NOISE CONTROL IN MECHANICAL SYSTEMS

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Week: 10

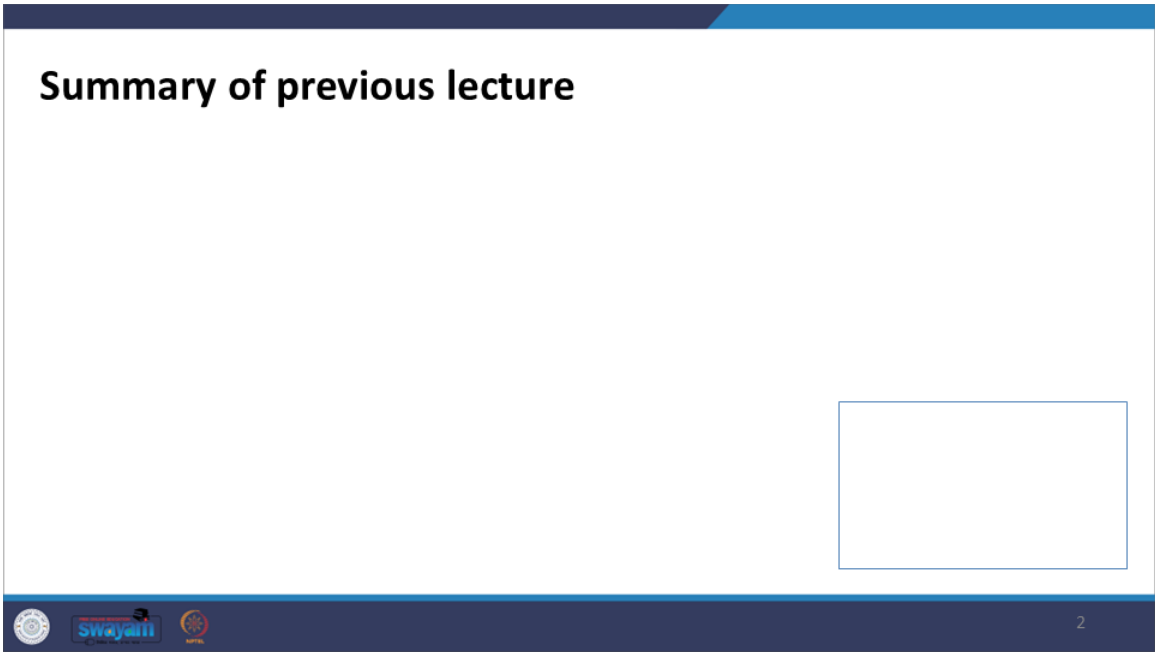
Lecture: 50

Lecture 50: Sonic Crystals 2

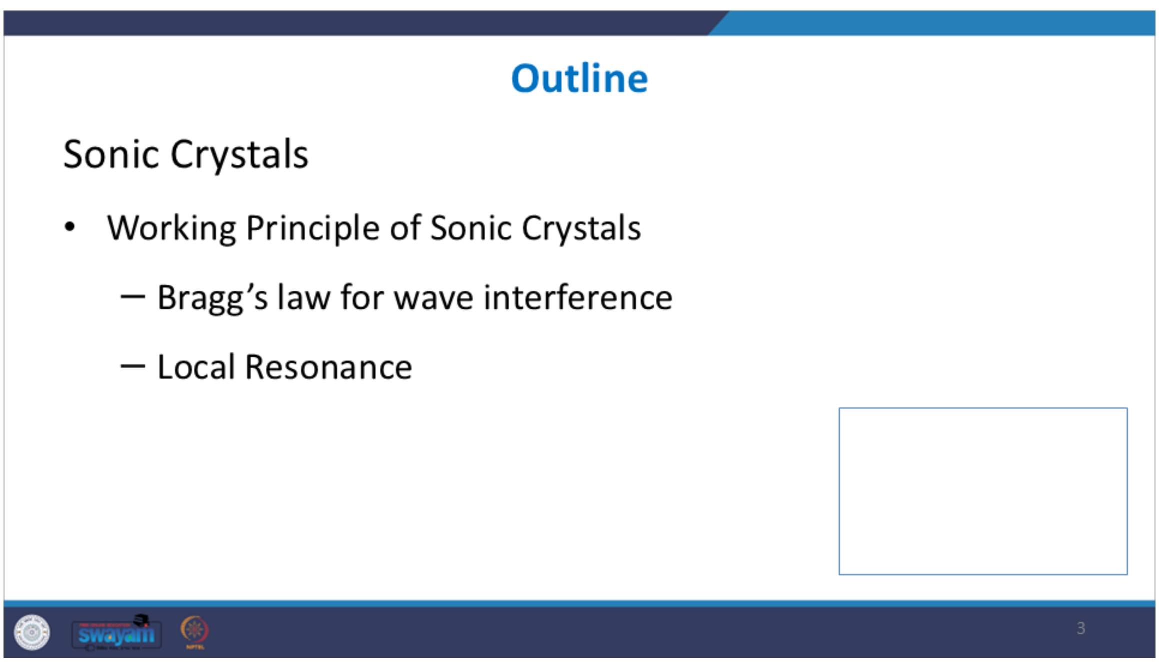
Hello and welcome to this lecture course on noise control in mechanical systems with myself, Professor Sneha Singh from IIT Roorkee.

Previously, we began the discussion on acoustic metamaterials and started discussing the sonic crystals as one of the acoustic metamaterials. Ok. We abbreviate it as SNC just for this lecture.

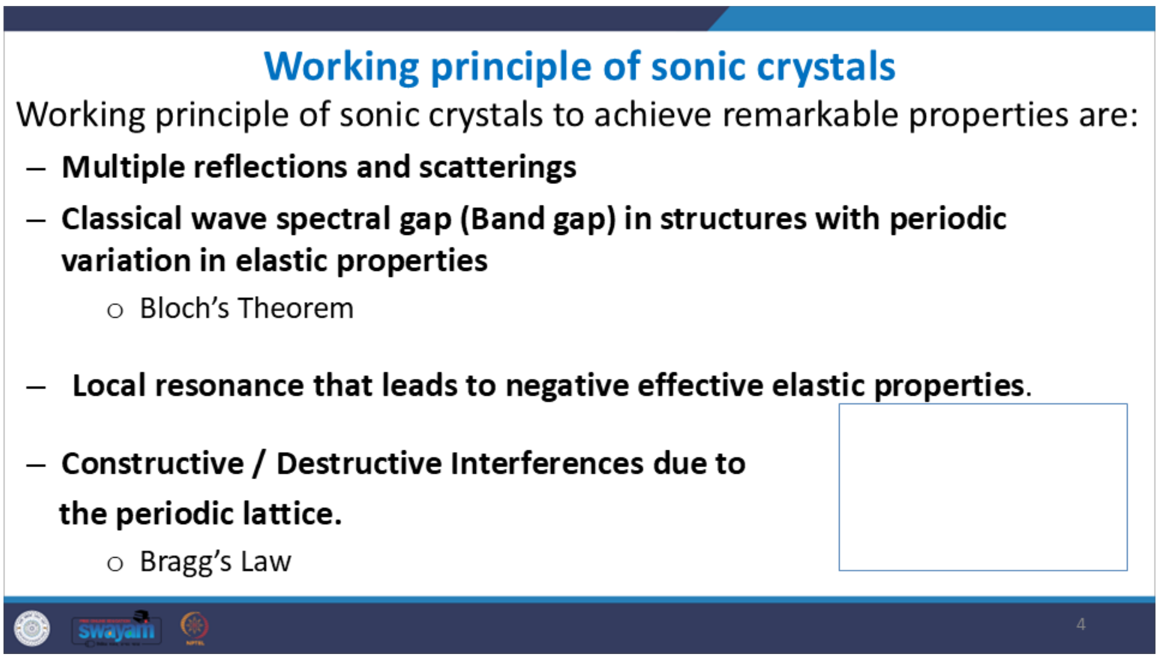
So, the sonic crystals are one of the very first acoustic metamaterials that were invented, and the concept started. What is a sonic crystal made of? It is made of an array or a periodic arrangement—we can say a periodic arrangement of scatterers in a host material. So, typically, the host material is a fluid acoustic medium, a light fluid acoustic medium such as air, water, etc. And the scatterers are these, you know, heavy or you can say high-impedance solid materials.



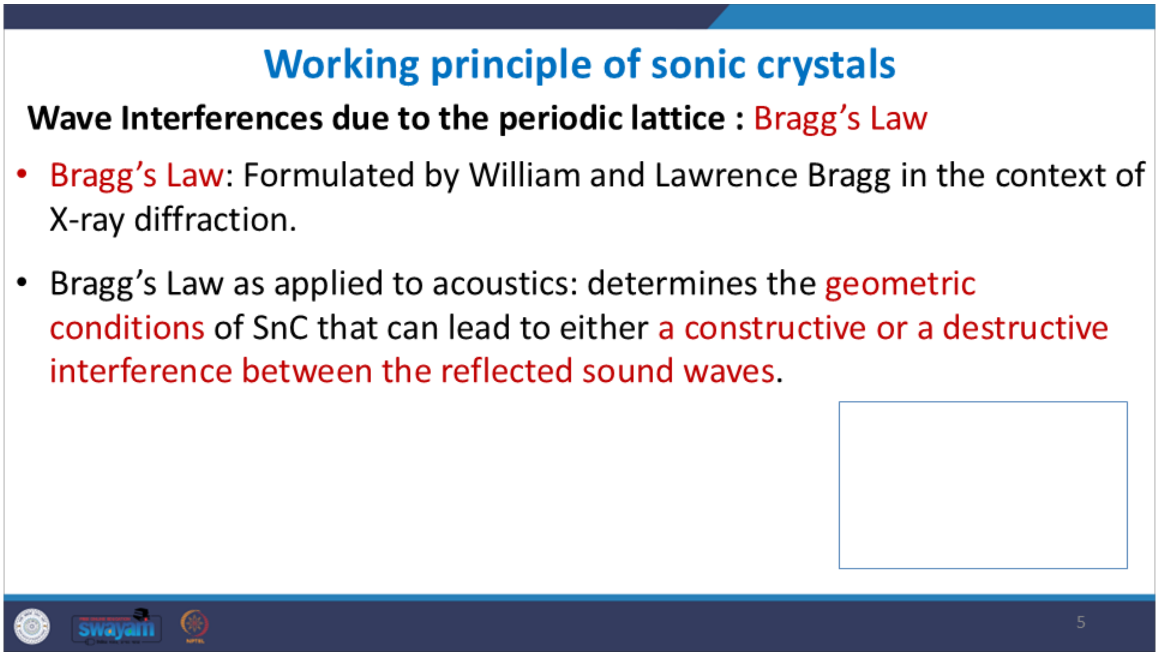
such as steel plates suspended in the air, aluminum solid cylinders arranged in the air, lead balls arranged in some cubic fashion—all of these will make a sonic crystal. So, today we will continue our discussion. So, we have been studying the working principles of sonic crystals. So, we already studied Bloch's theorem and will continue to the other working principles. So, this is for this lecture today.



Just to recap the working principle of the sonic crystals or the way they achieve you know the manipulation of sound waves and attenuation of sound waves is through the multiple reflections and scattering that happens. over each encounter with a layer of scatterer then the classic wave spectral gap or the band gap that results due to the periodic variations which is the Bloch's theorem. The Bloch's theorem governs that only the sound waves which have the same periodicity or whose wavelengths are of the same order as the periodicity of the crystal only those kind of sound waves exist within the material. So, this automatically makes that ensures that only you know certain discrete frequencies are allowable to pass through the sonic crystal.

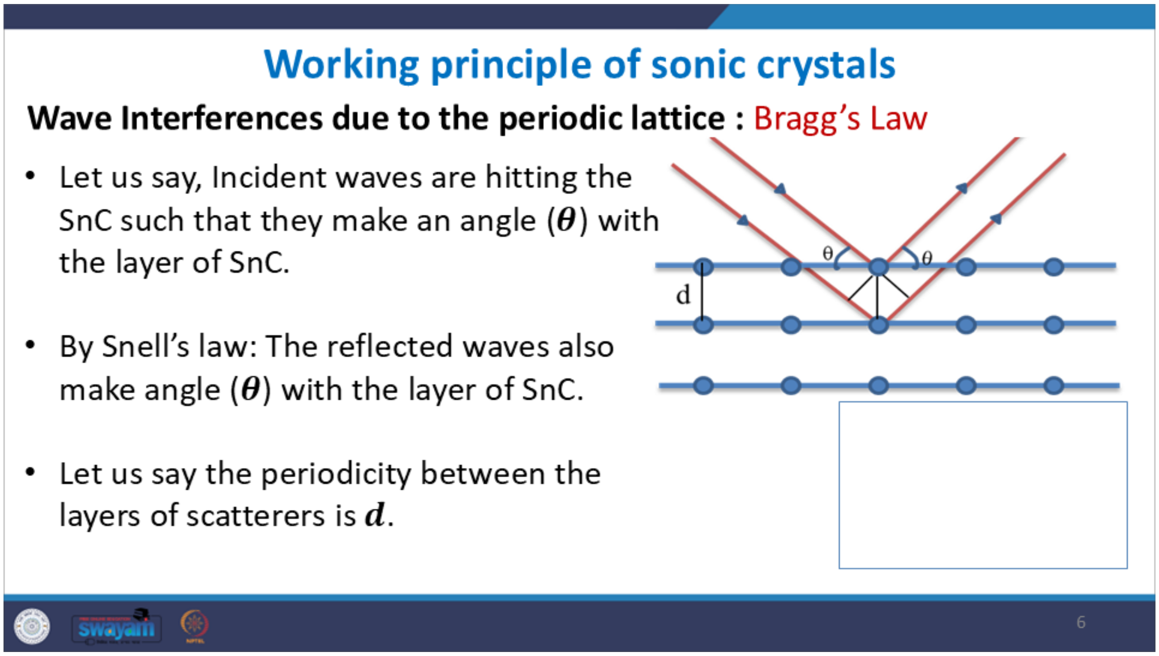


So, there are other phenomenon as well for example, the constructive and the destructive interference due to the periodic lattice which is given by the Bragg's law, the interferences that happen and then the local resonance which we will study today. Let us start with the interferences. So, wave interferences due to the periodic lattice and this is governed by the Bragg's law. okay so this law was you know formulated by you know William and Lawrence Bragg in the context of X-ray diffraction okay so the X-ray diffraction when you are bombarding you know a layer of material with X-rays the atoms or the molecules the heavier atoms or molecules they diffract the X-rays and based on the diffraction pattern the composition of the material can be estimated so similar concept is then extended to acoustics



So, if the sonic crystals are considered as you know just like I said that you know like we have a solid lattice where the molecules are arranged in some ordered fashion. Sonic crystal itself is like an arrangement of these solid scatterers in some 1D, 2D or 3D fashion and when you know the sound waves are incident on it. Depending on the arrangement, the sound waves will be reflected back and only some of it will transmit. So, depending on the pattern of the reflected waves, we can find out what is the periodicity. So, this Bragg's law is applied to acoustics and what it does here in case of the sonic crystal is that it can be used to determine what would be the geometric conditions.

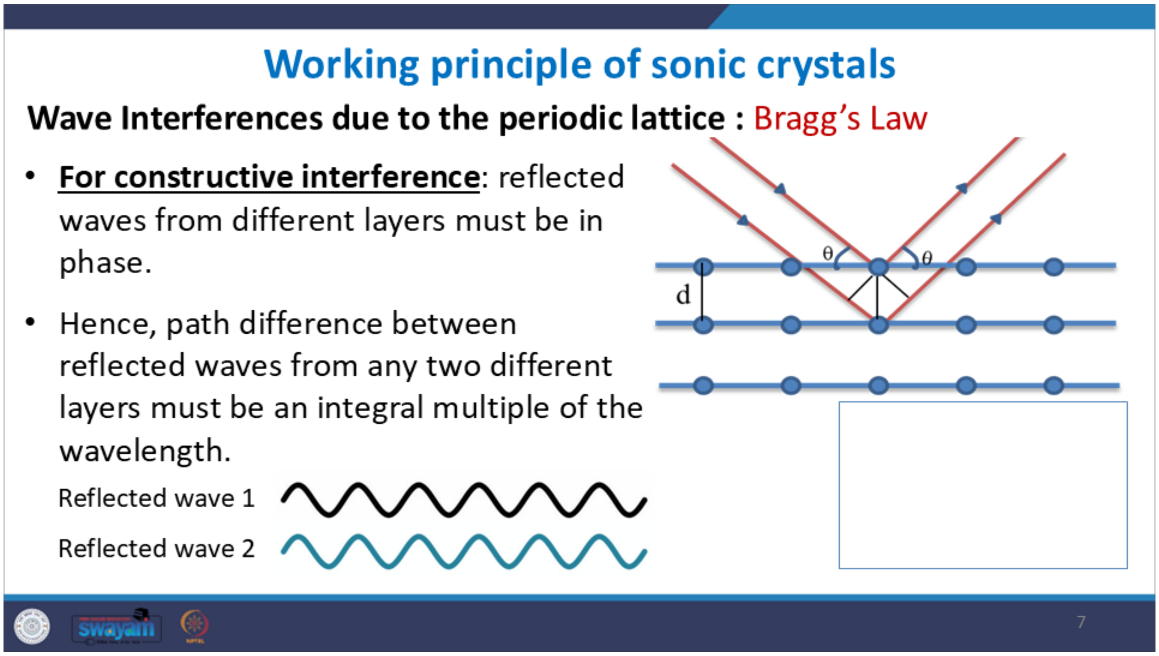
of a sonic crystal such that it can lead to a constructive or a destructive interferences between the reflected sound waves. So, let us see here. Let us say this is you know a top view of sonic crystal. These are your scatterers, okay. They are arranged in some kind of ordered fashion and this is the top view of the



sonic crystal, it is a 2D sonic crystal let us say. So, what is happening here is that let us say some incident waves are hitting this sonic crystal okay and they are making an angle of theta with the layer of sonic crystal this angle theta here. Then by the Snell's law okay in the acoustic fundamentals modules we have already seen that when the sound waves are reflected the angle of incidence is the same as the angle of reflection okay. And that is one part of the Snell's law.

And then there is another additional law which relates the angle of incidence with the angle of transmittance or transmitted angle. That is the second part of the Snell's law. So, by the Snell's law, the theta i is same as theta r so if the sound waves are hitting at an angle of theta with the layer of the sonic crystal here then some portion of it is going to be reflected back because of the change in the impedance there will be heavy amount of reflection because that's the purpose of the scatterers they are they are supposed to instead of transmitting the waves they are supposed to scatter or reflect back most of the waves that are hitting on it So, majority portion of the wave is then reflecting back and it is reflecting back with the same angle theta and let us say the periodicity between the layer of the scatterers this is d here this is same as the lattice vector in that direction okay.

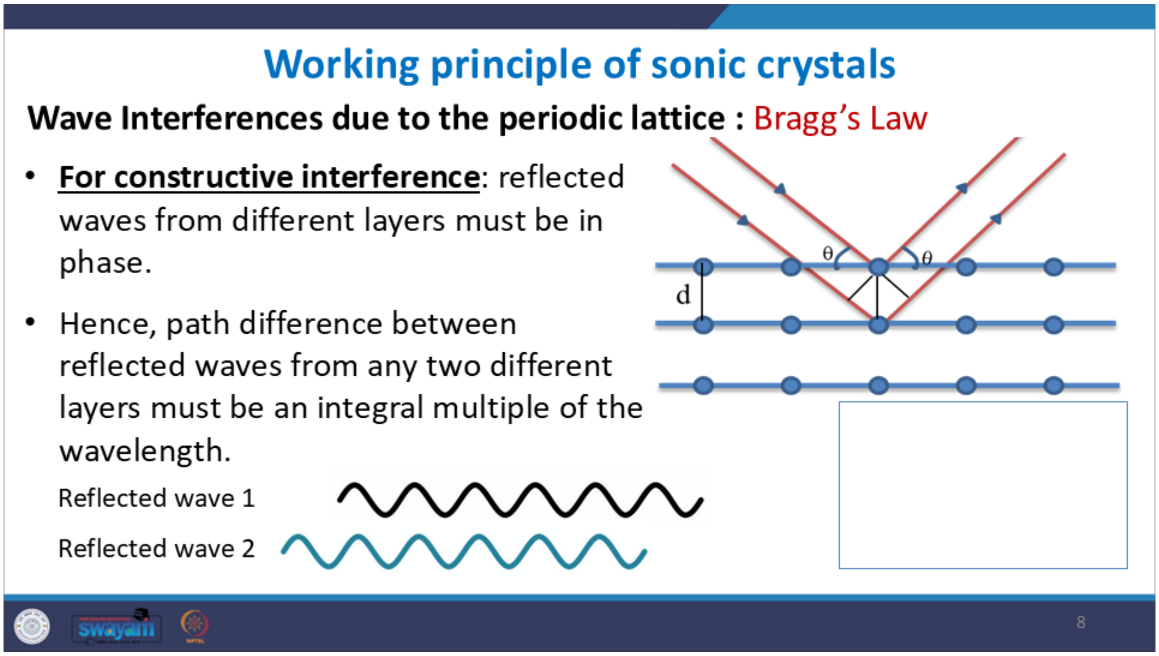
So, this thing here—the spacing or the lattice vector or periodicity, whatever you call it. Then, for constructive interference, suppose you know the waves are hitting and reflecting back. So, let us say this is the reflected wave 1. This is the reflected wave 2, and so on. Further waves will be hitting and getting reflected back from the third layer, and so on.



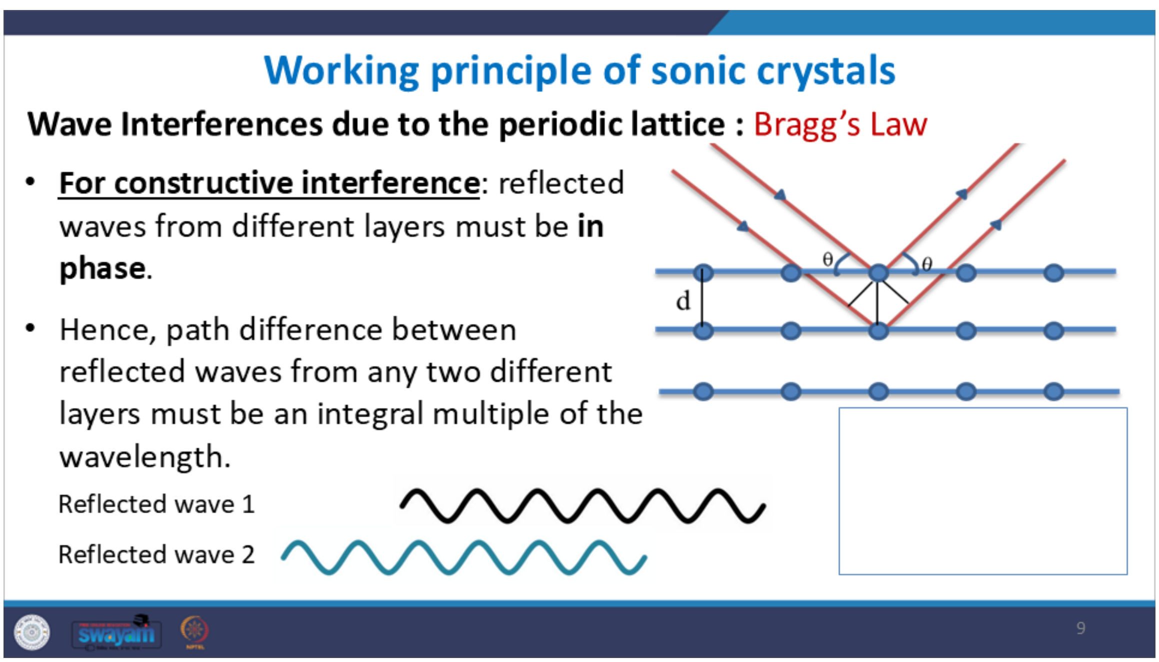
So, for the constructive interference to happen between these reflected waves, which are getting reflected from the different layers of scatterers, the net path difference has to be an integer multiple of the wavelength. To explain this, let us say this is the first wave. And this is the second wave. For complete interference, the peaks and the troughs of the wave have to match. So, that will give you constructive interference.

Over here, you can see the peaks and the troughs—they should match together. Spatially, and hence it will lead to constructive interference and a big wave like this, OK? So, for this—so here, what is the path difference? Here, you can see there is no path difference; the path difference is 0. But if you look here in this diagram, the reflected wave is following a certain path and reflecting back, but what happens to reflected wave 2? It is also taking that path, but it has to cover some extra distance because it is deflecting from the layer behind it. So, some extra distance it has to take.

So, hence there will be some path difference between R1 and R2, the first and the second reflected wave and so on. So, let us say the path difference is for a constructive interference, the peaks and the troughs have to match. So, the path difference can either be lambda. as you can see here that is the part difference between the 2 waves it is here the wave 2 is lambda times it is lambda behind the wave 1 ok because it is travelling late it had to cover an extra lambda distance and it is lagging behind the first wave but even though it had it is lagging behind the first wave. If the lag is only of lambda that is the extra path it had to cover was only lambda even then the two waves they will be able to constructively interfere with each other and the peaks and the crests will be matching.

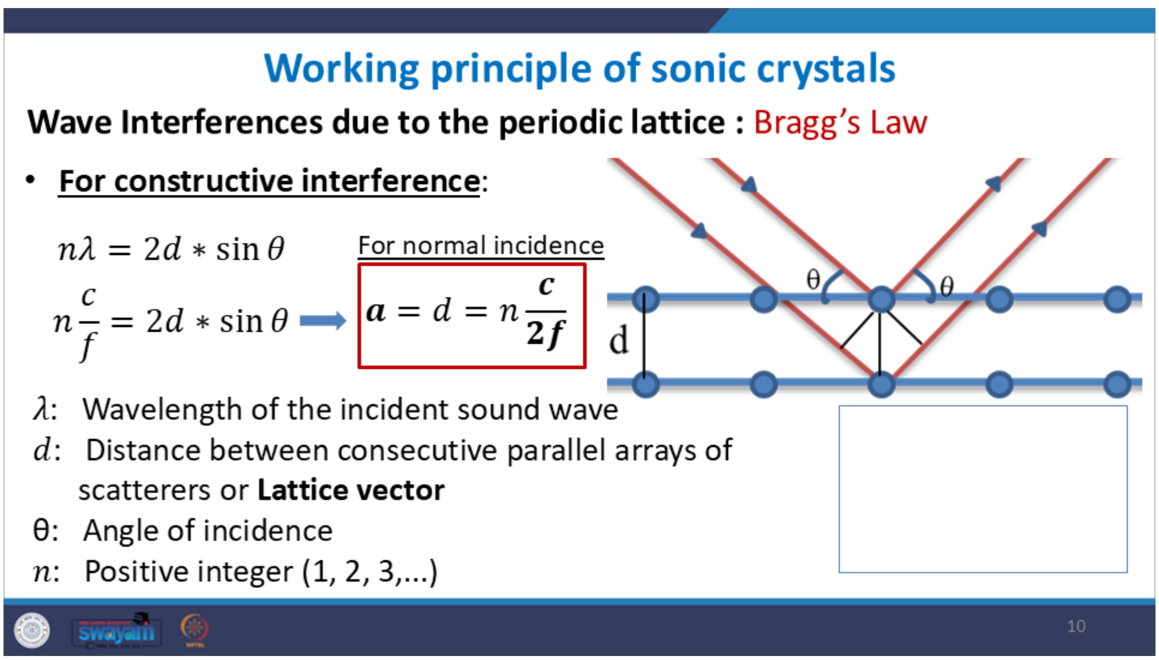


In the same way suppose you have the third layer or some additional path difference. So, either the path difference could be lambda what is the condition for the second you know what is the condition for another constructive interference the wave could be shifted even further like this. So, here you can see this is 1 lambda and this here is another lambda. So, here the net path difference. is 2 lambda here the path difference between the two waves is lambda.



So, even when the path difference is 2 lambda or the wave 2 it had to cover an extra 2 lambda path. So, that it lags behind by 2 lambda with the first wave even then the waves will constructively interfere the peaks and the crests will match together and there will be a constructive interference and so on. So, either lambda, 2 lambda in the same way, 3 lambda, 4 lambda like that the power difference will be then constructive interference will still happen. So, that becomes your condition that the part difference between any two reflected waves that are getting reflected from two different layers it must be some integral multiple of the wavelength ok.

So, for constructive interference to happen, the peaks can match with the peaks and the crests can match with the crests. Okay, the corresponding parts of the wave can match with each other. So, here, let us see in that case. Let us see here first an expanded view of this. So, let us say this is your first reflected wave 1, and this is your reflected wave 2. Here, let us say this is the path that it is taking, and here the same for reflected wave 2. This is the path. If you draw the parallel lines, you know this path is taken by this, and this is the path taken by the other wave. So, it is traveling the same distance here and the same distance here. But here, over here, this portion becomes your extra path that the reflected wave 2 has to cover. Okay.



I can make this extra path using this green light. So, this is the extra path it is covering, you know, the reflected wave 2. So, what is this extra path? If you see here, this is your theta.

So, let us derive what this extra path is. If this is theta, this angle also comes out to be theta, isn't it? This is the angle between these two lines, and this line is perpendicular to this, and this line is perpendicular to this line. So, they have the same angle theta, and in the same way, this should also be theta because this line is perpendicular to this line, and this one is perpendicular to this line, and the spacing between them is d. Okay. So, in this triangle here, if you see, this angle is theta, this part is d in length. Okay, in this triangle, and then it is a right-angle triangle because we have drawn this in that way to get the path difference. Okay.

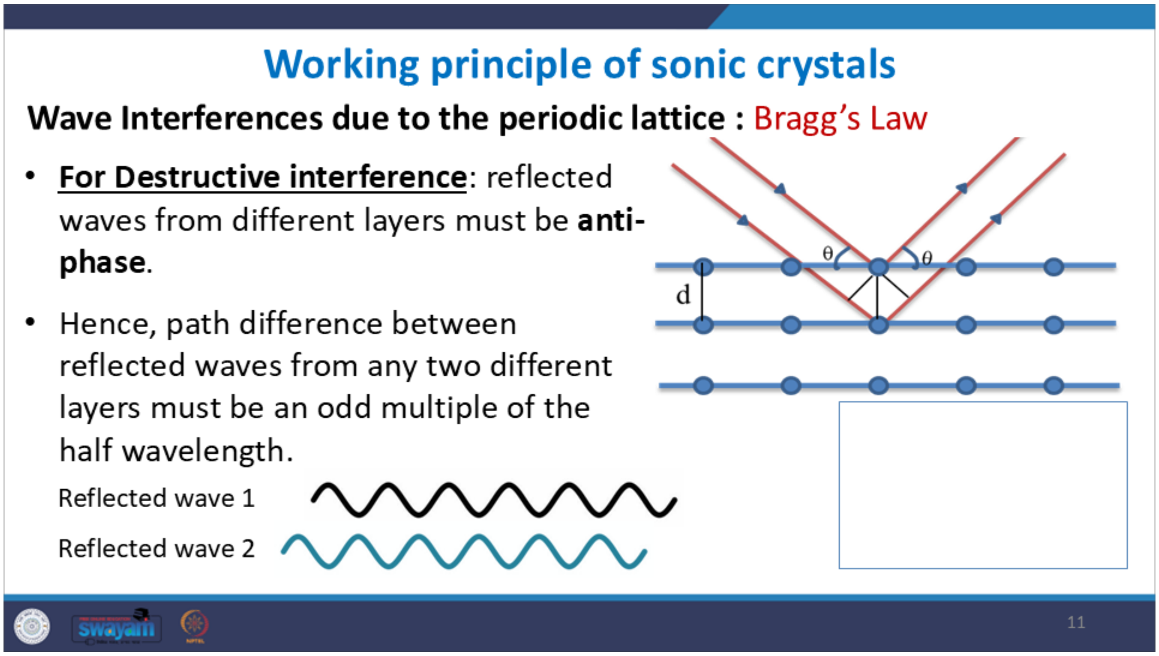
So, this is your d, this is your theta. So, what is this value here? This length here, this length comes out to be d into sin theta. Isn't it? d being the hypotenuse and the perpendicular to the angle would be d sine, the sine theta component of it.

So, it will be d sine theta. So, here this part is d sine theta. In the same way, this part is d sine theta. So, the extra path that the reflected wave 2 is taking becomes d sine theta plus d sine theta. So, it becomes 2 d sine theta, ok? This extra path.

So, this extra path has to be some integer multiple of lambda. So, that the waves can constructively interfere, ok? So, this becomes your condition for constructive interference. So, here this is for any generic wave with some incidence theta. So, what is lambda?

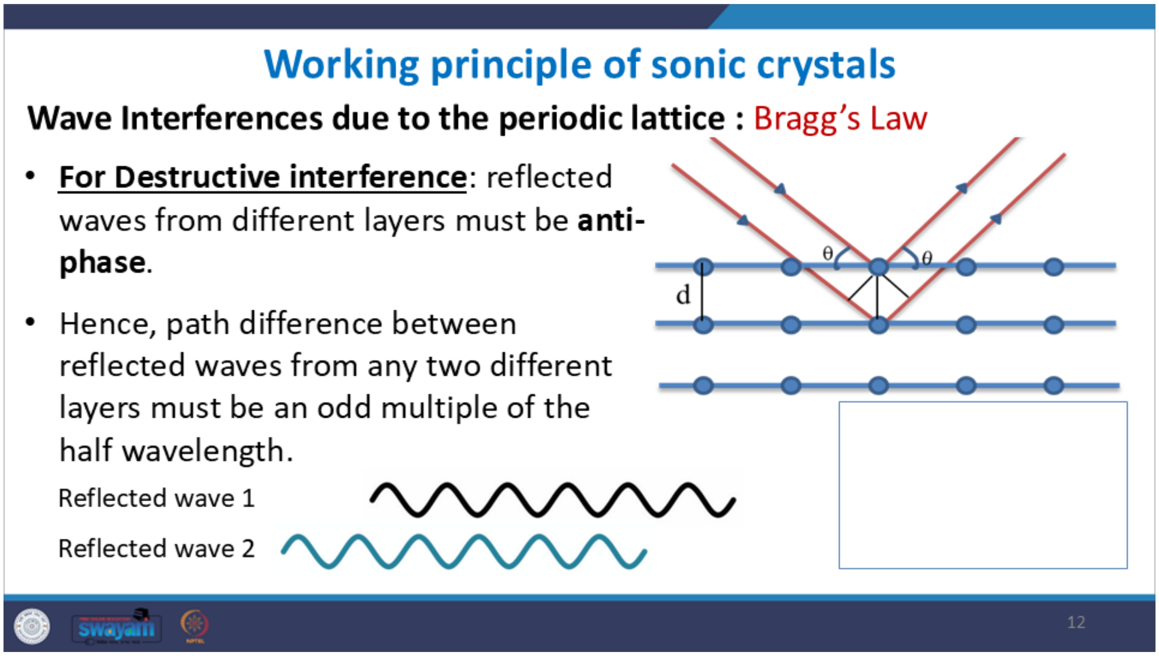
It is c by f the speed by the frequency of the wave you can replace it with this. So, n c by f is 2 d sine theta and suppose the normal incidence had to happen which means that theta was 90 degree then sine 90 would be 1. Then you can simply say the periodicity or the d the spacing is n times of c by 2 f. You can get from this relation for the case of normal incidence for a generic wave it would be a would be n times of c by 2 f sin theta and for normal incidence it will become n c by 2 f. So, which means that the spacing has to be some integral multiple of c by 2 f, where c is the speed of the sound wave in the host medium, f is the frequency of the incident sound wave. So, this is the speed of the sound wave and f is the frequency of the incident sound wave.

So, this is what happens then you know there is a constructive interference. So, that means that there is the reflected waves they are constructively interfering. So, which means that when you know the waves are incident the layer of sonic crystals they are behaving as perfect reflector and a large amount of reflected wave is going back into the medium. What would be the condition for destructive interference? In the previous case the waves they had to be in phase the reflected waves from the different layers.

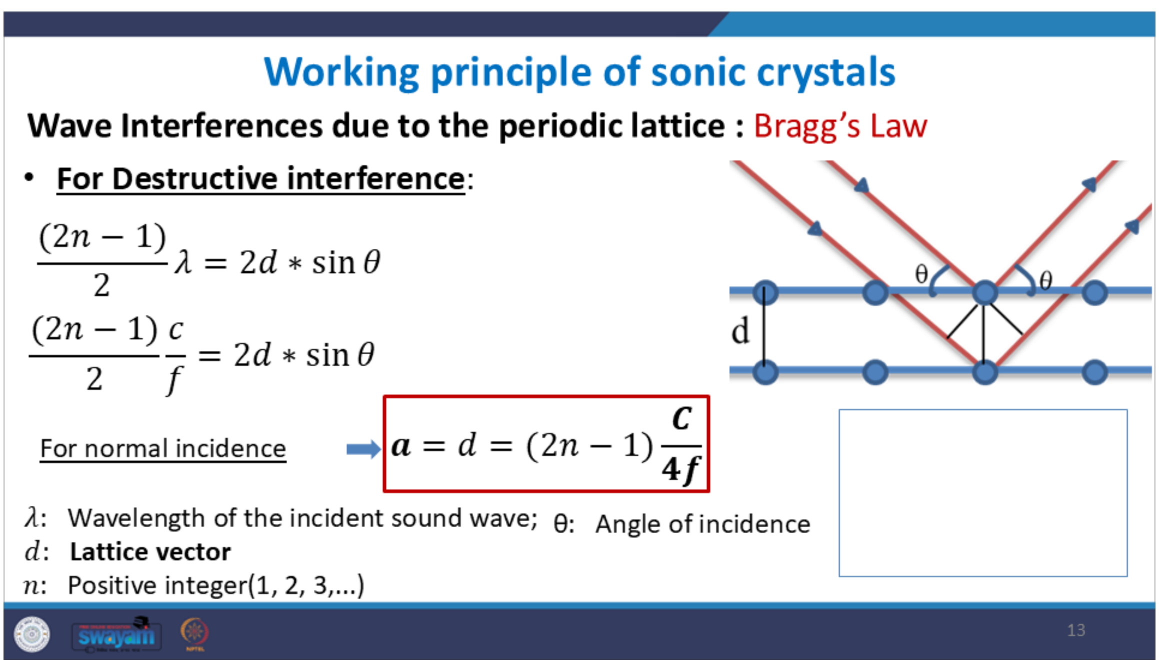


but for destructive interference now the waves coming out from the different layers they have to be antiphase so the peaks have to match with the troughs and the troughs have to match with the peaks of the first wave so something like this then you will have a destructive interference so let's see for example these two are let us say the reflected wave 1 and the reflected wave 2 from the two different layers and what is the path difference here if you see This is what? This is lambda by 2. So, here the path difference in this case is lambda by 2. We already know what happens when the path difference is lambda.

In that case, there will be constructive interference. So, for another destructive interference, what could be the further path difference? The wave could be further shifted. So, the path difference between the two waves could then be lambda plus lambda by 2, which is 3 lambda by 2.



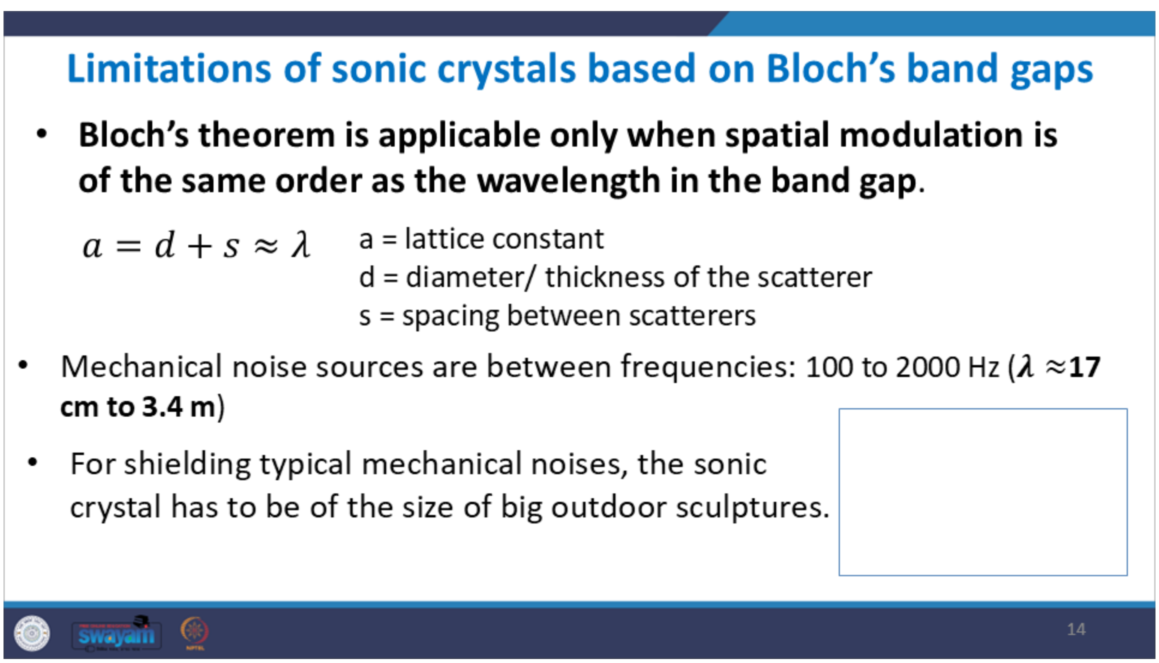
This would be the path difference, and so on. So, lambda plus lambda by 2, and then 2 lambda plus lambda by 2, which will be 5 lambda by 2. So, it has to be that ultimately, for destructive interference, the extra path that the second wave has to travel—so that there is a path difference between the two—has to be an odd multiple of the half of the wavelength. Okay. So, let us now mathematically express this: the extra path that the wave has to cover. Again, we have already defined that this is the extra path; this is the path difference between the two.



It was already derived for the case of constructive interference here—the 2d sin theta is the path difference. So, this path difference has to be some odd multiple of lambda by 2. So, again, it is (2n - 1) by 2, where n is any positive integer: 1, 2, 3, and so on. So, you can replace lambda by c by f. Then this is the condition you get.

Again for normal incidence the periodicity would be if you put it down it would be an odd multiple of c by 2 4 f okay. This is for normal incidence where theta is equal to 90 degree. For a generic case the periodicity would be an odd multiple of c by 4f sine theta okay so this is the condition for a destructive interference in that case what happens that the sound waves are incident and then the reflected wave is going back but it is ultimately getting cancelled so there is a perfect cancellation of the reflected wave so ideally there should be a perfect cancellation doesn't happen in reality because of various other unknown factors But, what it means is that you know suppose there is some sonic crystal here and some sound wave is getting incident at any angle then most of it is getting scattered, but the scattering there is a perfect cancellation.

So, the noise is not there ok. So, that was you know another working principle of the sonic crystals that at certain specific periodicities they can attain a constructive or a destructive interference. Accordingly, the sonic crystal can be used either as a perfect reflector to reflect sound waves or to cancel out the incident and the reflected waves. So, previous class we studied about you know the Bloch's theorem and what is the Bloch's band gap. So, from according to the theorem the wavelength that can exist within the sonic crystal this lambda has to be the same as the lattice constant.



Now, in most of the cases, so if you suppose a sonic crystal was to only operate on the working principle of the Bloch's theorem, then for most of the mechanical sources, you know that the frequencies are between 100 to 2000 hertz. We do not have very high frequency like 10,000, 20,000 hertz. The majority of the content is within 2000 hertz. So, if you take what would be the lambda corresponding to them, if you take 340 meters per second as the room temperature sound speed, then the lambda would be between 17 centimeters to 3.4 meters. And so, that means that to attenuate these kind of mechanical sources, the typical spacing, the typical periodicity has to be of this

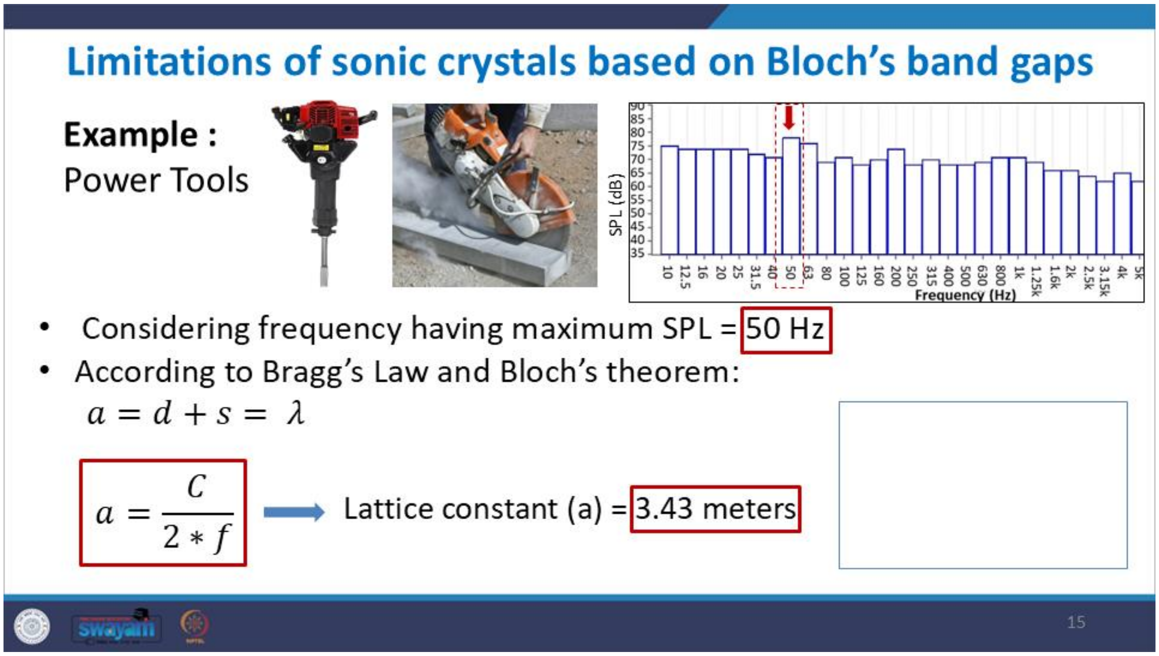
Of this range, this is not the entire dimension of the crystal. For a sonic crystal, we would need at least, you know, 4 to 5 arrays, let us say. Then that will mean that the overall dimension of the sonic crystal would be several magnitudes of lambda. So, the overall dimension of the sonic crystal would then become various times lambda, where n is a number greater than 1. Okay. So, it will be much larger than the wavelength because the individual spacing itself has to be maintained at the wavelength, and for low-frequency sounds, the wavelength can be very high, up to 3.4 meters also. And hence, very big outdoor sculptures may be required to manipulate these noise sources at these frequencies.

So, what to do about this? How to, you know, get some peaks or some, you know, how to get the sound manipulation at lower frequencies. For that, we can, just like in the case of the previous resonance-type metamaterials, I had made a comment. That a metamaterial should not just operate on one principle, but if you can have more principles like thermoviscous effects, resonance effects, frictional effects, and a lot of other things, then you can get better absorption at more frequencies. Otherwise, it will just be at sharp ranges. In the same way, What you can do is you can have additional mechanisms for sound wave manipulation, and hence, you know, you can have a new principle, which is the local resonance, which we will study about. Let us just see one practical example.

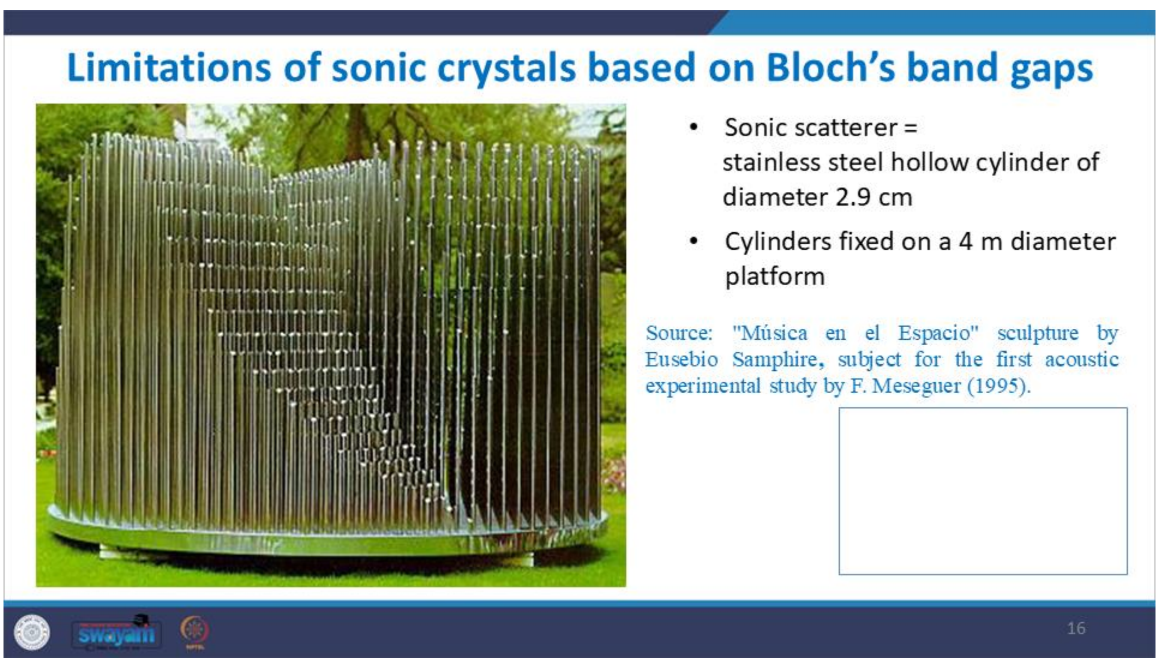
For example, this is the noise measured in our institute campus of a power tool machine. This is a surface-finding machine to cut these various blocks. And this is the typical noise spectrum that was measured for it. It is having a peak at, let us say, 50 hertz, and according to that, the lambda would come out to be almost it would be 6.86 meters and so on. And by Bragg's law also, C by 2f, this will give you 3.43 meters.

$$a = \frac{C}{2 * f}$$

Again, a very huge kind of sculpture would have to be built in order to tackle this frequency range, which would be several meters in length. And this shows the example of

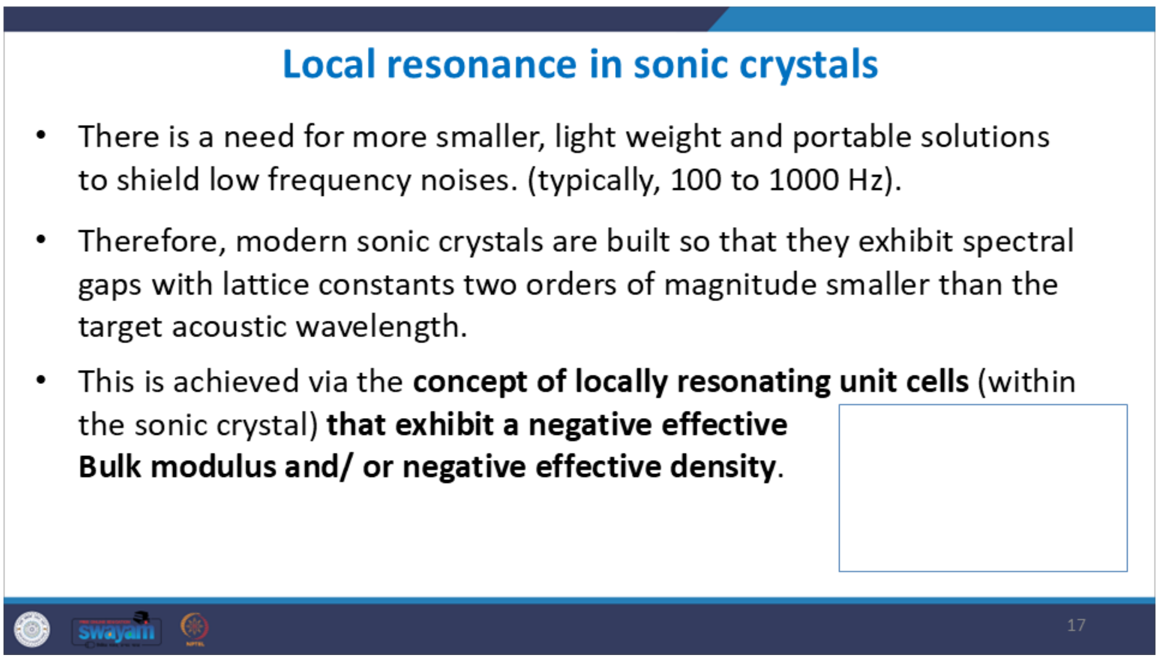


one such structure. This is one of the very first, you know, sonic crystals that was built for outdoors to bring down the outdoor noise sources, and it was quite overall a 4-meter diameter platform that was used. But we can't always use these structures, especially when we are dealing with, you know, these mechanical systems. In the environmental case, you can always in the environment, outdoors, have a big structure, but what if we



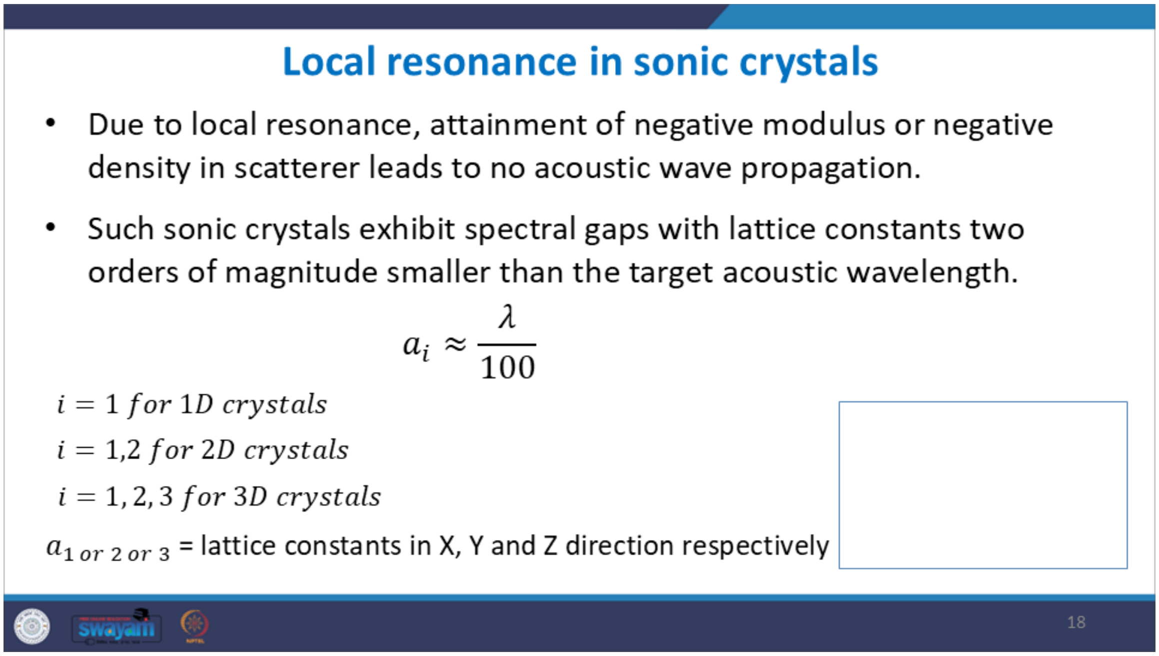
are dealing with some machinery and we need to have, we have some space constraints.

So, as I told you, some additional working principle should come into play, and one of those is local resonance. So, through the principle of local resonance, you can make sure that the attenuation happens at a lower frequency, and we are not just dependent on the Bloch wave's frequency. So, what is this principle, you know? So, here the concept of locally resonating unit cells within the sonic crystal that exhibit a negative effective bulk modulus and/or negative effective density. So, at the very beginning, when we were discussing, you know, metamaterials.



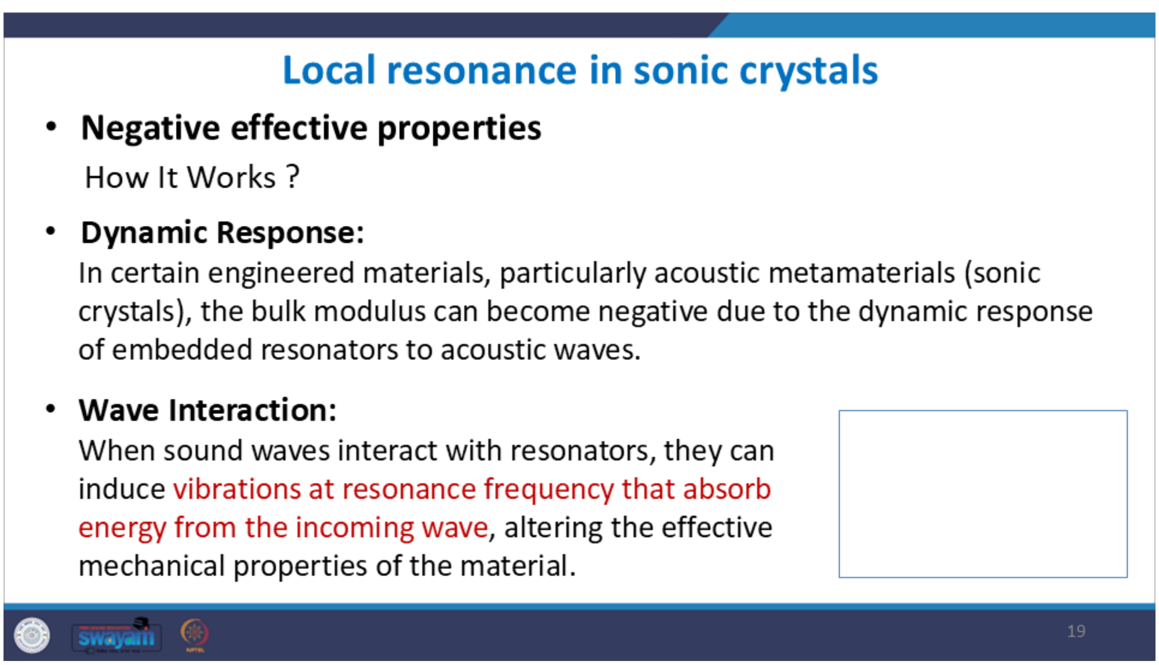
So, it was said that the principle is mainly that the metamaterials can achieve either a negative effective density or a negative effective bulk modulus, both of which will act to attenuate the sound waves. So, here also the same principle is there. So, instead of scatterers, now the scatterers are some resonating elements. So, the scatterers act as resonators. And at certain low frequencies, they start resonating, and at those frequencies, they attain either a negative density or a negative bulk modulus, which leads to the attenuation of sound waves, okay.

So, typically you know these sonic crystals which are based on local resonance they are designed these days such that according to the Bragg's law or the Bloch's theorem the spacing has to be of the same order as lambda it has to be lambda. But now with the principle of local resonance, we can attain some spacing which is lambda by 100 approximately, that is what is attained. So, we can bring down the overall size by 100, okay. We can reduce it by 100 times using the concept of local resonance, okay, where ai is the individual lattice vector for the sonic crystal, okay. For the 1D crystal, it will be just 1 lattice vector.



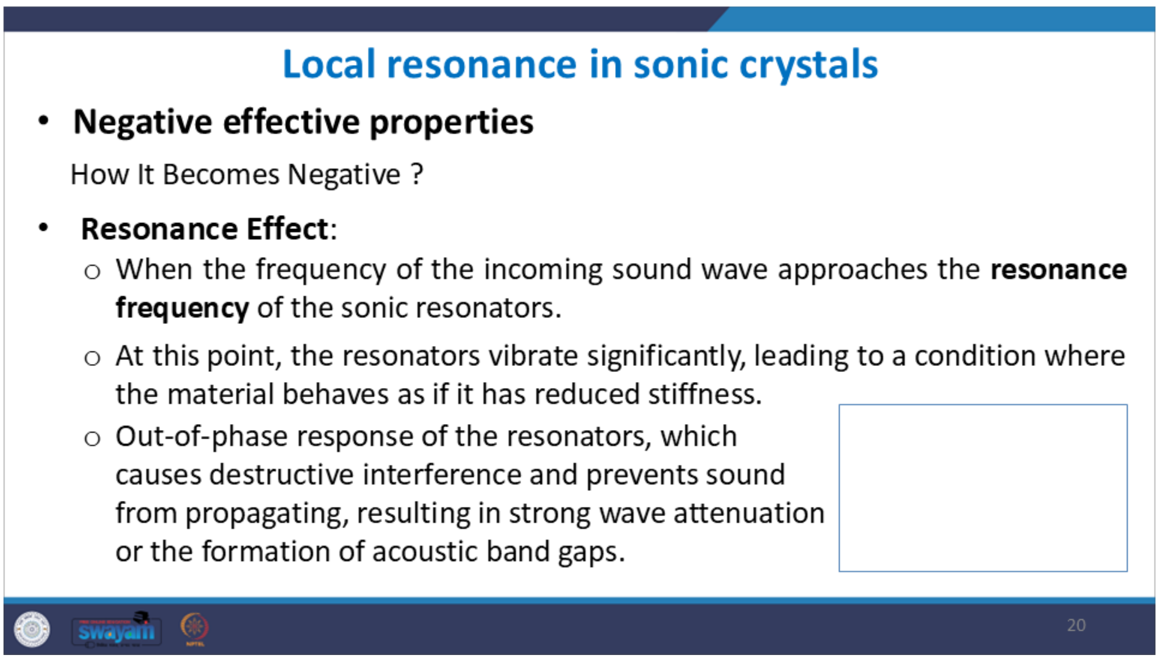
For 2D, 2. And for 3D, the 3 lattice vectors. So, each of the lattice vectors can be brought down by an order of 100. How does it work? So, it was already explained what is the effect of these negative properties.

So, first of all the bulk modulus becomes negative for these engineered materials due to some dynamic response of embedded resonators to the acoustic waves. When the sound waves they are interacting with these resonating. So, here the scatterer itself is a resonating element ok. It is some kind of resonator. Okay. That is the principle of



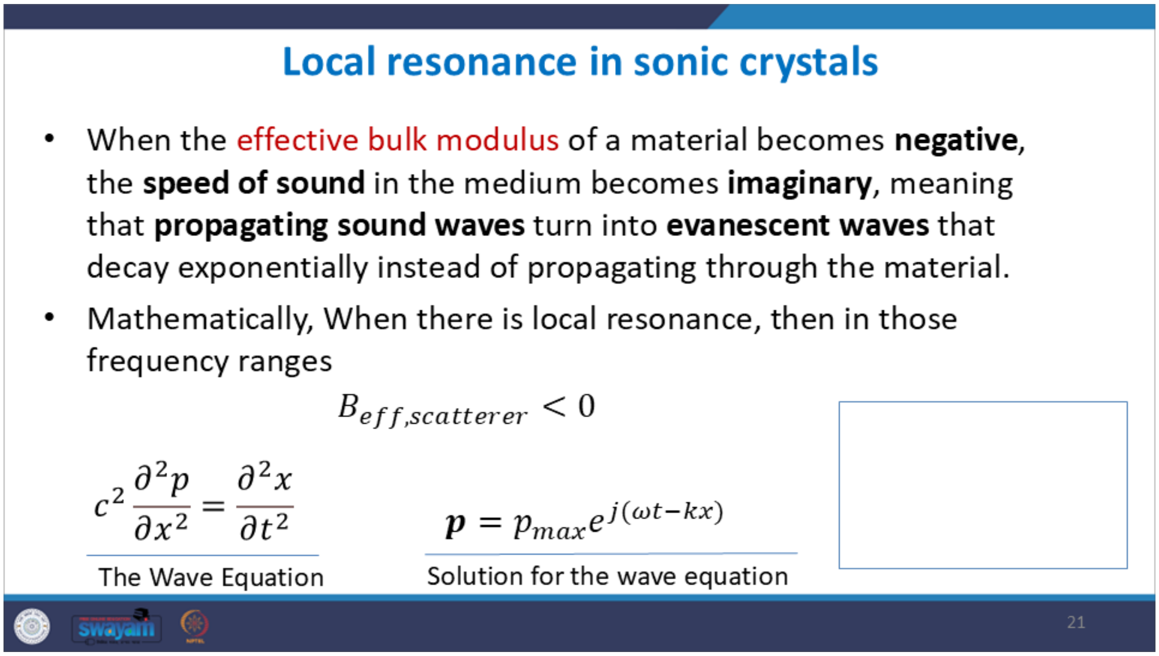
resonance.

You make the scatterer as a resonator and at its natural frequency the vibrations at the resonance frequency will absorb the energy from the incoming wave and it will also alter the mechanical properties. Okay. When the frequency is approaching the resonance frequency of the sonic resonators, the resonators vibrate significantly. Most of the energy gets used up in driving it into resonance and the material starts behaving as a material with much reduced stiffness. Out of phase response from the resonators can happen which can cause destructive interference and prevent the sound from propagating. This results in a strong wave attenuation.



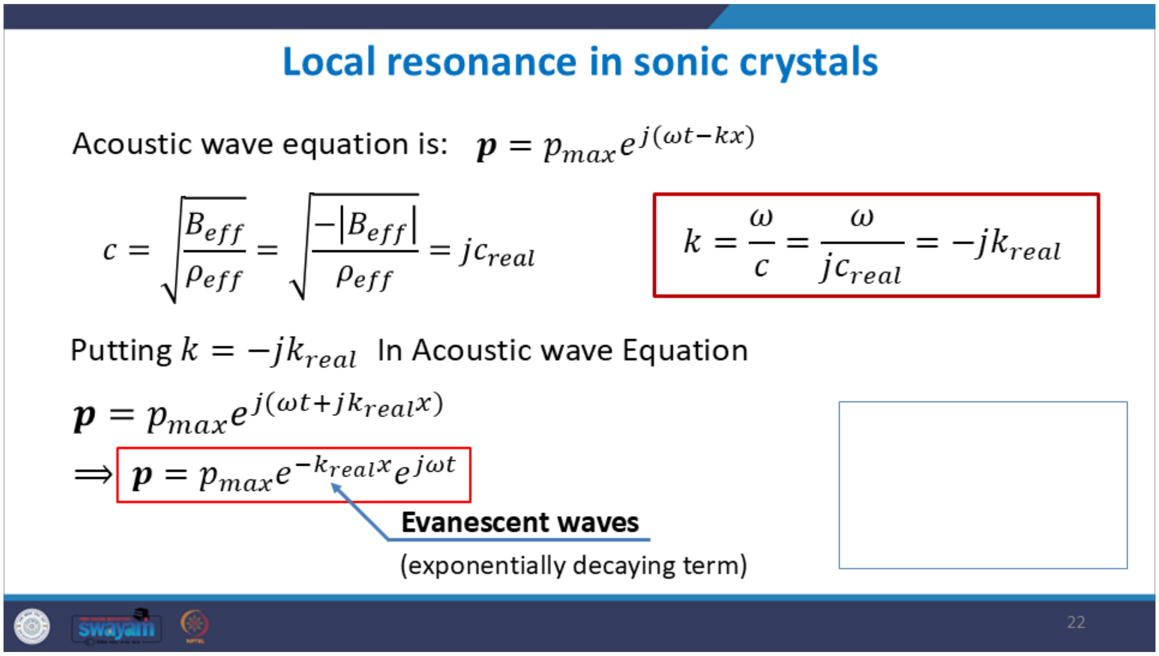
So, the concept of resonance was already introduced in the previous lectures that how when the sound wave is incident on some structure and the structure attains resonance, very simple concept. The sound wave is incident on a structure and the structure attains resonance, how it leads to attenuation at that resonance frequency. That concept has been utilized in Helmholtz resonator in the perforated and the micro perforated panel and also in the acoustic you know panels the general panel absorbers. All of these are resonance based absorbers so the instant sound wave is able to drive it into resonance or at very high vibrations and because of it you know there is loss in the sound wave energy.

So the same concept can be utilized. So previously you know the other concepts they were not based on resonance. So we can utilize the same concept. We have we already have a periodic array of scatterers why not make a scatterer as one of these resonant resonating element. So, if the scatterer itself becomes a resonator then individually at the local level of the scatterer we are having this resonance phenomenon and hence the term local resonance because individually at the localized zone of the resonator or that the localized zone of the scatterer this resonance phenomenon is happening okay so what is



the effect of having a negative bulk modulus or a negative mass density so this is what has already been covered but just a quick recap At resonance, the effective bulk modulus of the scatterer is becoming 0, let us say, because of the resonance. So, at this frequency, if this becomes 0, then let us say this is the solution of the wave equation. It is e to the power j omega t minus kx. some kind of you know amplitude function.

Then if suppose this is negative then this would be what this would be some negative term. So, inside the square root of the thermodynamic speed of sound will have some negative term. So, this will lead to some j times of serial which is some kind of complex speed of sound we will obtain and hence the k vector or the k which is omega by c will be omega by some complex quantity and j square is equal to minus 1. So, this will become minus j k real again a complex quantity.



If you put this now in the acoustic wave equation you get P max e to the power this becomes plus j into j is j square. So, minus k real x into e to the power j omega t. So, what you get is this is the solution not of a propagating wave, ok. This wave is not propagating spatially, it is decaying spatially, ok. So, what is happening is that typically the sound waves are what?

These are pressure variations; these are pressure oscillations, okay. The pressure oscillations that are traveling through space and reaching our ear, let us say. This is our human ear, and they are traveling from the source and reaching the receiver via the means of these pressure fluctuations, okay. This is a propagating wave, but here it is not a propagating term. There is no sinusoidal component with respect to x, but e to the power minus kx.

So, this is like an exponentially decaying term. So, spatially, what is happening instead of propagating, the wave starts decaying and stops. So, it does not propagate through and does not reach the receiver. So, at local resonance also, if B effective or rho effective becomes negative, then we will get a non-propagating wave at that frequency. So, with this, I would like to close the lecture.

Thank you for listening.

Thank You