NOISE CONTROL IN MECHANICAL SYSTEMS

Prof Sneha Singh

Department of Mechanical and Industrial Engineering

IIT Roorkee

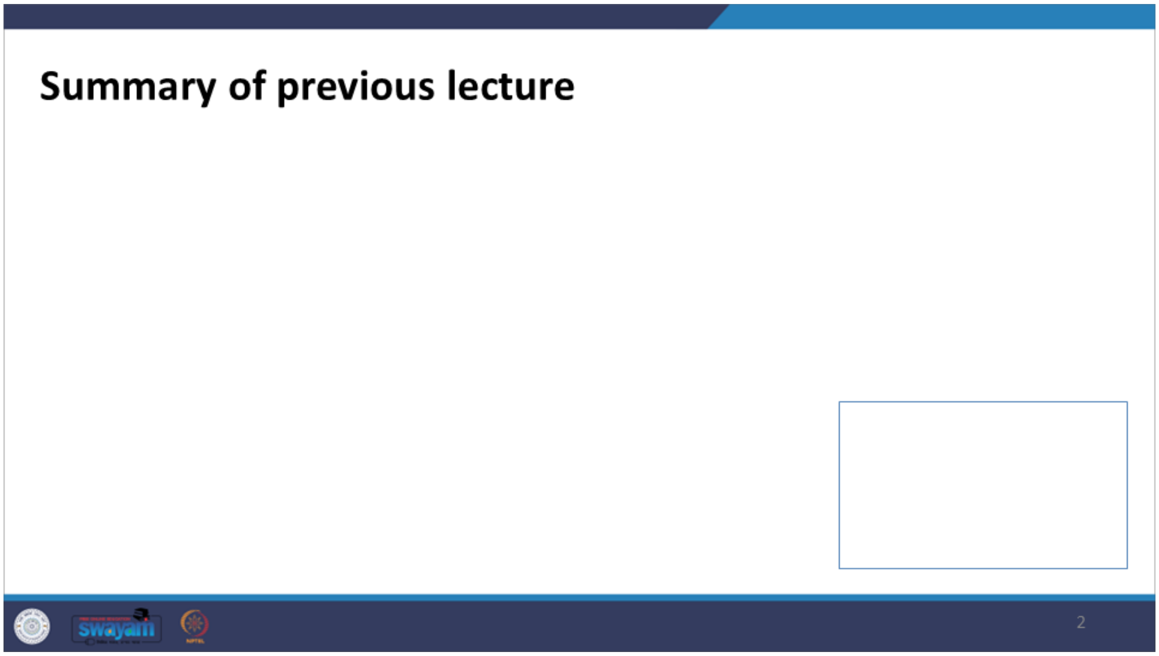
Week:10

Lecture: 49

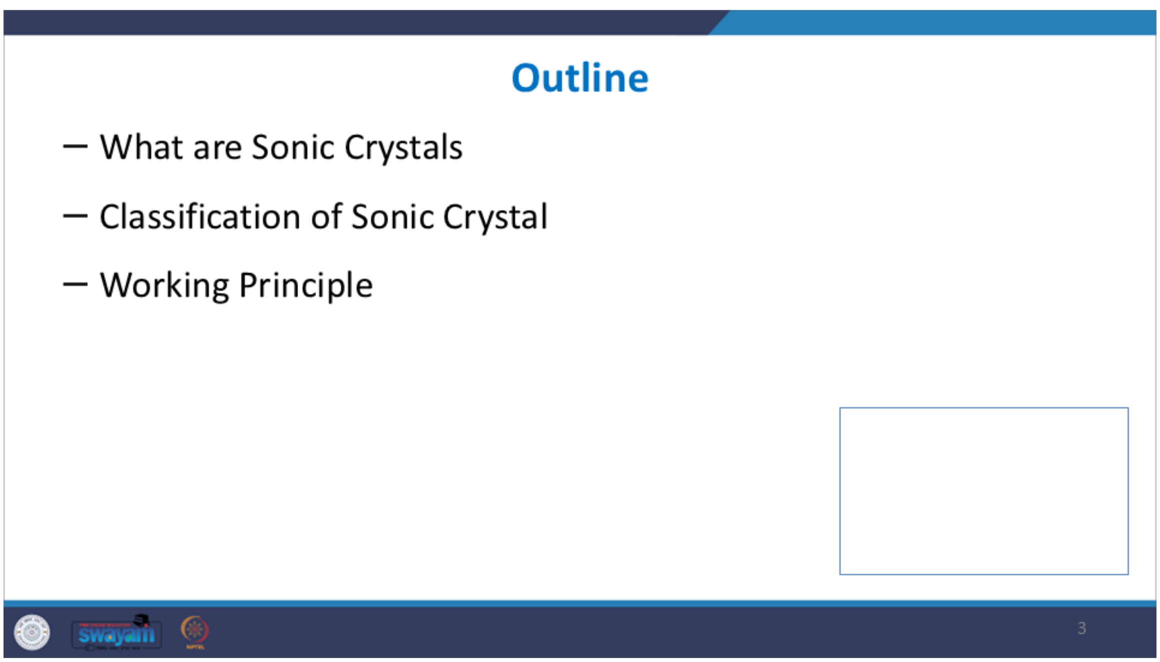
Lecture 49: Sonic Crystals 1

Welcome to this lecture series on noise control in mechanical systems with me, Professor Sneha Singh from IIT Roorkee.

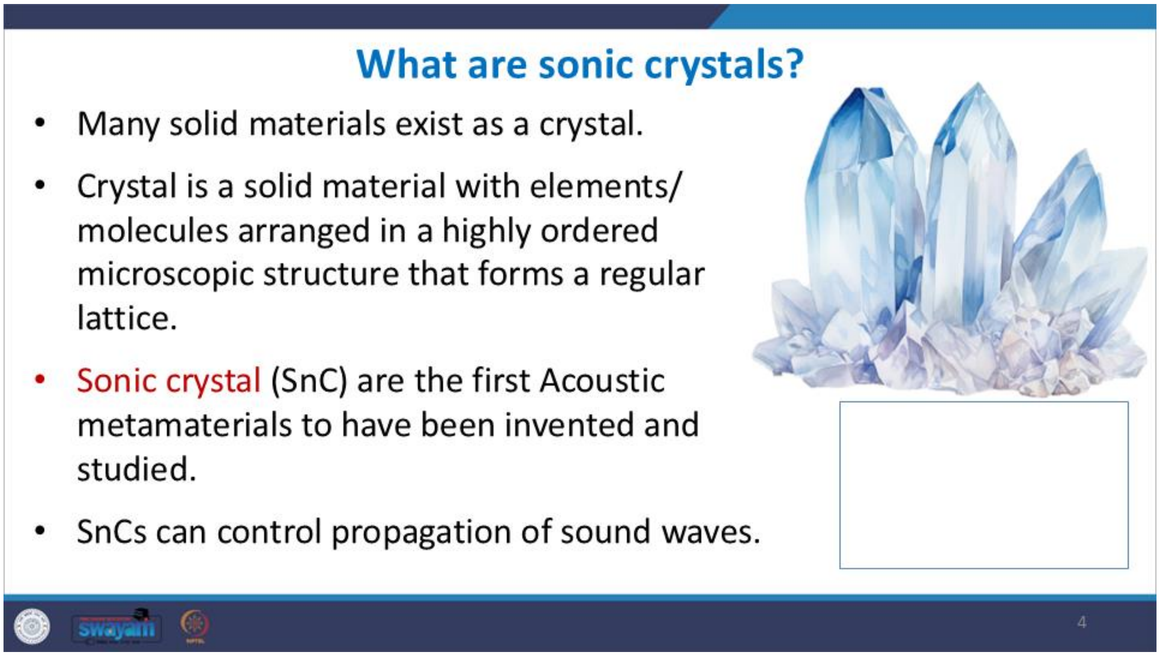
So, we have discussed some noise control strategies. We started discussing acoustic metamaterials as one of the materials used for sound absorption and sound wave manipulation, and then in that, we studied space coiling metamaterials—what the working principle of these metamaterials is, how they can be used to redirect and control sound waves, and how they can achieve attenuation of these sound waves.



Today, we will start with another kind of metamaterial called sonic crystals, and that is what we will be discussing in today's lecture. What are sonic crystals? What are the classifications of sonic crystals? Then, we will discuss the working principle of sonic crystals, and this topic will continue in subsequent lectures as well.



So, what are sonic crystals? Let us start with the word 'crystal.' You can see a crystalline structure here. If you have studied solid-state physics or even in the 10th and 12th classes about crystalline materials, you know that solids can exist in crystalline form or amorphous form. Okay. So, what is this crystalline form? What does the term 'crystal' generally mean for a solid material?

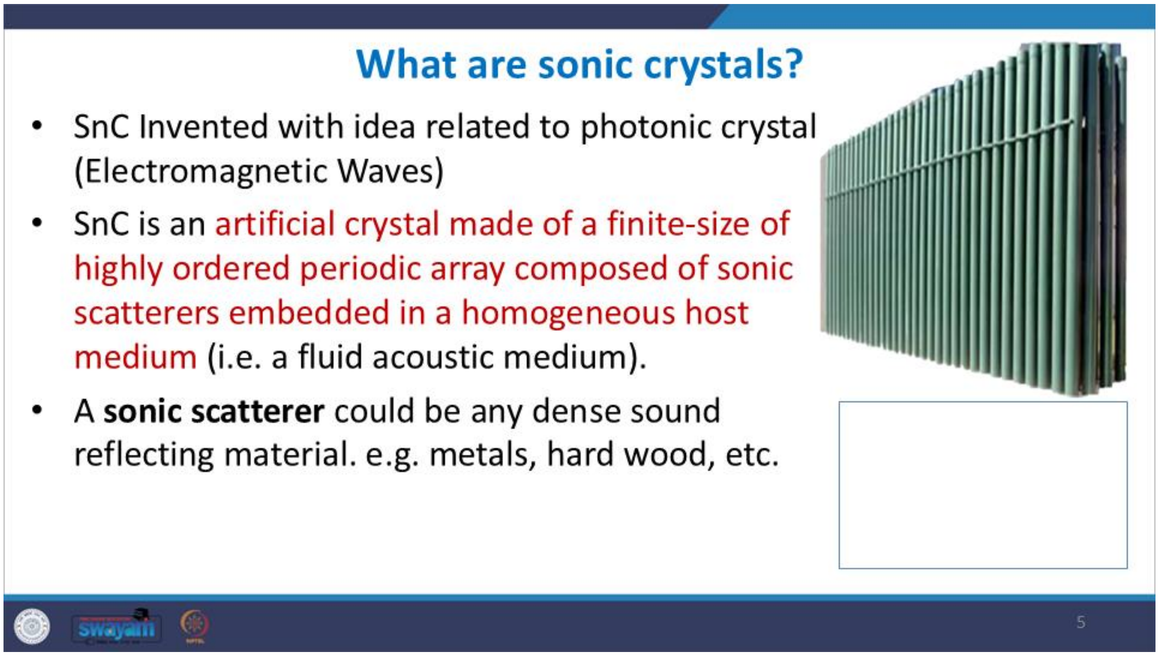


It means that a material or a solid material where its building block could either be its molecules or its atoms, but whatever is making up that solid material is arranged in a highly ordered microscopic structure that forms a regular lattice. For example, you have rock salt or NaCl. It is a crystalline structure, the Na atoms and the Cl atoms. Here, they are arranged periodically in various kinds of lattice structures. You have the face-centered cubic structure, the body-centered cubic structure, the cubic lattice, and so on.

So, let us say, for example, something like this: Na+ and Cl- arranged periodically in some kind of an array. In a 3D format, you can have these kinds of molecules arranged to form various kinds of cubic arrangements, face-centered arrangements, body-centered arrangements, and so on. And this arrangement of the molecules can continue in 3D. Due to this, the materials have a shiny appearance and sharp edges. That is the concept of a crystal, where the element or the molecule exists as a highly ordered set in three dimensions. This ordered matrix is called the lattice. It is the periodic, ordered layout of the molecules that is called its lattice. So, what is a sonic crystal?

It is, you know, some kind of periodic structure. So you might have guessed that it might be some kind of crystalline structure that can be used to control sound waves. So it is very similar. So what is it? It is one of the first acoustic metamaterials that was invented and studied, and it was found to control the propagation of sound waves.

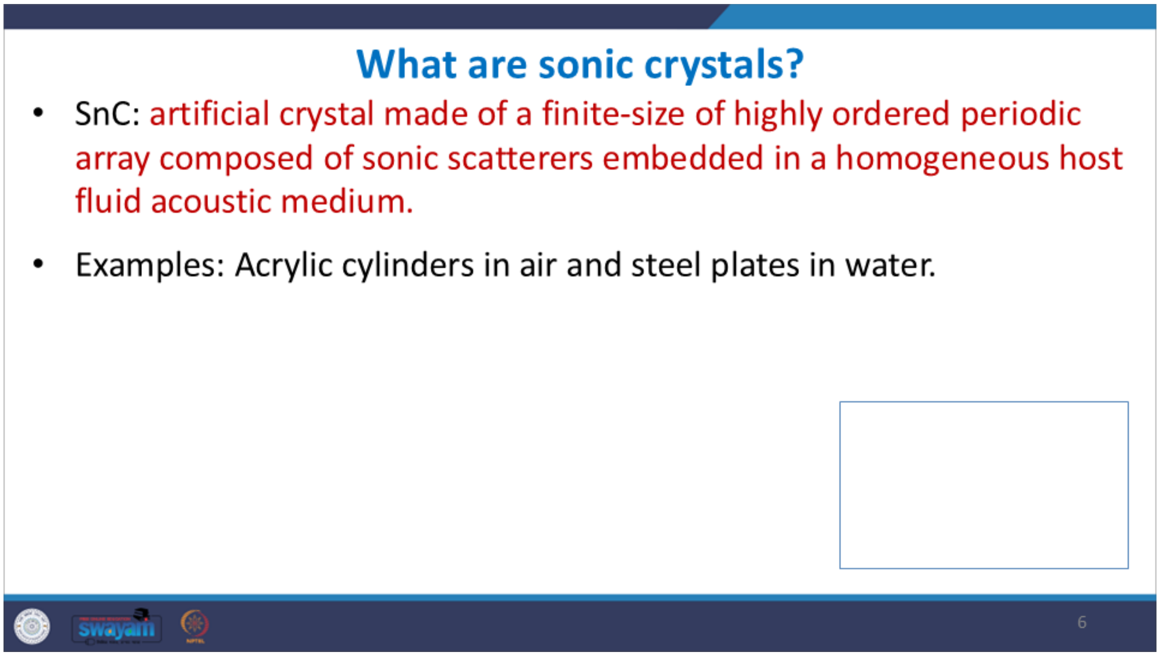
The study of acoustic metamaterials, you can say, very closely coincides with the invention and the formulation of the concept of sonic crystals. And this concept is derived from photonic crystals, which are crystalline structures or periodic arrangements. of materials used to control electromagnetic waves. So, what is a sonic crystal? Let us now straight away come to the definition.



It is an artificial crystal made of a finite size of highly ordered periodic array composed of sonic scatterers embedded in a homogeneous host medium. Do you know what a sonic scatterer is here? Let us see these terminologies. It is any kind of dense, sound-reflecting material. Usually, in the sonic crystal, the host medium is some kind of fluid acoustic medium.

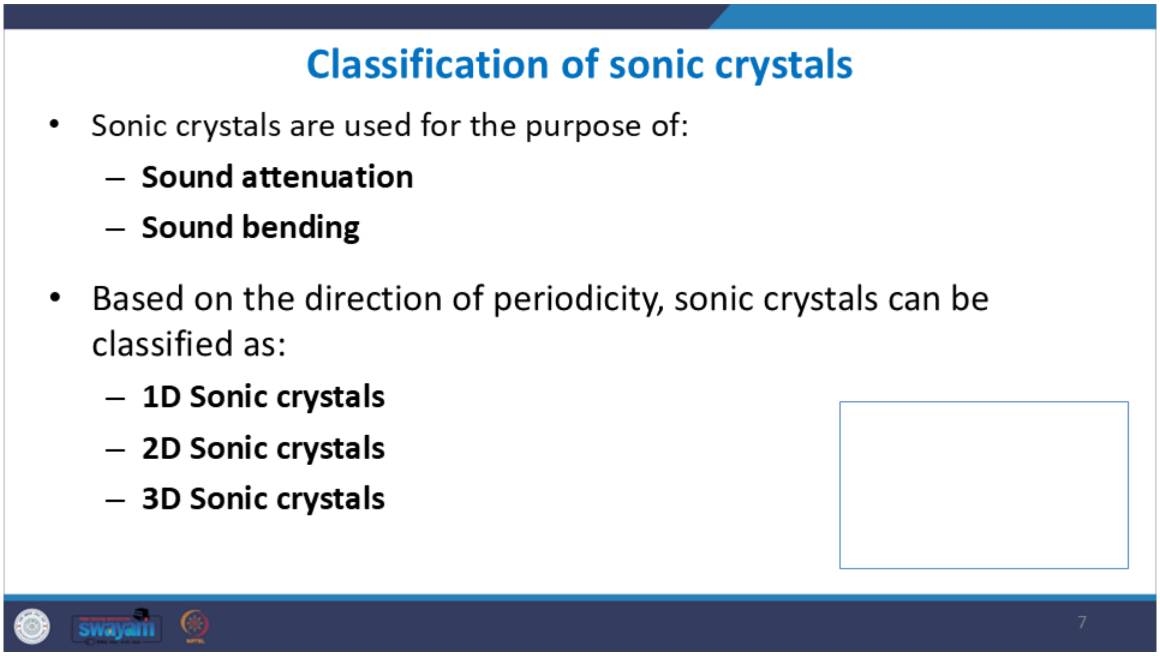
It could be air, water, or some kind of light fluid. And the scatterer is any kind of solid structure. It could have any kind of shape. And, it is the properties that the impedance of that should be much higher than the host medium. So, it acts as an acoustically rigid kind of structure to scatter the waves, okay?

So, it could be metals, hardwood, etcetera. So, let us see, you know, schematically, for example, let us take this definition here. It is an artificial crystalline kind of structure made by a finite size of highly ordered, periodic array composed of sonic scatterers in a homogeneous host fluid acoustic medium. So, for example, let us say, you know, our host medium is air, which is the most common acoustic medium, air.

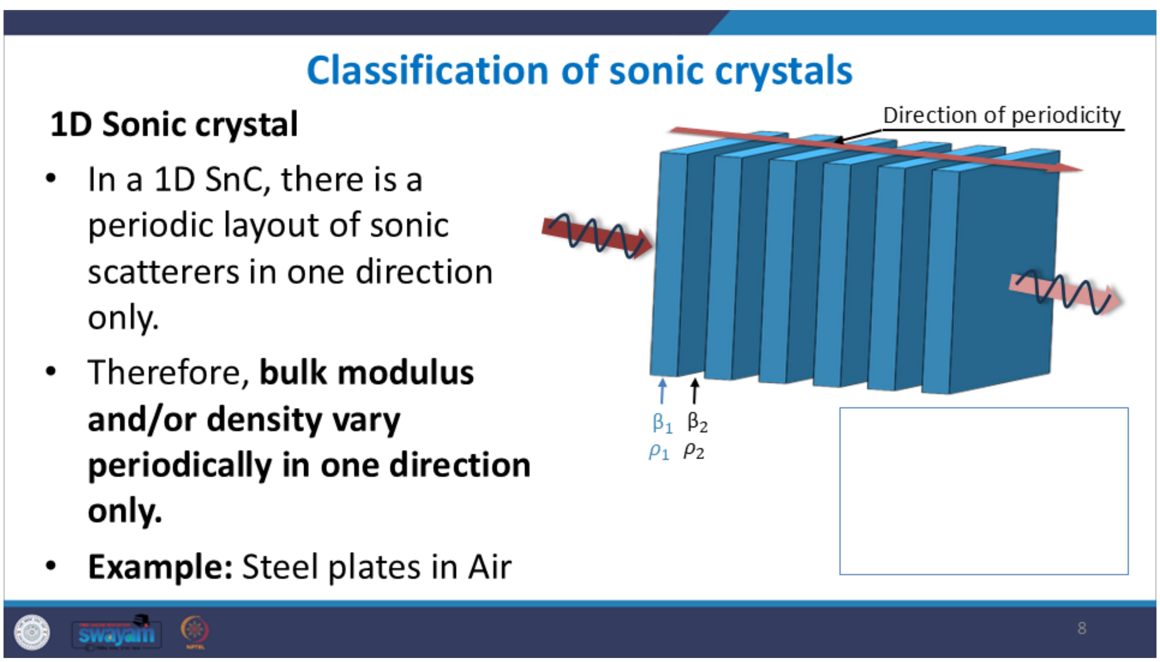


So, in the air, if you could just put cylinders, some acrylic cylinders in the air, and arrange them periodically with a constant gap. So, I am arranging these cylinders periodically in a 2D layout, making them, okay, and maintaining a constant gap. So, some kind of lattice arrangement I am creating. So, here my building block is this acrylic cylinder, which is arranged in a 2D periodic format. Okay, and at a fixed periodicity or in a regular ordered arrangement in the air medium, so this also is like a crystal because here the scatterer is behaving like the molecule of the crystal, which is arranged in an ordered layout within the host medium. Or you can have, you know, some other examples could be steel plates, so we can have large plates of steel arranged periodically

in the air and so on. So, all of these kinds of dense structures are arranged periodically in some acoustic medium or in an ordered lattice format in an acoustic medium; they can be called a sonic crystal. So, what is the classification? So, you know that first of all, sonic crystals are used for attenuating the sound and for bending the sound waves, and based on—because here what is happening, you know—you can boil down a sonic crystal to simply any kind of dense scattering material structure that is arranged in an ordered periodic layout in air or any kind of fluid medium.



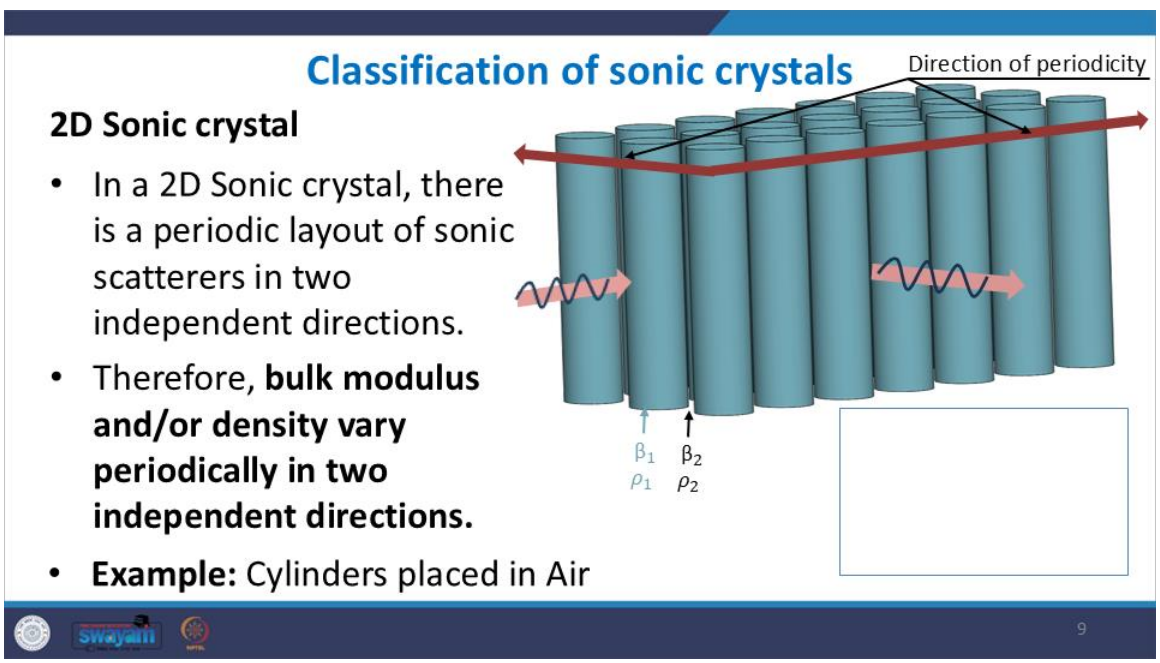
In the fluid medium, we have some scattering structure and we are arranging it in a periodic layout. So, depending on what is this periodic layout or what is the direction of periodicity, the sonic crystals can be classified as you know 1D, 2D and 3D sonic crystals. So, let us see a 1D sonic crystal here you know you can see this in this figure here. We have arranged the steel plates in air and we are maintaining a constant gap between two steel plates.



Let us say you know the gap between this and this. And this and this is some constant gap and in an ordered fashion not haphazardly but in an ordered fashion okay. So the gap is maintained constant and we are arranging it okay this is some constant gap we can say. gap we are maintaining between the steel plates then what you see is that here there is a periodic layout in only one direction ok the periodicity is in this direction here pointed given by this arrow this gives you the direction of periodicity. So, what you observe here is that let us say you know beta 1 and rho 1 are the bulk modulus and the mass density of the steel and beta 2 and rho 2.

So, this is let us say for the steel here or the scatterer whatever scatterer you are using. In our case for example, we are using steel and this is for the so this is for the scatterer and this is for the host ok. then what you see is that if you observe it in this direction periodically the beta 1 and rho 1 is changing. So, when you know the sound waves are propagating from here ok, these sound waves they are propagating from here. So, first they encounter a medium with a different beta 1 and rho 1 and then they encounter a new medium.

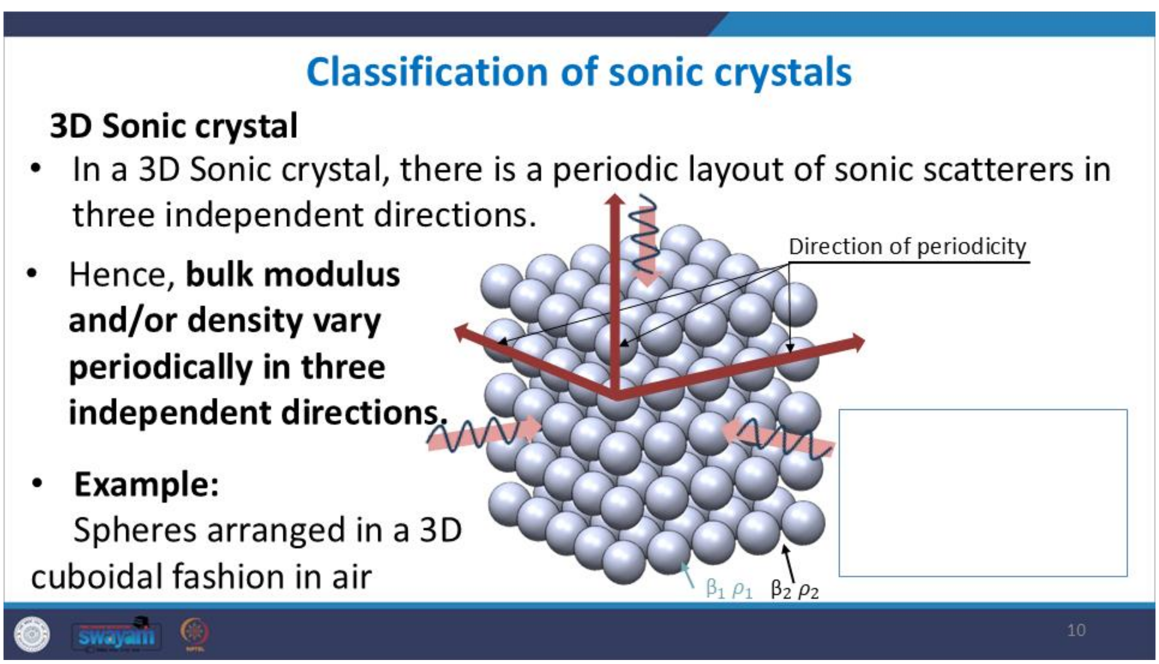
So, first of all we have a medium with particular beta 1 rho 1 and then we have another medium with a different beta 1 rho 1 and then we have the same medium again with a beta 1 rho 1 and then we have the airs So, in that way periodically the beta 1 and rho 1 which is the bulk models and density keeps varying periodically at a certain gap in this direction. In the 2D now same thing happens now in 2 different directions. So, for example, you have the solid cylinders, you can have the steel cylinders placed in the air.



So, what you see is that and they are placed in a 2D periodic layout. So, here the variation that is happening will be in two different directions. Ok. So, once again if you have this as the property of the scatterer that you are using and this is for the property of the host medium. They both have different bulk modulus and density. So, what you see is that if you take this as your two dimensions, ok this and this, what you see is that when the sound wave is propagating, there is a change in the these properties periodically in this direction. Ok.

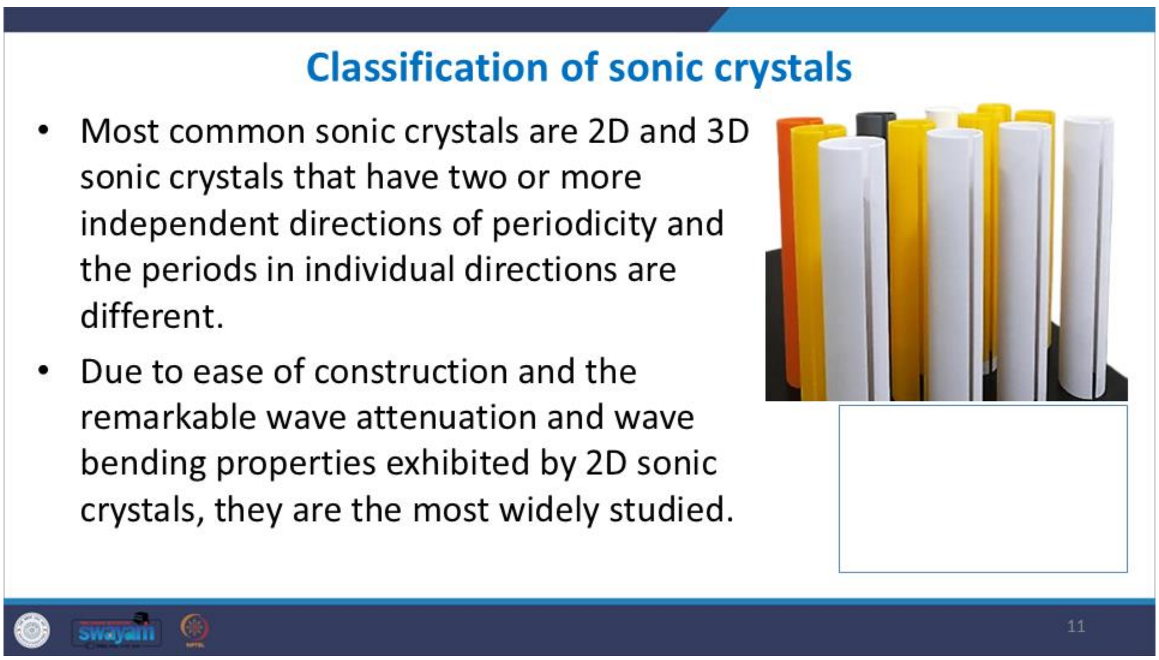
So, first they encounter the cylinder, they encounter the air medium, then again the sound waves will encounter the cylinder with a different row and then again they will encounter the air medium. and then they will again encounter the cylinder medium and so on. So, the B, rho keeps changing periodically over this direction because the cylinders are periodically placed in this particular direction. But the same thing happens here as well in the other independent direction. The sound waves here you know they are

This particular region has a particular beta rho, and periodically there is a variation: a new beta rho for the air gap, then again this beta rho for the steel cylinder, then again the beta rho for the air gap, and then the beta rho for the steel, and so on. So, periodically in the two directions, the properties keep changing, okay? And in the 3D model, you can now have the spheres of steel arranged in a 3D cuboidal fashion like this. And as you can see here, in this direction, the properties keep changing periodically, okay? The air gap—so beta rho for the steel—the sound waves will encounter, then the beta rho for the air gap, then again the beta rho for the steel, and then again the beta rho for the air gap.



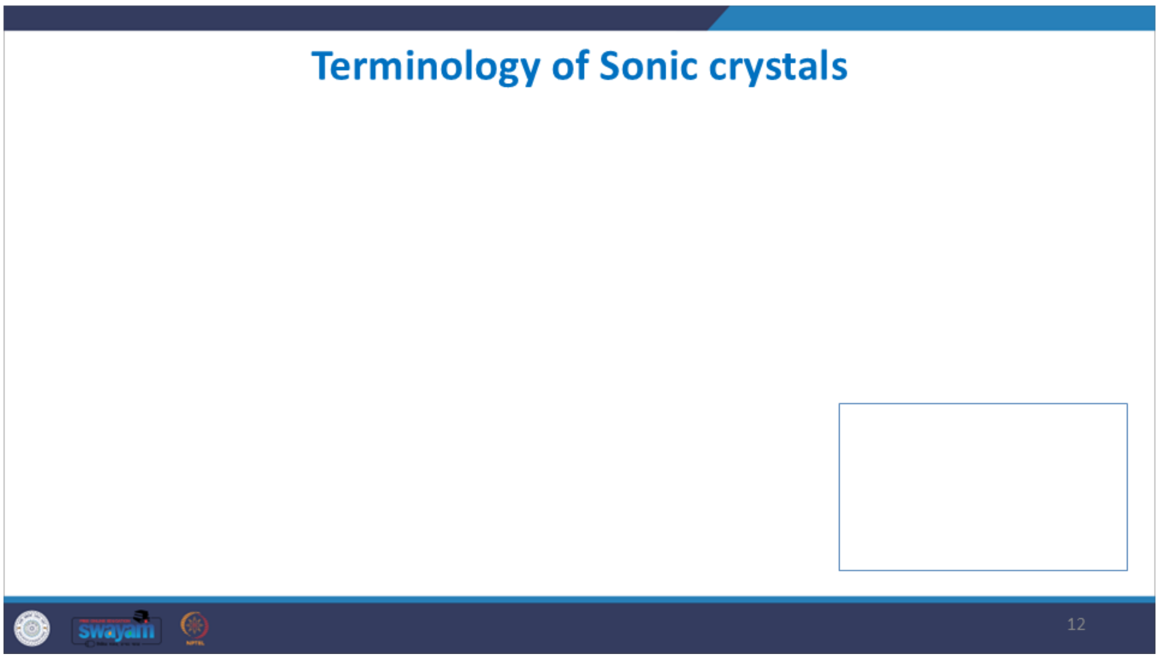
And then the steel and air gap, and so on. So, this is happening in this direction, and there is a periodic layout in 3D. So, even here in this direction, periodically, the properties keep changing. When the sound waves are propagating, okay? So, the red zone indicates the beta rho for the steel, and the yellow zone indicates the beta rho for the air medium, and so on.

And in the same way here, you can also have periodically changing properties, and here the directions of periodicity are independent of each other. So, three independent directions, and in the 2D and 3D sonic crystals, the periodicity may be different for the independent directions. It can be the same also, or it can be different like that. So, it can have the same periodicity, it can have a different periodicity, and so on. Okay. Now that we know these are the three types of classifications, the most commonly studied solid crystals are these.



Sonic crystals are 2D and 3D kind of sonic crystal. Again this shows a typical sonic crystal that was manufactured in a lab one of the research studies where we had these hollow solid cylinders arranged in a 2D format. These are some of the common sonic crystal studies are 2D and 3D because they are able to better manipulate the sound waves. And once and specially when you have independent you know you are giving them two or more independent directions of periodicity and specially if the periods you maintain as different you get. So that is the most commonly studied one because you are getting more unique properties and a better sound wave manipulation if you have different periods in the two different directions ok.

So different periods in the two different directions ok. So, out of the two, you know, 2D and the 3D sonic crystals, the 2D is more easier to manufacture, not just easier to manufacture in terms of the scatterer, but it is also easier to install and arrange, okay. So, the installation and the arrangement as well as the manufacturing is much easier for 2D and hence they are the most widely studied kind of sonic crystals. Let us just see a few terminologies related to sonic crystals. So, now we have firstly the scatterer which you already know.



I will indicate it using some. So, let us say I am looking from the top and we have an arrangement of crystals in a 2D format. So, let us say you know steel cylinders arranged in an ordered 2D layout in air. Ok. And here the steel is indicated by this black ball.

So, I am looking at the top layout of it, and the air is indicated by simply the white zone or fine I can, and the air could be represented by this. This kind of a dotted air medium, then this is the, you know, the top view of the layout, and let us define the various terminologies within this. So, remember that this is a 2D crystal. So, there will be two independent directions of periodicity and hence two different periodicities in the two directions. Okay. And so on, and the same cylinders arranged like this here, 2D layout.

Okay. And so on in this direction as well, and we have the air medium surrounding them. So, this is the air medium over which they have been embedded, and this shows the top view of this structure. Now, let us define some terminologies related to this. Here, our steel cylinder is the scatterer; here, this steel cylinder which is given by this thing here is your scatterer or the sonic scatterer, also you can say, because it is So, this is what this is—usually any kind of structure which has a high impedance compared to the host medium and can scatter the sound waves incident on it. So, if a sound wave is coming, typically when it is incident on it, most of it gets scattered; only a very small portion of it passes through. That is the definition of the scatterer: it is able to scatter, able to reflect or scatter most of the sound energy incident on it. Okay, that can scatter most of the sound energy incident on it. It can scatter or reflect back most of the energy; transmission is very low, the transmission.

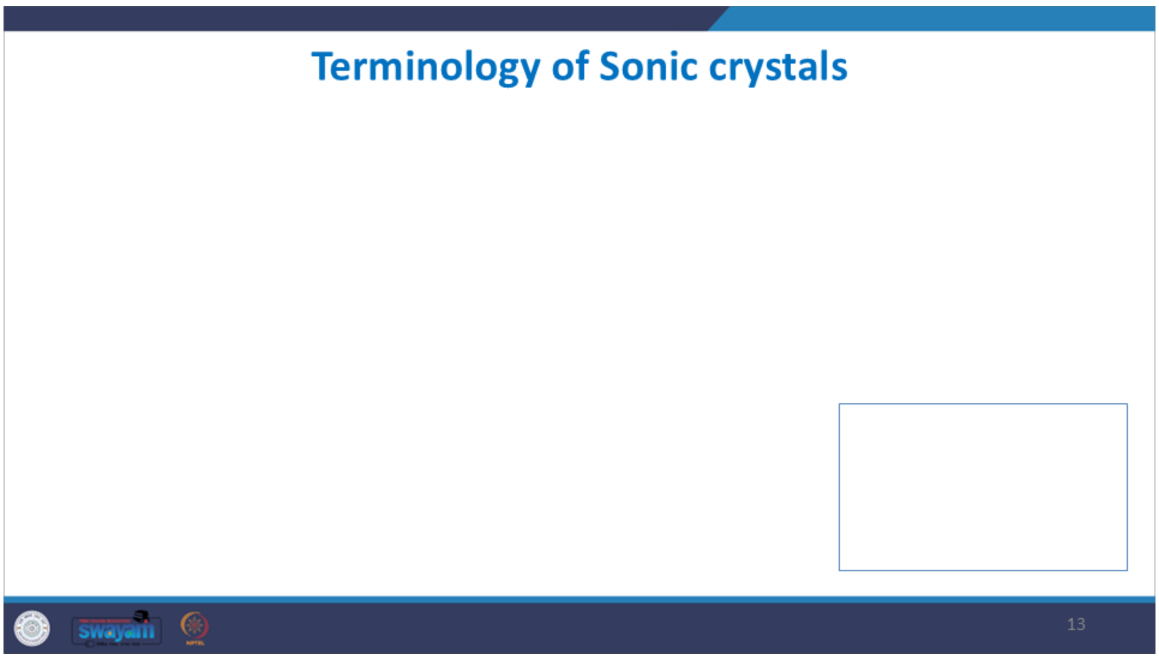
is very low for these scatterers, okay. So, how can this be achieved? You can have a higher acoustic impedance, much higher acoustic impedance of the scatterer compared to the host medium, then you would be able to get more reflections and scattering and very little transmission because of the higher difference in the acoustic medium, okay. So, they are able to reflect or scatter most of the sound energy incident on them, and the sound transmission is very low through these scatterers, okay. And here the host is what?

This is the air medium. Here, the air medium itself is the host. And what does it mean? It is the acoustic medium with much lower impedance than the scatterer. Okay. So, it is that acoustic medium which is in the background where the scatterer is embedded, and it has much lower acoustic impedance than the scatterer.

Now, let us see the periodicity. So, over here, what you observe is that, you know, in this direction. So, let us see these two independent directions of periodicity. Okay. Let us call it some direction x and y. Then, in the direction of x, what you observe is that. There is a constant gap between the crystals.

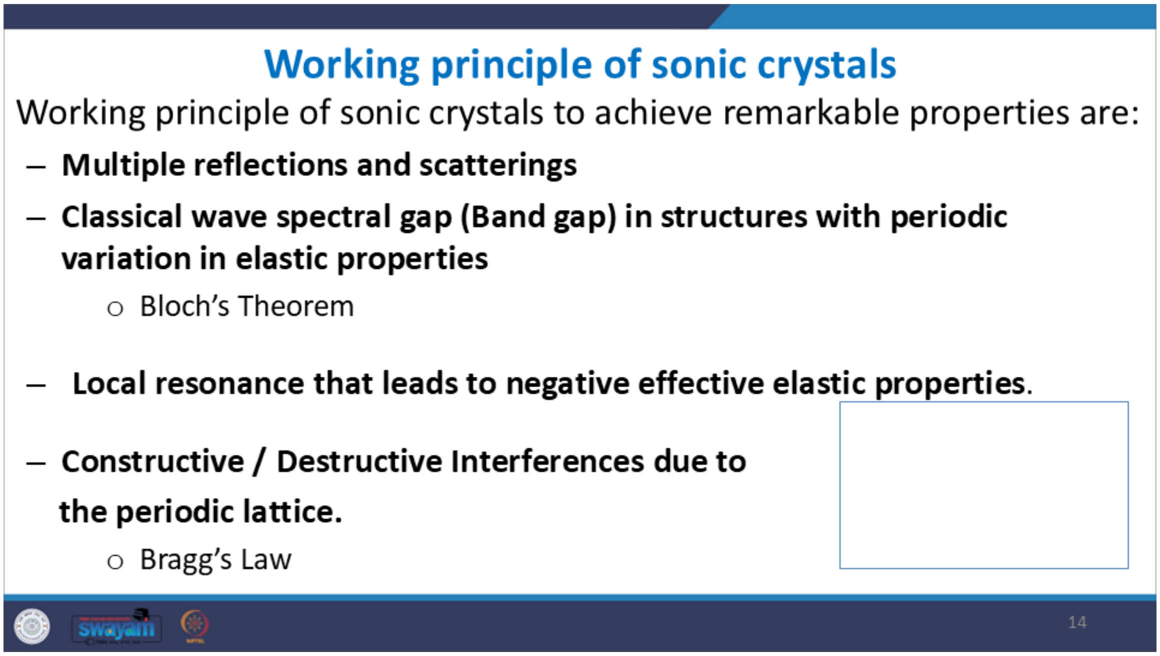
Let us call it a1 and so on throughout, and in the direction of Y, there is a constant gap of a2. And so on, ok. So, here, periodicity in the X direction is simply a1, ok, which shows, you know, how periodically it is changing. So, this is simply the spacing between the scatterers, ok. So, this is simply what this is. Even it can also be seen as the uniform spacing. In many of the recent research, you have sonic crystals with irregular spacing or continuously varying spacings as well, but primarily in the very basic sonic crystal structure that was proposed, usually, you know, the periodicity was maintained constant in the direction.

So, this is the uniform spacing bit measured between the centers. Of the two adjacent scatterers, ok, uniform spacing that was maintained between the centers of the two adjacent scatterers, ok. We can also call it as a vector, just like in the crystal, we have a lattice vector. If suppose these are your X and Y directions, we can assume a vector a1. There is a vector, you know, this vector here a1, and then this vector here a2. The a1 and the a2 vector then here a1 can be called as the lattice vector. It is a primary lattice vector. Okay, a vector has a magnitude and a direction, so here it is a vector. Its magnitude is the periodicity in that direction, is equal to the periodicity value in that periodicity value, and the direction of this lattice vector is the direction of the periodicity, okay. In the same way, you know, the periodicity in the Y direction is simply a2, which is the uniform spacing. This is for the X direction. This is the uniform spacing between the centers.



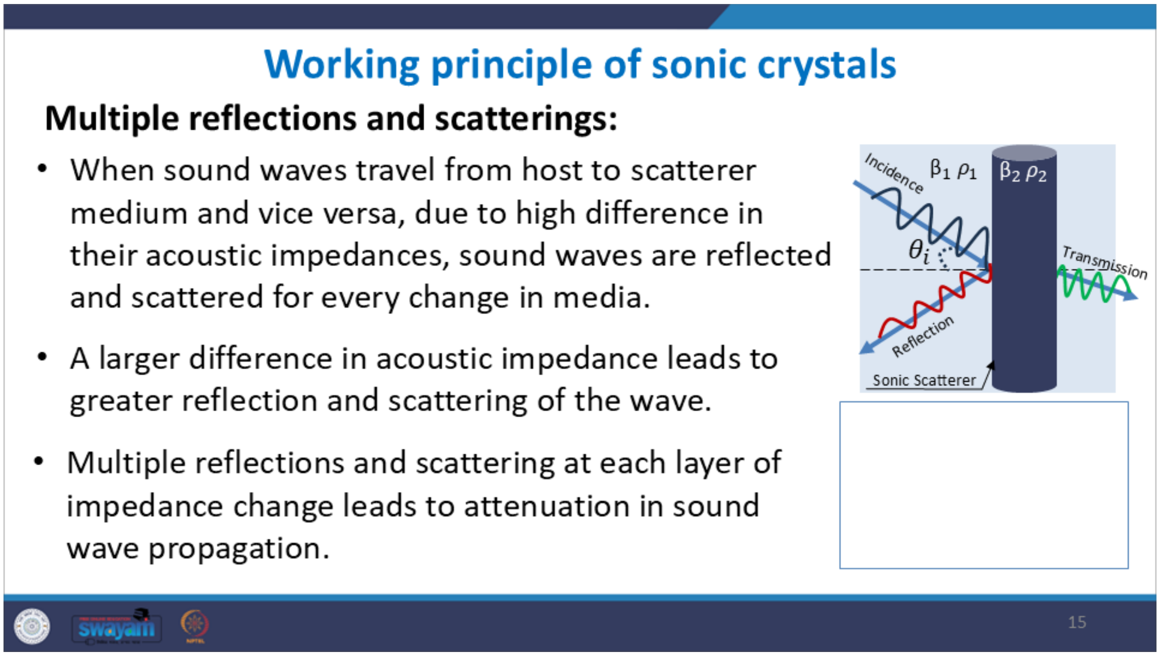
Of the two adjacent scatterers in the Y direction or the other independent direction, and the vector corresponding to that a2 vector is another primary lattice vector. So, for a 2D sonic crystal, we will have 2 lattice vectors, 2 independent lattice vectors. For 3D, we will have 3 independent lattice vectors, and for 1D, we will just have 1 independent lattice vector. So, this is another primary lattice vector, and because it is a vector quantity, what it means is that whose magnitude is simply a2 or the periodicity. So, the magnitude of this a2 is the. Value of periodicity or the spacing, and the direction is the gives the direction of the periodicity, ok.

So, these are some of the basic terminologies—these various terminologies we have: the lattice vectors or the primary lattice vectors. Then, the periodicity values, and then the scatterer and the host. These are some of the basic terminologies, and I have explained them using an example of steel cylinders arranged in a 2D layout in air. What do they mean? Now, let us see, because when I discuss the working principle, these terminologies might be used.



So, I sort of incorporated that. So, the working principle of sonic crystals can be defined—you know, there are four working principles, you can say. The first is multiple reflections and scatterings that happen while the sound wave is propagating. Then, you have the wave spectral gap that results, or the band gaps that result due to the periodic variation in the elastic properties, and the theoretical underpinning of this kind of phenomenon is the Bloch's theorem. Then, we have the local resonance that leads to various kinds of negative elastic properties.

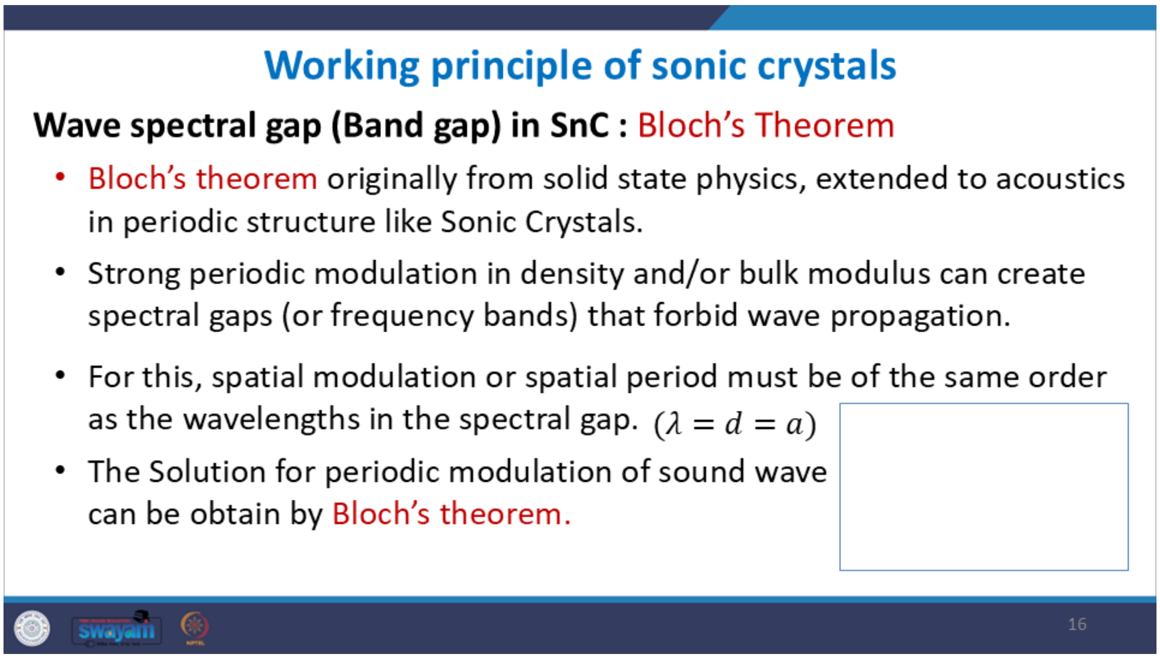
Then, we have some constructive and destructive interference that happens because of the periodic lattice itself. And here again, the theory is based on the Bragg's law of refraction. So, we will study them. So, in this class, we will just study the first two, and then in the next lecture, we will continue our discussion on the next two kinds of working principles. So, firstly, multiple reflections and scatterings.



Obviously, you know scatterer is what? It is a medium of much higher impedance compared to the host medium. So, the name itself says a scatterer. So, which means that when a sound wave is incident on it while passing through the host, most of it gets reflected or scattered away and this transmission is low. okay.

The reflection is high, the transmission is much low and the higher is the difference between the acoustic impedance of the two media, the more will be this reflection and scattering that will happen. But here we do not have just one scatterer that is leading to the reflection and the scattering. If you think about it here in this case, suppose some sound wave is propagating, it will first get scattered through the first layer and a small portion passes through then again there is some scattering and a very small portion passes through then again this one has some scattering and then and over as the you know the layouts continue the more and more wave is getting scattered and this is leading to the attenuation of the sound wave so you know by the first layer a certain portion of the energy is getting reflected back and the transmitted energy is lower and when the second round of scattering happens through the second layer of the scatterers then some more energy gets reflected back and even much lower transmitted energy passes through into the third layer of scatterers

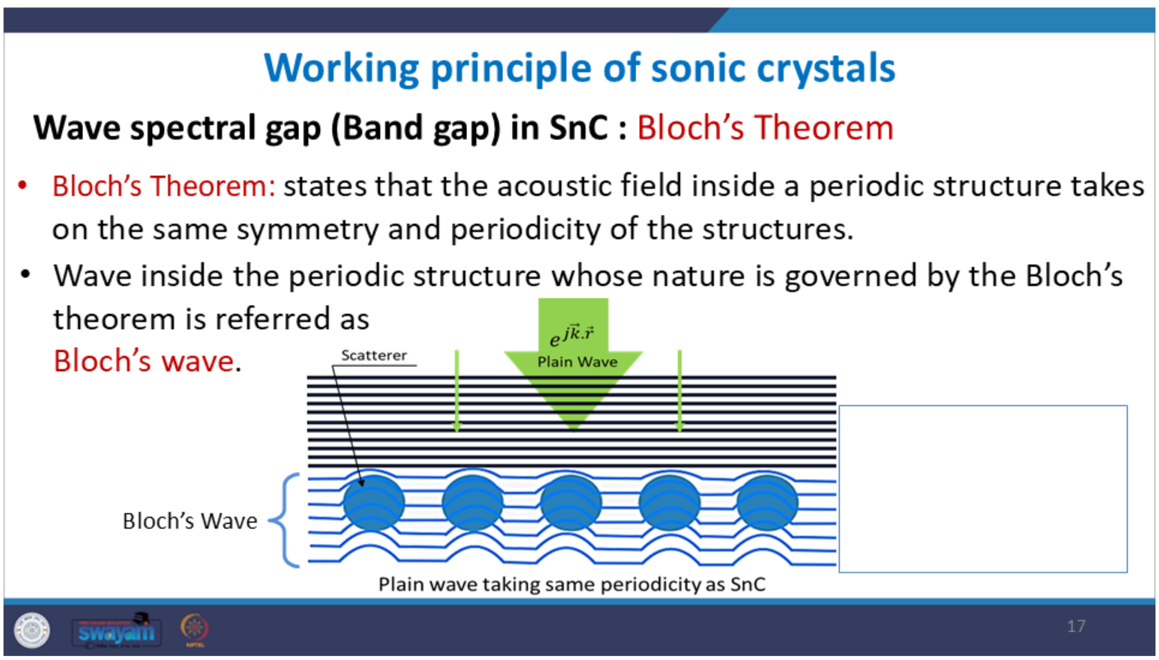
Again this phenomenon continues and by the time you know the sound wave is going through periodically through several layers of scatterers much at each layer you know reflection is happening scattering is happening and transmission is getting further reduced and this happens multiple times and due to this these you know multiple scatterings and multiple reflections happening at these multiple layers it leads to an overall attenuation in the sound wave propagation. Ok. Then the second one is the wave spectral gap or the band gap in the sonic crystals which is governed by the Bloch's theorem. Again the Bloch's theorem came first from solid state physics but then this theorem was extended to the acoustics of the periodic structures such as a sonic crystal. And this theorem is valid for any kind of periodic structure and by periodic structure in acoustics we mean a structure where the acoustic properties are varying periodically.



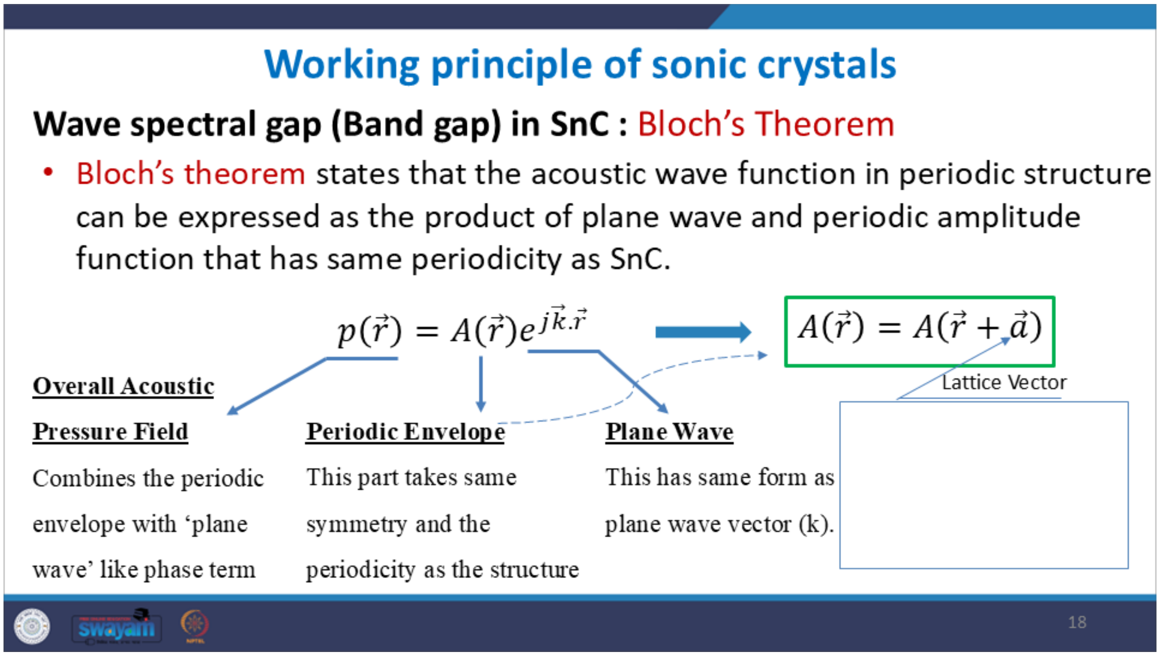
What are the major acoustic properties? The major acoustic properties are beta and rho, or the bulk modulus and the mass density, and these vary periodically. That will create a periodic structure, and in that kind of structure, Bloch's theorem will be valid. So whenever we have any structure where there is a strong modulation or a strong periodic modulation or variation in the mass density or in the bulk modulus of the structure, or both, it can create some spectral gaps of frequency bands where wave propagation is forbidden, okay.

So that is essentially what happens, and for this to happen, the spatial modulation or the period by which the properties vary should be of the same order as the wavelength in the spectral gap. So suppose you want to control the sound waves of lambda wavelength, then the gap or the periodicity that you will have to maintain in the structure is the same as the wavelength that needs to be controlled. Then what will happen is that those wavelengths will not pass through essentially, and they will be blocked. The regions or the frequencies which are getting blocked are also called the frequency band gaps. So, let us see how this happens, okay.

Why the periodic variation is stopping some sound waves from traveling through, such that the sound waves that are getting stopped have the wavelength which is of the same order as the periodicity of these periodic structures. So Bloch's theorem states that the acoustic field inside any periodic structure will have to take the same symmetry and the same periodicity as that of the structure itself. Okay. So, let us say waves that follow this Bloch's theorem are called Bloch waves. So, let us say we have some periodic structure and a broadband sound wave was incident on these periodic structures, but once the sound wave is passing through, due to Bloch's theorem, it says that once the sound waves enter into this periodic structure, only those sound waves which have the same periodicity as the structure itself will not pass through.

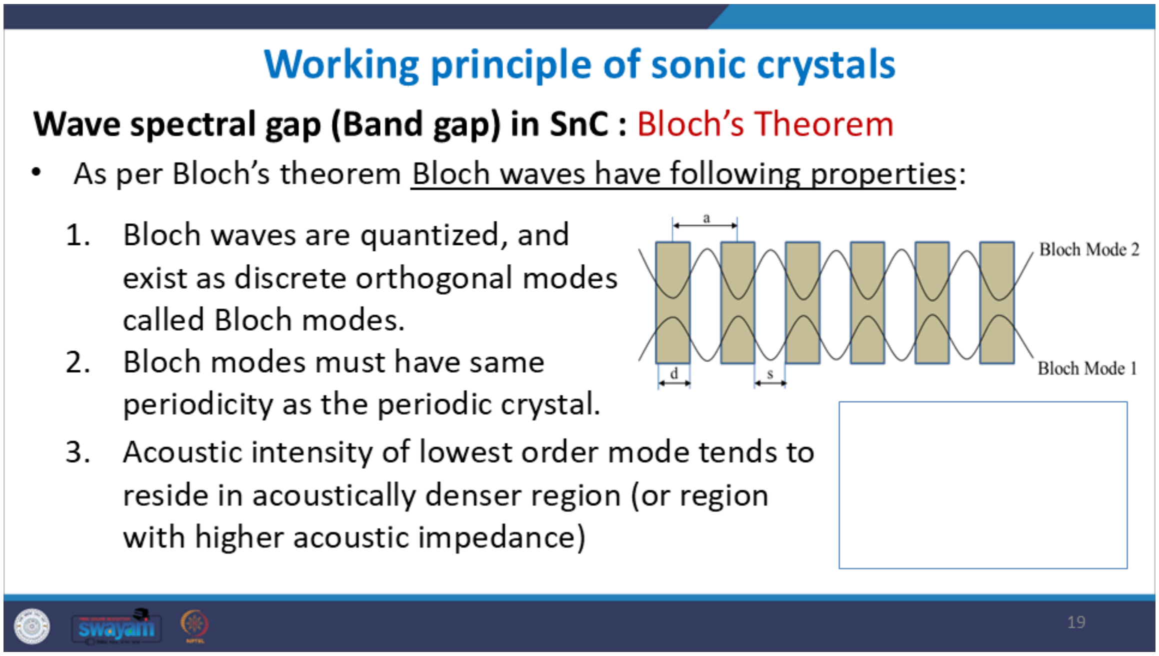


So, the sound field inside the structure can only exist in this format where it has the same periodicity as the crystal or the structure. And let us say we can represent the overall pressure field inside a sonic crystal using this formulation. So, here what we have is the overall acoustic pressure field; this is the periodic envelope, okay. So, this part takes the same symmetry and periodicity as the structure; it is given by this format. So, here the amplitude of the wave is varying periodically, and the periodicity is the same as the lattice vector of the sonic crystal.



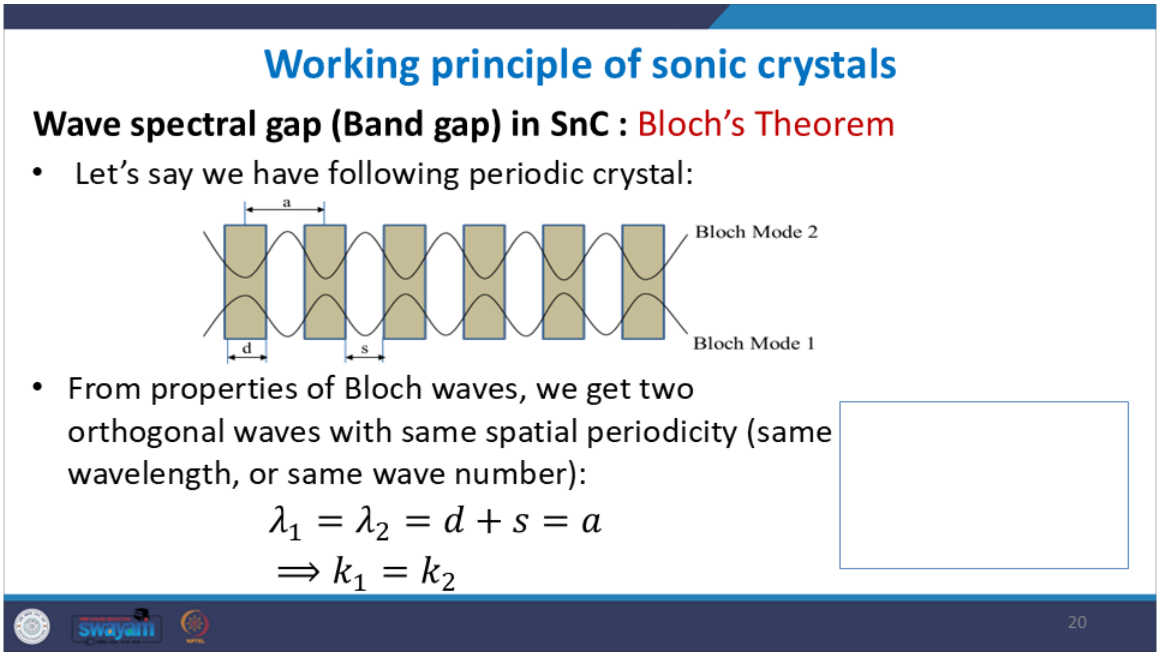
So, suppose We are considering a sonic crystal that has some periodicity of A, which means that the B and rho are changing after every distance a. Then the amplitude of the wave will also be periodic in nature and will change with every distance a. Okay. And because it is a plane-propagating wave, the nature of propagation will be maintained as a plane-propagating wave. But the overall amplitude will be periodically varying with the periodicity of the structure.

So, instead of having a simple constant amplitude into e to the power jk dot r, now we have a varying amplitude with a plane wave component. So, some of the properties when Bloch's theorem is solved in solid-state physics lead to the Bloch waves having the following properties. Once again, the derivation of this is not within the scope, but rather The Bloch theorem is a very extensive concept that you can study in physics and solid-state physics. But for acoustics, we are just taking the key conclusions of the theorem and applying it to see how sonic crystals behave.



So, the key concluding points of this theorem are that if the Bloch waves exist in this periodic structure, They are quantized and they only exist as discrete orthogonal modes, and these are called the Bloch modes. And these Bloch modes must have the same periodicity as the periodic crystal itself. And the acoustic intensity of the lowest-order mode should reside in the acoustically denser region, which means that the pressure maxima should occur in the medium of higher acoustic impedance. So, just keeping this in mind, let us start seeing one by one what happens.

So, let us say we have this periodic sonic crystal structure. So, within this structure, the sound waves can only exist as waves having the same wavelength as a. So, the very first thing is that Only the sound waves where lambda is equal to a—that kind of sound wave—can exist within this structure. So, the overall field will always have lambda equal to a. Let us say the first mode—this is one Bloch mode—is this, and this is another Bloch mode. When you go from a lower mode to a higher-energy mode, then the higher-energy mode has a pressure maxima in a lighter medium.

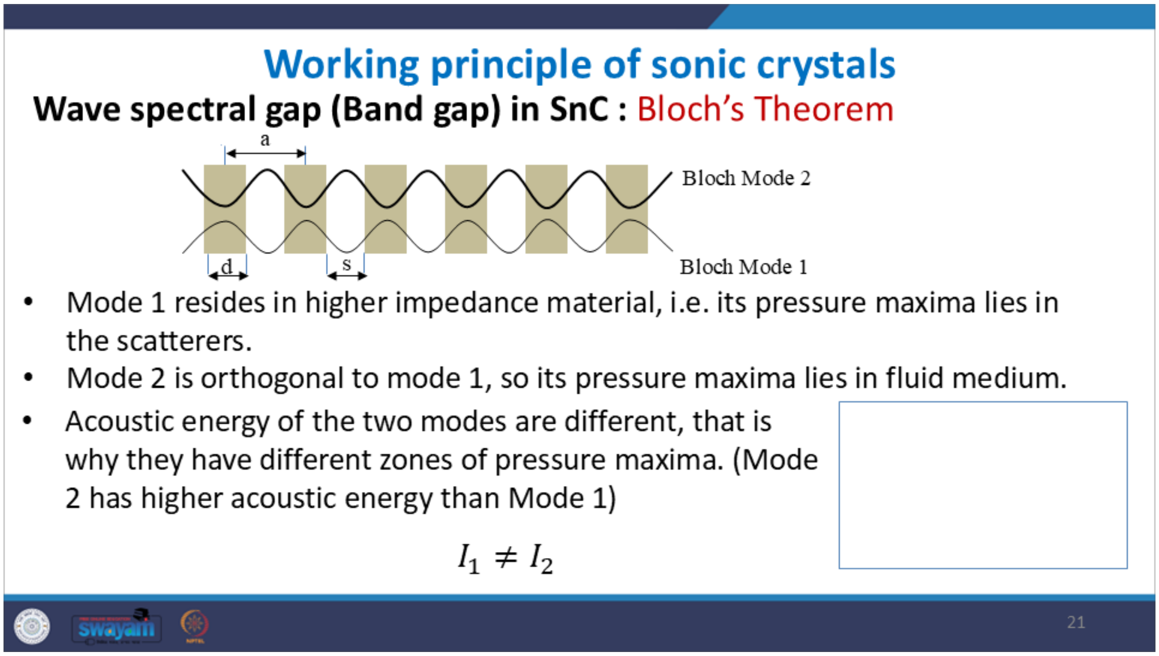


So, this higher-energy mode has a pressure maxima in the air, whereas the lower mode has the pressure maxima in the denser region. So, from the properties of the Bloch wave, we know that the waves inside this periodic structure, or the sound field, must maintain the same periodicity as the periodicity of the structure itself. So, which means that the wavelength—the spatial periodicity of a sound wave—is what? It is the wavelength. Okay. The temporal periodicity is the frequency. Okay.

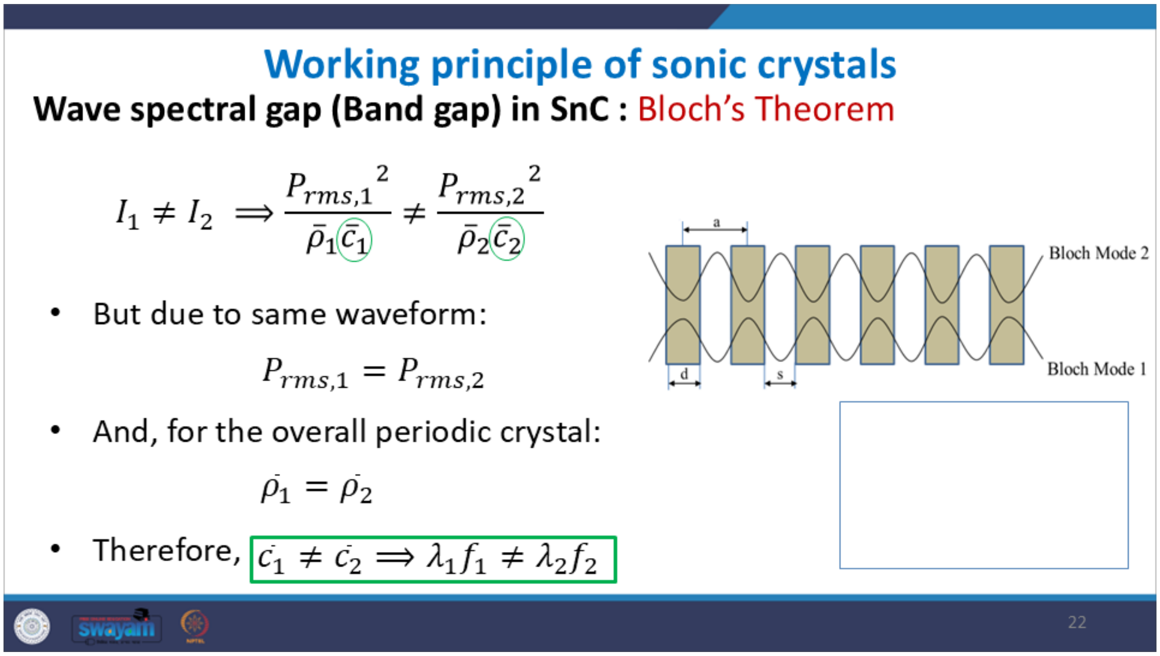
With time the periodicity of the wave is called as the frequency. Whereas, with space the distance over which the wave is repeating its pattern that is the wavelength. Ok. This is what? Wavelength is what? It is the spatial distance over which the wave is repeating its pattern.

So, basically By definition, the lambda itself is the spatial periodicity in the waves, okay? And the Bloch's theorem says that the spatial periodicity of the block waves must be same as the structure's periodicity. So, that means that if suppose sound waves are passing through a sonic crystal of where the spacing is a in between the scatterers, then the lambda also should be same as a, d plus s okay the lambda should be same as the periodicity of the structure and so the lambda of the structure is same k is what 2 pi by lambda so this means that k 1 and k 2 the k vectors is going to be same and the lambda is going to be the same as the structure itself now

these are the two modes. Now, because mode 1 is residing in a denser zone and mode 2 is residing the pressure maxima of it is residing in a lighter zone which means that the two modes they have different energies where mode 2 has a higher energy than mode 1. Again let me refer back to this to the property third property. So, by this third property, if suppose two modes have different energies, then their pressure maxima happens at the different regions. Whenever their energies are different, the pressure maxima or the peaks, okay, the crests and the troughs, they happen in the different spatial regions.



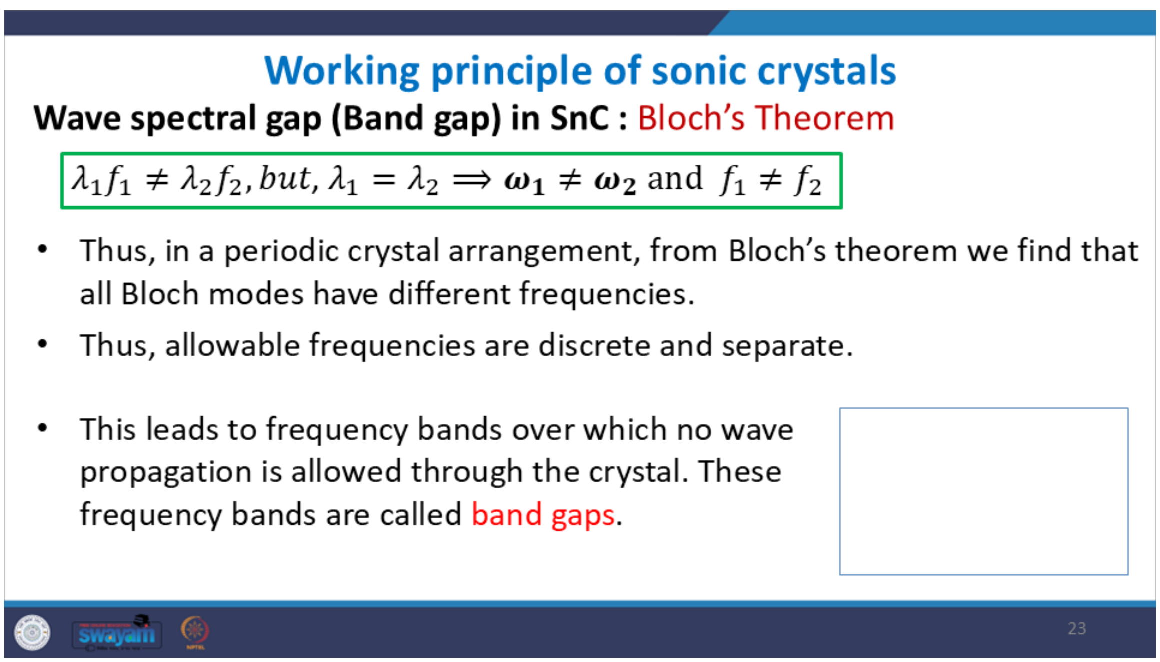
A more acoustically dense region occurs for a lower mode. So, here As you can see by the nature of this, mode 2 is higher because here the pressure maxima occurs in a lighter region, and in mode 2, the pressure maxima occurs in a denser region. So, these two modes have different acoustic intensities, and you know that acoustic intensity is what? For a plane wave propagation, it is p squared by rho c.



Okay, the square of the pressure divided by the characteristic impedance—in the acoustic fundamentals, we have seen this derivation.

$$I_1 \neq I_2 \Longrightarrow \frac{{P_{rms,1}}^2}{\bar{\rho}_1 \bar{c}_1} \neq \frac{{P_{rms,2}}^2}{\bar{\rho}_2 \bar{c}_2}$$

So this means that (Prms1)^2 / (rho1 c1) for the first mode should not be the same as (Prms2)^2 / (rho2 c2) for the second mode, okay? Because I = (Prms)^2 / (rho c) for a plane wave. So that is why this should not be the same, and hence. But we know that both



waves have the same waveform.

The RMS pressure is the same, and because if you consider not a small region but a much larger region of the sonic crystal, then overall, for a much larger region of the sonic crystal, you have the same number of scatterers and the same amount of air medium. So the overall density for both modes is going to be the same. The only conclusion we can draw is that if P is the same, rho is the same, then c must be different for the two modes, or the speed of sound must be different, which means that lambda1 f1 should not be equal to lambda2 f2. But we know that lambda1 is equal to lambda2 by Bloch's theorem.

So that means the frequencies must be different for the two modes. Thus, in a periodic crystal arrangement, by Bloch's theorem, we see that the sound waves only exist as these Bloch modes—these discrete Bloch modes, each of which has a different frequency. So, they have different frequencies. So, only some discrete frequency points exist where the waves can propagate. All other frequencies, the wave cannot exist, which creates some gaps, and these gaps where the waves cannot exist are called bandgaps.

So, not all frequencies can exist; only certain quantized discrete frequencies can exist. And all the in-between frequencies where the wave cannot exist in this structure. These are the band gaps of the structure. So, with this, I would like to close this lecture. Thank you for listening.