NOISE CONTROL IN MECHANICAL SYSTEMS

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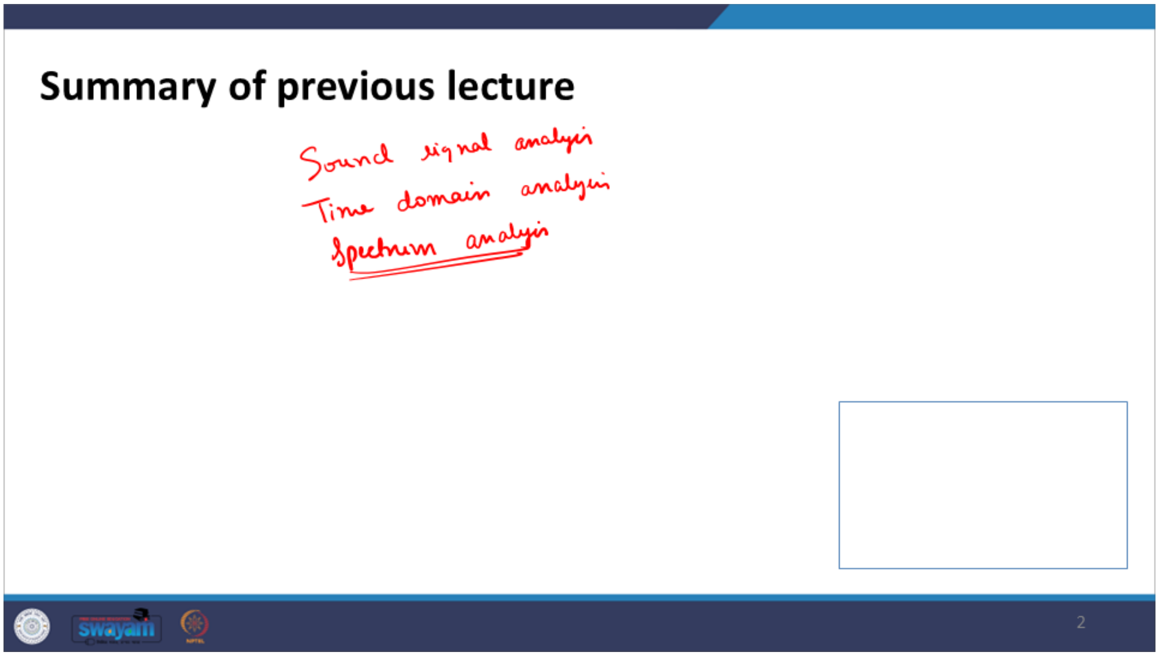
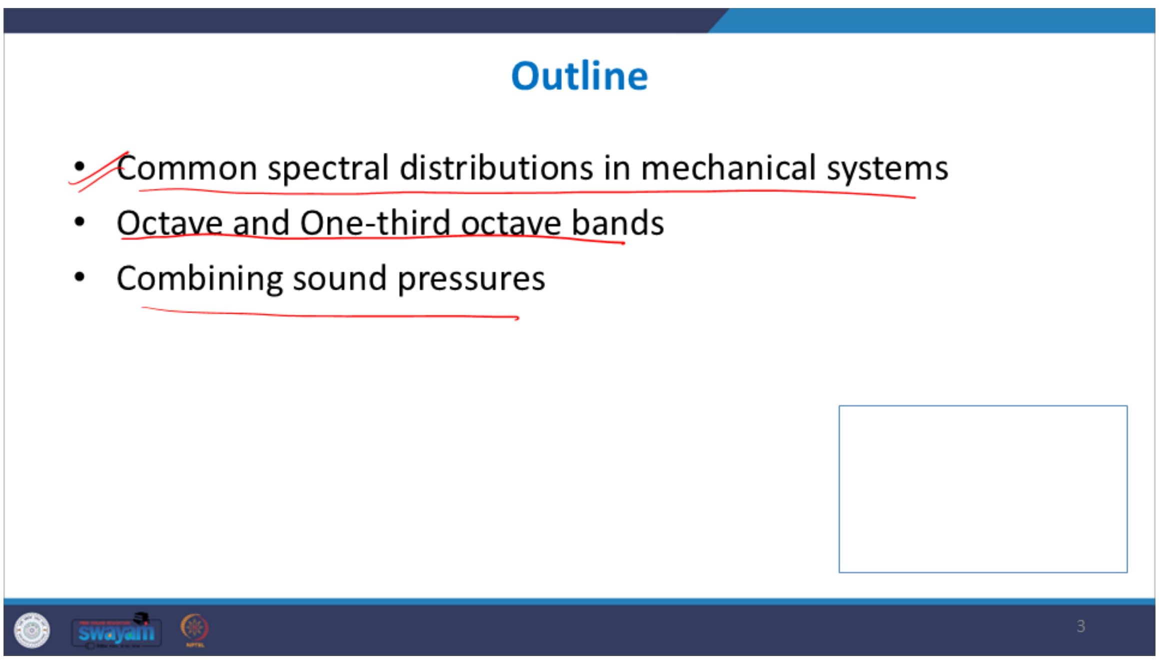
Week:4

Lecture:16

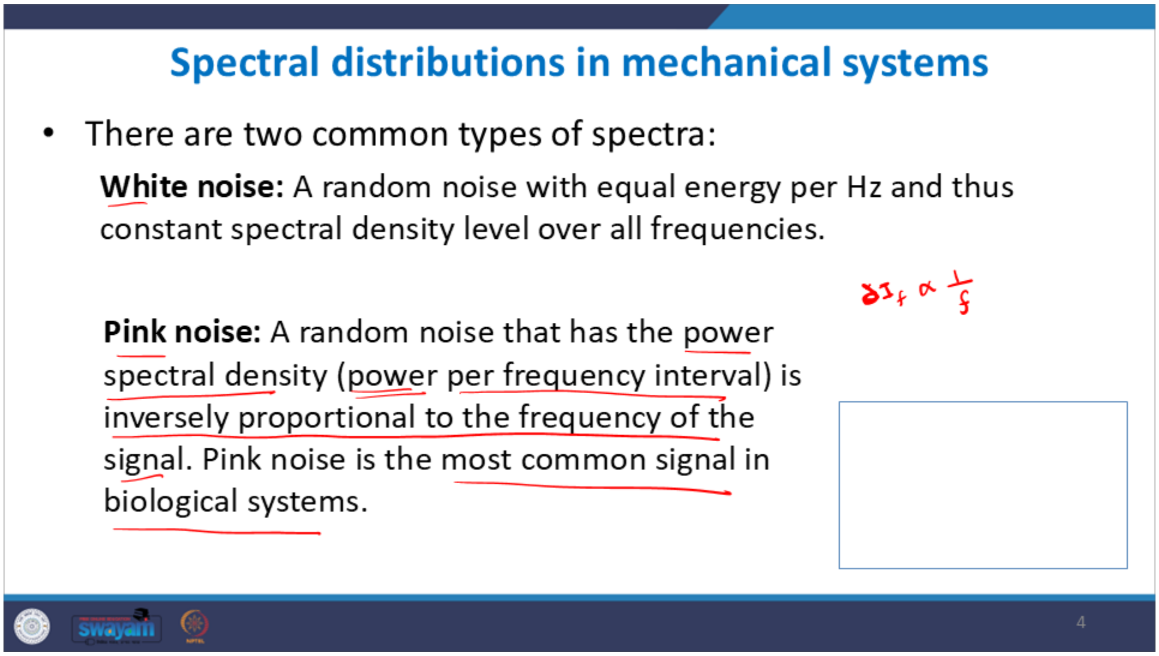
Lecture 16: Octave and one third octave bands

Hello and welcome to this lecture series on noise control in mechanical systems with me, Professor Sneha Singh from IIT Roorkee. So far, we have been discussing the module on sound signal analysis. And, in the sound signal analysis module, we have covered the time domain analysis. and then the spectrum analysis or the frequency domain analysis. So today, we will continue our discussion on the spectrum analysis. In particular, we will see two important ways of representing the sound waves. First of all, we will see the common spectral distributions in mechanical systems and the two important frequency spectrums that are used throughout the world, which are the octave and the one-third octave bands. And how do you combine the sound pressure levels in these bands? So, these are some of the topics to be discussed in this lecture.

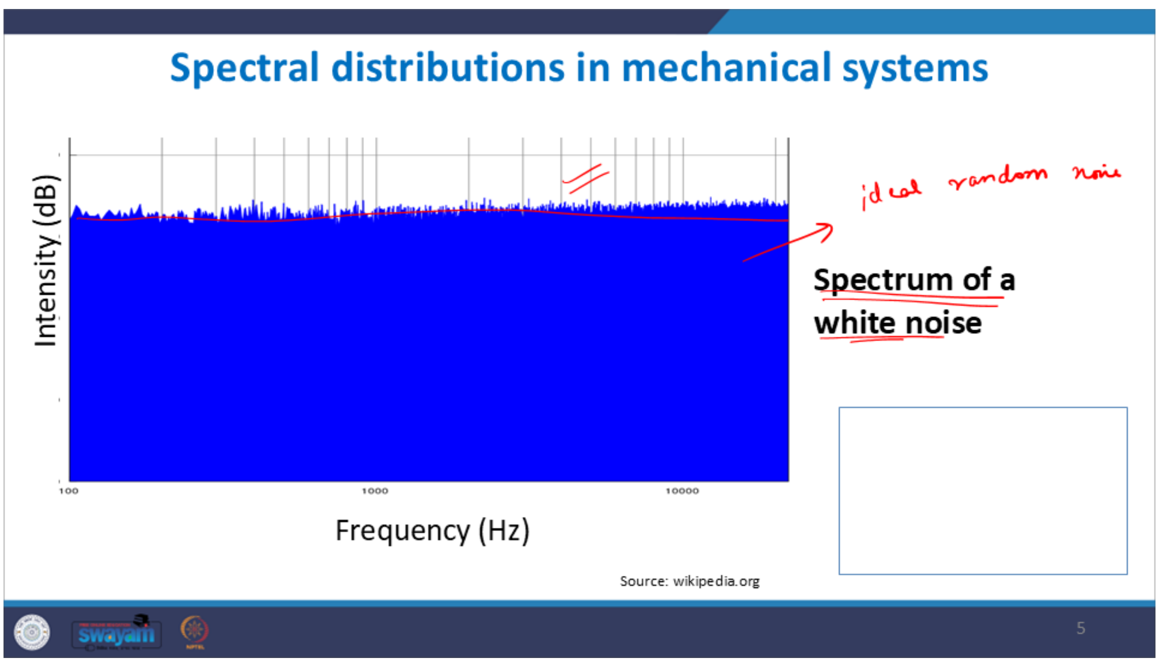




So, what are some of the common spectral distributions in mechanical systems? Some of the two most common types of spectra are white noise and pink noise. These are the two idealized spectra. So, what is white noise? When a random noise has equal energy per hertz, it has a constant spectral density level throughout its frequency, and that is called white noise.



So, here, what happens is that the energy content is the same for all the frequency bands, or it has a flat response throughout all the frequency bands. So, obviously, the spectrum of white noise is going to look like this.

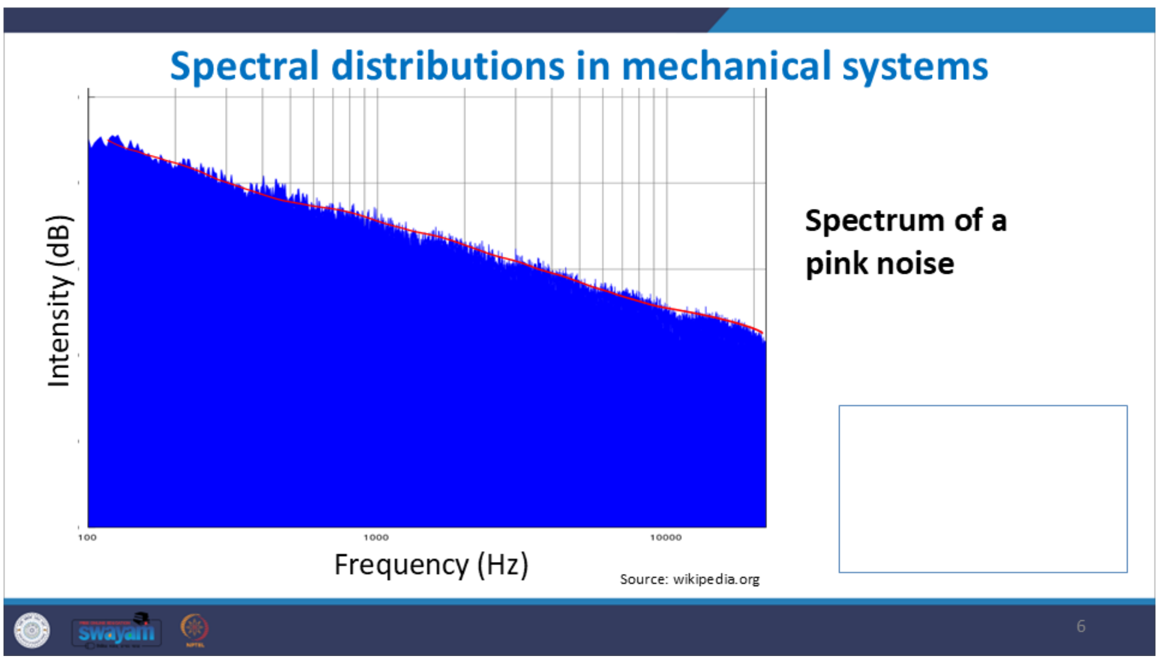


So, any random sound, okay, an ideal random noise which is a combination of various sources of various frequencies and intensities.

So, an idealized random noise can be written as white noise because it is unbiased and has a uniform distribution throughout the frequency. The other kind of common spectrum is pink noise. And here, what happens is that the power spectral density is not constant per hertz, but rather the power spectral density, or the amount of acoustic power which is present per frequency interval, is actually inversely proportional to the frequency of the signal. So, in the previous class, we have seen that the spectral density distribution is with respect to frequency.

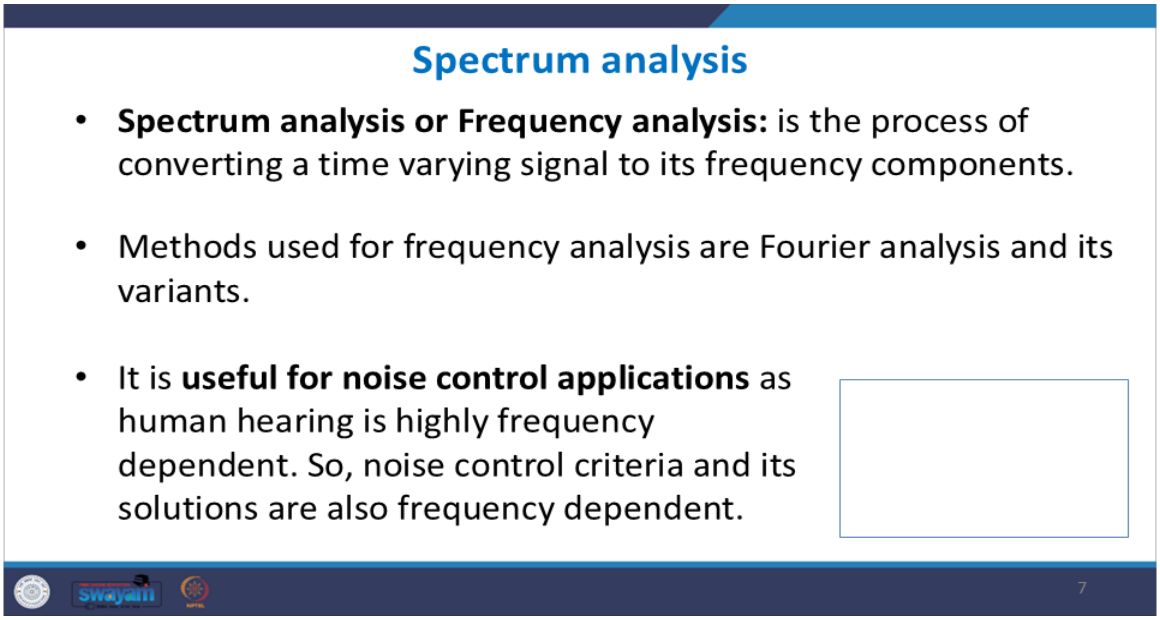
$$\delta_{If} = \frac{1}{f}$$

For every frequency, it is inversely proportional to that particular frequency over which that sound wave exists. So, pink noise is one of the most common signals that is available for biological systems. So, most biological systems emit pink noise-type spectra.

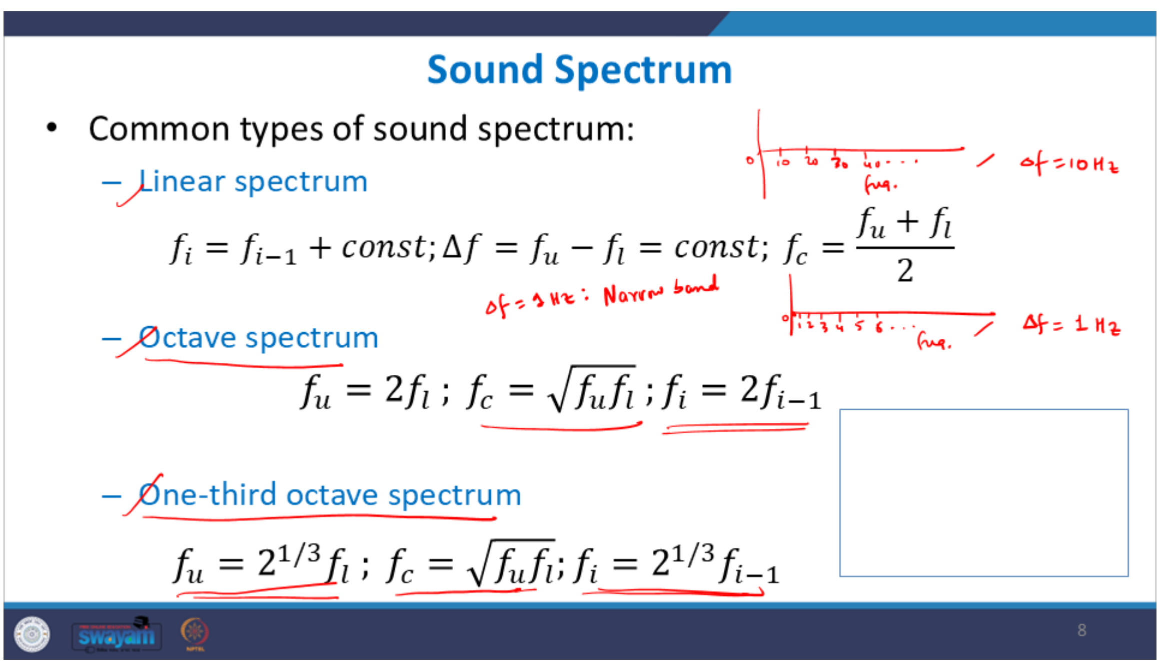


So, you can see the power spectral distribution of this pink noise, where the intensity is uniformly decreasing with respect to the frequency. It is, linearly, you can say, overall the envelope is almost like a line that is decreasing with frequency; a linear decay is happening here.

Whereas over here, it is almost a flat response or constant distribution. So, spectrum analysis, quickly a recap, is that spectrum analysis or frequency analysis is the process of



converting any time-varying signal into its frequency components. And the methods used are Fourier analysis and its variants. And it is very useful for noise control applications because our human hearing is highly sensitive to the frequency of the sound that is being heard. And hence, just the time signal, the time variation of the sound, is not that much important. It is of importance, but it does not give the complete picture. In order to understand how this sound will affect human beings and other living beings, the frequency component of that sound needs to be analyzed. And hence, spectrum analysis becomes important.



And some of the common types of sound spectrum are linear spectrum. Where you know the intensity of the sound is being represented in a linear scale of the frequency. Which means that every new frequency band is after a constant difference from the previous band. For example, you can either have from 0, 10, 20, 30, 40; this is a linear scale of the frequency. And the narrow band is when this

$$\Delta f = 1\ Hz$$

then we call it a narrow band. This is one type of the linear spectrum where you analyze for 0, 1 hertz, 2 hertz, 3 hertz, 4 hertz, 5 hertz, and so on. For every particular hertz, what is the frequency content? Both of them here, in both these cases, the frequency is in a linear scale, okay? It is linearly varying, and the interval of the scale is fixed or constant. Here, for example, the delta f is 10 hertz. This is the interval of every scale, okay? Or the band, and here the Δf is 1 Hz.

However, you know, sometimes given the nature of human hearing, we are sensitive to some lower frequencies, whereas sensitivity to some of the higher frequencies is still low. Given that, sometimes it is more useful to represent, the spectrum of a sound using a logarithmic kind of frequency band.

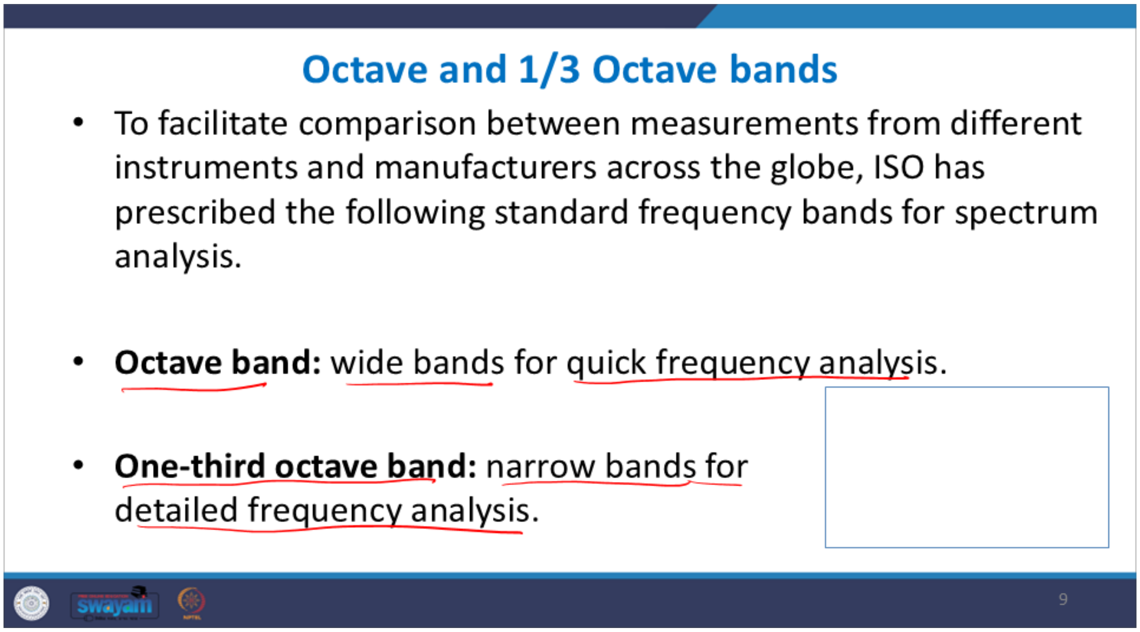
In that case, two types of frequency bands are mostly used, which are the octave spectrum and the one-third octave spectrum. In the octave spectrum, what happens is that every next frequency band is just double the first one. If f_l is the lower limit of one band and f_u is the upper limit. Then, the upper limit of the band is twice the lower limit, and the central frequency is the geometric mean of this upper and lower frequency limit.

In the same way, for the one-third octave spectrum, in every band, the upper limit of the band is

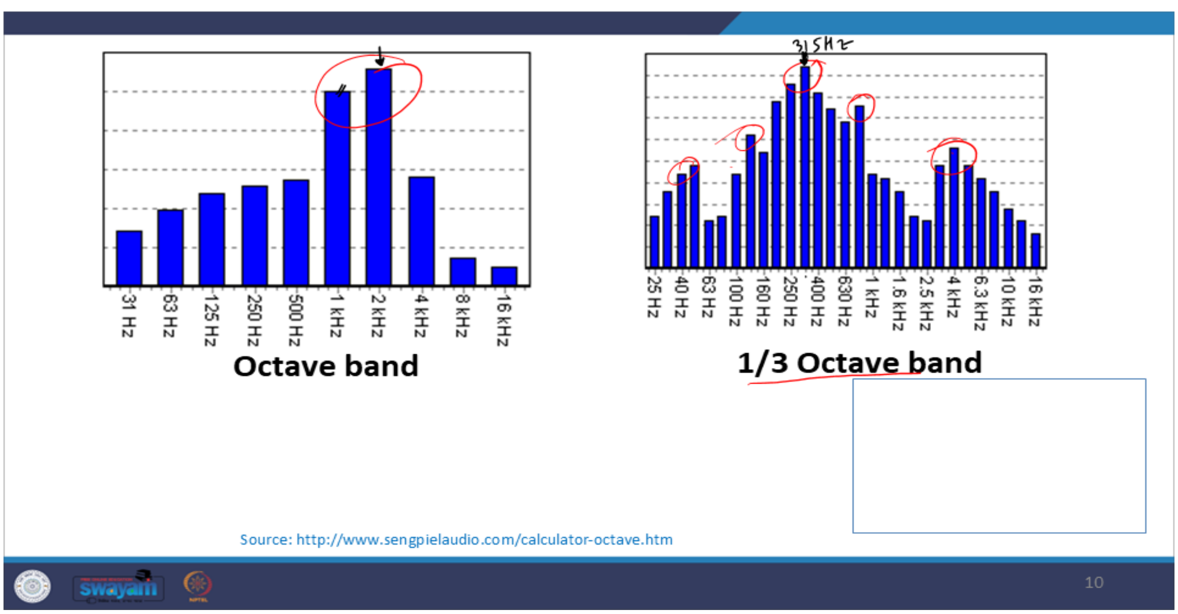
$$f_u = 2^{\frac{1}{3}} f_l\ ;$$

$$f_c = \sqrt{f_u f_l}; f_i = 2^{1/3} f_{i-1}$$

2 to the power of 1/3 times the lower limit of the band, and the central frequency once again is the geometric mean of f_u and f_l. These are some of the equations which are the generic equations followed by every frequency band within the octave and the one-third octave spectrum.

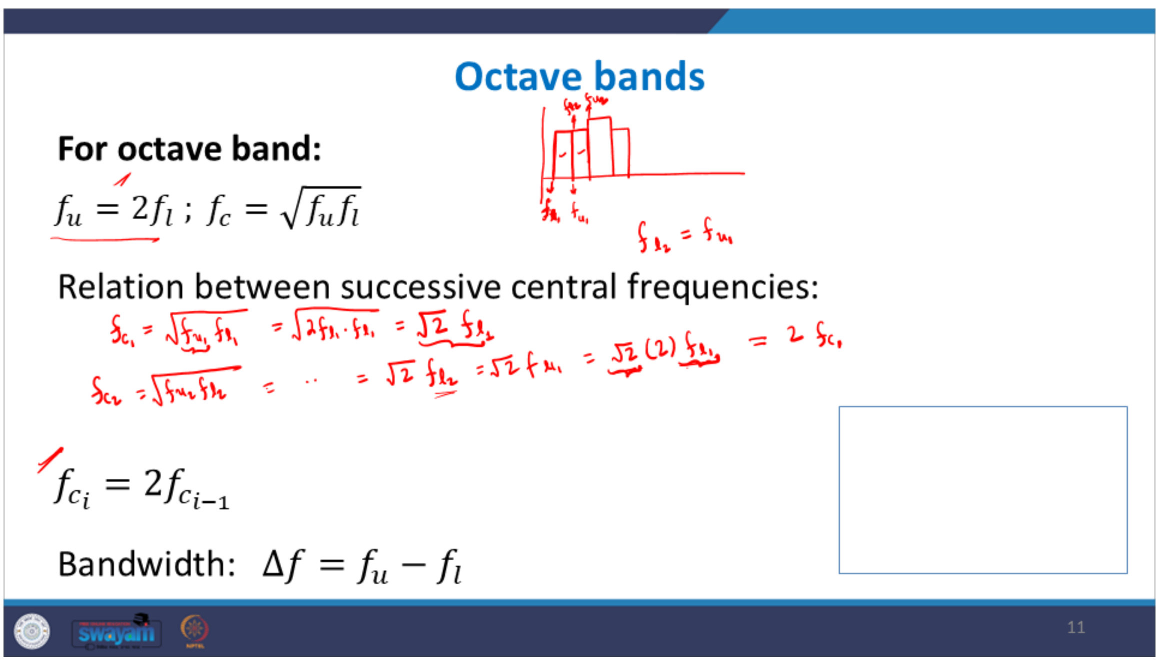


let us see. why we use this spectrum? We use it because it facilitates the comparison between the measurements from different instruments and manufacturers across the globe. In order to standardize the measurements and analysis of sound signals, the ISO has prescribed the use of these standard bands. Usually, the octave band, where every new upper limit of the frequency is twice the lower limit. It is a wider band, as you can see, and the one-third octave band where the upper limit of frequency in every band is 2 to the power of 1 by 3 times the lower limit. This is a narrower band. So, the octave band here is the wide bands which are used for some quick frequency analysis, and one-third octave bands are typically these narrow bands which are used for more detailed frequency analysis of a noise source. For example, you can see these two spectra here.



So, as you can see, this is not for the same noise source, but for two different noise sources, just two different octave and one-third octave bands are given to you. So, here, suppose you want to do a quick analysis. and quickly see what the frequency content is like, so you would come to know that the frequency is typically peaking around this zone and in the rest, it is decaying sharply. Suppose you want to go further for more detailed analysis, then here it shows a typical one-third octave band of a different noise source.

In more detail, you can see that there is a frequency peak here and then there is a small decay, then another frequency peak, then over here there is a frequency peak and then a decay. These are some of the shorter frequency peaks (Refer Slide 10). So, you can do a finer analysis, you can see the frequency content with a finer frequency resolution. You can see further peaks and more detailed spectrum using the one-third octave band. So, let us see for an octave band the generic relation between any band.



So, let us say this is your octave band, and then you have the second band, the third band, and so on. These are the various bands, and this is your, the f_{u1}, let us say, and this is f_{l1}, this is f_{u1} for the first band. So, the lower frequency limit and the upper frequency limit for the first band in octave, and then accordingly this is your, the lower frequency limit and the upper frequency limit in the second band. f_{l1}, f_{u1}, and f_{l2}, f_{u2}, and the generic relation for every such band is that the upper limit of that band would be twice of the lower limit, and the central frequency would be the geometric mean of the two. So, let us see what is the relationship between the successive central frequencies. So, what we observe here is that, let us say. The central frequency of some, let us do for the two successive bands 1 and 2, and see what happens. So, the central frequency of the first band would be the square root of the upper limit frequency multiplied by the lower limit frequency,

$$f_{c1} = \sqrt{f_{u1} f_{l1}}$$

and the upper limit frequency would be twice of the lower limit frequency. So, this we expand further.

$$= \sqrt{2f_l \times f_l}$$

$$= \sqrt{2} f_{l1}$$

So, in the same way, you know, the central frequency for the second band once again is,

$$f_{c2} = \sqrt{f_{u2} f_{l2}}$$

and by the similar derivation, it would be

$$= \sqrt{2} f_{l2}$$

And as you can see here that, the bands are successive in nature. There is no gap in between these bands. The bands are usually continuous in nature. So,

$$f_{l2} = f_{u1}$$

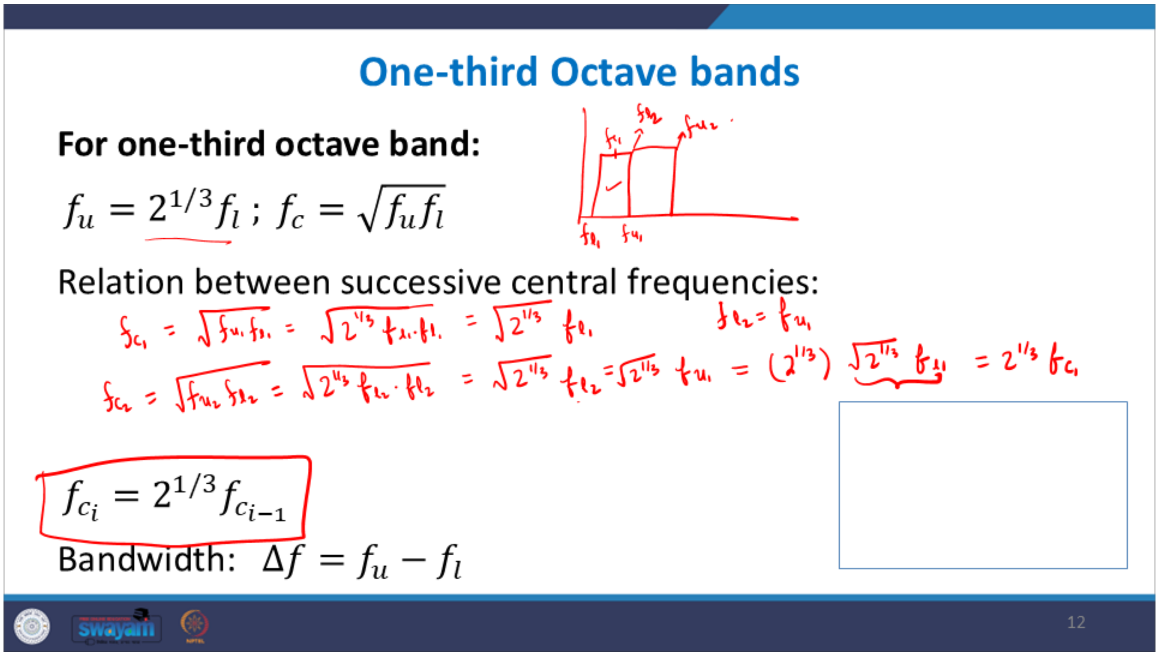
So, whatever is the upper limit of the first band, your second successive band starts from that frequency onwards. So, the upper limit of the previous band is now the lower limit of the next band. So,

$$= \sqrt{2} f_{l2}$$

$$= \sqrt{2}\,(2) f_{u1}$$

So, what you see here is that it is actually 2 times the central frequency of the first band because this was the central frequency of the first band, so twice of this. So, the central frequency also follows the same relation, which is the central frequency of any successive band would be twice of the central frequency of the first band. So that would be the relation between the central frequencies, and the bandwidth for each band would be the upper limit minus the lower limit frequency.

In the same way, you know, let us see the relationship between the one-third octave band.



So here the upper limit of any band, is

$$f_u = 2^{\frac{1}{3}} f_l \,;$$

$$f_c = \sqrt{f_u f_l}$$

And in the same way, suppose this was your first band f_{l1} and f_{u1} of this band is there, and there is something called f_{c1}, and then this another successive band, whose lower limit is f_{l2} and f_{u2} and so on. So, using the same analysis let us say the central frequency for this very first band would be

$$f_{c1} = \sqrt{f_{u1} f_{l1}}$$

and

$$f_{u1} = 2^{\frac{1}{3}} f_{l1}$$

In the same way, if you do it for the second one what you get is once again it could be represented using the same relations, which is, but f_{l2} is the same as the f_{u1}.

So, where the first band is ending, that frequency is the starting point for the second band; they are continuous in nature. So, hence when you replace this, f_{l2} is simply f_{u1}. So, which means if you solve it further,

$$f_{c2} = \sqrt{f_{u2} f_{l2}}$$

$$f_{c2} = \sqrt{2^{\frac{1}{3}} f_{l2} f_{l2}} = \sqrt{2^{\frac{1}{3}}}\, f_{l2} = \sqrt{2^{\frac{1}{3}}}\, f_{u1}$$

$$f_{u1} = 2^{\frac{1}{3}} f_{l1}$$

$$f_{c2} = \left(2^{\frac{1}{3}}\right) \sqrt{2^{\frac{1}{3}}}\, f_{l1} = \left(2^{\frac{1}{3}}\right) f_{c1}$$

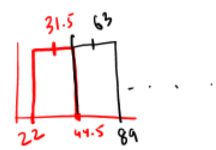
and this quantity is nothing but your central frequency for the first band. So, what it means is that just like the relation between the upper and the lower frequency, the central frequencies of the successive bands also follow the same relation. So, the central frequency of any ith band would be

$$f_{c_i} = 2^{1/3} f_{c_{i-1}}$$

just like here in this case, this was twice of f_{c1}. And the bandwidth, once again, is the difference between the upper and the lower frequency. So this shows the octave bands obtained.

Octave bands

Centre frequency	Lower frequency	Upper frequency
31.5	22	44.5
63	44.5	89
125	89	177
250	177	354
500	354	707
1000	707	1414
2000	1414	2828
4000	2828	5657
8000	5657	11313
16000	11313	22627



The lower frequency is shown. The upper frequency is shown. For example, this is your band. It starts from 22, so it starts from 22, ends at 44.5, and the central frequency is around this. The second continuous in nature, starts from 44.5 here, and goes till 89 and it has a central frequency around 63 Hz, and so on, the bands continue.

So, this shows the typical, the low frequency, upper frequency, and the central frequency of these octave bands. So, what it also means is that, suppose a graph is given of an octave band, let us say this typical graph, and a certain intensity has been reported for some band, let us say. We are trying to see in this graph, the peak is being reported around 2 kilohertz (refer slide 10). So, if you go back to this, so if suppose you are getting the highest peak in the 2 kHz central frequency band, which means that in the frequencies around this to this, we are getting the peak. So, we are representing the entire band, 1414 to 2828 Hz, by the single central frequency.

One-third Octave bands

Octave band centre frequency	One-third octave band centre frequency	Band limits	
		Lower	Upper
	25	22	28
31.5	31.5	28	35
	40	35	44
	50	44	57
63	63	57	71
	80	71	88
	100	88	113
125	125	113	141
	160	141	176
	200	176	225
250	250	225	283
	315	283	353
	400	353	440
500	500	440	565
	630	565	707
	800	707	880

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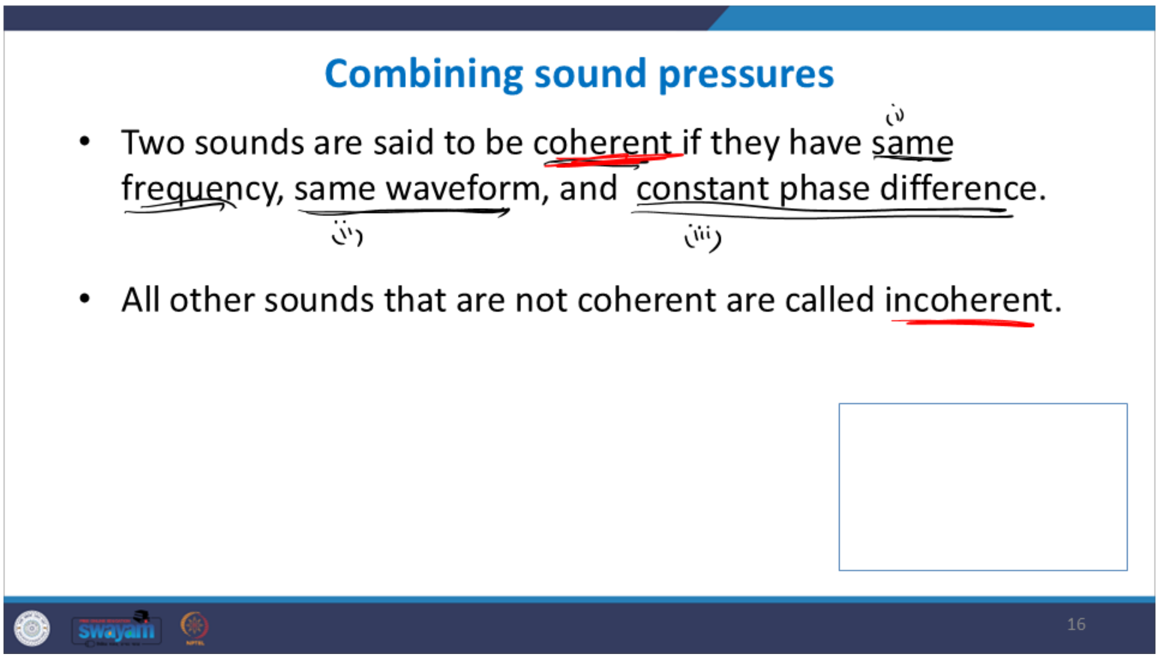
These are the octave band central frequencies and one-third octave band central frequencies. In the one-third octave band, one octave is divided into three parts. So, there are three individual central frequencies and three individual bands, which together comprise a single octave band, and so on. So, these are some of the tables that are already available on the internet, as well as in various other books.

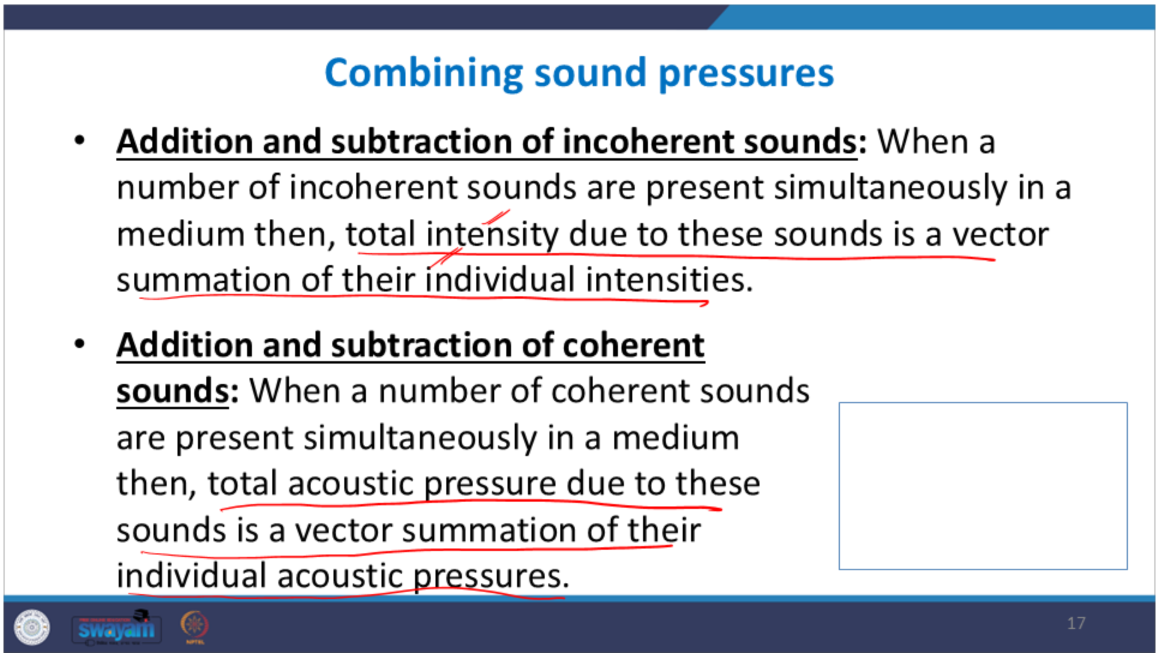
It is something which you can always keep in mind and memorize, so that interpreting any octave band or one-third octave band becomes easy. So, let us say, for example, In this graph over here (refer slide 10), we are getting a peak around this point, which corresponds to a one-third octave band of. It is between 250 and 400. It would be a 315 Hz peak.

What it actually means is that over here (refer slide14), in the frequencies between the range of 283 to 353, we are getting a frequency peak. And so on, these are the various values.



So, let us say now that suppose there are two sounds which are coming together and they are present in the same space. So, suppose we have two different sound sources and they are active together, and both of them have some individual waves. When they are active together, what would be the corresponding wave? So, suppose when the two sounds, which are two sound sources, are playing together, they are said to be coherent.

So, the sources could be coherent or incoherent. So, coherent sources are those which have the same frequency, the same waveform, and also a constant phase difference. So, the same frequency, the same waveform, and a constant phase difference are the three conditions that need to be satisfied. Any two sounds that do not satisfy each of these three conditions, which are the same frequency, the same waveform, and the third condition is a constant phase difference throughout time, will become incoherent in nature. So, let us see that if suppose there are more than one sound source present, how do we combine their sound pressures together, given that they are either coherent or incoherent.

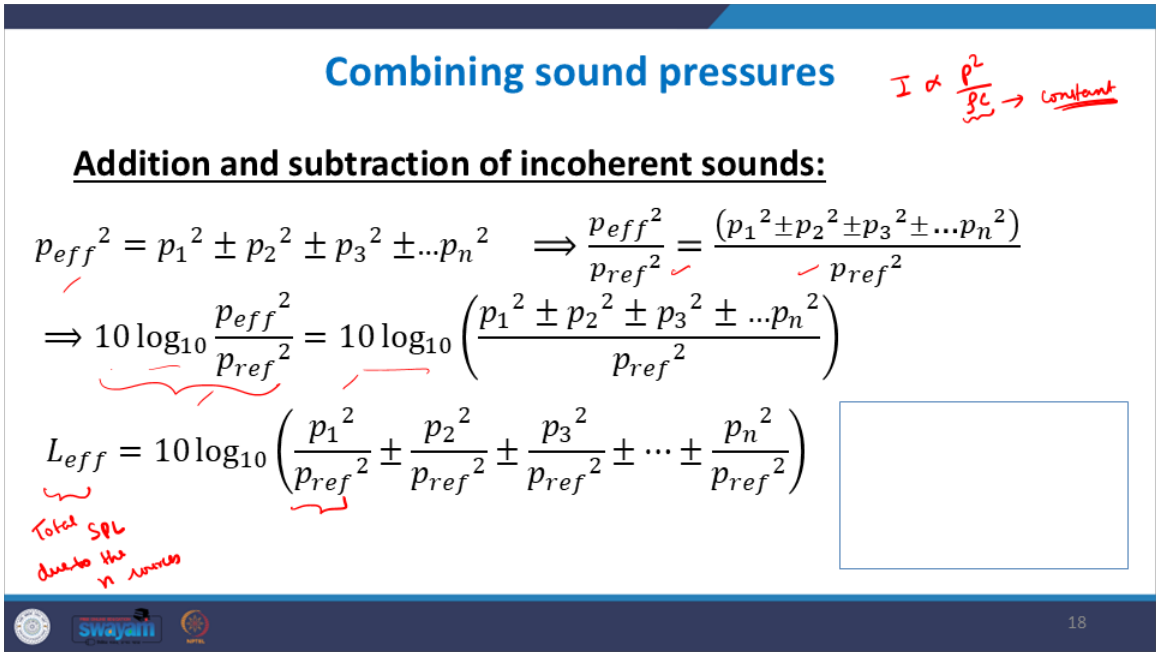


So, whenever incoherent sounds are present together, the total intensity due to these sounds is actually a vector summation. So, the total intensity is a vector summation of their individual intensities. So, the intensities add up, whereas when coherent sounds are present together simultaneously, the total acoustic pressure is actually a vector summation of the individual acoustic pressures. So, that is the difference. So, let us derive the combined sound pressure levels for the case when we have more than one sound source present in a medium and we want to find out the combined sound pressure level, given that the individual sound pressure levels are provided.

So, let us say we have got various incoherent sources, you know, P1 to N, N number of incoherent sources. And they are present together or simultaneously in a medium; then, the total intensity would be the summation vector summation of the individual intensities. So, which means that, and we know that the intensity is,

$$I \propto \frac{P^2}{\rho c}$$

because all of them are present together in a medium. So, this ρc is constant for all these sounds. We want to see what would be the combined level at a point.



So, ρc is going to be constant. So, which means that the intensity would be directly proportional to P^2. So, P effective whole square would be,

$$p_{eff}^2 = p_1^2 \pm p_2^2 \pm p_3^2 \pm \cdots p_n^2$$

So, here I have added plus and minus both signs. Suppose the sources are present together and you want to find out the combined sound level, you would use plus; but suppose if some source was already present and it has been now put off, so its sound pressure needs to be subtracted from the rest, then you will be using the minus sign accordingly according to the case. when all the sources are present, it would be plus; when some source was present initially, now it is absent, and its effect needs to be removed, you will use a minus sign. This is the general, you know, summation of the intensities of the various sources; if you divide the left-hand side and the right-hand side by the p reference whole square and

$$\frac{p_{eff}^2}{p_{ref}^2} = \frac{(p_1^2 \pm p_2^2 \pm p_3^2 \pm \ldots p_n^2)}{p_{ref}^2}$$

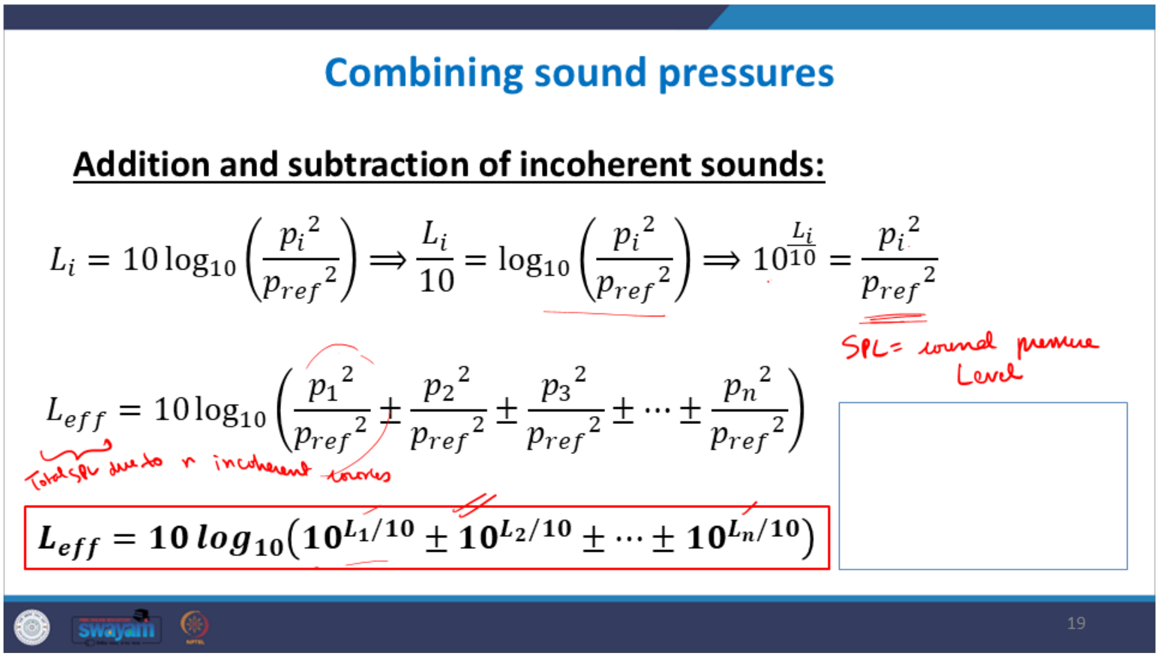
then do a 10log10 to both the sides, you would end up with this equation.

$$10\log_{10}\frac{p_{eff}^2}{p_{ref}^2} = 10\log_{10}\left(\frac{p_1^2 \pm p_2^2 \pm p_3^2 \pm \ldots p_n^2}{p_{ref}^2}\right)$$

Now, by definition, this is what this is. The sound pressure level, this is the total sound pressure level due to the n sources.

$$L_{eff} = 10\log_{10}\left(\frac{{p_1}^2}{{p_{ref}}^2} \pm \frac{{p_2}^2}{{p_{ref}}^2} \pm \frac{{p_3}^2}{{p_{ref}}^2} \pm \cdots \pm \frac{{p_n}^2}{{p_{ref}}^2}\right)$$

This is the total sound pressure level due to the n sources, and this is 10log10 of this big quantity here. let us see what these quantities are in terms of the sound pressure level of the individual sources.



So, if you take any,

$$\frac{{p_i}^2}{{p_{ref}}^2}$$

So, the level of the i-th source is

$$L_i = 10\log_{10}\left(\frac{{p_i}^2}{{p_{ref}}^2}\right)$$

So,

$$\frac{L_i}{10} = \log_{10}\left(\frac{{p_i}^2}{{p_{ref}}^2}\right)$$

and then

$$10^{\frac{L_i}{10}} = \frac{{p_i}^2}{{p_{ref}}^2}$$

and we know that the total SPL due to the n incoherent sources L effective is 10 log 10 of this big quantity, and each of these individual quantities here is simply

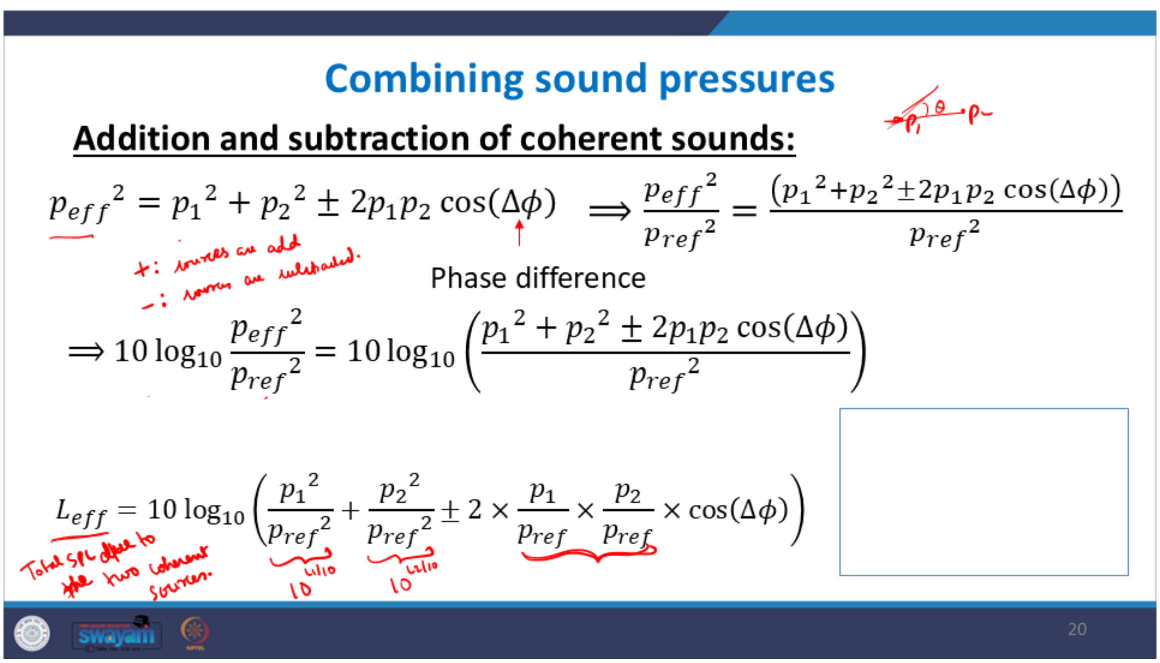
$$\frac{{p_1}^2}{{p_{ref}}^2} = 10^{L_1/10}$$

So, P1 square by P difference square simply becomes 10 to the power the SPL of the first source divided by 10 and so on,

$$L_{eff} = 10\, log_{10}\left(10^{L_1/10} \pm 10^{L_2/10} \pm \cdots \pm 10^{L_n/10}\right)$$

so ultimately you end up with this equation.

What it means is that the total SPL, let us say, due to the presence of n number of sources is 10 log 10, so it is logarithmically added up. So, you do the 10 log 10 of, and within the brackets, you have 10 to the power the SPL of the first source individually divided by 10 plus 10 to the power the SPL of the second source divided by 10, and so on, till 10 to the power the SPL of the nth source divided by 10. So this is how you can combine the individual SPLs to get the overall SPL. Where SPL is obviously the sound pressure level.



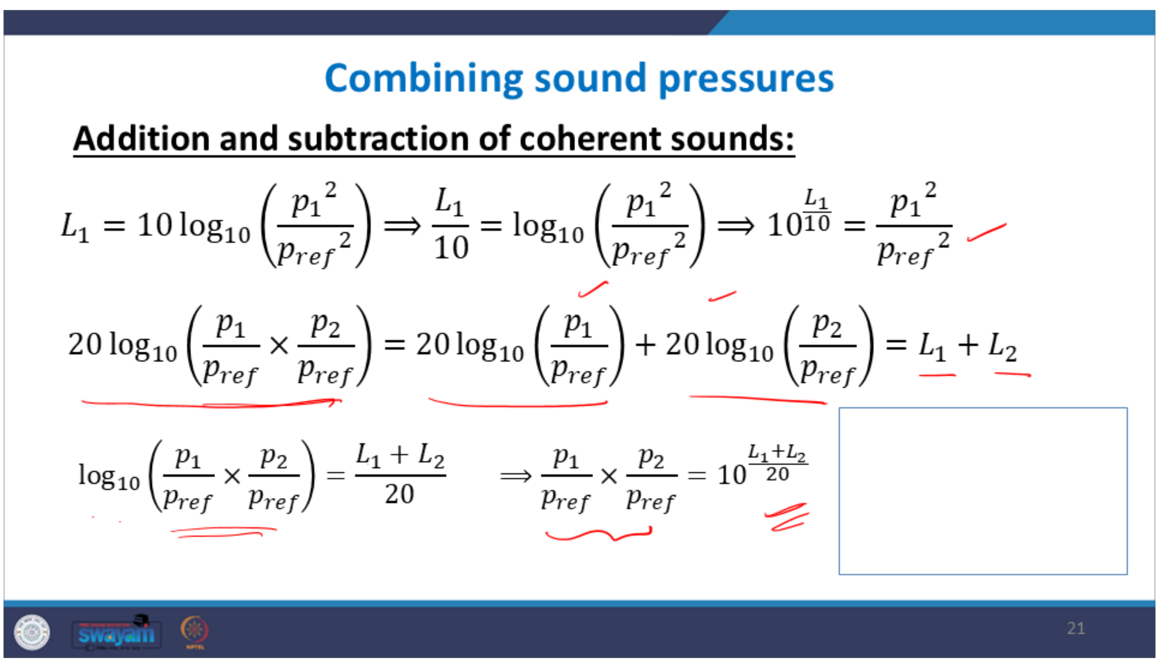
So, in the same way, suppose coherent sounds are present. So, in the case of coherent sounds, it is the acoustic pressure that gets added vectorially. So, you do the vector addition of the acoustic pressures. Here suppose you have an acoustic pressure p_1. And another acoustic pressure p_2, and the phase difference between them is (ϕ). Then the net pressure would be the vector summation of p_1 and p_2 with the angle between them as $(\Delta\phi)$, which is the phase difference. So, this is the formula:

$$p_{eff}{}^2 = p_1{}^2 + p_2{}^2 \pm 2p_1p_2\cos(\Delta\phi)$$

So, plus is used when sources are added to get the total source, and minus is used when sources are subtracted. one source is getting absent like that. So, again if you divide it by P reference and then do a 10 log 10, you end up with the L effective, where this is the total SPL of the two, here we have only derived for two. So, this is the total SPL of the two coherent sources,

$$L_{eff} = 10\log_{10}\left(\frac{p_1{}^2}{p_{ref}{}^2} + \frac{p_2{}^2}{p_{ref}{}^2} \pm 2 \times \frac{p_1}{p_{ref}} \times \frac{p_2}{p_{ref}} \times \cos(\Delta\phi)\right)$$

which is given by this big expression here. Now, we already know that this is simply 10 to the power L1 by 10, and we have already derived that this is $10^{L_1/10}$ (refer slide20). We have already derived this. Let us see what these are in terms of the individual SPLs.



$$L_1 = 10\log_{10}\left(\frac{{p_1}^2}{{p_{ref}}^2}\right) \Longrightarrow \frac{L_1}{10} = \log_{10}\left(\frac{{p_1}^2}{{p_{ref}}^2}\right) \Longrightarrow 10^{\frac{L_1}{10}} = \frac{{p_1}^2}{{p_{ref}}^2}$$

suppose you want to find out what is this quantity. So, if you do

$$20\log_{10}\left(\frac{p_1}{p_{ref}} \times \frac{p_2}{p_{ref}}\right) = 20\log_{10}\left(\frac{p_1}{p_{ref}}\right) + 20\log_{10}\left(\frac{p_2}{p_{ref}}\right) = L_1 + L_2$$

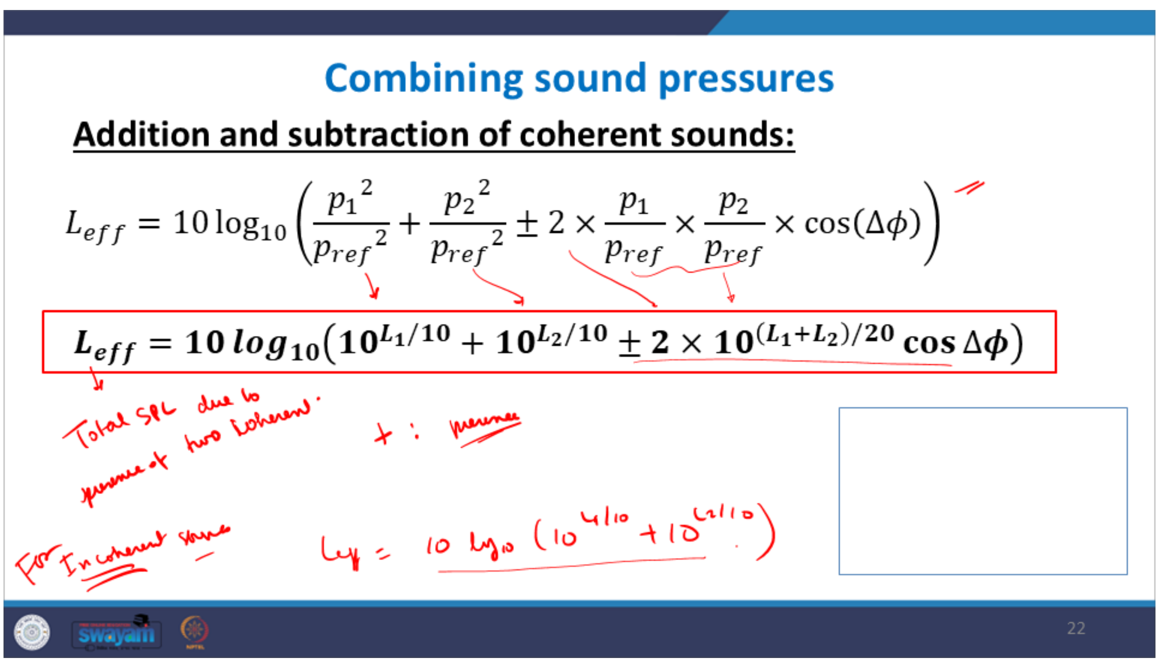
by the property of logarithm. So, ultimately this expression is the summation of the individual sound pressure levels of the two sources. So, now you remove the 20. So, basically what you do is, divide this expression by 20 throughout. So, you will get

$$\log_{10}\left(\frac{p_1}{p_{ref}} \times \frac{p_2}{p_{ref}}\right) = \frac{L_1 + L_2}{20}$$

So, ultimately this quantity would be what? It would be

$$\frac{p_1}{p_{ref}} \times \frac{p_2}{p_{ref}} = 10^{\frac{L_1+L_2}{20}}$$

and this is our total sound pressure that we had represented.



$$L_{eff} = 10\log_{10}\left(\frac{{p_1}^2}{{p_{ref}}^2} + \frac{{p_2}^2}{{p_{ref}}^2} \pm 2 \times \frac{p_1}{p_{ref}} \times \frac{p_2}{p_{ref}} \times \cos(\Delta\phi)\right)$$

$$L_{eff} = 10\,log_{10}\left(10^{L_1/10} + 10^{L_2/10} \pm 2 \times 10^{(L_1+L_2)/20} \cos\Delta\phi\right)$$

So, this is a slightly more complicated expression for the total sound SPL due to the presence of two coherent sources. Plus sign for presence. And suppose subtraction you have to do, then you will use the minus sign. So, this is a more complicated expression if you compare it with the addition of incoherent sources.

Here (refer slide 19), this was our expression: Suppose you were, there were only two incoherent sources present, then what would our expression be?

It would be 10log10, and within the brackets, we have 10 to the power of L1 by 10 plus 10 to the power of L2 by 10. Okay, if suppose it was incoherent sources. So, the same thing for incoherent sources would be 10 log 10 of this thing value plus 10 to the power of L2 by 10, and it would end here (refer slide 22).

But if the sources are coherent, then in that case, you have this quantity, and inside that, you have some additional quantity. So, which means that when the sources are coherent together, their combination gives a higher sound pressure level compared to when the sources are incoherent in nature. I would like to close this lecture. Thank you for listening.