

NOISE CONTROL IN MECHANICAL SYSTEMS

Prof. Sneha Singh

Department of Mechanical and Industrial Engineering

IIT Roorkee

Week:4

Lecture:17

Lecture 17: Spectrum analysis Numerical



The slide header features a blue and white color scheme. At the top, there are three logos: IIT Roorkee, Swayam (Free Online Education), and NPTEL Online Certification Course. Below these logos, the title "Noise Control in Mechanical Systems" is displayed in a large, bold, black font. Underneath the title, "Lecture 17" is written in a smaller, bold, blue font, followed by "Spectrum Analysis: Numerical" in a bold, blue font with a red checkmark icon. Below the title, the name "Dr. Sneha Singh" and her affiliation "Mechanical and Industrial Engineering Department" are listed. At the bottom of the slide, there is a photograph of the IIT Roorkee main building, a large white structure with a central dome and multiple columns. A small number "1" is visible in the bottom right corner of the slide.

Hello and welcome to the lecture 17 in this series on Noise Control in Mechanical Systems. I am Professor Sneha Singh from the Mechanical and Industrial Engineering Department at IIT, Roorkee. And in the lecture 17, we will do some numerical problems based on the frequency domain analysis and the spectrum analysis of sound signals. So, let us begin. So, to summarize what we will be doing? we will be doing the numerical problems on these topics. So, let us see the very first problem.

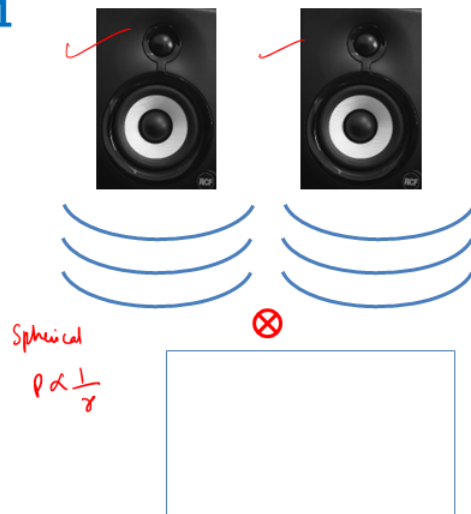
We have got two baffled loudspeakers here, and by the any time it is mentioned as baffled loudspeakers as already told if it is a baffled speaker they behave as an acoustic monopole source. And hence they are giving away spherical wave front. So, here the wave front is

Outline

- Numerical problems on Spectrum analysis and Octave analysis

Problem - 1

You have purchased two identical and **coherent** baffled loud speakers with a phase difference of 45° from the market. When a sound signal is played from either of these loud speakers it creates a total SPL of **60 dB** at a reference point. If both speakers play the same sound signal each individually generating 60 dB of SPL, what is the total noise level at the reference point?



spherical and P is inversely proportional to R and these two identical and coherent baffled speakers are there and their phase difference is 45 degrees. Each individually when played generates a sound signal which makes a total SPL of 60 dB at a reference point. Now, both are placed together side by side and they both are run each individually contributing 60 decibels, then what would be the total noise level due to both the speakers running simultaneously at the phase difference of 45. So, here this is about addition of noise due to

two coherent sources where the phase difference is given to you. So, what do you do? We have already seen the derivation to find out that.

Solution - 1

$$\begin{aligned}
 \textcircled{1} \quad L_{eff} &= 10 \log_{10} \left(10^{60/10} + 10^{60/10} + 2 \times 10^{\frac{60+60}{20}} \cos 45^\circ \right) \\
 &= 10 \log_{10} \left(10^6 + 10^6 + \frac{2 \times 10^6}{\sqrt{2}} \right) \\
 &= 10 \log_{10} \left(10^6 \times (2 + \sqrt{2}) \right) \\
 &= 10 \log_{10}(10^6) + 10 \log_{10}(2 + \sqrt{2}) \\
 &= 60 + 5.33 = \underline{\underline{65.33 \text{ dB}}}
 \end{aligned}$$

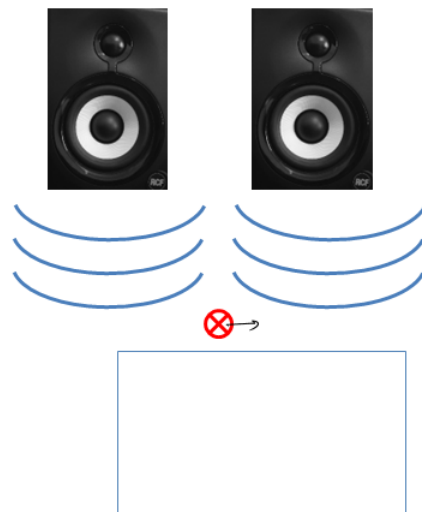
So, in this case, the L effective is simply

$$L_{eff} = 10 \log_{10} \left(10^{60/10} + 10^{60/10} + 2 \times 10^{(60+60)/20} \cos 45^\circ \right)$$

So, if you solve this (refer slide 4), so once you solve it this they overall contribute when present together are 65.33 decibels of sound waves. If you do this simple mathematics and calculation, now here you are getting an addition of 5.33 decibels when the two sources they are playing together which are coherent in nature and having a phase difference of 45.

Problem - 2

You have purchased two identical and **incoherent** baffled loud speakers from the market. When a sound signal is played from either of these loud speakers it creates a total SPL of **60 dB** at a reference point. If both speakers play the same sound signal each individually generating 60 dB of SPL, what is the total noise level at the reference point?



Now what happens? when we have the same problem, the same two speakers, which are baffled which means they are generating spherical wavefront, and they are incoherent in nature and then the sound signal is being played where individually it generates a 60 db at this point and if both are now present simultaneously what should be the overall noise level? so in the previous case as you saw if it is a layman and he or she is suppose given this problem that one source gives 60 other source gives 60 then they would think okay 60 plus 60 120 but that is not the case When it is two sources with 60 plus 60, the actual summation leads to 65.33 which is not much larger than 60 decibels itself.

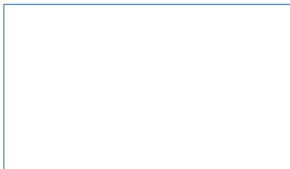
Now, over here let us solve. We use the equation for incoherent sound sources.


Solution - 2

② Incoherent sounds

$$\begin{aligned}
 L_{eff} &= 10 \log_{10} (10^{L_1/10} + 10^{L_2/10}) \\
 &= 10 \log_{10} (10^6 + 10^6) \\
 &= 10 \log_{10} (2 \times 10^6) \\
 &= 10 \log_{10} (10^6) + 10 \log_{10} (2) \\
 &= 60 + 3 = \underline{63 \text{ dB}}
 \end{aligned}$$

④ Two equal amplitude incoherent sources lead to 3 db increase in overall SPL.




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So, what you see here from the previous equation is that we get this term, this term and this term and there is no additional term due to the phase difference because of the incoherence in the source (refer slide 4). So, over here the overall level will rise in the overall level will be smaller as compared to the coherent source which is given like this. So, you use the logarithmic property to sort of separate the two factors.

So, this comes by definition becomes 60 and this if you calculate it comes out to be 3. So, you get a total of 63 decibels which is smaller than 65.33 decibels. So, here if suppose the sources are incoherent then that is the net addition if they are equal in nature. So, let us

give one conclusion here to equal amplitude incoherent sources lead to a 3 dB increase in overall SPL when obviously playing simultaneously at the same point.

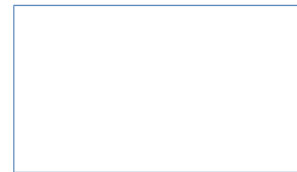
Problem - 3

Monopole - spherical wavefront

Noise level from a baffled loudspeaker at 1 m is 60 dB. Two such incoherent speakers are purchased and placed side by side in a free field environment. What is the total noise level at 3 m?

$$\Downarrow$$

$$p \propto \frac{1}{r}$$



Now, let us see the third problem here in the noise level from a baffled loudspeaker again this is baffled. So, it is a monopole source with spherical wave front and at 1 meter it is giving us 60 dB. Now, two such incoherent speakers are now purchased and they are placed side by side in a free field environment which means that in the free field the waves there are no reflection. All the acoustic pressure is because of the direct sound field and it follows this law where pressure is inversely proportional to the distance of the measurement.

Solution - 3

③ SPL at 1 m: $L_1 = 60 \text{ dB}$ due to 1 source

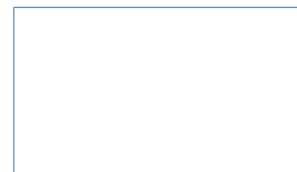
SPL at 3 m: $L_3 = ?$

$$p \propto \frac{1}{r} \quad p_3 = \frac{1}{3} p_1$$

$$L_3 = 20 \log_{10} \left| \frac{p_3}{p_{ref}} \right| = 20 \log_{10} \left| \frac{p_1}{p_{ref}} \times \frac{1}{3} \right|$$

$$= 20 \log_{10} \left| \frac{p_1}{p_{ref}} \right| - 20 \log(3)$$

$$= 60 \text{ dB} - 9.5 \text{ dB} = 50.5 \text{ dB}$$



Now, you have to find what is the total noise level at 3 meters?

Let us begin like this. So, you have here let us call the you know the level SPL at 1 meter as L_1 and this is given to be 60 dB due to 1 source. one incoherent source now let us see over here you have to find out the SPL due to both the incoherent sources at 3 meters. Let us first find out what is the spl at 3 meters due to just one source individually, so L_3 what is that? Now think about it the

$$P \propto \frac{1}{r}$$

which means that

$$P_3 = \frac{1}{3} P_1$$

right the pressure would be decreasing inversely proportional to R and L_3 is what by definition it is $20 \log_{10}$ whatever is the pressure RMS pressure at 3 meters by P reference which we can write as

$$\begin{aligned} L_3 &= 20 \log_{10} \left| \frac{P_3}{P_{ref}} \right| \\ &= 20 \log_{10} \left| \frac{P_1}{P_{ref}} \times \frac{1}{3} \right| \end{aligned}$$

So, this again becomes

$$= 20 \log_{10} \left| \frac{P_1}{P_{ref}} \right| - 20 \log_{10}(3)$$

So, the pressure is obviously going to decrease and hence the SPL is going to go down. So, this is given to us this is what this is L_1 by definition.

$$= 60 \text{ dB} - 9.5 \text{ dB}$$

So, individually each source is contributing 50.5 dB at 3 meters. So, now what would be the net effective for these incoherent sources at 3 meters acting together at 3 meters?

L effective at 3 again by the formula is this.

$$L_{eff,3} = 10 \log_{10} \left(10^{\frac{L_1}{10}} + 10^{\frac{L_2}{10}} \right)$$

We know that two same forces acting together ultimately it boils down to a 3 dB increase. the way we have solved it in the previous case

$$10 \log_{10}(2) = 3$$

Solution - 3

For incoherent sources acting together at 3 m:

$$\begin{aligned} L_{\text{eff},3} &= 10 \log_{10} (10^{L_3/10} + 10^{L_3/10}) \\ &= 10 \log_{10} (10^{L_3/10}) + 10 \log_{10} (2) \\ &= 10 \log_{10} (10^{5.05}) + 3 \text{ dB} \\ &= 50.5 + 3 \text{ dB} \\ &= \boxed{53.5 \text{ dB}} \end{aligned}$$

which we have calculated before.

$$10 \log_{10}(10^{5.05}) = 50.5$$

So, which is the original decibel by each individual source and the conclusion that we had drawn last time that two incoherent sources acting together will increase the overall level by 3 decibels. The total effective level at 3 decibels is coming out to be 53.5 and in this way you solve.

Problem - 4

- Noise level generated by a uniform traffic flow on a building **A** located 20 m away from the highway is 75 dB. Assuming the traffic as a line source, what is the noise level generated in another building **B** located 10 m away from the same highway road?
- How will the noise level change in building **B** when the traffic density doubles?

Fine so let us solve another problem these are short numericals on this particular topic so in this problem what is happening the noise level is being generated by a uniform traffic flow and on the building A which is 20 meters away this uniform traffic law is creating a noise which is 75 dB.

So, whenever such things are mentioned by default you can assume that here this 75 dB is due to this uniform traffic flow and there is no significant contribution from other source. If it was, it would have been mentioned in the problem. So, we ignore the contribution from the background, the people living etc. and the sole contribution or the significant contribution from the uniform traffic flow is coming out to be 75 dB.

And assuming that this traffic which is uniformly flowing is a line source. So, in our lecture on cylindrical wave front, I have introduced the concept that when is a cylindrical wave front generated, it is generated in the case of line source. And some of the examples could be a train moving at a constant speed or a traffic moving uniformly with a uniform traffic density. This could be example of constant line source and they generate a cylindrical wave front. Now, this traffic you have to find out first of all what is the noise level that it will generate in another building B which is located 10 meters away from the same highway road. Let us first solve the part A of this problem.

Solution - 4

a) Uniform Traffic flow = Cylindrical source \Rightarrow cylindrical wavefront

$$p \propto \frac{1}{\sqrt{r}}$$


$$\frac{p_A}{p_B} = \frac{\sqrt{10}}{\sqrt{20}} \Rightarrow \bar{p}_B = \sqrt{2} p_A$$

SPL at B, $L_B = 20 \log_{10} \left| \frac{p_B}{p_{ref}} \right|$

$$= 20 \log_{10} \left| \frac{\sqrt{2} p_A}{p_{ref}} \right|$$

$$= 20 \log_{10} \left| \frac{p_A}{p_{ref}} \right| + 20 \log_{10} (\sqrt{2})$$

$$= L_A + 3 \text{ dB}$$


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Here the traffic flow uniform is treated as a cylindrical source. It gives a cylindrical wave front. It is giving a cylindrical wave front and what happens in the cylindrical wave front?

It does not follow the same law as the spherical one. Here, the pressure is inversely proportional to root of the measurement of distance,

$$P \propto \frac{1}{\sqrt{r}}$$

And it is given that the P at point A, and where is point A located? It is 20 meters from the source and point B is located 10 meters from the source. So, the P at point A by the P at point B should be in the inverse ratio of the distance B is at 10, A is at 20.

$$\frac{P_A}{P_B} = \frac{\sqrt{10}}{\sqrt{20}}$$

$$P_B = \sqrt{2}P_A$$

because it is inversely proportional to the root of the distance and hence the pressure at the building B because of the same source would be root 2 times the pressure at the building A. So, now we have this and we have to find out the SPL at the location B for the same case and this one is given as 75 dB.

SPL at B or simply L of B is given as by definition the definition remains the same it is the pressure at that point by the reference pressure and this in terms of A we can write. Now if you remember in the previous sessions, whenever we were discussing about the spherical wave front. So, whenever double the distance the intensity the SPL reduces by 6 decibels as you double the distance and the SPL increases by 6 decibels as you half the distance, but here it is not the inverse dependence here it is root over. So, the SPL actually increases or decreases by 3 decibels. In the case of a cylindrical source, so here if you do this, this by definition becomes the L at point A plus 3 dB.

So, this becomes the level at point B becomes 75 which is the level at A plus 3 decibels. So, only a 3 dB increase as you half the distance from the source and that becomes your first answer.

Let us solve the second part of this problem. How will the noise level change in building B when the traffic density doubles in this case? So, let us say here that you know each the traffic overall is contributing the same amount of energy. Now, if you increase the density of the traffic which means within the same area or within the same length now you have double the cars or double the noise sources therefore overall, the energy is doubling up so if you say that the density is doubling here because it is acting as the source. The surface density of the source so for the line acoustical source. The energy radiated would be directly

Solution - 4

b) For the line acoustical source:

$E \propto \text{surface density} \rightarrow \text{mass flow rate}$

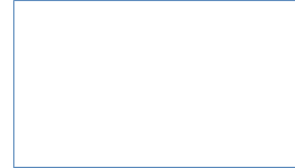
Traffic density double, for equivalent cylindrical acoustic source,
mass flow rate doubles

\Rightarrow Energy radiated doubles

SWL = doubles

$$\frac{W_{\text{new}}}{\text{Area}} = \frac{2 W_{\text{old}}}{\text{Area}}$$

$$\Rightarrow I_{\text{new}} = 2 I_{\text{old}}$$



proportional to the surface density of that source, because that would determine the mass flow rate. you can refer to the lectures on acoustical sources to know the concept behind it because these sources they radiate the sounds by the means of this you know expansion and contraction of their masses so here the energy that is radiated would be directly proportional to the mass flow rate so as you increase the surface density the mass flow rate is going to double so once the traffic density doubles. for the equivalent cylindrical acoustic source or the line acoustic source, the mass flow rate also doubles because it is the density multiplied by the speed and considering the surface velocity is same. So, mass flow rate also doubles. Mass flow rate doubles which mean the energy being radiated doubles. So, now what it means for our question at hand is that now the net sound power level is doubling up. The source is say having double the strength.

So, the sound power level doubles. okay so which means that the power of the new traffic source should be twice of the power of the old traffic source now if you divide this by the area and by the area of measurement considering both are being measured at the same reference point that all this also means that the intensity due to the new source is twice of the intensity due to the old traffic source. What has been asked is that in this problem that when the traffic density is doubling how will the level at the building b change so the let us say the I_{new} at B is given by definition So, it can either be given as $20 \log 10$ of P at B by P reference and you know that SPL and SIL are same. So, in the same way it can also be written as



$$\begin{aligned}
 L_{new} &= 10 \log_{10} \left| \frac{I_{new,B}}{I_{ref}} \right| \\
 &= 10 \log_{10} \left| \frac{2 \times I_{old,B}}{I_{ref}} \right| \\
 &= 10 \log_{10} \left| \frac{I_{old,B}}{I_{ref}} \right| + 10 \log_{10}(2) \\
 &= 78 + 3 = 81 \text{ dB}
 \end{aligned}$$

Solution - 4

$$\begin{aligned}
 L_{new,B} &= 10 \log_{10} \left| \frac{I_{new,B}}{I_{ref}} \right| \\
 &= 10 \log_{10} \left| \frac{2 \times I_{old,B}}{I_{ref}} \right| \\
 &= \underbrace{10 \log_{10} \left| \frac{I_{old,B}}{I_{ref}} \right|}_{L_{old,B}} + \underbrace{10 \log_{10}(2)}_{3 \text{ dB}} \\
 &= 78 \text{ dB} + 3 \text{ dB} = \underline{\underline{81 \text{ dB}}}
 \end{aligned}$$

This is twice of the intensity due to the old traffic at the building B divided by I reference. So again using the property of the log to separate the two terms okay and this again comes out to be 3 decibels and this by definition becomes your L old at B. So, you get we have already found in the last case that it was 78 decibels for building B. So, 78 decibels plus 3 decibels we get 81 decibels as the new sound level at the building B and this solves our problem 4.


Let us see one final problem for this topic. here a value is given so we have studied about octave and one-third octave spectrum here you are given an octave spectrum, and this is the central frequency of the spectrum. let us say this is the SPL so just mark this change here so this is the SPL at that frequency so the various SPL values are measured.






SPL (dB) Problem - 5

Centre frequency	Lower frequency
31.5	30 ✓
63	33 ✓
125	44 ✓
250	56 ✓
500	63 ✓
1000	64 ✓
2000	64.5 ✓
4000	X = ? → 62.6 dB
8000	56 ✓
16000	45 ✓

Find the missing SPL in dB in the following Octave Band, given that the total SPL of the noise spectrum is 70 dB.






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so basically, we are getting these values you know the various values at the various octaves has been measured and it is given that the overall noise level of this spectrum is 70 decibels you do not know what is the noise at 4000 band so you have to find this missing noise let's see how to solve this.

Solution - 5

⑤ Combining of individual levels of individual bands.

$$70 = 10 \log_{10} (10^{L_1/10} + 10^{L_2/10} + \dots + 10^{L_n/10})$$




$$70 = 10 \log_{10} (10^3 + 10^{3.3} + \dots + 10^x + 10^{4.5})$$

$$\frac{70}{10} = \log_{10} (\dots)$$

$$10^7 = \cancel{10^{70}} (8181482.9 + 10^x)$$

$$10^x = 1818517.1$$

$$x = \log_{10} (1818517.1) = 6.26 \approx$$




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Again, here in this problem combining of individual levels of the individual bands that formula we have to use which was given before. So, individual we are trying to combine the levels at the individual frequency bands to get the total L equivalent.

Here the total L equivalent is given by

$$70 = 10 \log_{10} \left(10^{\frac{L_1}{10}} + 10^{\frac{L_2}{10}} + \dots + 10^{\frac{L_n}{10}} \right)$$

and so on till whatever the levels you are trying to combine so over here what you do is these become these individual levels which we try to combine over till here (refer slide 18) in which one of them is an x. so this becomes 30 by 10 the second one becomes 10 to the power 33 by 10 and so on So,

$$\begin{aligned} &= 10 \log_{10} \left(10^{\frac{30}{10}} + 10^{\frac{33}{10}} + \dots + 10^{\frac{X}{10}} + \dots 10^{\frac{45}{10}} \right) \\ &= 10 \log_{10} (10^3 + 10^{3.3} + \dots + 10^x + \dots 10^{4.5}) \end{aligned}$$

and then in the middle we have one more term that is 10 to the power. Let us say this is capital X then let us say X is capital X by 10 then 10 to the power X by 10 would become 10 to the power small x. So, here this term comes 10 to the power small x till the last term you will be summing it up which is 10 to the power 4.5.

So, let us find what this value is coming out to be. So, let us negate these 10 logs 10 let us do this first of all take the 10 here.

$$\frac{70}{10} = \log_{10} (10^3 + 10^{3.3} + \dots + 10^x + \dots 10^{4.5})$$

$$10^7 = (8181482.9 + 10^x)$$

$$10^7 - 8181482.9 = (10^x)$$

10 to the power x ultimately comes out to be this number when you calculate.

$$1818517.1 = (10^x)$$

So, x comes out to be

$$x = \log_{10}(1818517.1)$$

$$x = 6.26$$

Solution - 5

$$x = \frac{X}{10}$$
$$\Rightarrow X = x \times 10 = 6.26 \times 10 = 62.6 \text{ dB}$$

$$x = \frac{X}{10}$$

$$X = x \times 10$$

$$X = 62.6 \text{ dB}$$

So, 62.6 decibel was a missing dB from that table over here.

So, with these problems, I would like to conclude this lecture. So, hopefully you learnt a lot on how to solve problems related to decibels. Thank you for listening.

Thank You