NOISE CONTROL IN MECHANICAL SYSTEMS

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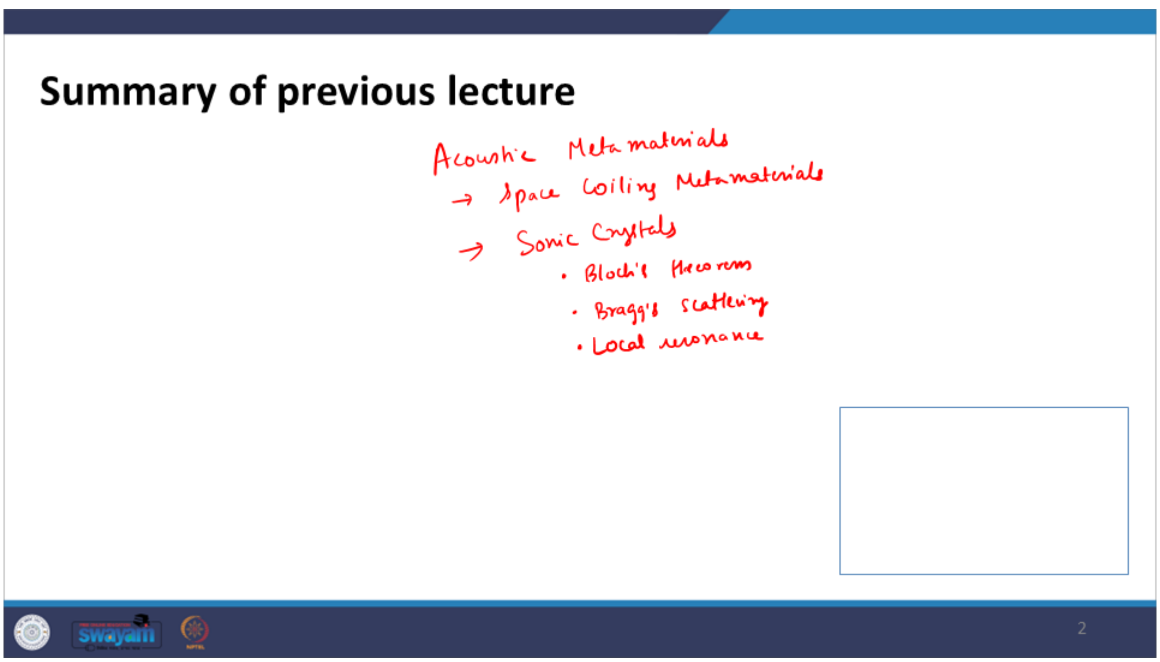
Week:11

Lecture:52

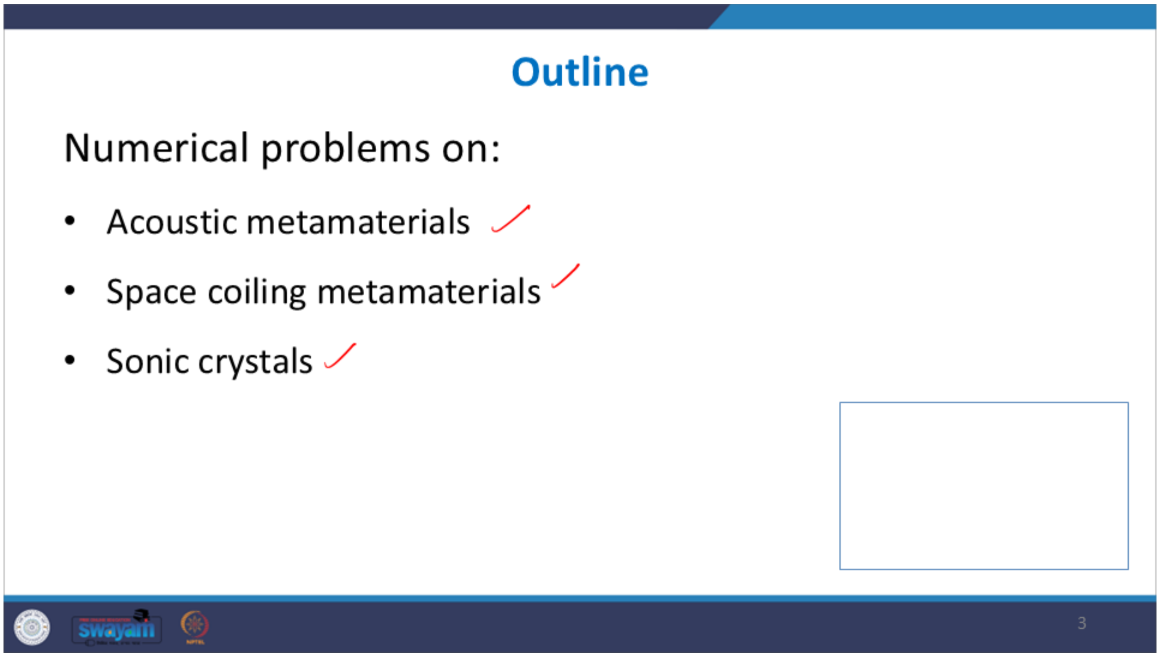
Lecture 52: Acoustic Metamaterials Numerical Problems

Hello and welcome to this lecture course on noise control in mechanical systems with myself, Professor Sneha Singh from IIT Roorkee. So, in the previous classes, we have studied about acoustic metamaterials. These are engineered materials that are used to control the properties of sound waves. They can sometimes achieve extraordinary manipulation of sound waves and could be very handy for low-frequency noise control. Among the acoustic metamaterials, we studied space-coiling metamaterials and sonic crystals.

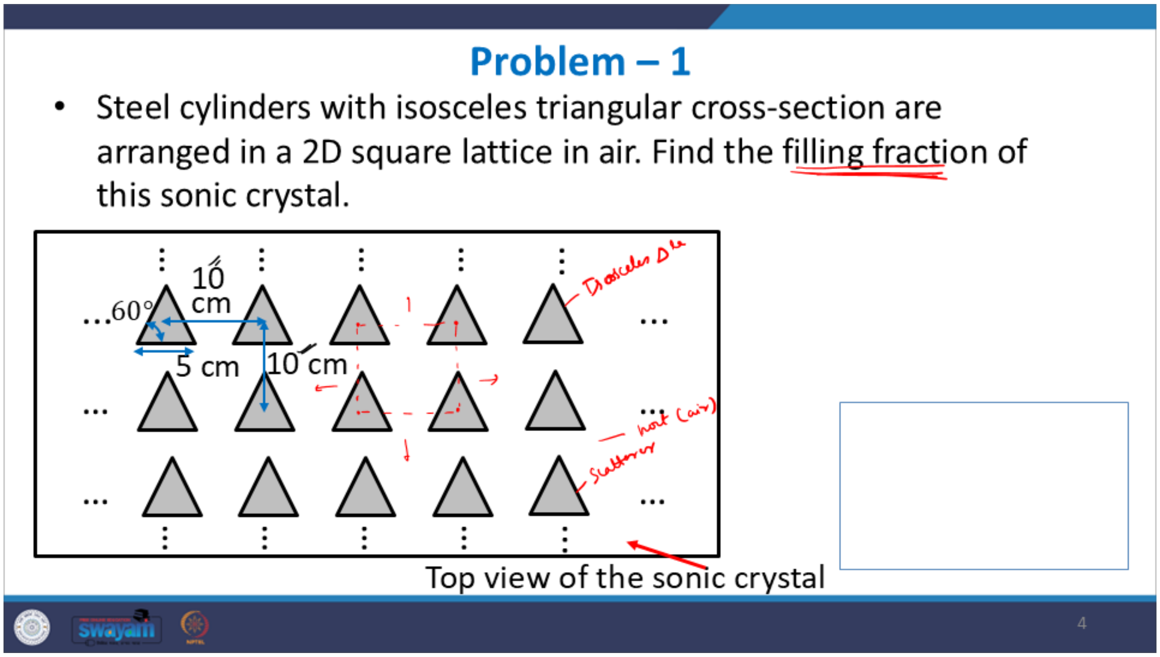
So, these are the two kinds of metamaterials we studied. What is a space-coiling metamaterial? It is used to manipulate sound waves by making them go through elongated pathways or maze-like channels within a small constrained physical dimension, thereby slowing down the sound waves, decreasing their propagation, reducing their speed, and delaying their phase. Then we had sonic crystals, which are a periodic arrangement of some scatterers. Scatterers are solid materials that have high impedance and can scatter most of the sounds incident on them. So, they are high transmission loss materials. Most of the sound waves get reflected and scattered by these scatterers. Scatterers. That is the name of it. So, it is a periodic arrangement of scatterers embedded in some kind of acoustic medium, such as air. You can have like steel cylinders placed periodically in air, steel plates, then aluminium rods, etc. placed periodically. They can all form a kind of sonic crystals and they also can be used to control sound waves to various phenomenon such as the Bloch's theorem, then we have the Bragg scattering effect that leads to constructive and destructive interference among the reflected waves. Then we have the local resonance phenomenon that also contributes to a high transmission loss when the sound waves pass through these sonic crystals ok.



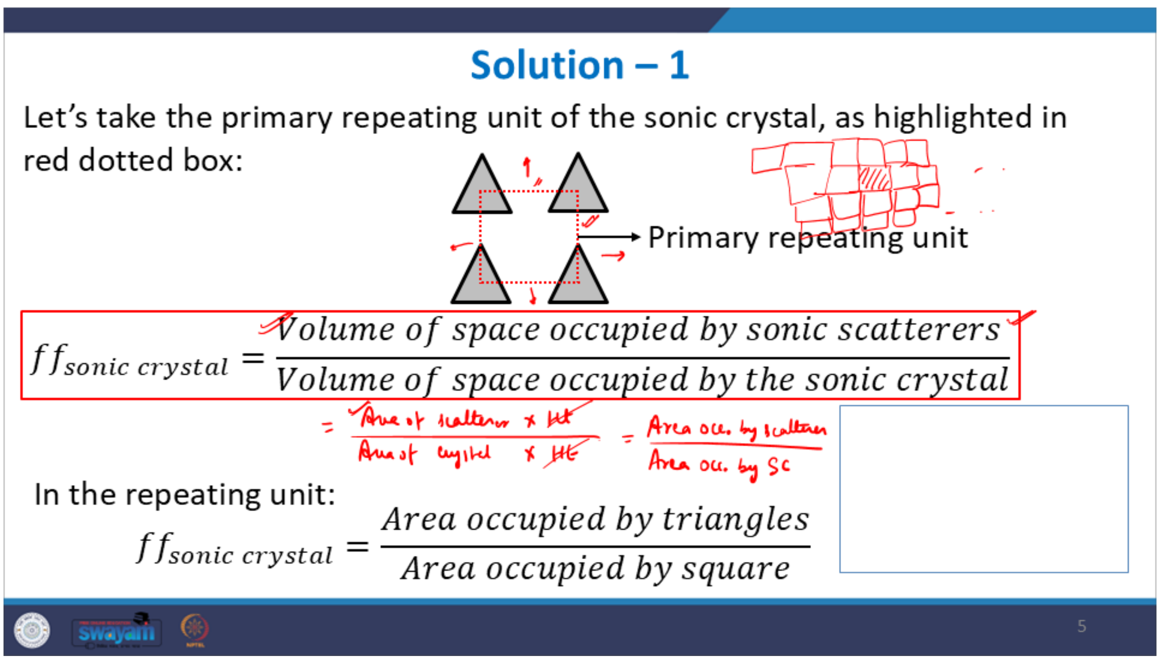
So, today we will see some numerical problems based on the concept studied in acoustic metamaterials specially you know space coiling metamaterials and sonic crystals.



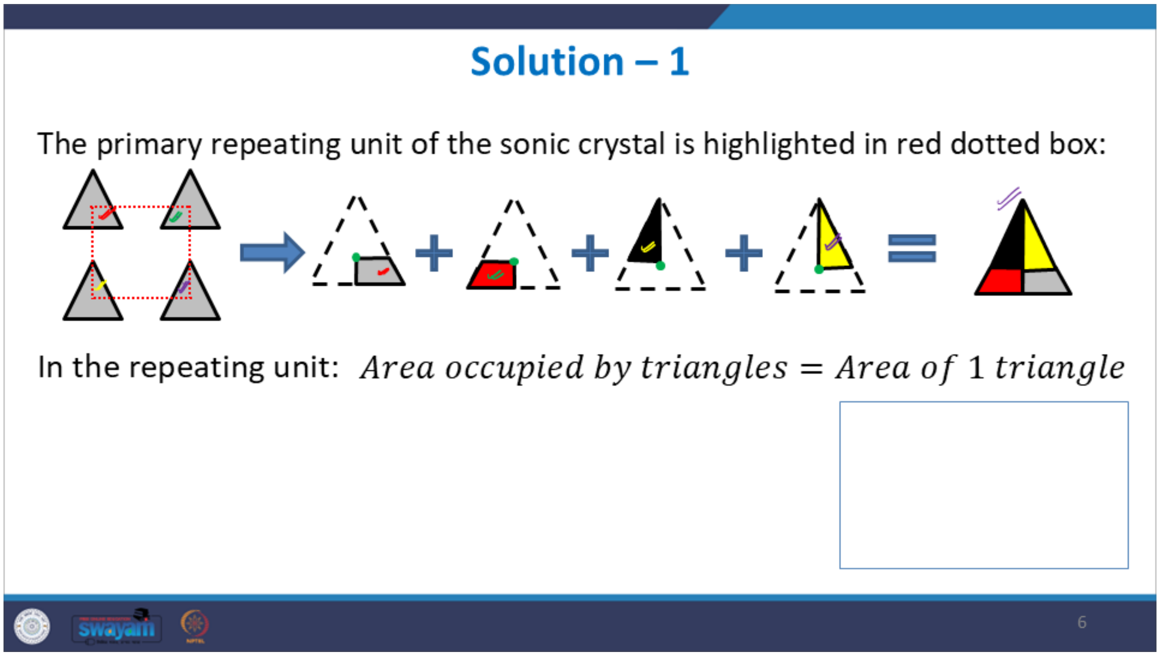
So, the first problem here is that you know You have got this is a sonic crystal. It shows a three top view or the layout of the sonic crystal where this is your scatterer and it is embedded in this host medium which is air. So it is steel cylinders with isosceles triangular cross section. So this shows a isosceles triangle. This is an isosceles triangle. This is the cross sectional view of the steel cylinders as seen from the top. It has an isosceles triangular cross section arranged in a 2D square lattice in the air. Find the filling fraction of this sonic crystal. So, just like in a crystal—any kind of crystal—you have the filling fraction, or in any kind of composite material, this filling fraction is a term that is used. This term can also be used to define a sonic crystal. What is meant by filling fraction is the net volume occupied by the scatterer divided by the net volume of the crystal. So, if the height of the crystal is the same, then the volume can be represented by the area for a 2D sonic crystal.



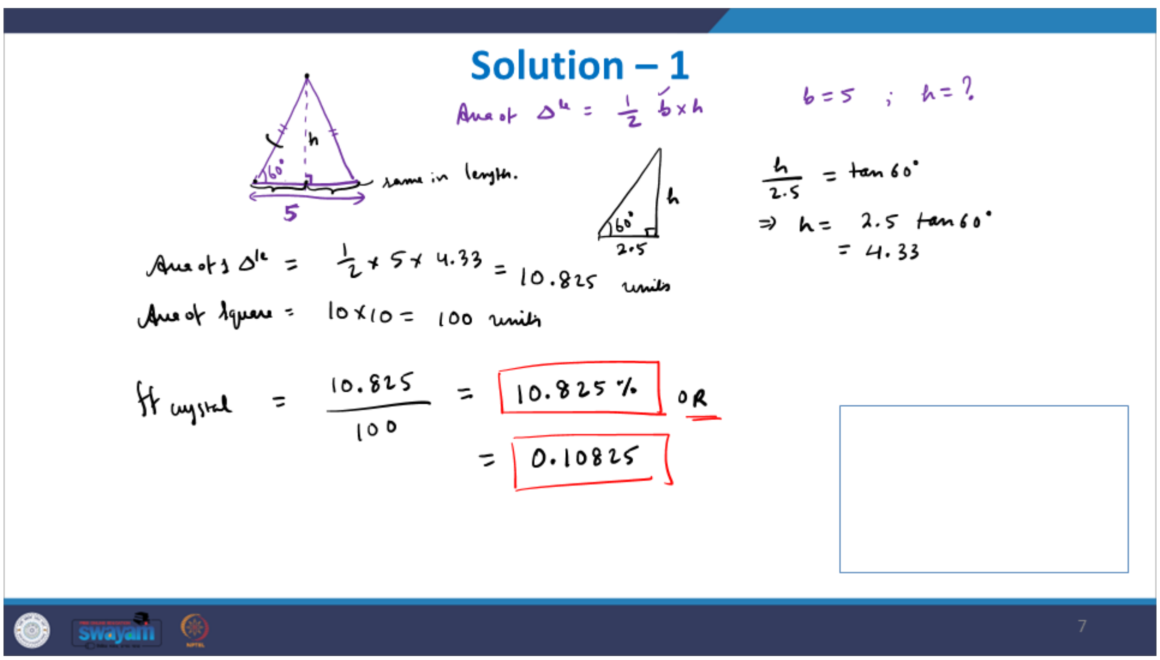
So ultimately, the volume of space—you have to find the filling fraction. For a sonic crystal, it is defined as the volume of space occupied by the sonic scatterers divided by the volume of space occupied by the sonic crystal. So for this one, you can take a primary repeating unit. So, just like how you solve for the porosity of a perforated panel, in the same way, you can have a primary repeating unit in the sonic crystal, which is a really large—almost an infinite—array of these scatterers, and it can be boiled down to a repetition of this repeating unit. This is the primary unit. If you repeat it in the two directions, you will end up getting the complete sonic crystal. So, if you can find out the filling fraction of this repeating unit—okay, the one that is enclosed within the red dotted square. If you repeat this in these two directions—okay, if this unit is repeated in the two directions like this—you end up getting the complete sonic crystal, okay. You repeat this unit in the two directions, and you end up getting the complete sonic crystal, and so on. So, if you can find the filling fraction of one single unit, okay, the same goes for all the other units. So, you can find out the filling fraction of the overall sonic crystal. Let us see what happens in this one repeating unit. So, here is your sonic crystal, and we have taken out a repeating unit, which is made by joining the centroids of the four scatterers like this. This is your repeating unit. When this is periodically repeated in the two directions, you end up getting the complete sonic crystal. So, here in this repeating unit, what is it? You are given the volume with the height. So, this is a 2D layout. You can extrude it in 3D. So, the height of the scatterer and the height of the fluid medium would be the same. So, the volume would be the cross-sectional area multiplied by the height, and because the height is the same for both the scatterer and the crystal, the height is the common factor, which gets nullified from the numerator and the denominator of this equation. So, ultimately, what you get is the area of the scatterer multiplied by the height of the scatterer and the area of the crystal multiplied by the height of the crystal, which is the same for both. So, the filling fraction then becomes the area occupied by the scatterer divided by the area occupied by the crystal, okay. So, it is simply the area occupied by scatterer for a 2D crystal, this is the case, not for a 3D or a 1D crystal. But for a 2D crystal, this becomes area occupied by the whole crystal, whole sonic crystal. So, in this case, you know, it is the area occupied by the triangles. These triangles represent your scatterer and the white portion represents your host medium. So, the area occupied by the triangle divided by the area of the square because this is the area of the repeating unit. So how much area of the triangle in this unit divided by the total area of the square.



If you see here you are joining the centroids of these four triangles in the first unit. This is the first unit which is this portion of the triangle from the second unit here. From this triangle we are getting this one. Similarly from this triangle we are getting this unit. Similarly from this triangle we are getting this unit and from this triangle here we are getting this unit. So, within a square, these 4 units, these 4 portions of a triangle are present. When you combine these 4 portions out of symmetry, what you see is that together they make up a 1 full triangle. So, the overall area occupied by 1 triangle is simply the sum of these portions, which is ultimately the complete triangle in itself. So, there is one triangle present per primary unit, and this is an isosceles triangle. You can find out its area. Let us find it out.

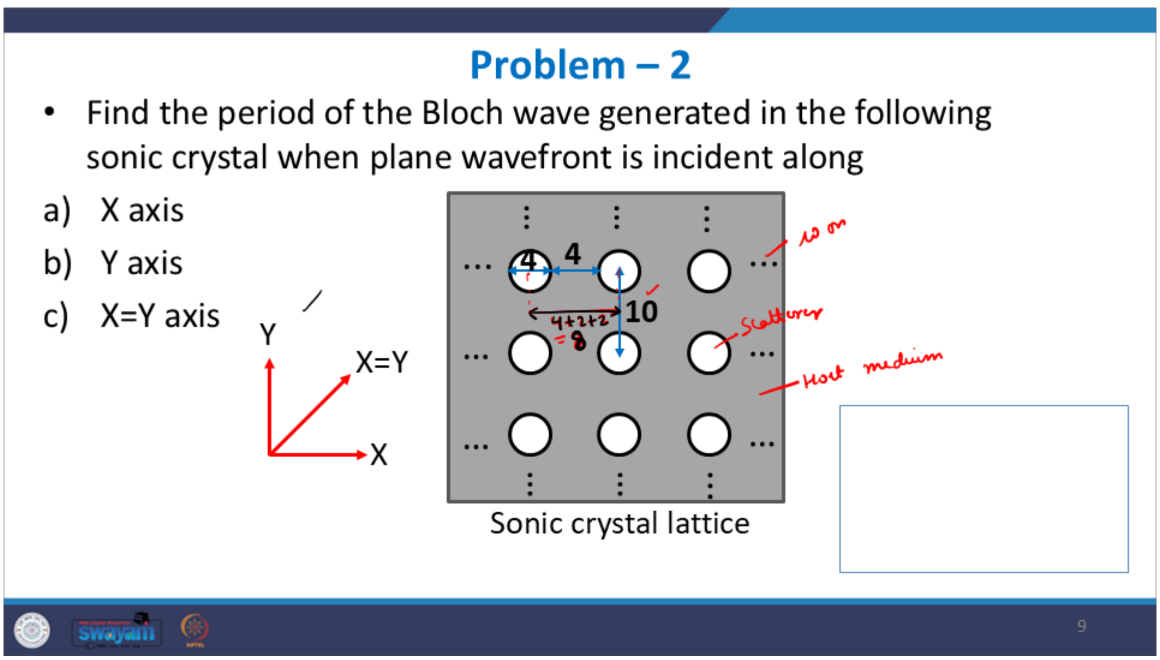


Let us say this is our isosceles triangle, and the dimensions given to you for this one are: 5 centimeters is the base, and the equal angle is 60 degrees. So, let us come here. This is 5. Okay, right now I am not writing the units because the units will get canceled in the filling fraction formulation. So, 5 is the base, 5 units. So, half into base into height—the area of this triangle would be what? It would be half into the base into the height of the triangle. The base is already known, which is 5, okay. Let us find out what the height of this triangle is. This is given to be 60 degrees, okay. So, in this triangle here, the property of the isosceles triangle says that when you drop a perpendicular to the base—okay, the base is the one which is the unequal side of the isosceles triangle, okay. So, if you drop a perpendicular from the opposite vertex to the unequal side of the equilateral triangle, then it is a perpendicular bisector—it will bisect the side into half. So, basically what it means is that the length of this portion and this portion would be same, okay, by the property of isosceles triangle. So, if you take any one triangle here, any one right angle triangle, so what you get, this is your h, okay, what is h? So, let us see this triangle here. here this height is h and this is b by 2 which is 2.5 and this angle is 60 degree, this is a right angle triangle. So, h by 2.5 is tan of 60 degree, okay, the property of the trigonometrical property. So, that means that the height of the triangle is 2.5 times of tan 60 degree which if you calculate comes out to be 4.33 units. So, now the area of the triangle area of one triangle is half into the base which is 5 into the height which is so you get this as units and the area of the square of the reporting unit okay over here if you see what would be the area of the square you see that the square is made of a side of 10 centimeters okay so simply 10 into 10 will give you the area 100 units. So, the filling fraction would be what? For this sonic crystal, it would be the area of this triangle which is this divided by the area of the 1 square. So, this gives you the area of the scatterer by the area of the whole crystal which is in percentage you can write it or you can write it as a fraction as 0.10825. Both ways the answer is correct. So, this becomes your answer.

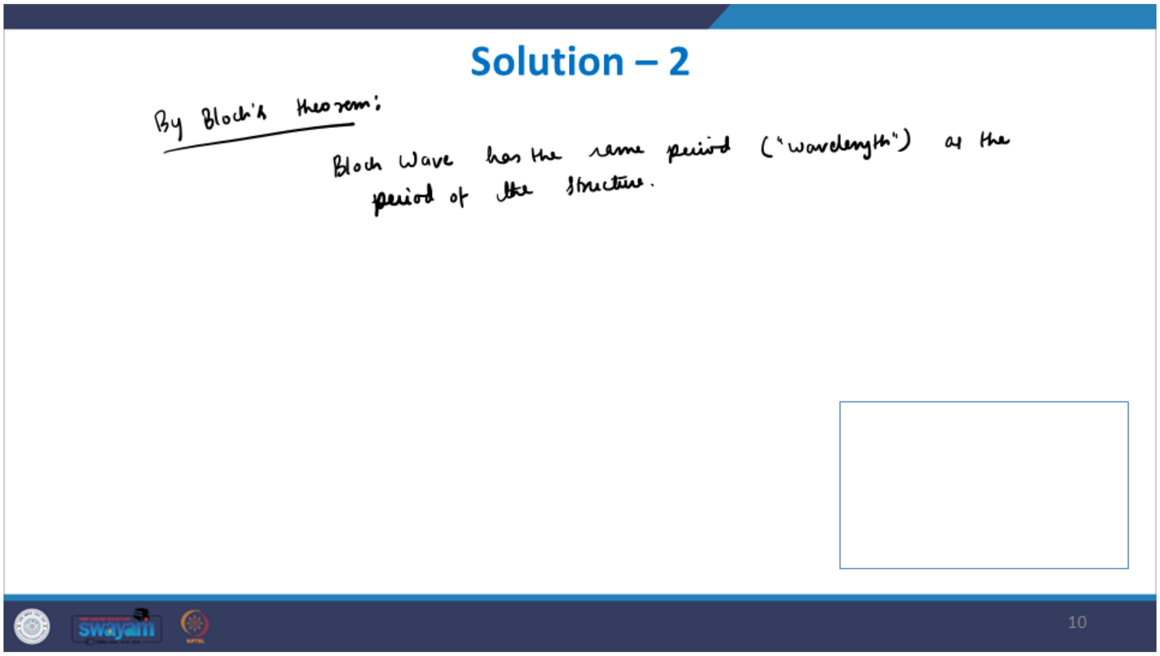


So, let us see another problem here you know you have to find the period of the block wave generated in the following sonic crystal when the plane wave front is being incident. So, now there are three conditions given. So, this is your sonic crystal this dotted line means so on. So, just a portion of the sonic crystal is given you can imagine that this is extending in the 2D in a large format. So, here what is given is that it is made up of circular scatterers.

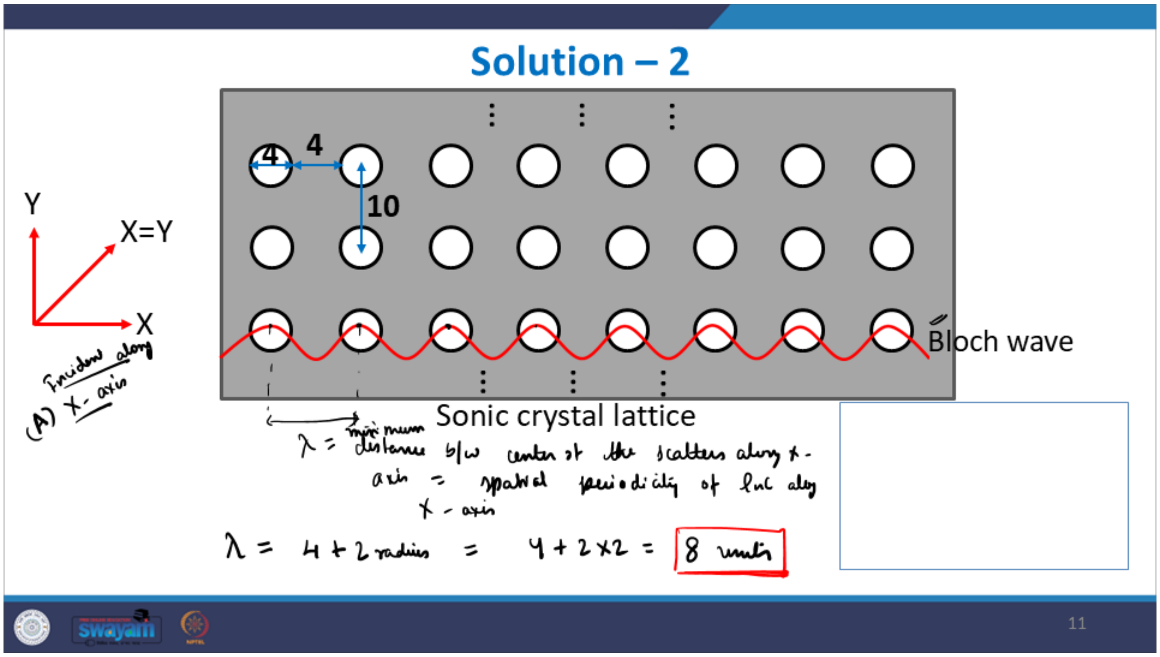
These are the scatterers of the sonic crystal and this is your host medium. The scatterers are circles of diameter 4 units and the spacing is given as that the spacing in the vertical direction between the centers of the 2 scatterer is 10 units that is given to you. Now, you can find out what would be the spacing between the centers of the 2 scatterers here. It would be 4. plus half of this plus half of this, 4 plus the radius of one scatterer plus the radius of another scatterer or 4 plus twice of r which would be 8 units, okay. So, the horizontally the distance between the center of the two scatterers is 8 units and vertically it is how much? It is 10 units, okay. So, this becomes your 8 units, 4 plus twice of the radius, okay. Okay. So, when this plane wave front is incident, you know, by Bloch's theorem, you know that Bloch waves are generated in a sonic crystal. You have to find out the period of this Bloch wave when the wave is being incident along the x-axis is your first case, then y-axis and xy-axis, which is given here. Let us find one by one.



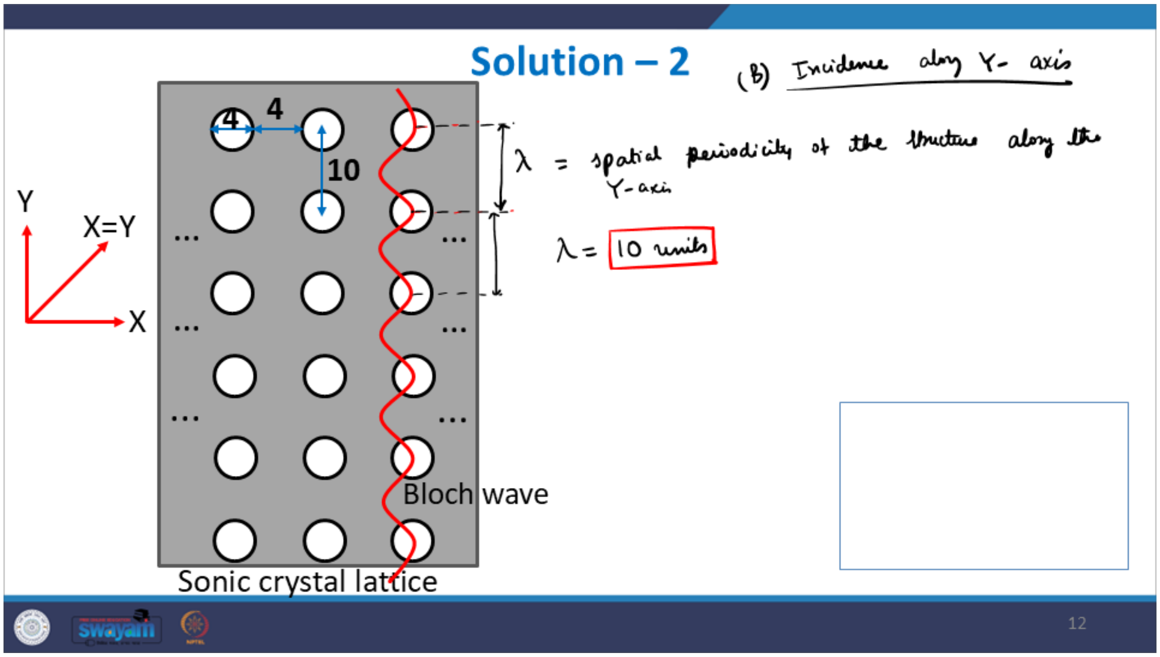
So, for the part A. Okay. So, over here for the part A, because the plane wave front is incident. So, by Bloch's theorem, Bloch's theorem says that you know the Bloch wave which is created inside a periodic structure has the same period which means it has the same wavelength. The spatial period is the wavelength. So this Bloch wave has the same wavelength or the same periodicity spatially as the period of the structure. how periodically the you know properties of the structure are changing in the same way that same period will be there of the wave in itself. So, let us see what is the period of the structure in the x direction.



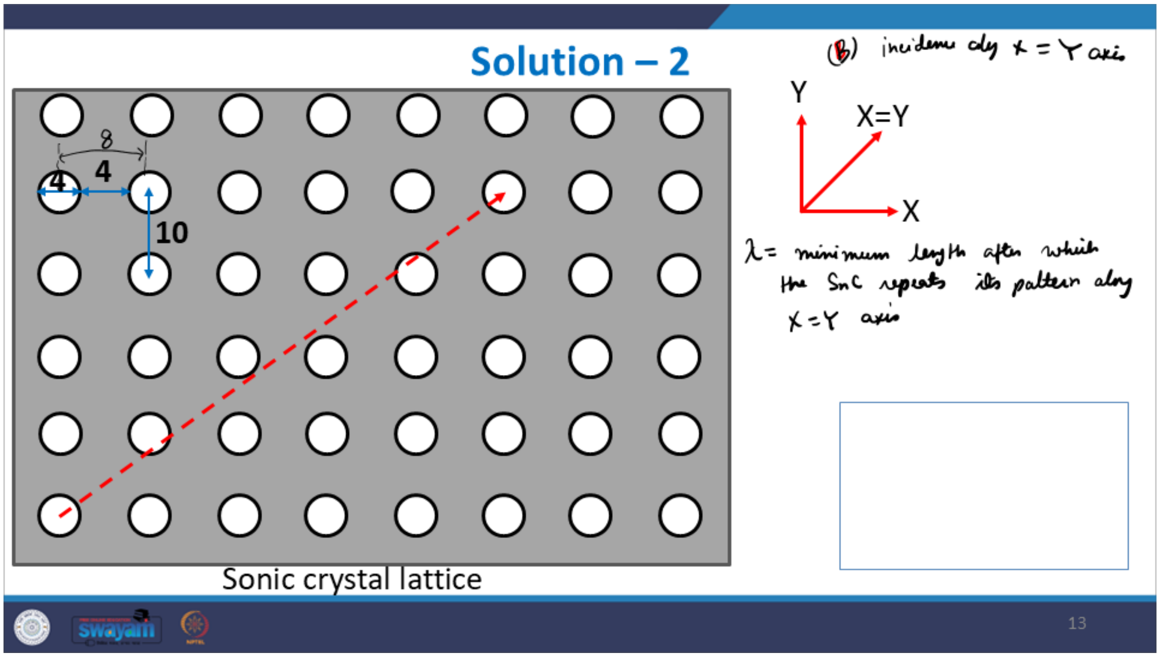
So, when the wave front is incident along the x axis which is your part A when the incidence is along incidence along the x axis. So, the wave that would be created when it is passing through would be this shows the wave along the x axis. So, here as per the Bloch's theorem the wave will have the same period as the periodicity of the crystal which means that This could be one such wave, okay? It is repeating its pattern in the same way as the structure is repeating its pattern in its x-axis. So, as you can see here, one such wave would have all the peaks on the center of the scatterers. So, this λ would be the periodicity. This λ would be what? It would be simply the distance between the center of the scatterers along the x-axis because that is what gives you the spatial periodicity of the structure of the sonic crystal along the x-axis. So, along the x-axis, the structure repeats its pattern every 8 units. So, this wavelength for the Bloch wave would be what? It would be 4 plus twice the radius of the crisp scatterer, which will be 4 plus 2 times 2, which is 8 units. This will become the period of the Bloch wave in the x-direction.



In the same way, if the wave was incident not along the x-axis but along the y-axis, then this shows one portion of the sonic crystal. This is how the wave will be generated. Once again, if you talk about the wavelength for this wave, let us see what the period of this Bloch wave is along the y-axis. Okay, so what will it be? This λ for the y-axis would be what? It would simply be the spatial periodicity of the structure. along the y-axis. So, what is the net period after which the structure repeats its pattern along the y-axis? And as you can see, after every 10 units, the structure repeats its pattern. That is the periodicity. This is your periodicity, okay? The structure repeats its pattern after every this distance. So, this λ would be 10 units, okay? So, here you had 8 units as the wavelength of the block wave along the x-axis. The wave only propagated along the y-axis. The wavelength would be 10 units because it corresponds to the unit after which the structure repeats its patterns.

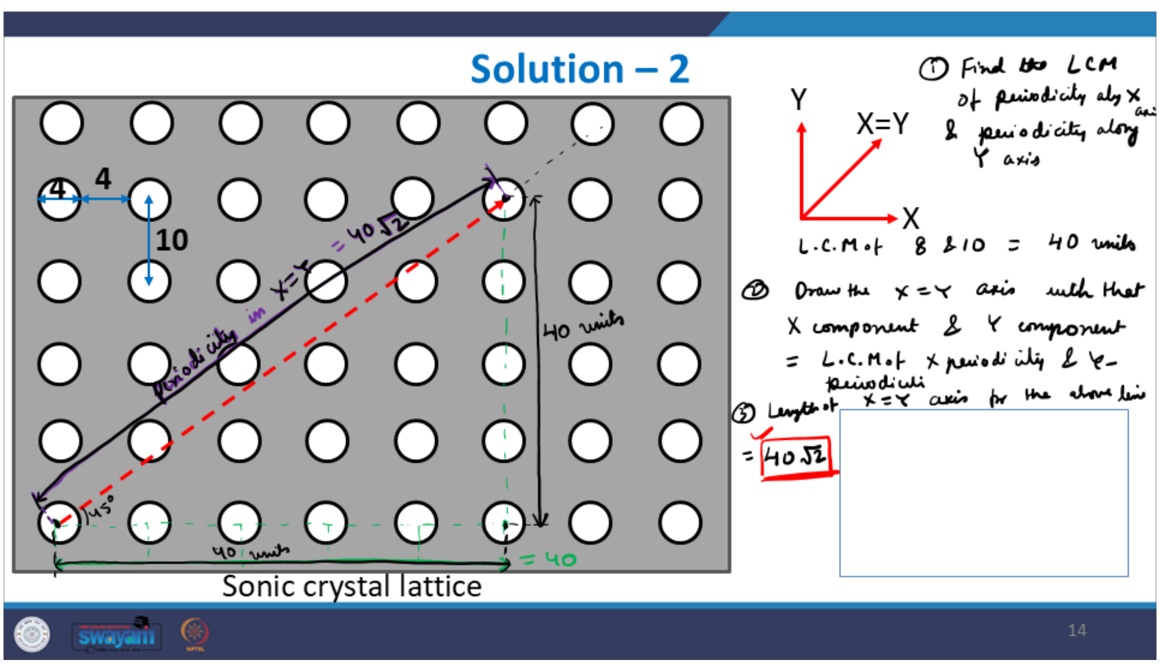


In the same way, suppose when the wave was incident along the x equals y-axis, part B incidence. along x equals y axis. So, now again you have to find basically what you have to find is that the λ would be the minimum length or distance after which the sonic crystal repeats its pattern along the x equals y axis. So, just like in the previous case, you were seeing after what distance the structure repeats its pattern—what is the minimum distance after which the sonic crystal is repeating its pattern along the x-axis or the y-axis? Here also, it would be the minimum length. So, just to clarify here, okay, spatial periodicity—and this was a spatial distance or the minimum distance. After which the structure is repeating its pattern. So, in this case, if you see here, the structure does not repeat the pattern because here this spacing is 10, whereas this spacing here, is 8. So, the structure does not repeat if you draw the x is equal to y axis—this is your x is equal to y axis—it does not repeat the pattern because here, how would you solve this question? Let us find out the least common multiple of 8 and 10. So, find out the LCM of 8 and 10, which is the first thing to do.

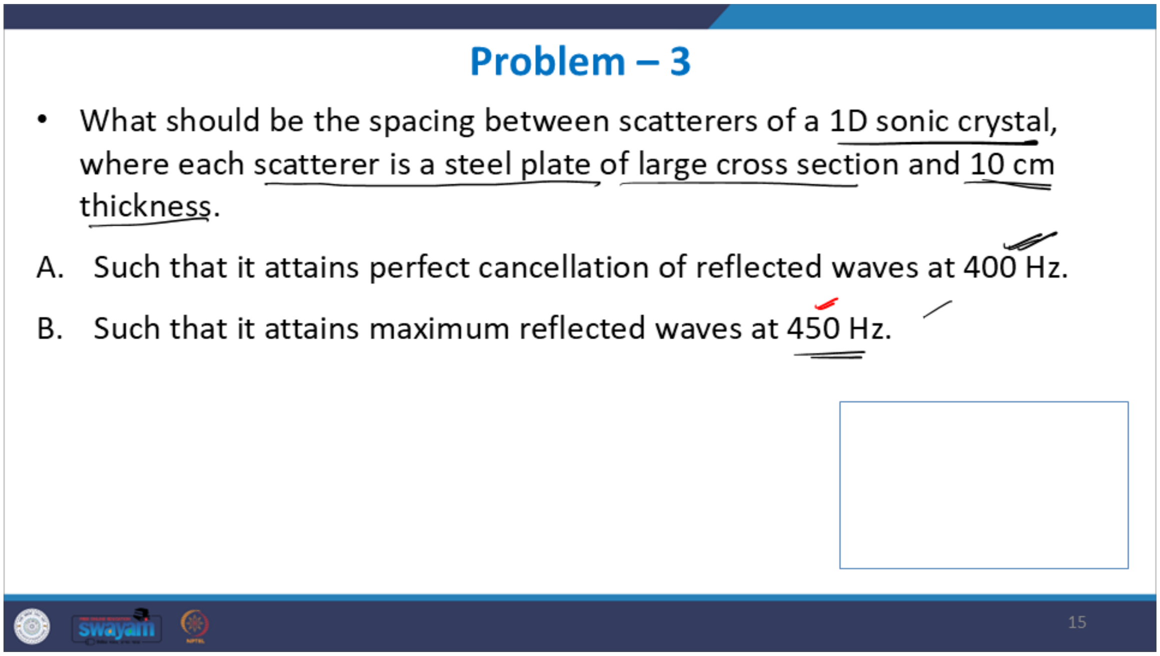


Step 1: find the LCM or the least common multiple of the periodicity along X and periodicity along Y. So, what is the common multiple of the periodicity along the X and the Y axis? So, here, what is the answer? The LCM of 8 and 10—that is your periodicity—which would give you 40 units. So, now what you do is you draw the X is equal to Y—draw the X is equal to Y axis such that the horizontal and vertical component, which is the x and the y component. So, you have to draw the x is equal to y axis such that the x component and the y component is equal to the LCM of, you know, the two periodicities you have found—the x periodicity and the y periodicity. So, what it means is that essentially from here, let us say we start from this point and we measure a length of 40. We will reach up to here because here 40 means what? It is 8 units. So, which means that 1, 2, 3, 4, 5. After 5 repetitions, you will reach 40. This is the net length of 40. This is equal to 40. This distance is what? This is 40 units. In the same way, the y component is equal to 40 units from here to here. As you can see, this distance again is what? 40 units, okay? So, 40 units here in this triangle and 40 units here. So, both the x and y components have 40 units as the length, which is the LCM of 8 and 10. And then you draw the line joining the two. You will get an x is equal to y axis, obviously, because the x and the y component are the same. So, it is an x is equal to y axis. And what you see is that we started from the center of the scatterer, okay? And then we covered a hold pattern we are not seeing a repetition in the pattern because here it has passed from the center of the scatterer here over here. And then it is passing from somewhere below then

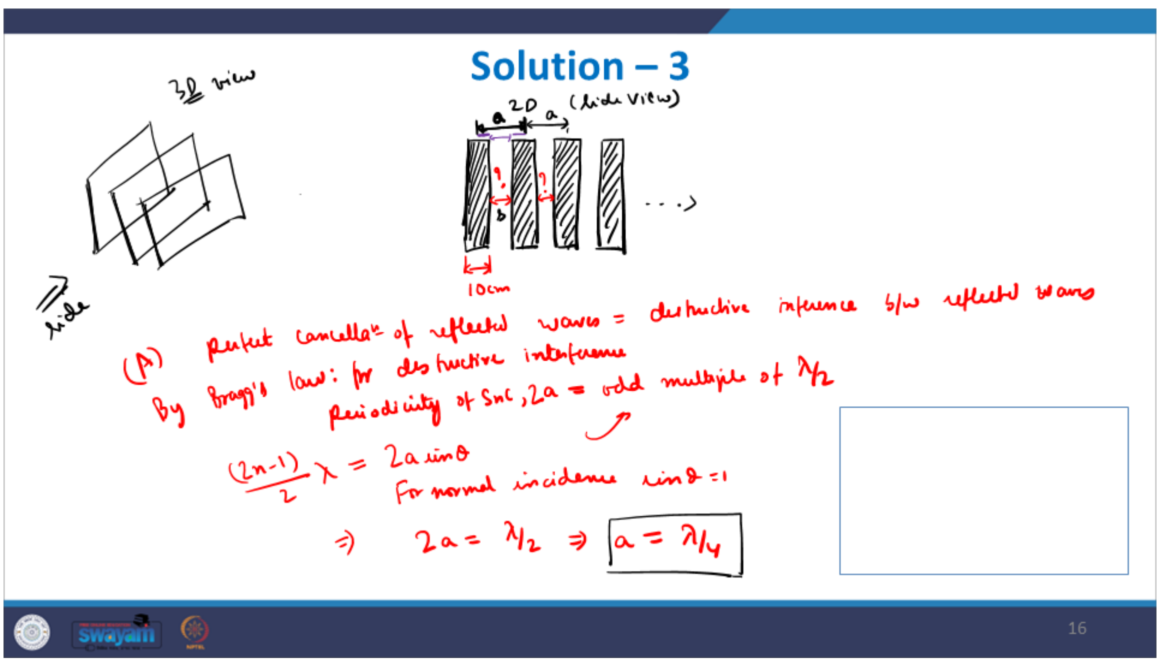
exactly below then somewhere above and then finally when you reach here it again passes from the center of the scatterer. and then the whole thing will repeat and so on. So, again after you know these length the pattern will repeat. So, it started from the center of the scatterer and then went through the different kind of variations and once again it reaches the center of the scatterer here. So, when we have drawn that what will be the answer? The answer would be simply then the length of the x is equal to y axis for the above line will give you the periodicity. So, in our case, what it would be? It is a right angle triangle with this being 45 degree. Why? Because it is an x is equal to y axis. So, it would be 40 into root 2 by the Pythagoras theorem. This would be your spatial periodicity. 40 root 2 units. So, that is your periodicity. So, what you see here that after every this distance, The structure repeats this is the periodicity in x is equal to y which is 40 times of root 2 ok. So, this is the length after which the structure is repeating its pattern along the x is equal to y direction. It started from the center of one circle or the center of a scatterer and after the various kind of non-changing patterns, it again reaches the center of the scatterer and then once again the pattern repeats in this direction. So, this is how you get the 40 root 2 that is the periodicity in the x is equal to y axis. So, what you do here you find the least common multiple and then you draw a triangle such that the x and the y component is equal to that least common multiple and then you draw the x is equal to y axis and the length of that you know, hypotenuse or the length along the x is equal to y axis such that the x and the y component become the LCM will give you the periodicity in this direction. So, as you can see here, the pattern starts repeating from center of one scatterer, it started and once again it reaches the center of another scatterer.



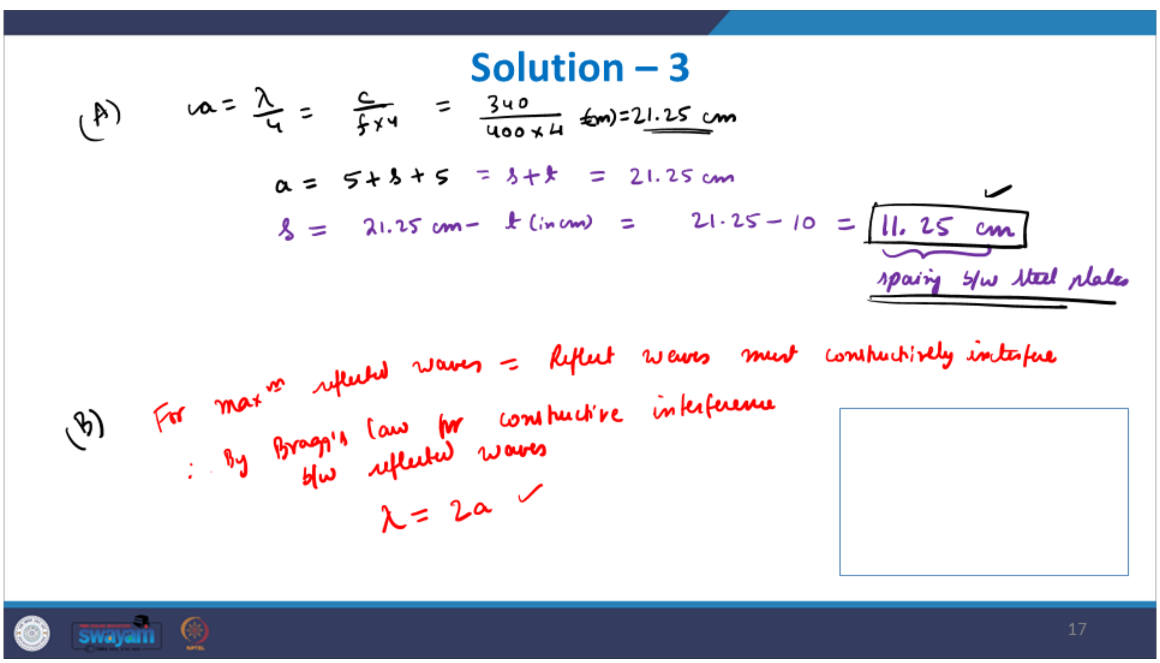
So, let us see the problem 3 which is you know what should be the spacing between scatterers of a 1D sonic crystal where each scatterer is a steel plate. of large cross section and 10-centimeter thickness such that it attains perfect cancellation of reflected waves at this frequency and then second part is attains maximum reflected waves at 450.



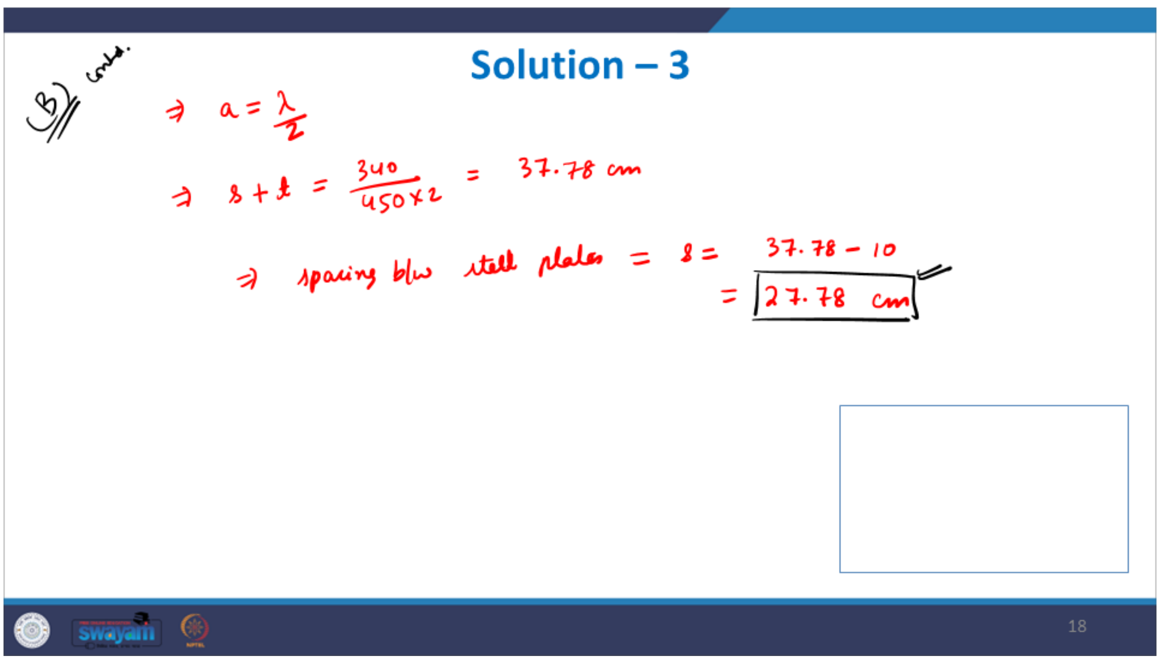
So, let us start here. First of all, let us see what this scatterer looks like. So, what should be the spacing between scatterers of a 1D sonic crystal made of a steel plate with a large cross-section and 10 centimeters as the thickness? So, let us say this is your side view I am showing you of this. So, this would be the 3D view. These are the plates and so on, which are arranged in the 3D view. But in 2D, what would it look like if you look from this side here? This side view would look like what for the sonic crystal? It would look like this. You know, these are steel plates of a certain thickness. Which is given to be 10 centimeters. So, you would have a 10-centimeter steel plate. This is your scatterer that I am showing with the red, and the white portion represents the air or the host material. So, periodically these plates are arranged. And so on in this direction. So, here this length is given as 10 centimeters, but this spacing is not given to us. What is this spacing? That is not given to us. And that is what you have to find: what should be the spacing between them? What is the condition? So, in the first case, Case 1. There is a perfect cancellation of reflected waves at 400 Hz. So, perfect cancellation of reflected waves. What does it mean? It means that there is destructive interference between the different reflected waves from the different layers of the scatterers. So, by Bragg's law, For destructive interference, what is the condition? We have already studied this in the previous lectures. For destructive interference between these reflected waves, you know the periodicity. The periodicity of the sonic crystal, which is A, should be odd multiples of λ by 2 to achieve this. For normal incidence, sin Θ is equal to 1. So, ultimately the condition that you get is that that twice of A is an odd multiple. of λ by 2. So, the lowest case we will consider. So, for lowest case, the 2a should be equal to λ by 2. This means that the periodicity should be λ by 4. That is your condition so that there is a destructive interference between the reflected waves and they get cancelled out. So, a is equal to λ by 4. There is a destructive interference. So, a is λ by 4 and you are achieving this at 400 hertz.



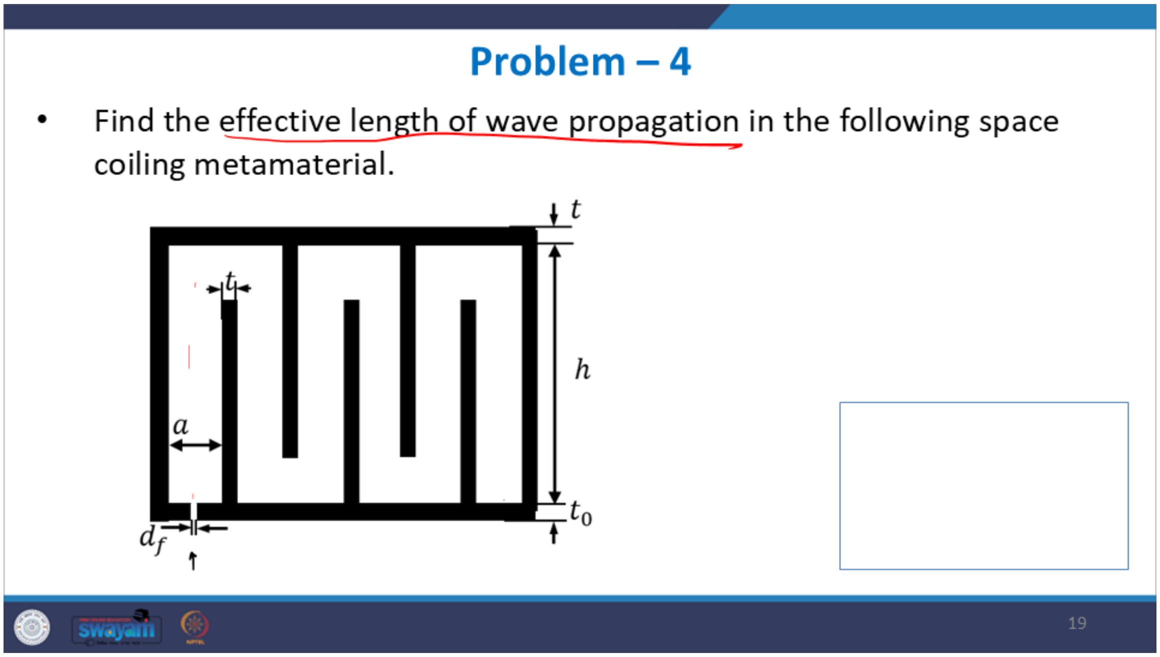
So, let us see what is this for part a the periodicity should be λ by 4 which is c by f into 4 λ is c by 4. So, taking a 340 meters per second as your speed of sound in the air at room temperature if you solve this you will get the answer as 21.25. So, this is in meters okay? This is in meters. So, answer you will get is in centimeters 0.2125. So, that is your periodicity, okay? So, what does it mean? What is the periodicity here? The periodicity is this value. This is your A, okay? After every A units, the crystal repeats its pattern. So, if you see here, again this is your periodicity. So, in this, what is A? A is simply the spacing S plus 5 plus 5, okay. So, A is what? It is 5 plus the spacing between the cylinders plus another 5, okay. So, 5 units here, okay. So, 5 units here and then some spacing S and then another 5 units, that gives you the net periodicity. So, it is S plus the thickness, okay. Overall, or simply you can say S plus the thickness of the plate, and this should be 21.25 centimeters. So, you have to find the spacing. It would be 21.25 centimeters minus the thickness in centimeters, which is given to us as 10. So, ultimately, the spacing would be 11.25 centimeters. That is what the spacing is. Between the steel plates, that is what has been asked of us: what is your spacing? This becomes your answer for the first case. In the same way, you can solve it for the second case. For the second case, it has been asked what the spacing would be if the sonic crystal is able to attain maximum reflected waves at 450 Hz for maximum reflected waves. Okay, let us solve again. For maximum reflected waves, the reflected waves must constructively interfere. So, for maximum reflected waves, if they are constructively interfering, then the amplitude is going to shoot up; it is going to rise. That will be the condition for maximum reflected waves. Therefore, by Bragg's law, once again, for constructive interference between the reflected waves, What should be the condition? It is given as λ has to be twice of A, where A is the periodicity of the sonic crystal.



So, this implies that your A has to be—okay, λ has to be here. λ by 2 was—A was λ by 4 in the previous case; here, it is λ by 2, okay. So, λ has to be twice of A, which means that A has to be λ by 2, okay. This means that A is what? It is your spacing between the cylinders plus the thickness, which is λ by 2, which is 340 divided by the frequency. The frequency in this case is 450 Hertz for the second part, multiplied by 2. So, this value comes out to be, if you solve it, 37.78 centimeters. This implies that the spacing Between the steel plates for this condition would be 37.75 minus the thickness of the cylinders, which is 10. So, which is 27.78 centimeters. This is your answer for the second part. Part B. So, like this, you can find out the various conditions for attaining constructive interference or destructive interference between the reflected waves from the sonic crystal.

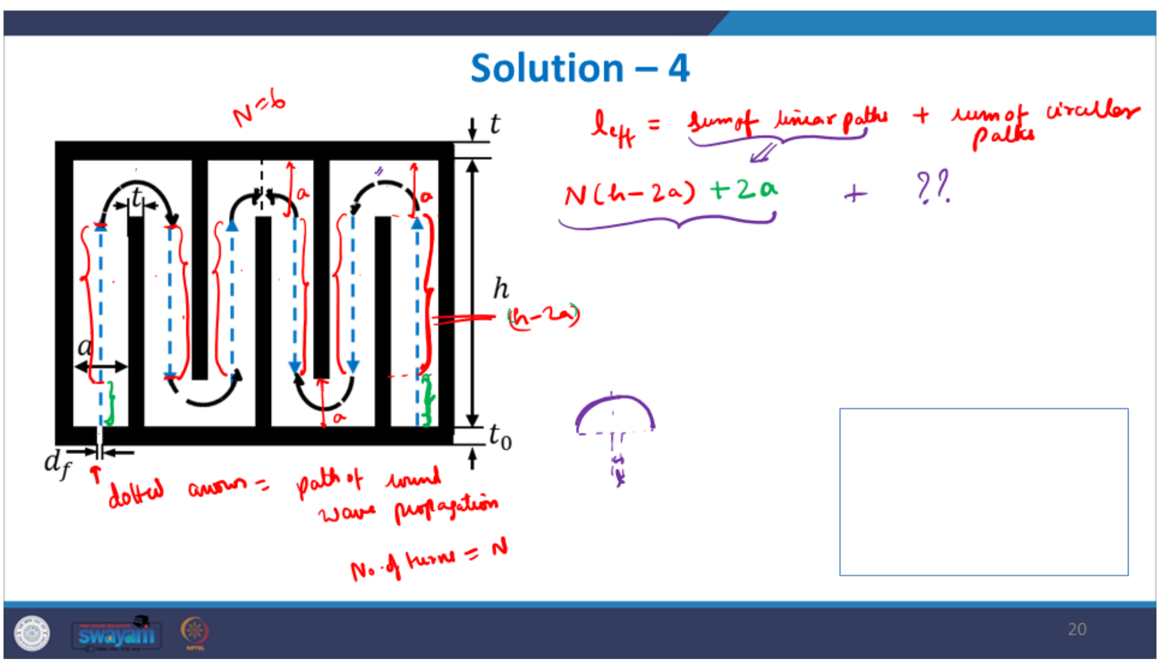


Let us solve the last question for this lecture here. A cross-sectional view of a space-coiling metamaterial is given to you. And this shows the cross-sectional view. The wave enters from this end, goes through this elaborate pathway, and finally reaches here. So, that is the pathway it is taking. What you have to find is the effective length of the wave propagation along the sonic crystal. So, let us see what kind of pathway the wave takes. You can refer to one of the papers published by us, where an iso-surface analysis was done to see that the wave goes like this and then takes an elliptical or circular path along the turn and simply reaches here.



So, this is the pathway the wave takes; it starts from here. So, once the wave is incident on this, the dotted arrows They are showing the path of the sound wave propagation. So, the sound wave enters from this particular end and then it takes this pathway in the space-coiling metamaterial. So, let us see what this would be. It would be so here—the total length. The effective length of wave propagation inside this would be the summation. So, this is going to be the summation of the linear path and the circular path. So, you know, it will be the sum of the linear paths plus the sum of the net path. So, we have to find the length of this total path. So, we can break it down into the linear paths plus the circular paths. The sum of the linear paths and the circular paths. Let us find the linear path first. So, here, let us see what this length is. This is the common length that is repeating everywhere—this length. This length is repeating how many times? If you see here, 1, 2, 3, 4, 5, 5 and 6 okay so here so for a generic you know number of turns is equal to n. So, here the number of folds are 1, 2, 3, 4, 5 and 6. In this case, n is 6. So, it is repeating 6 times for every turn. So, for a generic case, let us find out for a generic case for n number of turns, this is going to repeat n number of times. So, what is the length of this particular thing? If you see here, this total length is h and then the width, this distance here, is a, it is

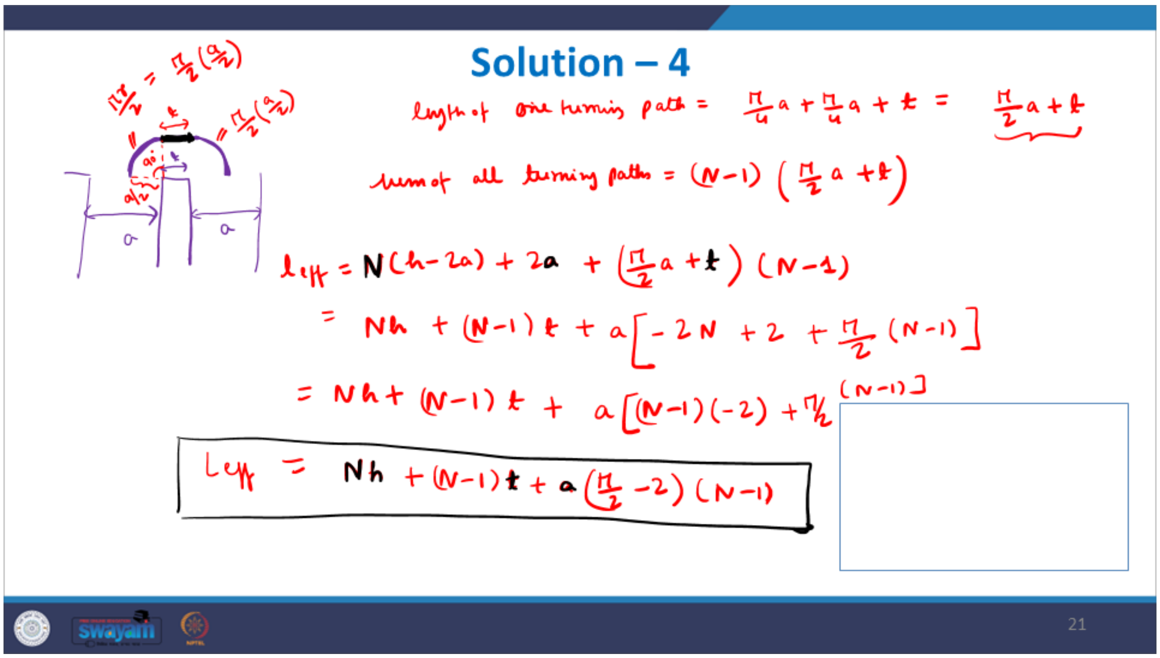
a. So, a units from here, so h minus 2 a is going to give you the length of this. Total h minus twice of a, a units removed from both the ends. So, n times of h minus 2 a will give you the length of this particular repeating linear path. is the length of this and it is repeating n times. So, n times of that plus this is the remaining pathway this green thing over here. So, this is the portion remaining. So, we have already accounted for all the linear paths which is n times of h minus twice a which is this length here. Then, we have to see what is the length of this one from here till here, this path, which occurs twice, okay? Once at both the extreme ends, this extra length is happening. So, it should be twice. So, plus twice of, what is this? This is A unit. So, twice of A. So, the red portion and the green portion together, they make up your, the linear paths, okay? So, the sum of the linear path comes out to be these two terms here. Now, let us find out the sum of the circular paths. You see how many turns it takes. Suppose it was n number of channels, then between two channels we have one turn. So, it would be 1, 2, 3, 4, 5, 5 such turns and 5 such turns are happening. So, for a generic case, it is n minus 1 turns. So, n minus 1 times this kind of circular turn is happening. So, this is n minus 1 times. Let us see one circular path and see what is its total length. This is your one circular path. So, here this thickness is t. This is a thickness t. So, let me give you an elongated view. This we have to find now. Let us see the sum of these circular paths.



Let us see an elongated view. This was your slab of thickness t, okay? And the wave is turning like this, and between two slabs, this is your a, okay? So, what you see here is that if we can divide this into three portions. The first portion is a circular sector, then we have a linear line like this, okay? And then again, we have that same circular sector, okay? So, what is this? If you see here, Here, this part has a length where this angle is 90 degrees, or it is the quarter of a circle. So, the net circumference of a circle is 2 π r, 2 into π into the radius. So, this is a quarter of that. So, this should be π r by 2. Is it not? It should be π r by 2 or simply π by 2 times the radius of this circular thing. And what is the radius here? This radius is a by 2. So, this length would become then π by 2 times a by 2. That would be your sort of the thing here. And this again would be the same thing. This also would be π by 2 into a by 2. And what would be this length? This would simply be t. The length of one circular path—one turning path, I would say—because it is made of circular as well as straightaway paths. The length of, you know, one turning path. The path is π/4 a plus π/4 a plus t, or, totally, what you can say is it is π/2 times a plus t. This becomes the length of one turn, and how many turns do we have? We have n minus 1 turns. So, the sum of all turning paths would be n minus 1 times this thing. So, this we have found. So, the net Leffective then would become what? The sum of the linear paths, which is this thing, plus this, is your net Leffective, OK. Suppose these are your varying, you know, parameters. This is your parameter n, this is your parameter a, this is your parameter t. So, we can separate it into these parameters and see. So, if you solve it further, you can either leave it here or see how it changes with respect to n, a, and t. Then what you see here is that if you solve it, you can end up getting this answer. So, n plus h plus—let us see what its variation is. So, its variation with respect to h is simply n times h, and with respect to t, it is n minus 1 times t. Now, let us sum up all the terms with respect to a. So, from here, we have minus 2 times n plus 2 plus π/2 times n minus 1, OK. So, what you end up getting is If you take out the common factor as n minus 1, from this what you get is minus twice, minus 2 of n minus 1 plus pi by 2 times n minus 1. So, ultimately your answer would become n h plus n minus 1 times t plus with a you will have pi by 2 minus 2 into n minus 1, ok. So, that would be the net effective length,

$$\boxed{L_{eff} = Nh + (N-1)t + a\left(\frac{\pi}{2} - 2\right)(N-1)}$$

and this is how it varies with respect to h, t, and a value, ok. So, what you see from here is that you can change the length of the pathway by simply increasing the number of folds or increasing the number of n, or you can increase the thickness of the slab, you can increase A. Similarly, you can increase the h parameter, and this is how you can change the effective length and accordingly change the frequency of resonance and tune the performance of a space-coiling metamaterial.



So, with this, I would like to close this lecture. Thank you for listening.