NOISE CONTROL IN MECHANICAL SYSTEMS

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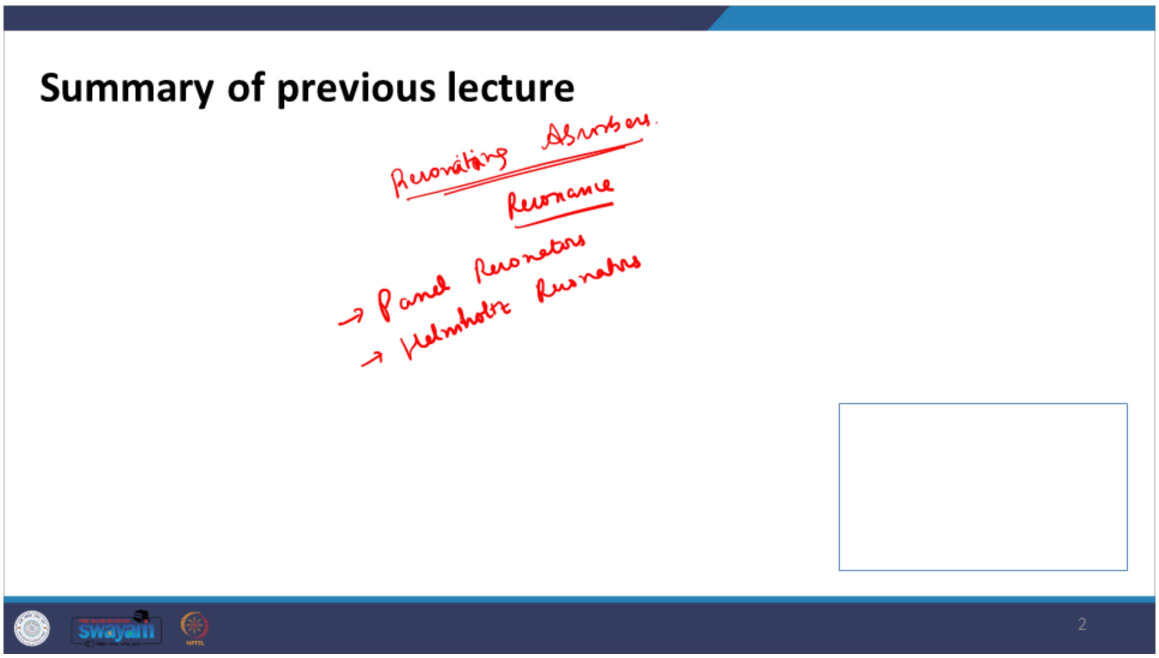
Week:9

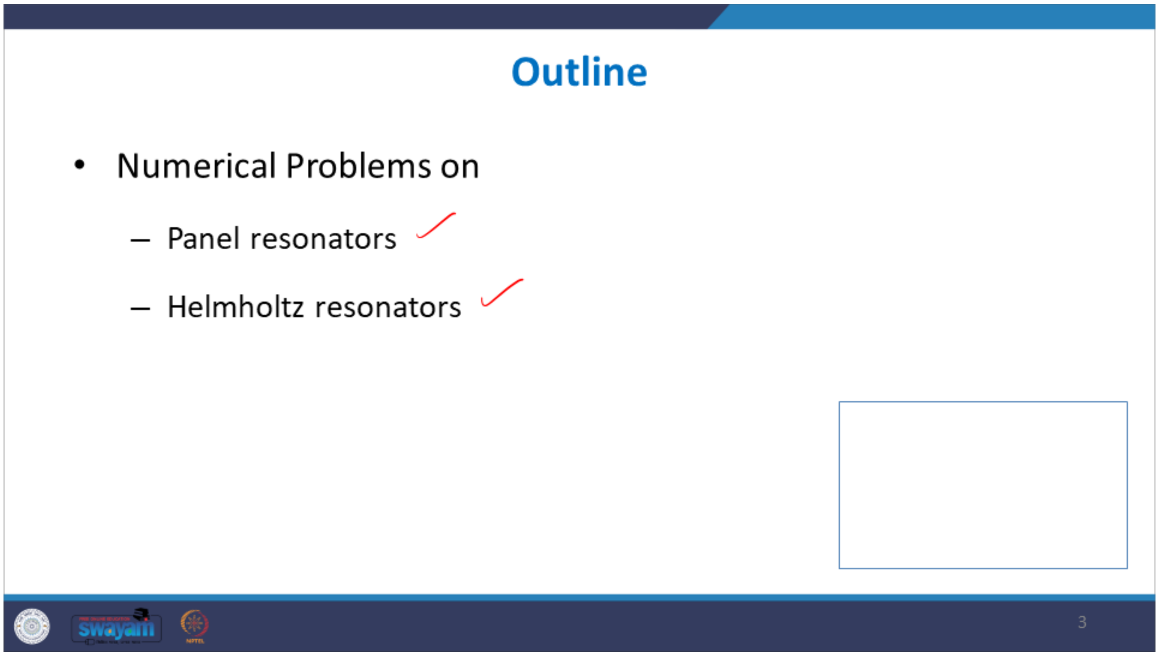
Lecture:41

Lecture 41: Numerical Problems on Resonators

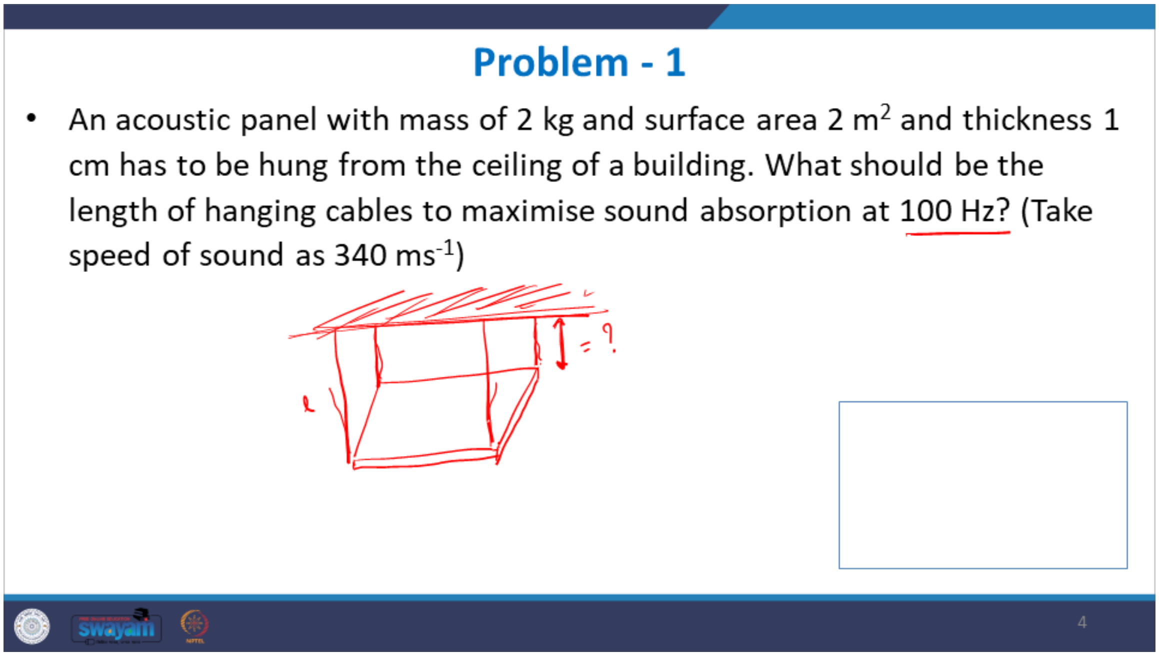
Hello and welcome to this lecture series on noise control in mechanical systems with me, Professor Sneha Singh from IIT Roorkee. We have been discussing in the previous classes the resonating absorbers. So, basically, the absorbers which absorb sound by means of the resonance phenomenon. So, out of these, we studied the panel absorbers or the panel resonators and the Helmholtz resonators.



So, let us today, in this lecture, solve some numerical problems based on these two resonators that we have studied.



So, the very first problem is an acoustic panel with a mass of 2 kg, a surface area of 2 meters square, and a thickness of 1 centimeter has to be hung from the ceiling of a building. So, this is a hung kind of acoustic panel, and the mass, the thickness, and the surface area—all these dimensions of this panel are mentioned, and it has to be hung from the roof of the building. What should be the length of these cables? So that we can maximize the sound absorption at 100 hertz, taking the speed of sound as 340 meters per second. Now, over here, this is like a trick question. A lot of information is thrown at you, but you actually need just one piece of information, which is this. Because you know that there is a panel hanging from the ceiling of a building. What would be the optimum location of that panel, or what would be the optimum gap between the panel and the rigid boundary? So, when we were studying these panel absorbers,

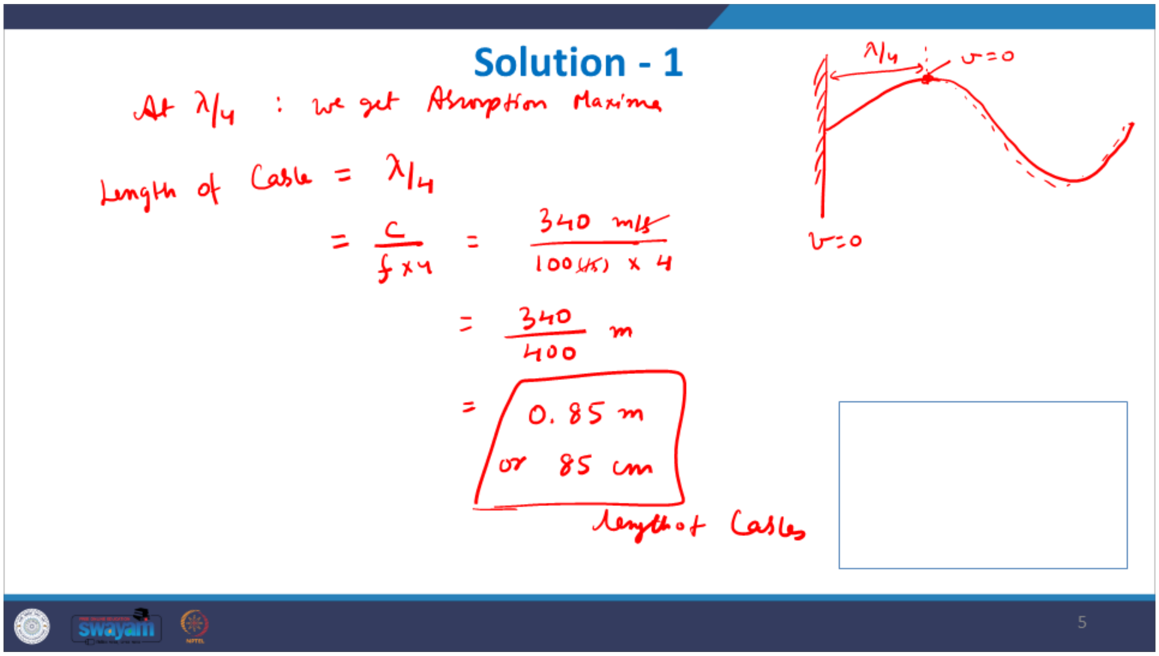


I had introduced to you that at λ by 4, we get absorption maxima because any kind of room which is surrounded by rigid boundaries—at the rigid boundary, our acoustic particle velocity is going to be 0; that is the condition. So, here it has to be 0. So, whatever be the wave front like this or you know. So, the first kind of maxima will occur at λ by 4 distance ok. This is your this is your full λ. So, at λ by 4 this particular distance you will get the first velocity maxima. So, that is the place where the sound waves will hit with the maximum velocity which will just optimize the vibrations of the panel itself. So, the distance between a rigid boundary and any kind of panel If it is kept at λ by 4, there you know the absorption gets maximized. So, for a maximum absorption at 100 Hertz, we need to maintain a λ by 4 distance. And what is this distance here? Here you have got this roof panel and some cables which are sort of hanging it. So, the length of the cables should be equal to λ by 4, okay, because that will decide the distance between the panel and the rigid roof. So, the length of the cables should be equal to λ by 4 for the maximum absorption and λ is what it is

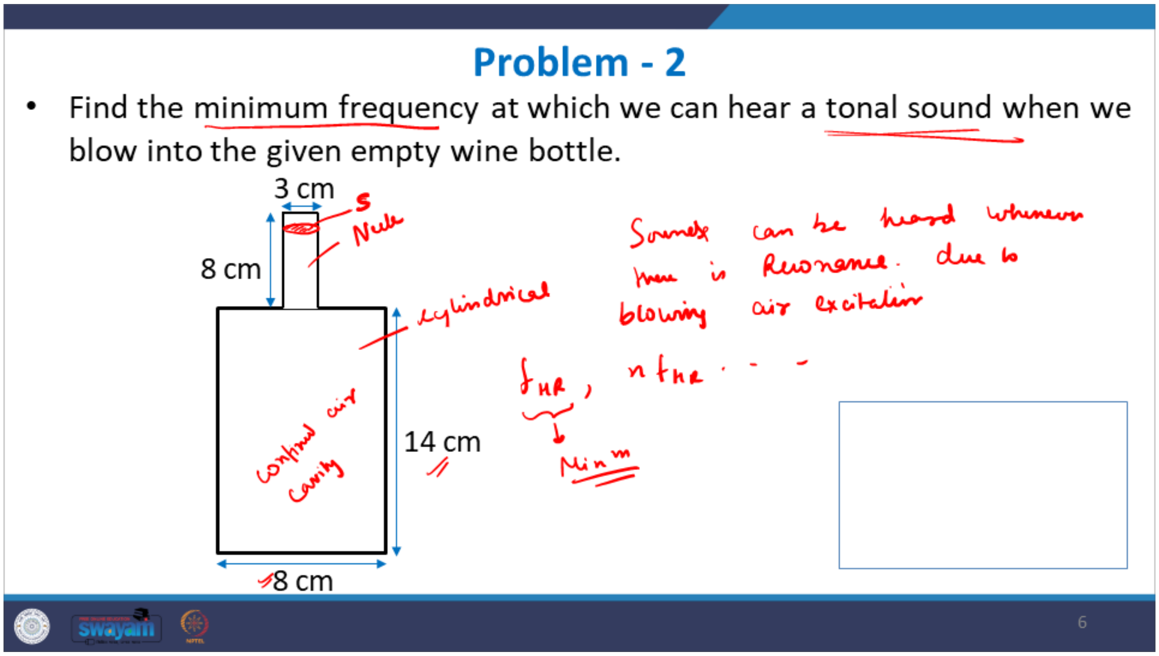
$$\lambda = \frac{c}{4f}$$

c is 340 meters per second and the frequency of interest is hertz or per second and this multiplied by 4. So, we get 340 by 400 meters. If you solve this, you get 0.85 meters or

85 centimeters. This should be your length of the cables to maximize the absorption at 100 hertz. You do not need any other information for this question.



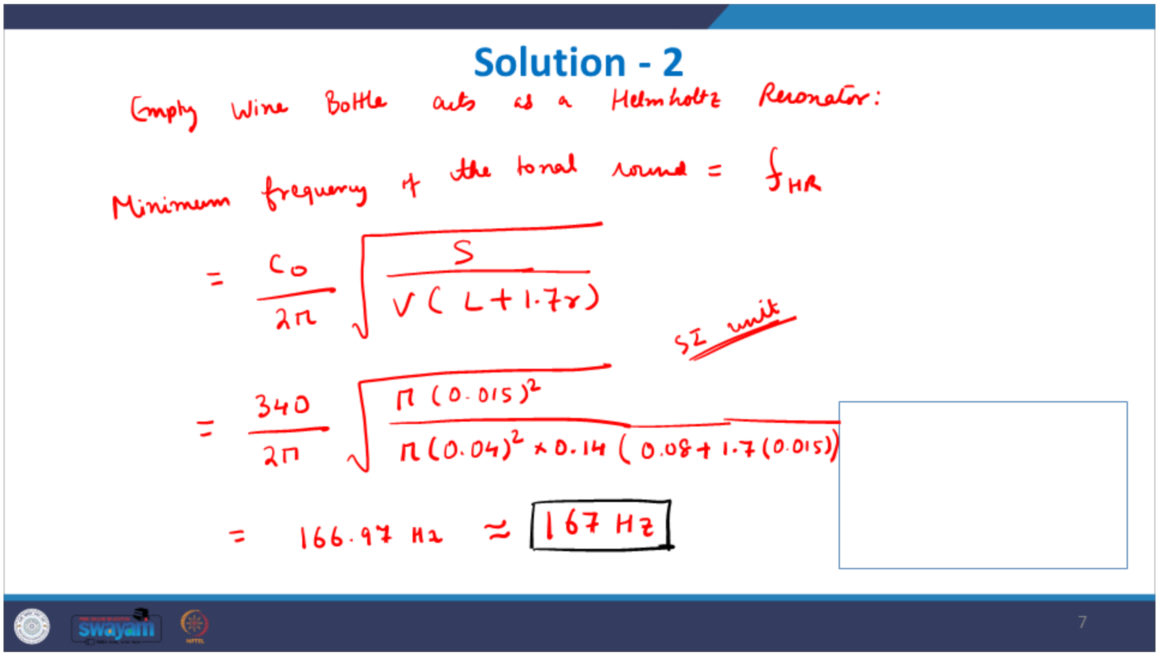
Let us see the second problem here. It's been asked what is the minimum frequency at which we can hear a tonal sound when we blow into a given empty wine bottle. So, usually if you see a wine bottle it usually has a very narrow and long neck and then a big bottle containing the wine. So, here this kind of bottle it actually acts as a Helmholtz resonator ok. This is a confined air cavity. This is the confined cavity. It is an empty bottle. So now it is containing air. So this becomes a confined air cavity and this acts as the neck of this cavity.



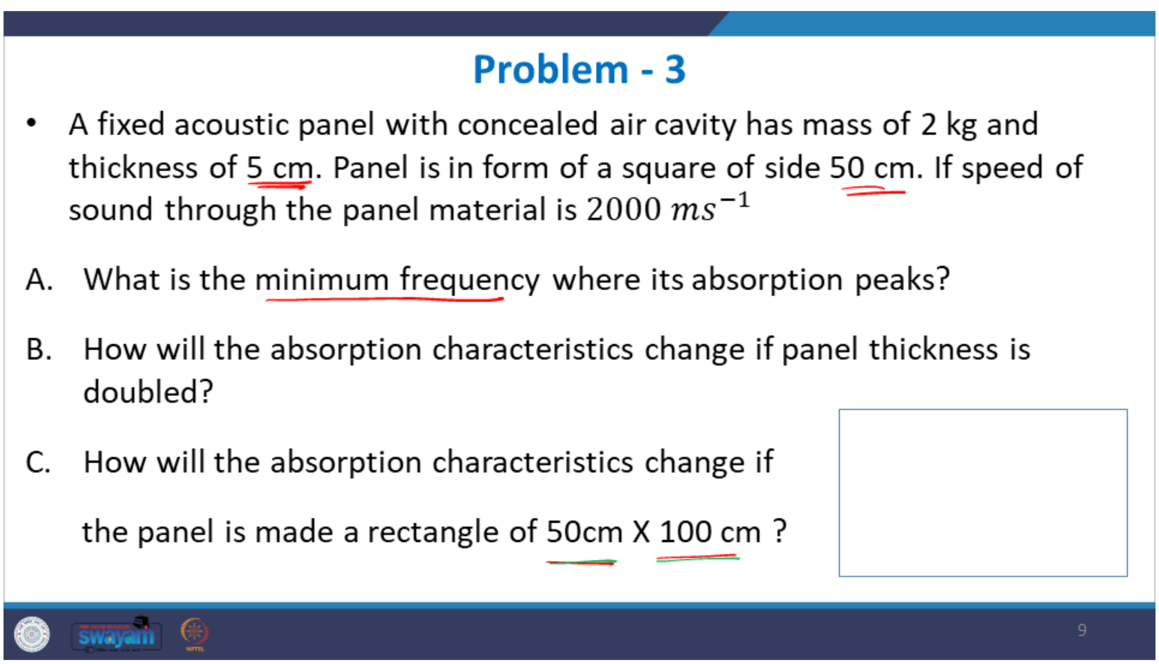
So this becomes you know here the empty wine bottle acts as a Helmholtz resonator So, sort of observe it in your real life whenever you have got any kind of empty bottle and you blow into it like that. You can hear if you are blowing and doing several attempts you will hear you know some kind of tonal sound or whistling sound that comes from this empty bottle. Why it happens? The way it happens is that we are blowing the air you know. continuously into this bottle or at the top of the bottle when you are blowing the air we are sort of creating the various you know flow of the air and the air particles at a broadband you know varied varied frequencies and at the frequency so these air particles are trying to excite the particles of the neck of this particular bottle. So, At the point where the frequency of our excitation, so we are blowing the air, we are giving some kind of excitation to this air particle. So, at the frequency at which you know the blowing air is matching with the fundamental frequency of the empty bottle. At that frequency you know the resonator will tune in. And at that frequency, the vibrations is going to maximize. So a tonal sound is going to come out. And that will match with the frequency at which the maximum vibrations have happened and that will be the frequency of this resonator. So, basically the tonal sound would be due to the resonance happening at the resonators fundamental frequency. So, now it is asked what is this minimum frequency at which we will hear the tonal sound. Now, sounds can be heard in this resonator whenever there is resonance due to the blowing air. blowing air, excitation. So, it could be at the fundamental frequency of the resonator and its further higher harmonics. So, this would be the minimum frequency. So, that is what has been asked. So, the minimum frequency of the tonal sound is simply the fundamental frequency of the Helmholtz resonator, which is this empty wine bottle here. So, this is going to be the formula for this.

$$\frac{c_o}{2\pi}\sqrt{\frac{S}{V(L+1.7r)}}$$

It is okay. So, where S is the cross-sectional area of the neck. So, let us start putting those values. So, here let us take air at room temperature and put this value in: 340 meters per second, the rounded-off value of the sound velocity in air at room temperature. You can also take 343 or 346; I am just taking 340 for ease of calculation. So, this is the velocity of sound waves in air at room temperature. What is the S here? It should be this cross-sectional area. of this bottle. This should be your S. So, here the diameter is given as 3 centimeters. So, the cross-sectional area should be $\pi D^2/4$ or simply πR^2. So, we can put π into R^2, where R would be point. So, it is 3 centimeters. Now, I am going to write everything in SI units. So, it will be $(0.015)^2$. And the volume again, what should be the volume? This is like a wine bottle; they are cylindrical in nature. So, this is a cylindrical cavity, OK. So, the wine bottle—this is the length of this. So, $\pi R^2 L$ should give you the volume, OK. So, πR^2, which is going to be half of 8 centimeters. So, 4 centimeters is your radius. So, in meters, I am writing like this: pi. So, all of this is in SI units; I am writing. So, $\pi R^2 L$ should give you the volume of the cylinder. L is this, and then the length of the neck is 8 centimeters, which I write here. Plus 1.7 times the radius of the neck, which is this. So, once you solve this particular expression, what you get is this thing here or you can just say it is 167 Hz. So, as you can see here, with just a typical, you know, this kind of bottle having a very easy construction—you do not have any kind of very small dimensions or like that—just regular dimensions, and you are getting a very low-frequency absorption characteristic. You are getting resonance at 167 Hz, which is a very low frequency.



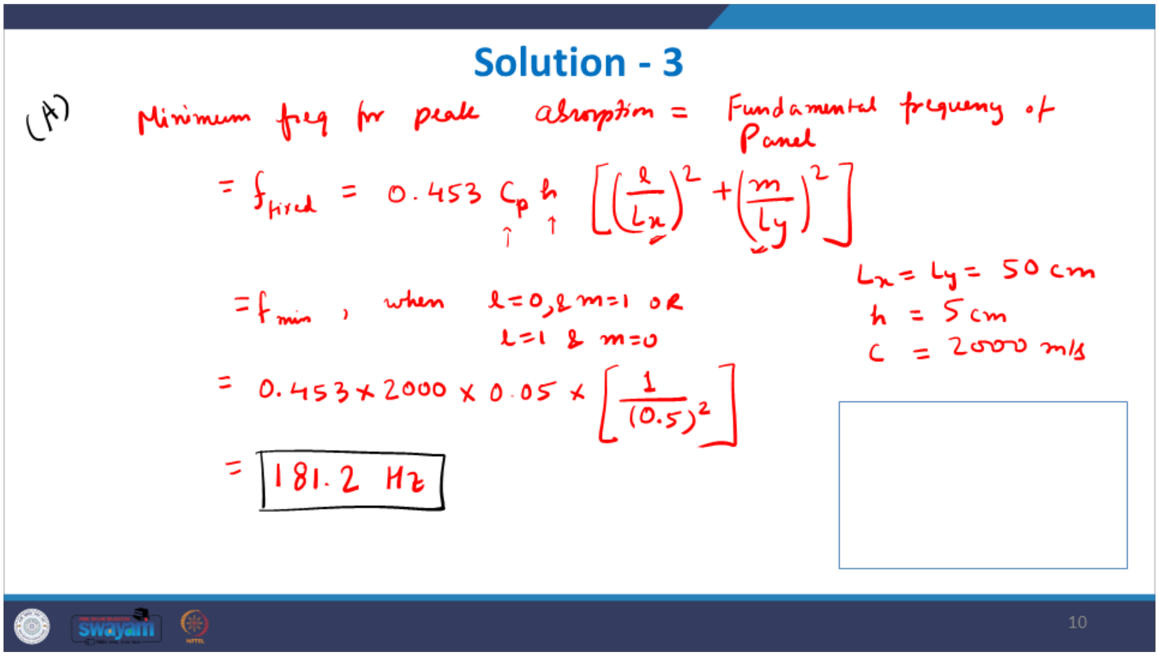
Let us see the third problem. So, here what we have is a fixed acoustic panel with some concealed air cavity, and it has a mass of 2 kg and a thickness of 5 centimeters. It is in the form of a square of side. So, the panel is like a square with a side of 50 centimeters. And the speed of sound through the panel material is 2000 meters per second. So, first of all, what we have to find is the minimum frequency where its absorption peaks.



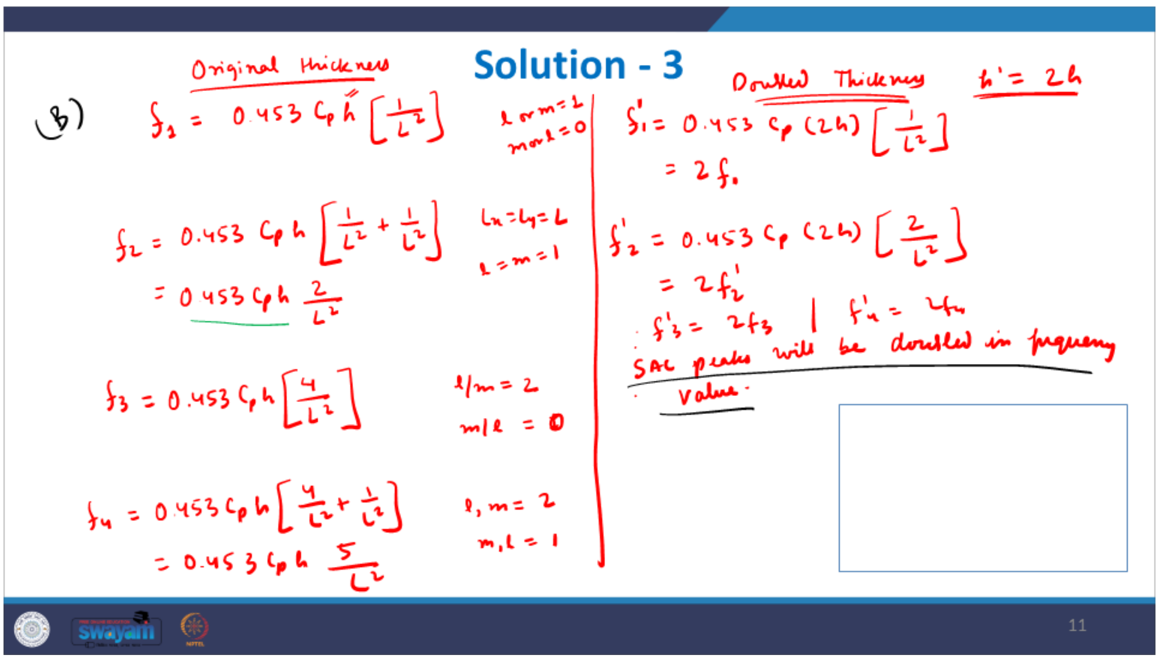
So, part A. Let us solve this. So, here the minimum frequency where the absorption will peak should correspond to the fundamental frequency of the panel. So, both the panel resonator and the Helmholtz resonator will have their absorption peaking at their fundamental frequency. So, the absorption always peaks at the natural frequency of these resonators and their successive integral multiples, okay? So, the minimum frequency should be. So, the minimum frequency for the peak absorption is the same as the fundamental frequency, which is the first natural frequency of the panel. Now, this is a fixed kind of panel; this is not a hung panel, and an expression was given to you for this fixed panel frequency. And that was,

$$f_{fixed} = 0.453\, C_p\, h \left[\left(\frac{l}{L_x}\right)^2 + \left(\frac{m}{L_y}\right)^2\right]$$

Like this. So, here, this is the speed of sound in the material of the panel; this is the thickness of the panel, and Lx and Ly are the dimensions along the x and y axes. So, the two dimensions of the surface area of the panel. So, it is already given to us that the panel is in the form of a square of side. So, which means that Lx and Ly are the same, which is 50 centimeters, and h, which is the thickness of the panel, is given as 5 centimeters. And the C value for the panel is 2000 meters per second. So, all of this is given to us. Let us punch everything in SI units and solve for the frequency. So, what you will get is, here, because both Lx and Ly are the same. So, think about it: $f_{minimum}$ will occur when l is equal to 0 and m is equal to 1 or l is equal to 1 and m is equal to 0. So, one of them is 0 and the other one is 1; that is where we will get the first value, and as you keep increasing the l and m values, the frequencies will keep increasing. Because here, both this and this value are the same, it does not matter; any one of them can become 1, and the other one is 0; we will get the very first frequency peak. So, let us See what happens when you put these values here, and then it is going to be 1 divided by 0.5 whole square, OK. So, when you solve this, you will get somewhere this as your answer. So, this gives you the minimum frequency where the absorption due to the panel is going to peak. So, that is our first. Now, you have been asked how will the absorption characteristic change if the panel thickness is doubled,



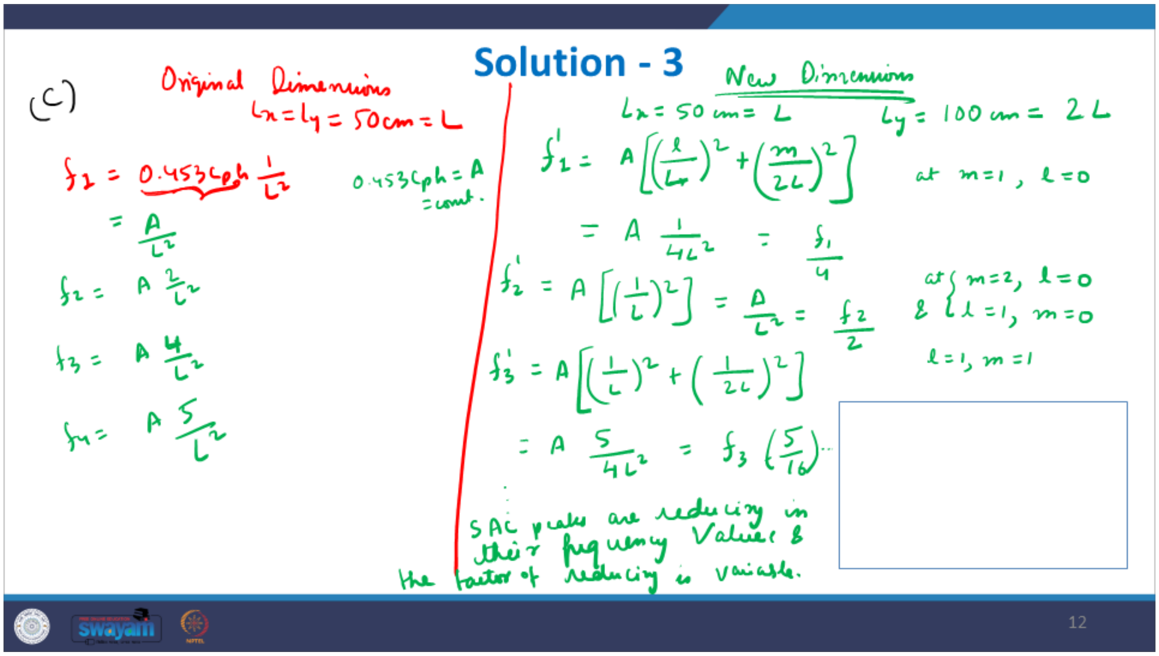
Part b. How will this absorption characteristic change? Let us just examine the first three frequencies and see what happens. So, the very first frequency we call it as f_1 is this thing here. These are constant, only h is going to change. So, first scenario we have you know original thickness and here when thickness gets doubled and we will see how the first three frequencies are changing. Some kind of Lx, here Lx and Ly are same let us call them. L simply so 1 by L^2 this is your first frequency the second frequency is when both reach a value of 1 okay and this is your second distinctive frequency which is this value here 2 by L square Let us see the third frequency where one of them becomes 2, one of these l or m value becomes 2. So, l slash m becomes 2 whereas the other one stays as 1 as 0 actually. So, if one of them becomes 2 and the other one stays 0 what you will see? 2 by L whole square. So, it becomes 4 by L square and the other one stays 0. In the same way, let us examine the fourth case. Here, one of them becomes 2 and the other one becomes 1. Let us examine these cases and see. 4 by L square plus 1 by L square. So, you get Now, let us see what happens to these. These are the first four frequencies of the panel's resonance or the first four natural frequencies of the panel, where this corresponds to either l or m becoming 1 and the other one becoming 0. Here, this corresponds to when both l and m are 1, and this corresponds to one of them being 2 and the other one being 0. The fourth case is where one of them is 2 and the other one is 1. Let us see what happens if the thickness doubles. So, here f_1, just the thickness is doubling. So, now the h' is twice h. So, whatever values are there, it will just be the same, just 2 h times this whole thing here. So, this would mean the new f'_1 should be twice f_1. Similarly, the new f'_2 everything else is the same, keeping the same values—just we are doubling the thickness. So, again, this is twice f'_2. And so on. So, what you observe is, you know, everywhere all the other expressions are the same, only this h is changing to twice of h. So, how will the absorption characteristic change? The SAC peaks will be doubled in frequency value. So, wherever you are getting the first frequency, just double it, and now the new frequency would be double of your previous frequency. So, all the successive frequencies would be double of their corresponding frequencies. So, the same way, f'_3 should be double of this, and f'_4 should be double of f_4. So, all the frequencies would be double of the original value. This is how the absorption characteristic is going to change.



Let us see the third case. How will this absorption characteristic change if the panel is made as a rectangle of 50 cross 100 centimeters? This is more tricky. So, in the previous case, the SAC peaks were simply doubling in their frequency value when the thickness was doubling. But for the third case, now what is happening is that, without doing original, let us take the original dimensions. Lx is equal to Ly, which is equal to 50 centimeters, okay. So, what is happening is that is 50 centimeters; let us call this value as L, okay. This is our first case or the original dimensions case. So, we have already solved. We will call it L. This is our first case. So, we have already solved for it in the previous example. What are the frequencies we were getting? f_1 was, if you think here, this particular value. Let us call this some constant because this is now going to stay constant for all our conditions 0.453 Cp h. Let us call this as some constant A, which is a constant. So, the first fundamental frequency is A by L square. The second frequency comes at this into 2 by L square. So, A into 2 by L square. The third one comes at A into 4 by L square. And the fourth one comes at A into 5 by L square. So, A by L square, 2 by L square, 4 by L square, 5 by L square like that. Now, let us see what happens when the dimensions are changing. So, for the new dimensions, how will these frequencies change? When Lx is same as 50 centimeter, which is your L value, but the Ly, one of them is doubling up, it has become a rectangle of size 100 cross 50. So, let us say Ly is changing and it is becoming 100 centimeter, which is twice of your L. So, one is L and the other one has now become 2L. So, what will the first frequency be? it would be same you know A into 1 by L square. So, the L value corresponding to the it would not be the same actually if you think about it how will this change where will you get the minimum value if you see

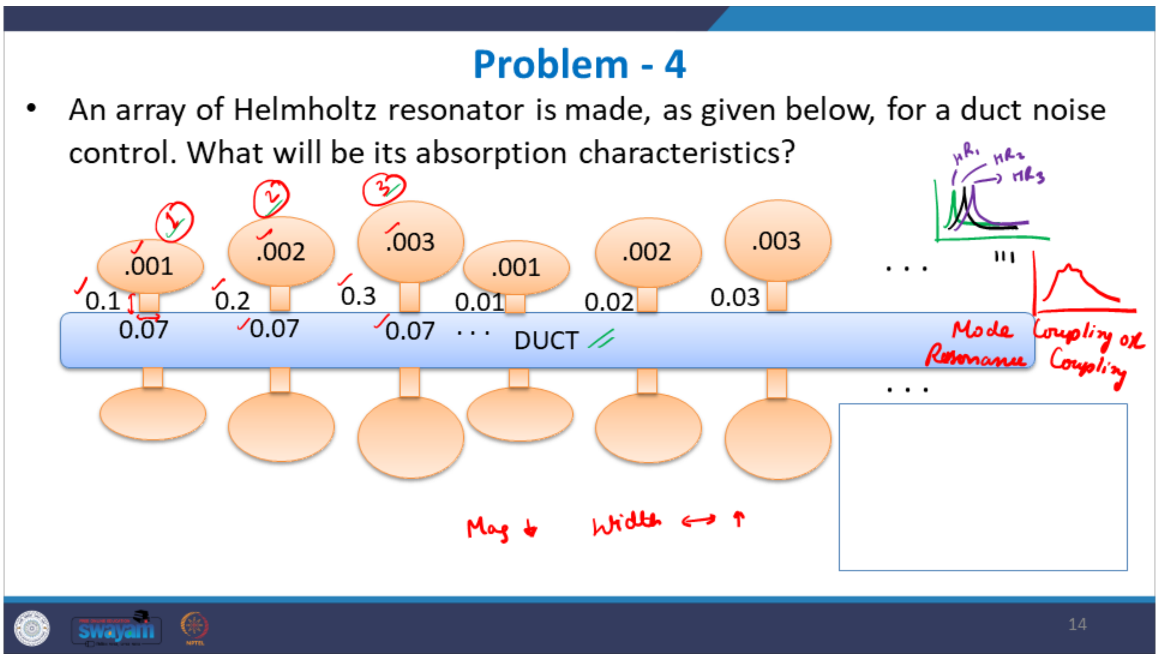
$$f_1' = A\left[\left(\frac{l}{L}\right)^2 + \left(\frac{m}{2L}\right)^2\right]$$

this becomes our new expression. So, here the minimum value will be obtained at m is equal to 1 and L is equal to 0. So, it would be A into m is 1 and L is 0. So, it would be A into 1 by 4 L square. So, the frequency the first frequency has reduced. So, this is the original frequency by 4. Then the second harmonic you get. Again, let us see what happens when both of the values become 1. Let us see what happens when m is equal to 2, you get. So, the second will happen when m is equal to 2 and l is still 0. What happens? Let us see. This is 0 and m is 2. So, 2 by 2 l. So, 1 by l whole square. And this would be same as l is equal to 1 and m is equal to 0. So, you get A by L square which is your second harmonic by 2. The third one which you will get should be when you have you know. So, when here l is becoming 1 and let us say m is l is becoming 1 and m is also becoming 1 what happens let us see. So, it is 1 by L square plus 1 by 4 L square. So, 4 L 5 A times of 5 by 4 L square. So, this is what? This is almost. So, it is this new frequency is how much? If you divide it by this, it is quite a bit. 5 by 4 and 4. So, it should be 4 by 5 by 4; it should be 16 by 5 of the previous frequency. Sorry, the other way round—it should be 5 by 16 of the previous frequency, and so on. So, almost one-fourth. So, like that, you can do it. So, what you see is that, in general, the SAC peaks are reducing in their frequency values, and the factor of reduction is variable in nature. It has to be individually calculated like that.



Okay. So, now let us come to the last problem of this lecture. You have got an array of Helmholtz resonators like this. This is your duct, and you have the side branches, which are the Helmholtz resonators, and they are being used for this duct noise control. You have to find out its absorption characteristic. Now, usually in noise control, because you know that Helmholtz resonator has got a unique frequency. So, we have sharp peaks at a particular frequency and then at its multiples. So, suppose you want to create a broadband absorption sometimes or slightly widened absorption. Sometimes, it is a practice that noise control engineers line up an array of Helmholtz resonators. with slightly varying magnitudes. So, their individual frequencies are different but close by. So, the individual fundamental frequencies of each of these varying resonators are close by but have different values. So, suppose the first frequency was coming here and the second frequency due to some other resonator was coming like this, and let us say due to the third resonator is like this. So, this is, you know, some resonator 1, resonator 2, and resonator 3 in the array with varying dimensions, each of them having their unique kind of characteristic. Then, when they are combined periodically, what results is that when these are combined periodically, the kind of overall thing which we get is something like this. This is called mode coupling or resonance coupling. So, what is happening is that

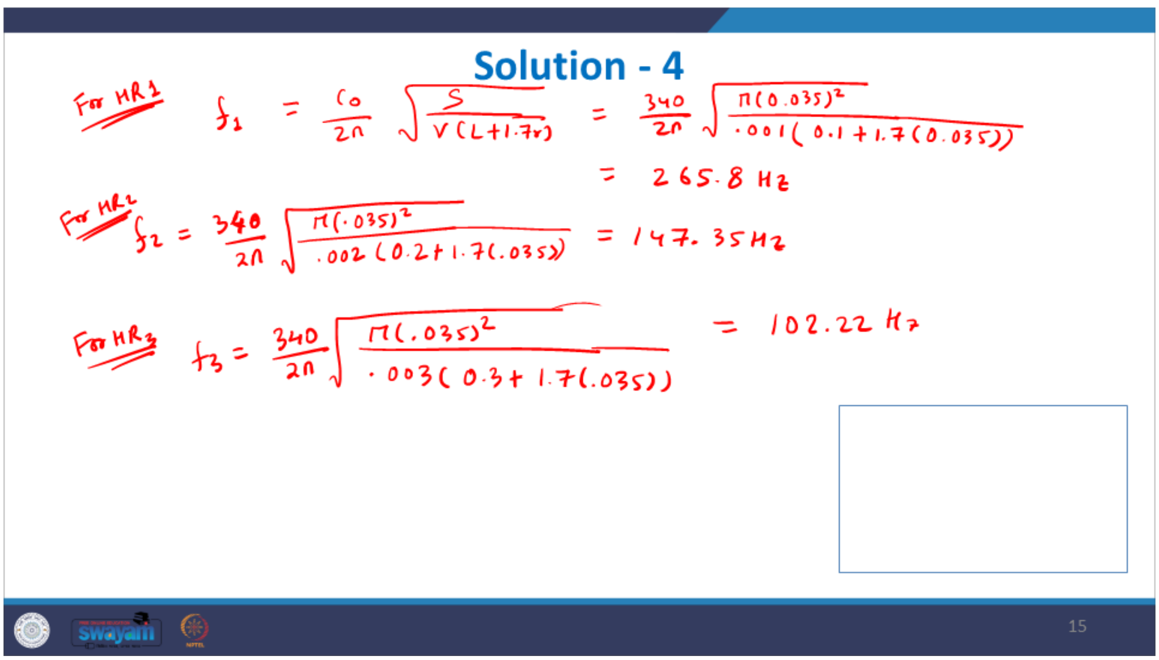
The various peaks are combining together to give a unified peak where the magnitude of the absorption is being compromised, but the width is being broadened. So, here for the unified peak, the magnitude goes down, but the width broadens up or increases, okay. So, this is what you get. So, let us see the individual fundamental frequencies of these resonators 1, 2, and 3. and 3 and see how their individual peaks look like and then we can guess a combined absorption characteristic.



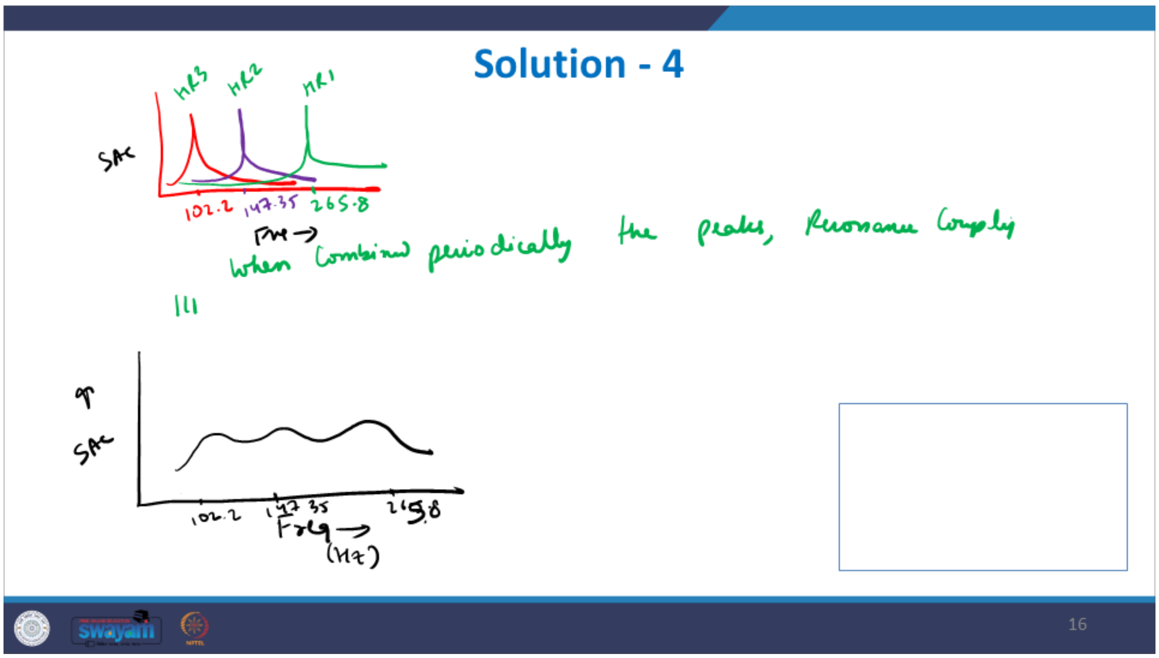
So, for HR_1 what is the fundamental frequency for this one let us say we put the various values. So, we have been given these various dimensions this is the volume of the cavity this is the neck of the this is the length of the neck ok and this is the diameter of the neck. So, these values are given for the different Helmholtz resonators. So, you can put it and find out the value.

For HR1 $$f_1 = \frac{c_0}{2\pi}\sqrt{\frac{S}{V(L+1.7r)}}$$

So, quickly in this one we will be putting the various things. So, in the very first case So, 0.07 so that is the diameter. So, the radius becomes 0.035. So, this becomes pi r square becomes your surface area volume becomes it is already directly given to you 0.001 and the length is 0.1 plus 1.7 times the radius. ok when you punch all of this value you will get a frequency at this place in the same way you can calculate for the second resonator in this series again you punch those values so let me directly put the values in the equation here the in the second case what is happening is that the Radius remains the same, the length of the neck and the volume of the cavity are changing. So, this is the same, but the volume of the cavity is doubling up and the length is also doubling up. So, here the overall frequency should go down. So, here if you calculate this, this is what you will get and for the third Helmholtz resonator, like that 0.02 and 0.002 then 0.03 and 0.003. 0.3 and 0.003. So, if you calculate this, this is the value you should be getting. So, their peaks are close by, but not same.



So, this is what, so one, so individually this is what the peak should look like. So, let us say for the first resonator, sorry for the third one, somewhere here we are getting a peak. For the second one, let us say somewhere here 147 we are getting a peak, sharp peak. And for the third one, we are getting a peak somewhere at 265.8. So, this is your HR 1, HR 2 and HR 3. So, this is what if they were present individually, these are the three individual peaks. but when combined periodically you know the peaks they have this resonance coupling so the peaks they combine together and decrease in magnitude. So, it should look something like this. The combined characteristic should look like this. If this is your SAC versus frequency and SAC versus frequency. So, you should get something like this. If this is your 102.2, this is your 147.35, this is your 265.8—these values are in hertz. So, the peak should be something like this. Like this. You can say something like this. This looks like a peak. So, this is the combined SAC where the magnitude is decreasing, but the width is increasing.



Okay. So, with this, I would like to close the lecture. Thank you for listening.