department of Mechanical and Industrial Engineering

IIT Roorkee

Week:03

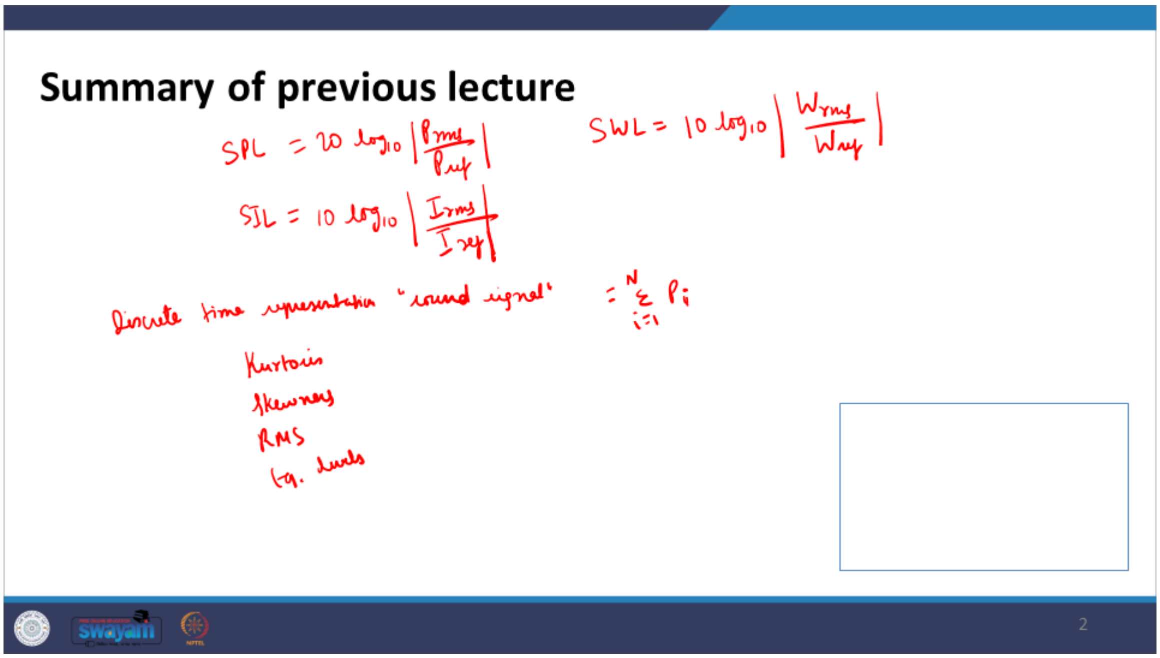
Lecture:14

Lecture 14: Time Domain Analysis of sound signals: Numerical



Hello and welcome to Lecture 14 in this series on noise control in mechanical systems with me, Professor Sneha Singh. In the last class, we studied the time domain analysis of sound signals. So, let us solve some interesting problems based on what we have learned. To summarize, the things we learned were sound pressure level, sound intensity level, and sound watt level or sound power level.

Then, we also saw what a discrete time representation of a sound signal is. So, sound as a signal collected from some instrument becomes a collection of these various P_i points, n number of P_i points, and then what kind of analysis we do to find out kurtosis, skewness, RMS, and the various kinds of equivalent levels. These are some of the important time domain features or statistical features of the sound signal in time.



So, let us do the numerical problems based on what we have studied.

Outline

- **Numerical Problems on:**
  - Sound Pressure level, intensity level and power level
  - Time domain analysis of sound signal
  - Equivalent noise levels

So, the first one is acoustic pressure being generated by some source; it is a point source. At a distance of 20 meters from that, we get the acoustic pressure measurement to be 0.5 pascals. Now, over here, I would like to say that the pressure is not instantaneous pressure; this is actually the RMS pressure. So, whenever we report the pressure, from now on, whenever you say that acoustic pressure is generated and measured at a point, we usually refer to the RMS pressure because the instantaneous pressure could be a fluctuating value. So, usually, the value that makes sense is the RMS pressure. So, this is the P_{rms}. You have to find the SPL at 20, SIL at 20, and then 10 and SWL of the source.

Problem - 1

- Acoustic pressure generated by a point source at a distance of 20 metres from the source is 0.5 Pa. → P_{rms}
- Find the following:
  - SPL at 20 m
  - SIL at 20 m
  - SIL at 10 m
  - SWL of the source

Let us go one by one. So, the first part is SPL at 20 meters, which is what we have to find. So, what is it? So, I will represent it as L_{20} because L means the sound pressure level and 20 represents the distance. L_{20} would be

$$\mathbf{L_{20} = 20\log_{10}\left|\frac{p_{20}}{p_{ref}}\right|}$$

which we already know is 0.5 pascals at 20 meters. This is the reference pressure. If you solve it, you will get it as 88 decibels. So that becomes our very first answer. Now, SPL at 10 meters. Is it SPL or SIL? It is SIL at 10 meters and 20 meters. Now, last class I told you that SPL and SIL essentially are the same.

Let us see how we can calculate this. So, the SIL at 20 would be

$$\mathbf{L_{20} = 10\log_{10}\left(\frac{I_{20}}{I_{ref}}\right)}$$

and I at 20 would be I is directly proportional to P^2. So, we get it as

$$\mathbf{L_{20} = 10\log_{10}\left(\frac{{p_{20}}^2}{{p_{ref}}^2}\right)}$$

again the same thing you will get it as 88 decibels.

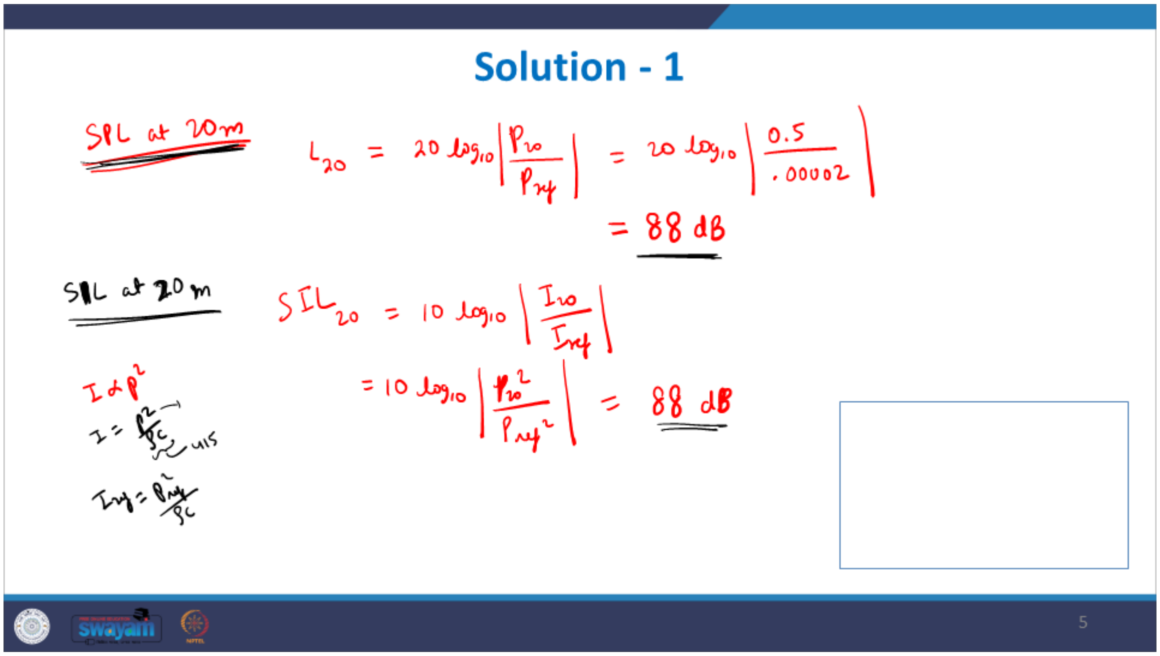
As a home exercise, what you can do is why don't you have this I as

$$\mathbf{I} = \frac{P^2}{\rho C}$$

input this $\boldsymbol{\rho C}$ as 415 for the air and the pressure whole square so you get the actual intensity and in the same way

$$\mathbf{I_{reference} = \frac{Preference^2}{\rho C}}$$

and you can calculate the actual values and see for yourself you will get it as 88 decibels.



Now we have to calculate the sound intensity level at 10 meters which is before that it was 20 meters now over here if you see this is a point source so if it's a point source and assuming its far field kind of measurement assuming far field measurement. What we should get? We should get a propagating; this should give us a propagating spherical wave front because it is a point source. So that means that in this case intensity should be inversely proportional to the square of the measurement distance. So let us say I_{20} is the intensity at 20 meters and I_{10} is the intensity at 10 meters. So, I_{20} by I_{10} is how much?

$$\frac{\mathbf{I20}}{\mathbf{I10}} = \frac{\mathbf{10^2}}{\mathbf{20^2}}$$

Okay So, if you go here and see the SIL at 10 should be

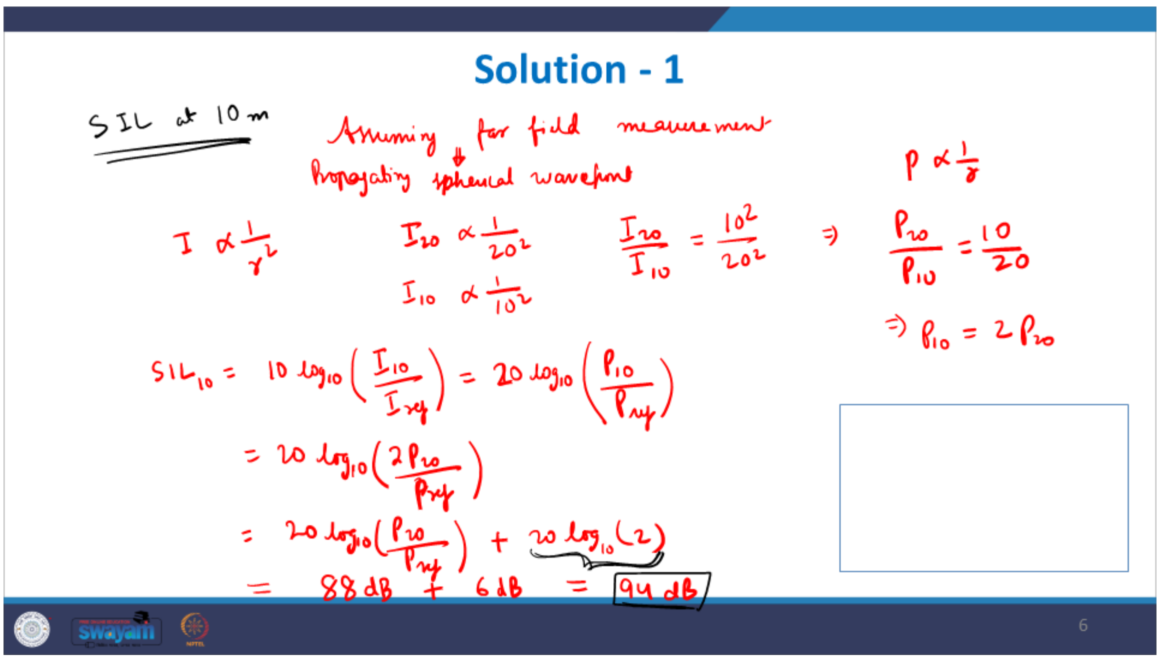
$$\mathbf{L_p = 10\log_{10}\left(\frac{I_{10}}{I_{ref}}\right)}$$

Okay, which now I_{10} you can write in terms of I_{20} because you do not know the I_{10} yet; you only know the I_{20}, and you know the pressure at 20. Okay, so you can write it in terms of pressure. Let us write it in terms of pressure because we know the pressure at 20. P would be inversely proportional to r. So, in the same way, P at 20 by P at 10 would become 10 by 20. So, we can write this like this in the same way because it is directly proportional as P_{10} by $P_{reference}$.

So, which means 20 log 10, and P_{10} is what? It is 2 times P_{20}. You are closer to the source; then the pressure you are getting is double of it, okay. You are halfway. So, distance half, the pressure doubles. So, you have 2 times P_{20} by $P_{reference}$, okay. If you use the logarithmic properties, P_{20} by $P_{reference}$ plus 20 log of 2

It is a very important term.

So, whenever the distance halves, this is the net increase in the SPL. How much is this? So, we already know, we have already calculated it in the previous section. This comes out to be 88 decibels. And what is 20 log 2? It comes out to be 6 decibels. Okay. So, you get an overall of 94 dB.



Okay. So, one conclusion we can draw from here is that with every doubling of distance for a spherical wave front or any point source, you know the SPL reduces by 6 decibels and vice versa, which means if you halve the distance, it increases by 6 decibels. Okay. Now, let us see the last part, which is the sound watt level of the source. Now, the sound watt level would be independent of whether we make the measurement at 20 or whether we make the measurement at 10. Okay. Of the source, we can use any point: 10 meters or 20 meters. So, how do you calculate that?

So, you know that power is given by the intensity multiplied by, for spherical, assuming that there is no barrier and it is emitting the noise source. Let us say we make a measurement at some point r, ok. It is emitting, radiating uniformly, and at some distance r, we are making the measurement and finding out the intensity and pressure there. Then power is given by

$$\mathbf{I = 4\pi r^2}$$

assuming no obstruction and a fully spherical wavefront. So, it is intensity multiplied by the total measurement area of the measurement surface. So, for that, it means that the power would be, you can either take 10 or 20 as you use.

$$\mathbf{So,\ P = I_{20} * 4\pi\ (20)^2}$$

This becomes your power level W_{rms}, we can call that, then the watt level is what it is given by. of we should be. Now, how do you calculate that? You can do it in this way. So, let us do this. Let us find out, let us represent it in the form of pressure. It would be

$$\mathbf{W_{rms} = \frac{(P20)^2}{\rho C} *(4\pi * (20)^2)}$$

So, we are decomposing intensity in the form of pressure. This is confusing, so I will just write this. Okay, so what happens is that now let us say $\boldsymbol{\rho C}$ for air at room temperature is 415. Then let us calculate that. So, this would be

$$\mathbf{SWL = 10\ log_{10}\ (\frac{0.5*0.5*4\ \pi\ 20*20}{415*10^{-12}})}$$

But now you know here that this is

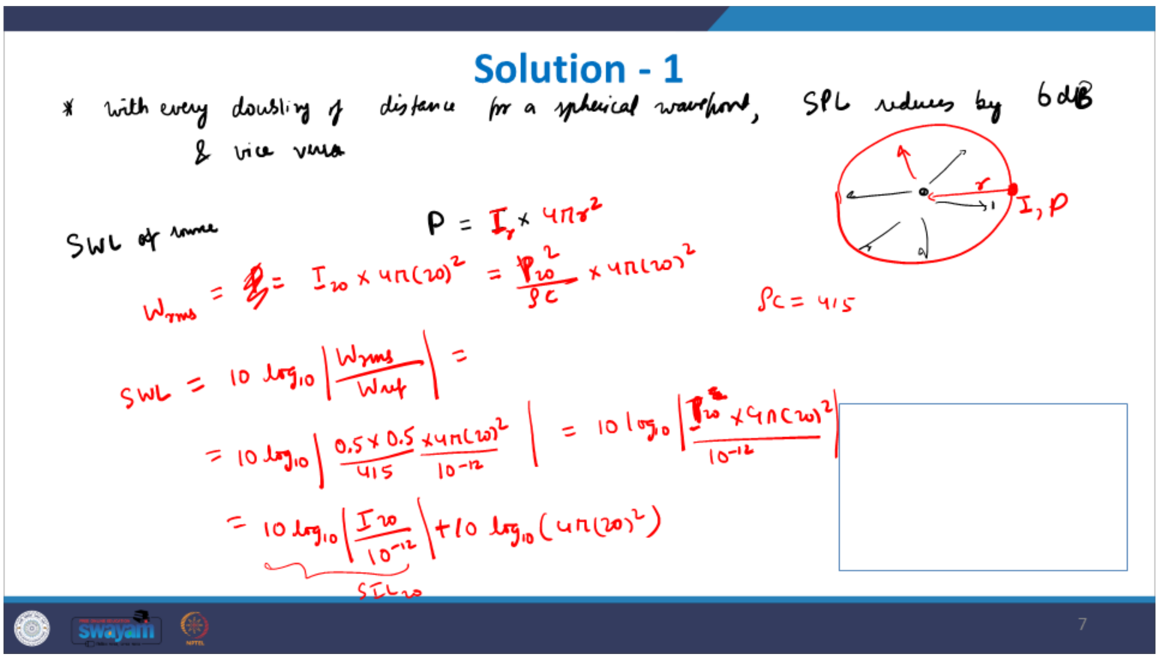
$$\mathbf{I = \frac{P^2}{\rho C}}$$

$$\mathbf{SWL = 10\ log_{10}\ (\frac{I_{20}*4\ \pi\ 20^2}{10^{-12}})}$$

$I_{reference}$ and $W_{reference}$ are the same. So, essentially, we already know what this quantity here is.

Now, this we already know by the definition. It is 10 $\log_{10}$ I_{20} by $W_{reference}$, which is 10 to the power minus 12, but the $W_{reference}$ and $I_{reference}$ are the same. So, this becomes your SIL at 20.

$$\textbf{SWL} = 10 \log_{10} \left(\frac{I_{20}}{10^{-12}}\right) + 10 \log_{10} (4\ \pi\ 20*20)$$



$$\textbf{SWL} = \textbf{SIL}_{20} + 10 \log_{10} (4\ \pi\ 20*20)$$

$$\textbf{SWL} = 88 + 37 = 125\text{dB}$$

So, this would be 88 decibels plus, if you find out this, you will get the answer is 37 decibels. So, overall, it comes out to be 125 decibels as the sound power level, which is true because you know The power level will always be higher than the pressure level and the intensity level of the source.

Solution - 1

$$SWL = SIL_{20} + 10\log_{10}(4\pi \times 400)$$
$$= 88\ dB + 37\ dB = 125\ dB$$

Let us move on to another problem. So, for the same conditions as given in the first problem, we had a point source in the first problem. Let us now see what if the source is cylindrical in nature and we have the same things given, which is the P at 20 is given to be 0.5 pascals, where this is the RMS value. Okay, find the frequencies also given; we might not use it. So, find the SPL at 20.

Problem - 2

- For same conditions as problem 1, if the source is cylindrical source and frequency is 200 Hz.

$P_{20} = 0.5\ Pa$ (↓ rms value)

- Find the following:
  - SPL at 20 m
  - SPL at 10 m

So, now here the SPL at 20 or L_{20}, again the same formulation,

$$\mathbf{SPL_{20} = 20\log_{10}\left|\frac{p_{20}}{p_{ref}}\right|}$$

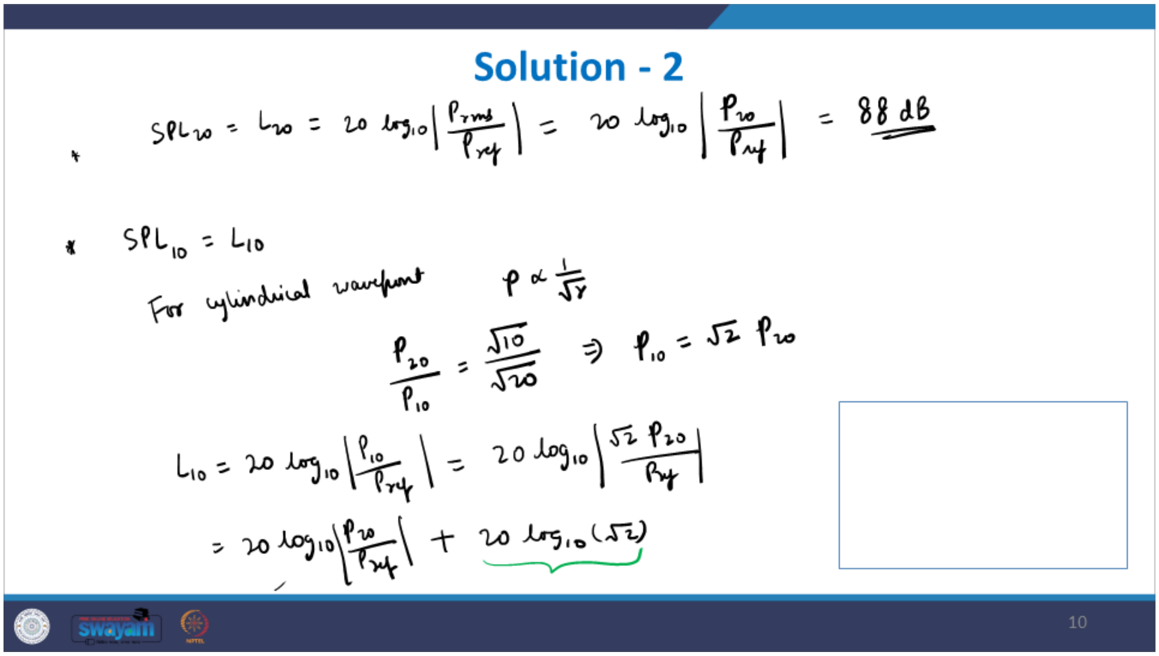
We already know we have calculated this; this was the same 88 dB as in the last case, the previous problem. Now, you have to find what is the SPL at 10, L_{10}. So, now let us derive the relationship between 20 and 10; it cannot be the same as what was there in the case of a spherical wavefront because there the relationship was different. So, for cylindrical waves, or cylindrical wavefront, the pressure is inversely proportional to the root of r, not directly proportional to r but the root of r.

$$\mathbf{\frac{P_{20}}{P_{10}} = \left(\frac{10^{0.5}}{20^{0.5}}\right)}$$

So, P_{10} should be what? It should not be twice of P_{20} but rather root 2 times P_{20}. So, that's one change here. So, now let's calculate again what is the L_{10}? L_{10} is simply.

$$\mathbf{L_{10} = 20\log_{10}\left|\frac{p_{10}}{p_{ref}}\right| = 20\log_{10}\left(\frac{1.414 * P_{20}}{P_{ref}}\right) = 20\log_{10}\left(\frac{P_{20}}{P_{ref}}\right) + 20\log_{10}(1.414)}$$

This again is another important term; this means that as you halve the distance, this is the increase in the SPL when it is a cylindrical source.



If you calculate this, the answer comes out to be 88 decibels for this one plus 3 decibels. So, you get 91 dB. So, another important conclusion is that with every doubling of distance for a cylindrical wavefront, SPL reduces by 3 dB. So, if you go into this one over here, with every doubling of distance, SPL reduces by 6 decibels, and here it is reducing by 3 decibels, and vice versa, which means that if you halve the distance, then it will increase by 3 dB. So, this is for cylindrical. Now, think about it: what would be the case for a plane wavefront? For a plane wavefront, think about it: the relationship between the pressure and P is independent. In fact, P_{rms} is constant with the distance of measurement with r, right? So, even if the distance doubles or reduces, the pressure remains the same, and if Prms remains the same, then 20 log10 P_{rms} by $P_{reference}$ would also be constant with the measurement distance for an ideal harmonic wave. So, what is the takeaway? The SPL of a pure harmonic plane wavefront does not change with measurement distance. Okay.

Solution - 2

$SPL_{10} = 88\,dB + 3\,dB = 91\,dB$

* with every doubling of distance for cylindrical wavefront, SPL reduces by 3 dB & vice versa

For plane wavefront: p is independent; p_{rms} is constant with 'r'

$20 \log_{10} \left| \frac{p_{rms}}{p_{ref}} \right|$ constant with measurement distance

* SPL of pure harmonic plane wavefront does not change with measurement distance

So, let us go and solve another problem. Let us see here. The following indicates noise levels recorded at fixed durations in a residential area throughout the day. So, in the 24 hours of monitoring a residential area at certain fixed long-time intervals, we are getting these various values. It is a total of 11 values we have got at different fixed durations. Now, you have to find from these values what is L_{10}, L_{50}, and L_{90}. So, basically, you have to find the 10th percentile, the 50th percentile level, and the 90th percentile level. I will tell you a very short way of doing this. Let us arrange them in the reverse order.

Problem - 3

- Following indicate the noise levels recorded at fixed durations in a residential area throughout the day (in dB):

  30, 35, 40, 45, 50, 55, 60, 65, 70, 75, 80

- Find the following:
  - L_{10}
  - L_{50}
  - L_{90}

So, let us do like this. Arranging values in reverse order. So, 80, I think it is an even 80, 75, 70, 65, like that, so 80 total 11 values, all in decibels. Now, for the L_{10}, which means that 10 percent of the times only the signal was above that level. So, you find out first of all what is the 10th percentile of these number of values; n is the number of observations. So, what is going to be the 10 percent of that?

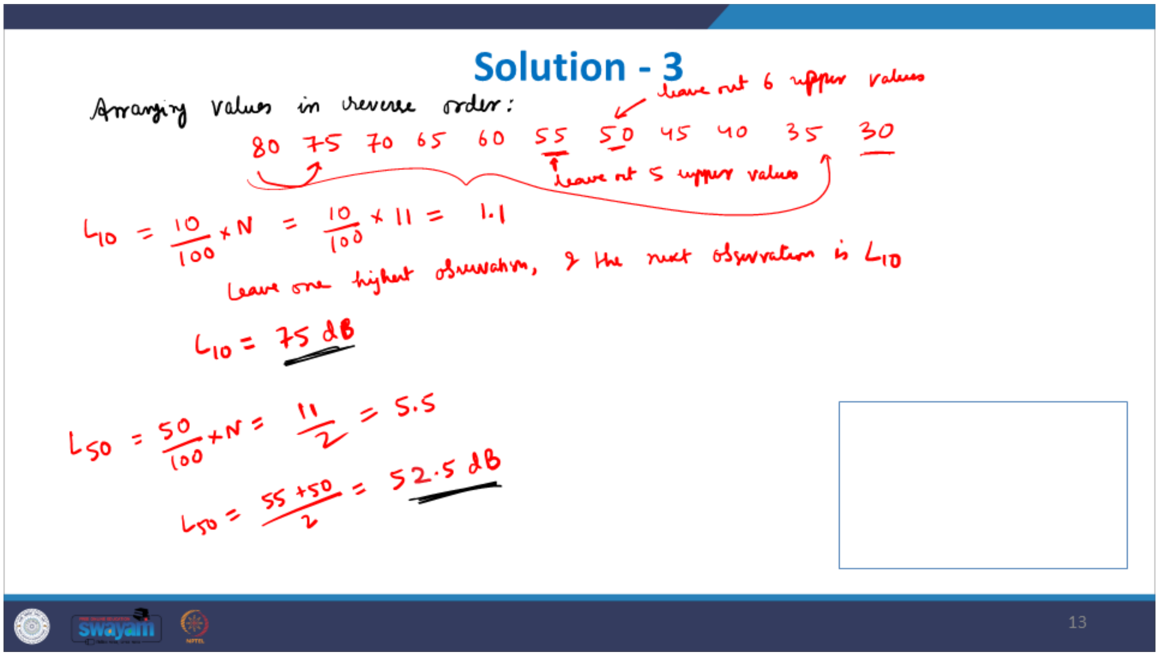
$$L_{10} = \frac{10}{100} * 11 = 1.1$$

So, which means that you go only first from the left. So, you leave only one observation, and the next observation is the L_{10}. So, here L_{10} becomes 75 decibels. So, which means that 10 percent of the times the level was above 75. It makes sense because 10 percent is one observation. So, only 10 percent of the times, which means that only one observation was above this level, which is 75, okay. So, only one observation, or 10 percent, was above 75 levels.

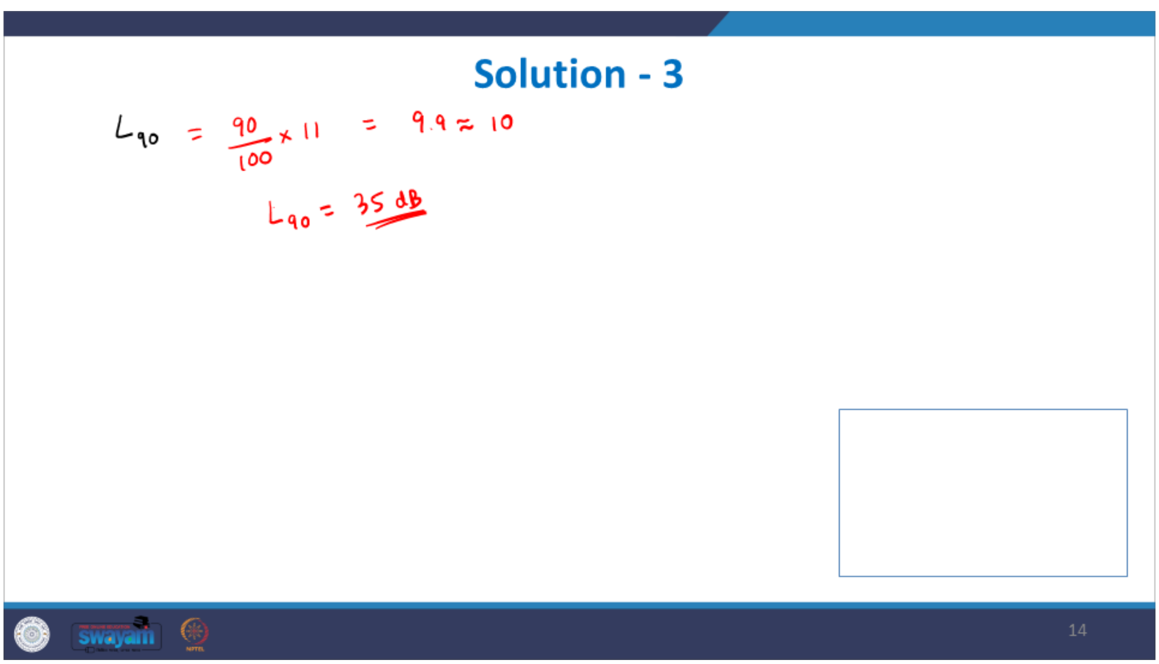
In the same way, if we have to find for L_{50} again, what you do is find out how many observations should be above this percentile.

$$L_{50} = \frac{50}{100} * 11 = 5.5$$

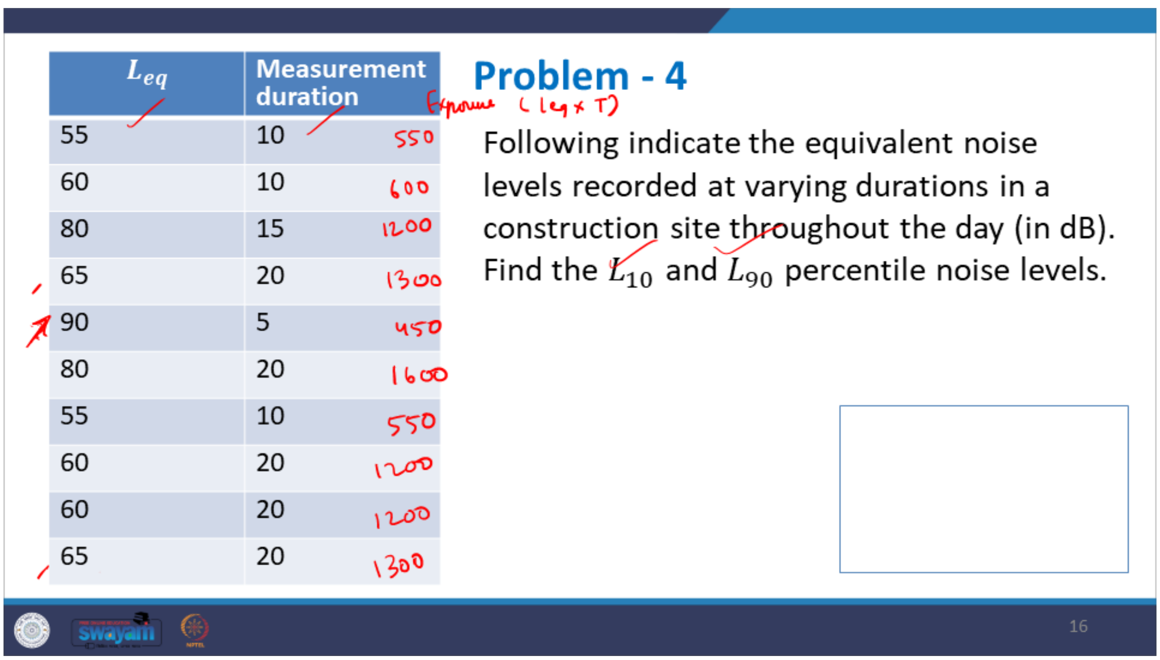
So, 5 and a half observations should be above it. If you want to be more precise, you can find out the midpoint. So, basically, it should be a midpoint between 5 and 6. So, let us say you leave out 5 observations. 1, 2, 3, 4, 5. So, you get to this level, which is 55, and then this is when you leave out 5 upper observations or upper values, and this is what you get when you leave out 6 upper values. And it has to be 5 and a half. So, you can go for a midpoint between them. So, L_{50} should be like 52.5 dB. So, this is how you are getting the various values.



Now, if you have to say L_{90}, again, you find out how many observations above you have to leave. Because 90 percent of the time, the recorded values have to be above that L_{90} level. So, what is 90 percent of 11? It is coming out to be 9.9, which means almost 10. So, out of the 11 values, 10 values have to be above that. So, what does it mean? You leave out the upper 10 values. So, leave all these upper 10 values, and this is the level you get to. You go to the 10th value. So, you have to, basically, it should be 35 decimals, not 30. Okay. So, you get to the 10th observation. So, you get L_{90} as 35 dB.



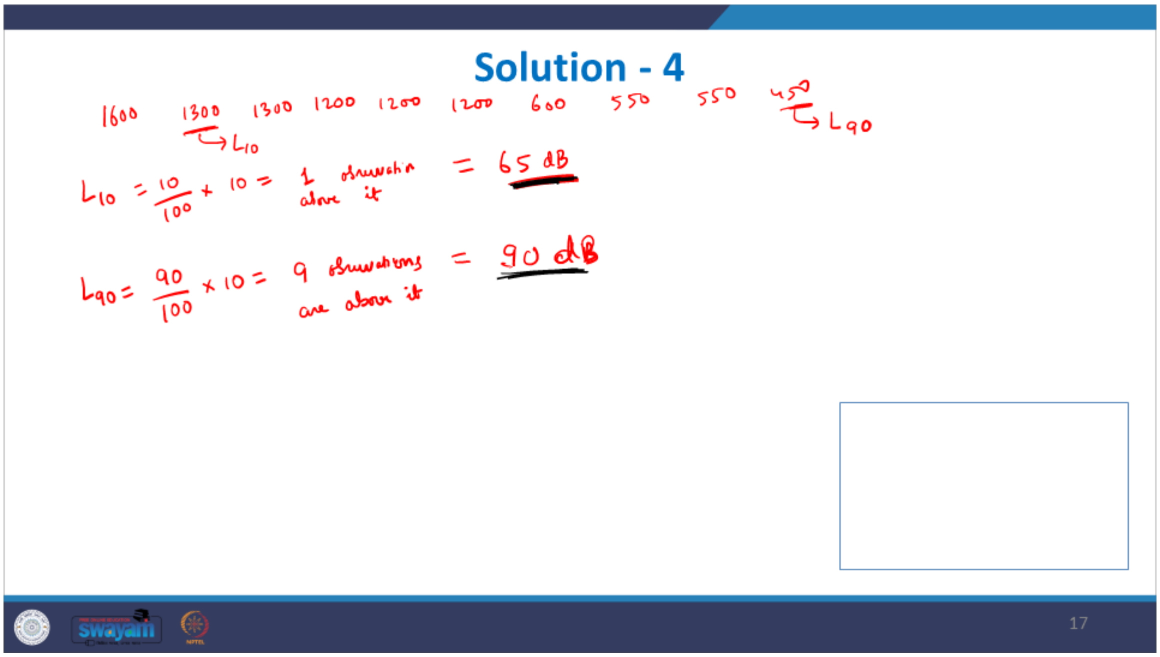
Let us go and see another problem where what you have is you have got the following observations done, and you got the Lequivalent. But here, in the previous case, we had a fixed measurement duration. Now, we have got changing measurement durations. So, the measurement duration is given, and the Lequivalent is given to you. Again, you have to find out the 10th and the 90th percentiles. You will follow the same process. But here, because the duration is different, one way could be to find out the total exposure, which means the level multiplied by the duration. So, let us find out and arrange them in terms of their exposure. So, the same things can be written as you can find out the exposure of this. So, you have 550, 600, you are multiplying these things, then 1200, which is $L_{equivalent}$ multiplied by time. Now, let us arrange them in ascending order and use the same formulation.



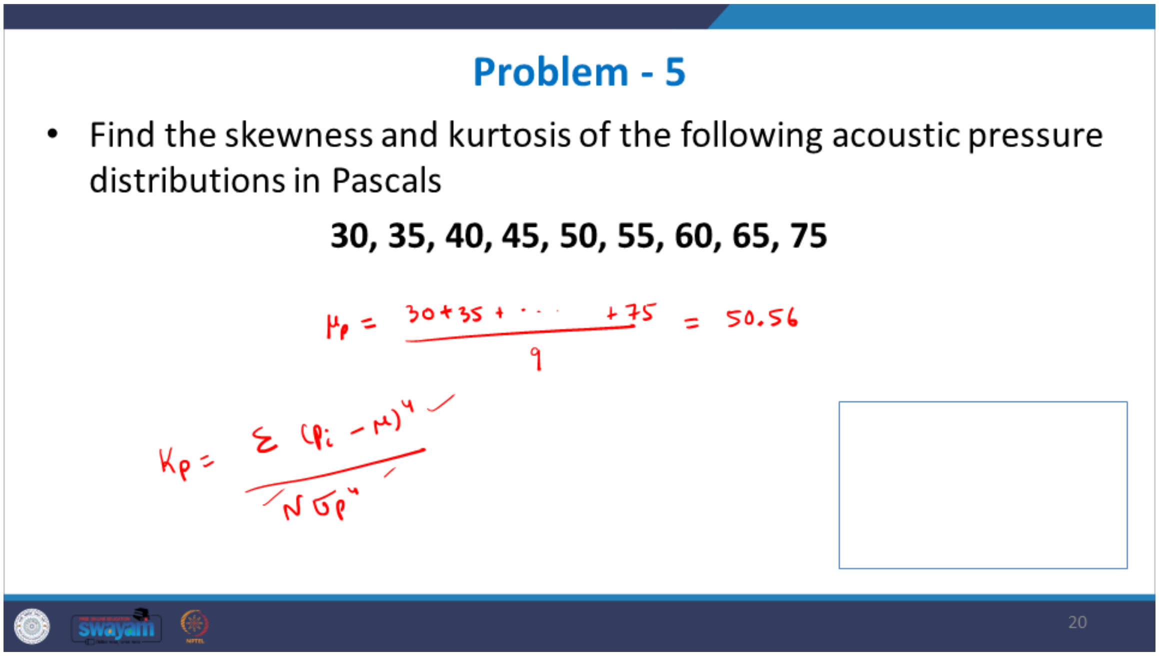
L_{eq}	Measurement duration	Exposure ($L_{eq} \times T$)
55	10	550
60	10	600
80	15	1200
65	20	1300
90	5	450
80	20	1600
55	10	550
60	20	1200
60	20	1200
65	20	1300

So, how many values are there? I think 10 values. So, we have got 10 values. Now, the L_{10} should correspond to one observation. So, leave out the first observation, and the second one would be your L_{10}. This should. So, which is this 1300? This corresponds to this 65 in both cases, it is 65 and 65. So, L_{10} becomes 65 dB. Now, let us see L_{90}. So, 9 observations are above it. So, the very last observation, which is this one, should be your

L_{90}. So, here only one observation was above it in L_{10}, and here all 9 observations are above it. So, this corresponds to 450, which is 450 over here. It is this one. So, you get this as. This is one observation above it, and here, nine observations are above this level. So, this becomes 90 decibels. Okay. So, I think these were the two values we had to find. Which we have found: 65 and 90.



So, let us say now that you have the skewness and the kurtosis of the following acoustic pressure distributions in pascals. You have to find out the skewness and kurtosis of this data. So, let us first find out for the first case.



So, here, first of all, find out the μ_p. What is the mean of this distribution? If you sum it all up together. And so on, divided by. 9. So, you would get it as 50.56, the mean of these pressure values. So, now, K_p is given by

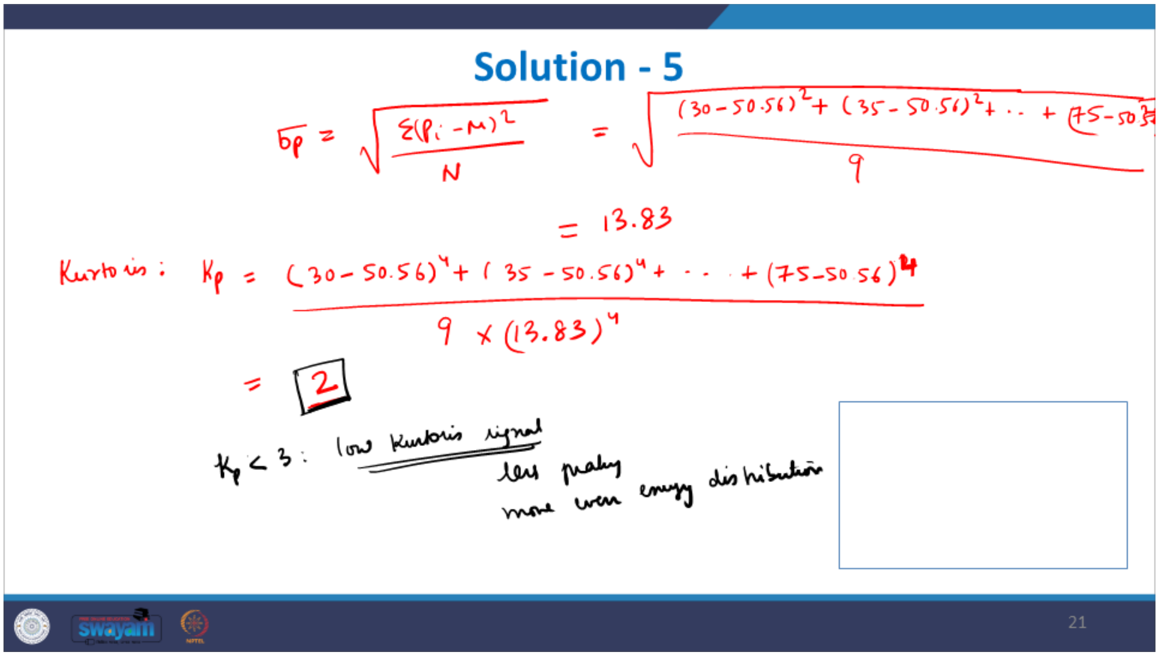
$$\mathbf{K_p} = \frac{\Sigma\,(\mathbf{Pi} - \boldsymbol{\mu})^4}{N\,\sigma p^4}$$

σ_p is what? It is given by, so σ_p is simply given by the standard deviation, the typical standard deviation formula

$$\boldsymbol{\sigma_p} = \left(\frac{\Sigma\,(Pi - \mu)^2}{N}\right)^{0.5}$$

So, if you do it, and so on till the last term which is 75 whole square by the number of observations, you should get it as 13.83. So now we have the σ_p and the μ_p for this distribution. Let us find out the kurtosis k_p, which would be you sum up these values to the power 4. All of this is much easier being done in Excel, or you can generate your own formulation to solve such problems, ok. So, this is how we are doing it, also this one also to the power 4. So, now you are summing up the fourth power of their deviation from the mean. divided by 9, which is n by the previous case. So, we have found this, this, and this. So, divided by 9 into the fourth power of the standard deviation.

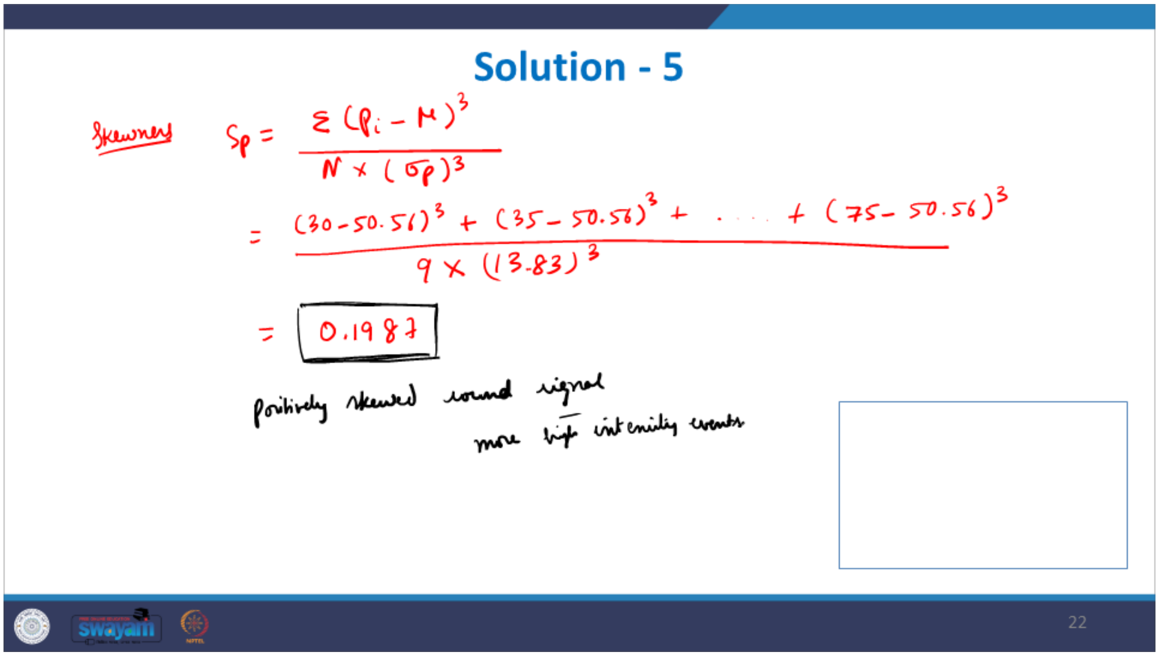
So, solving all of this, you should be able to get the value of 2 As you see here, the kurtosis, you have found it as 2. And you know that here K_p is smaller than 3. So, this is a low kurtosis signal, more even energy distribution.



Now, let us find out the skewness. The formulation for that is

$$S_p = \frac{\Sigma\,(Pi - \mu)^3}{N\,\sigma p^3}$$

So once again, you do this and so on for all the pressure values, okay? Just like here we did, and then divide by the number of observations into the third power of the standard deviation. you should be able to get a value of 0.1987. So, you see here that it is almost close to 0, slightly positively skewed, you can say. So, it is a positively skewed sound signal. So, what does it interpret? So, for the low kurtosis, it means more even distribution, less peaky, more even energy distribution over the signal, and here it means more high-intensity events. compared to the mean level. Okay.



So, I think with this, we will close this lecture, and I hope you enjoyed it. Thank you for listening.