NOISE CONTROL IN MECHANICAL SYSTEMS

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Week:02

Lecture:06

Lecture 06: Intensity and Power

Hello and welcome to Lecture 6 in this lecture series on noise control in mechanical systems. I am Professor Sneha Singh from the Department of Mechanical and Industrial Engineering at IIT Roorkee. So, this is Lecture 6, whose title is Intensity and Power. So, here we will be discussing two important terminologies with respect to sound wave propagation: sound intensity and sound power.

So, to quickly sum up the previous two lectures, we studied the harmonic plane wave and found the solution for a harmonic plane wave using the linear acoustic wave equation and the solution for any generic wave. That is traveling in any generic direction in the 3D

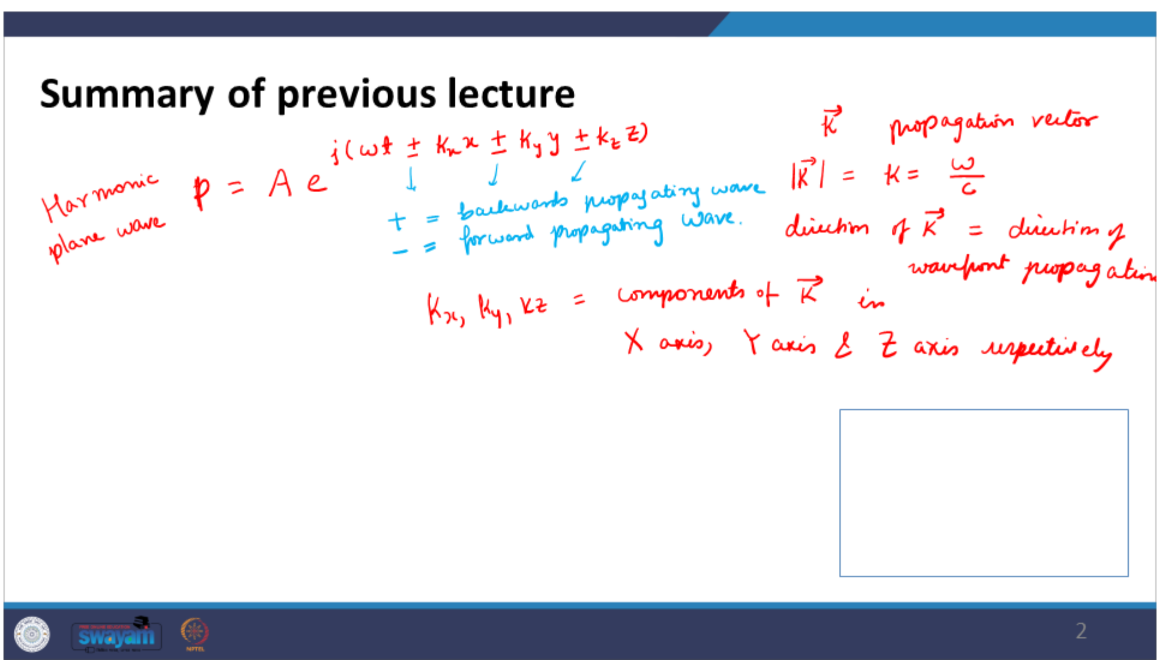
Cartesian system, which is given by the solution where P is the acoustic pressure of the harmonic plane wave, and this acoustic pressure P is given by some amplitude A, whose magnitude can be found based on the boundary conditions in any problem. And then

$$P=Ae^{j(\omega t \pm k_x x \pm k_y y \pm k_z z)}$$

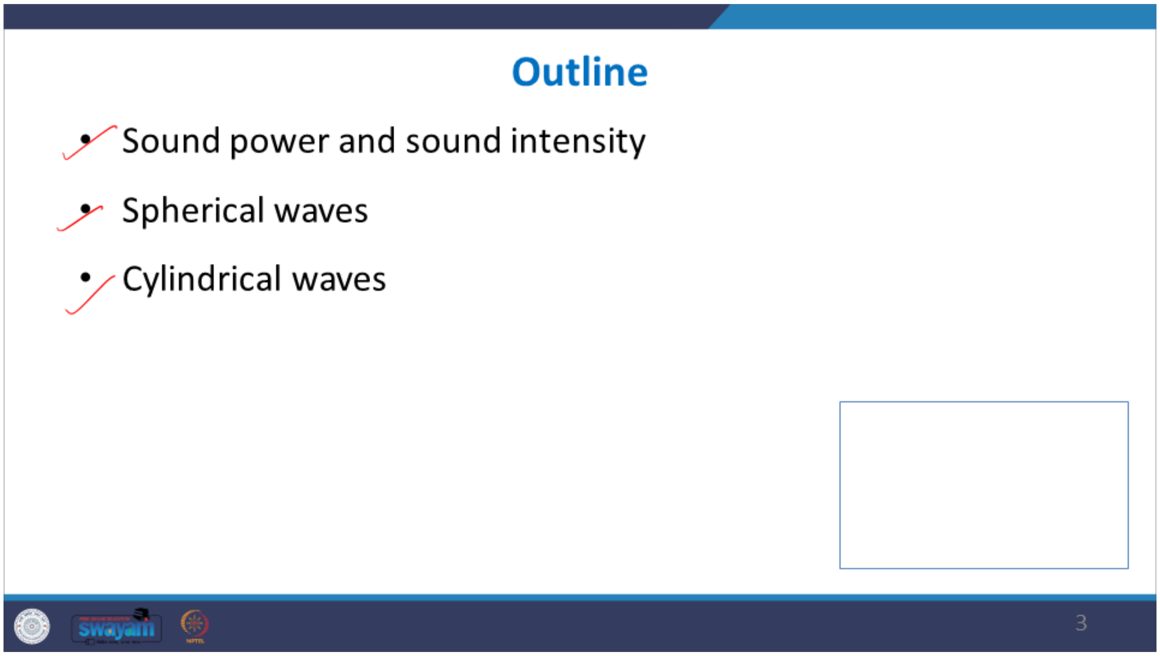
where we also studied the propagation vector k. The magnitude of this propagation vector, because it is a vector, has a magnitude and a direction. The magnitude is simply the wave number k, which is

$$k = \frac{\omega}{c}$$

This relationship has been derived. This gives you the direction of the vector k, which is the same as the direction of the wave front propagation. So, here k_x, k_y, and k_z are simply the components of the propagation vector k in the x-axis, y-axis, and z-axis, respectively. Okay, so this is the generic equation, and here in all these equations, if we see a plus sign, it means a backward-propagating wave. If it is a minus sign, it means the wave is propagating in the forward direction, which means it is propagating in the positive axis direction or the forward-propagating wave.

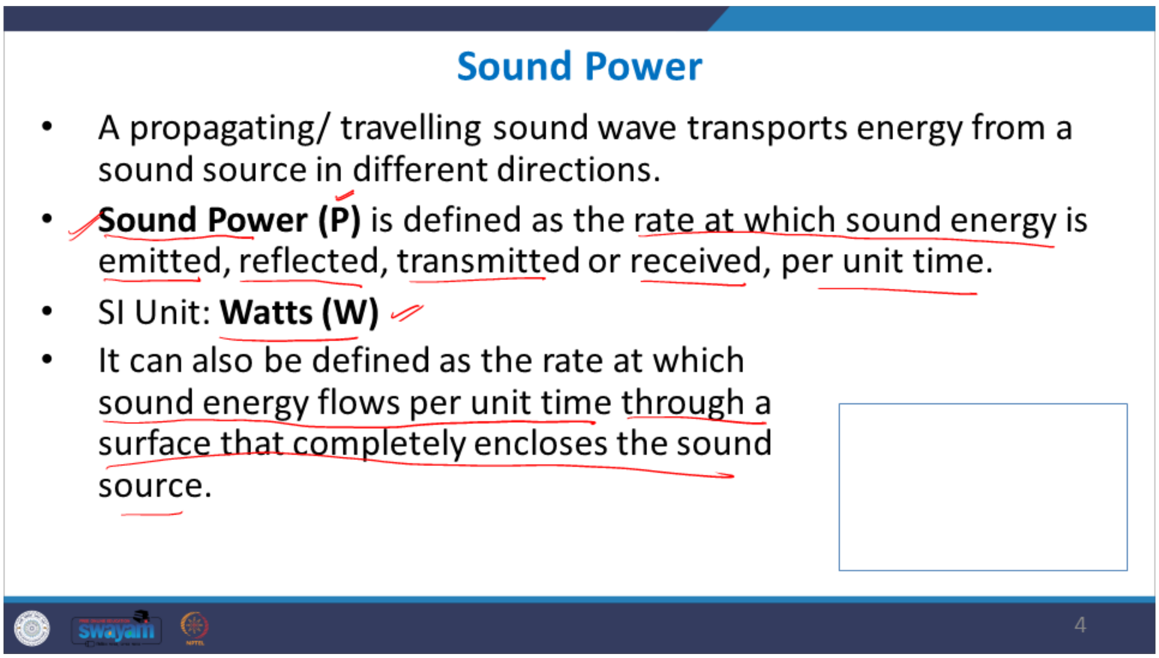


Okay, so with this summary, let us quickly start Lecture 6, where we will discuss two important quantities for wave propagation: sound intensity and sound power. Then, we will also discuss two new types of waves, which are very commonly encountered in the problems on mechanical systems. We have the spherical waves and the cylindrical waves.



So, what do you mean by sound power? So, at the very beginning, when we were introducing sound, So, what is sound? It is a form of energy, and this energy is in the form of the pressure variations or the disturbance or oscillation in the pressures that is transported through the medium. Okay. So, if it is any propagating or traveling wave, As the disturbance or the acoustic pressure, it propagates forward, or as the disturbance or oscillations in the particles, they propagate forward in space. What are they doing?

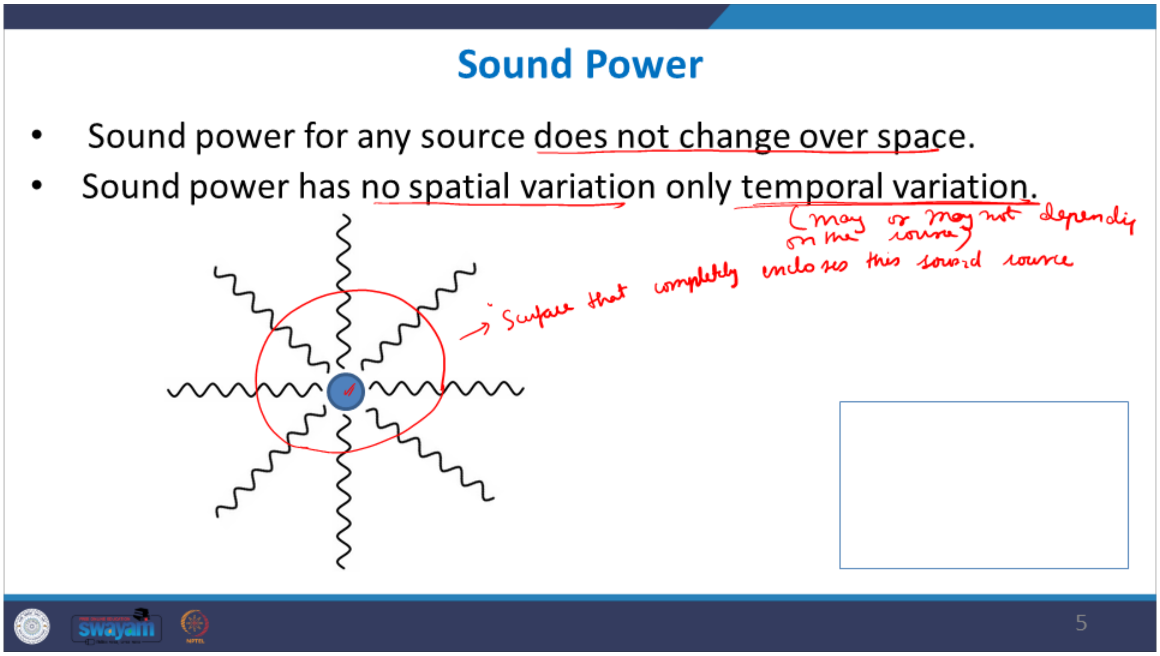
They are also transferring the energy from one location to another location in space. So, how is this energy being stored? It is stored in the form of the kinetic energy of these particles, which are set in motion when a sound wave propagates through. So, there is some energy that is propagating through when the sound wave is propagating. So, obviously then, there will be some power associated with it, some intensity associated with it. And just like in other fields such as in the field of heat or in the field of electricity, you have the same definition for power and intensity. So, what is power? It is the energy per unit time or the work done per unit time. So, here sound power P. That is the notation we will be using, is defined as the rate at which sound energy is being emitted, reflected, transmitted, or received per unit time. So, the rate at which this energy, depending on the context in which we are measuring it, is either emitted, reflected, transmitted, or received per unit time, and the SI unit is watts. Energy per unit time. It can also be defined as the rate at which the sound energy flows per unit time through a surface that completely encloses the sound source. Now, this particular definition would remind you of the heat flux that we have studied in various other mechanical courses.



So, suppose you have some source here, and it is emitting. Now, the source is there; it is radiating or emitting the energy all throughout the medium that surrounds it. So, suppose we had any surface that was enclosing this; let us draw a surface a closed surface that completely encloses this source or this sound source. So, whenever in these lecture series I refer to source, it would mean the sound source. So, over here what you see is that there is a source which is like a small ball; it is emitting the energy, and you have some kind of surface completely enclosing it.

So, it is receiving all the energy that is radiating. Suppose this surface had holes; then some energy would go through, or if it was not completely enclosing and only partially enclosing, then also in that case. Not all the energy from the source will be captured. Some of the energy will radiate away and won't be captured entirely by that surface. So, it's important that the surface we're talking about completely encloses the source. And then, the rate at which that surface will be receiving the energy per unit time will then become the sound power of that source. Now, it is obvious if you think about it logically: the sound source is radiating energy into the fluid medium or the fluid surrounding it. So, whatever energy it is radiating, we have a fixed source. So, no matter at what distance we measure, it is the same energy. And it is flowing per unit time.

So, it should not change over space. So, sound power does not change over space. There is no spatial variation. There could, however, be temporal variation, or not, depending on the source. Source. So, let us say, for example, I will take the analogy of light energy. We have a light bulb; we switch it on and off, on and off over time. So, it is radiating energy into space, then it stops radiating the light energy, then again it radiates and then stops radiating the light energy, depending on how we are switching it on and off. So, over time, the energy from that bulb is flowing. And changing over time but not over space, okay? So, whatever energy is radiating at, suppose some energy is radiating at the time t_1 to t_2, it should stay the same from one location to another, provided there is no blocking surface or anything. So, the energy should stay the same in the same way if, suppose, we have a noise source or machinery. Now imagine, and we are switching it on, switching it off, then switching that machinery on again, we are switching it off. So here, the energy from the source is varying over time. Or let us say we have got some mixer or some vacuum cleaner; we are changing its speed settings over time, then depending on how fast these machineries are, in general, you observe that the sound level rises. So, the more speedy they are, they are, in general, radiating more energy. So, their emitting of the energy may change over time. Or suppose we have a fixed machinery; we are doing nothing with it, just we are running it continuously, then the energy will stay the same. So, depending on how the source is running, the energy may or may not change over time, but this energy should not change over the location of the fluid medium.

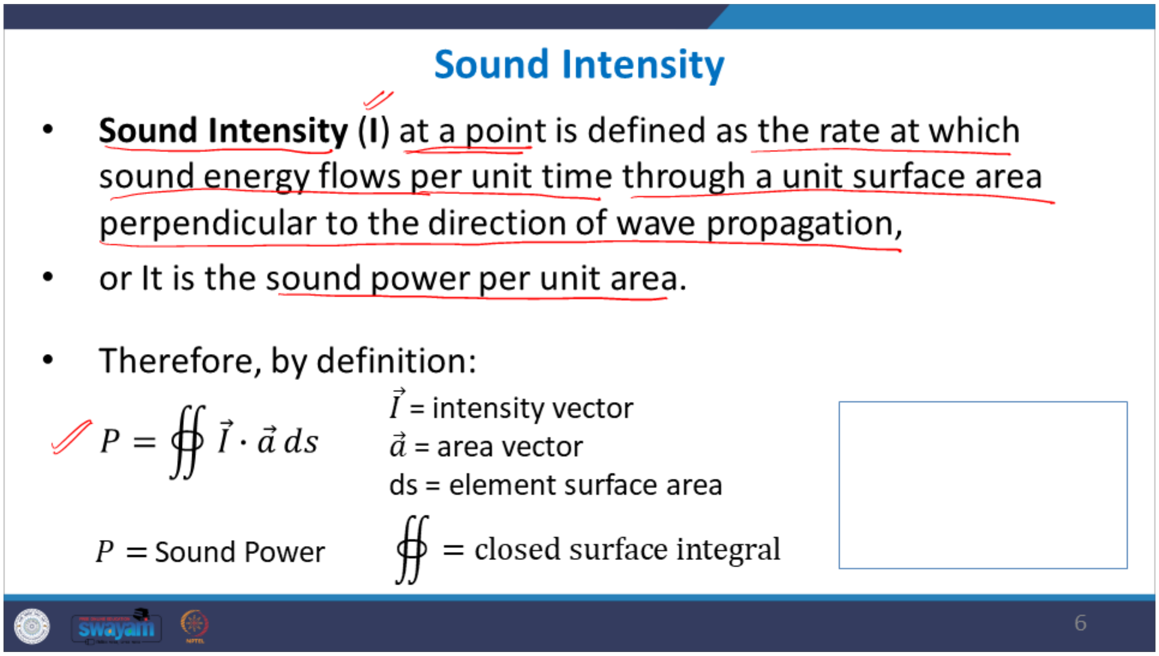


Now, we think about sound intensity. So, it is the energy. So, what was power? It was energy per unit time. Intensity becomes energy per unit time per unit area and intensity. So, just like sound power becomes a characteristic of the source, not of the location in which it is being measured. So, it depends entirely on how the source is running. Sound intensity becomes a characteristic both of the source and of the location at which it is being measured.

So, it is always defined at a certain point, not randomly. We cannot define intensity randomly; we have to specify where exactly we are measuring that intensity. So, it has to be linked to a particular location. We will denote this intensity term by the I. It is defined as the rate at which sound energy flows per unit time through a unit surface area that is perpendicular to the direction of wave propagation, or simply sound power per unit area. So, by definition, it should be the surface integral. The power we can obtain by doing this surface integral of intensity with the area over a particular surface that is enclosing the source.

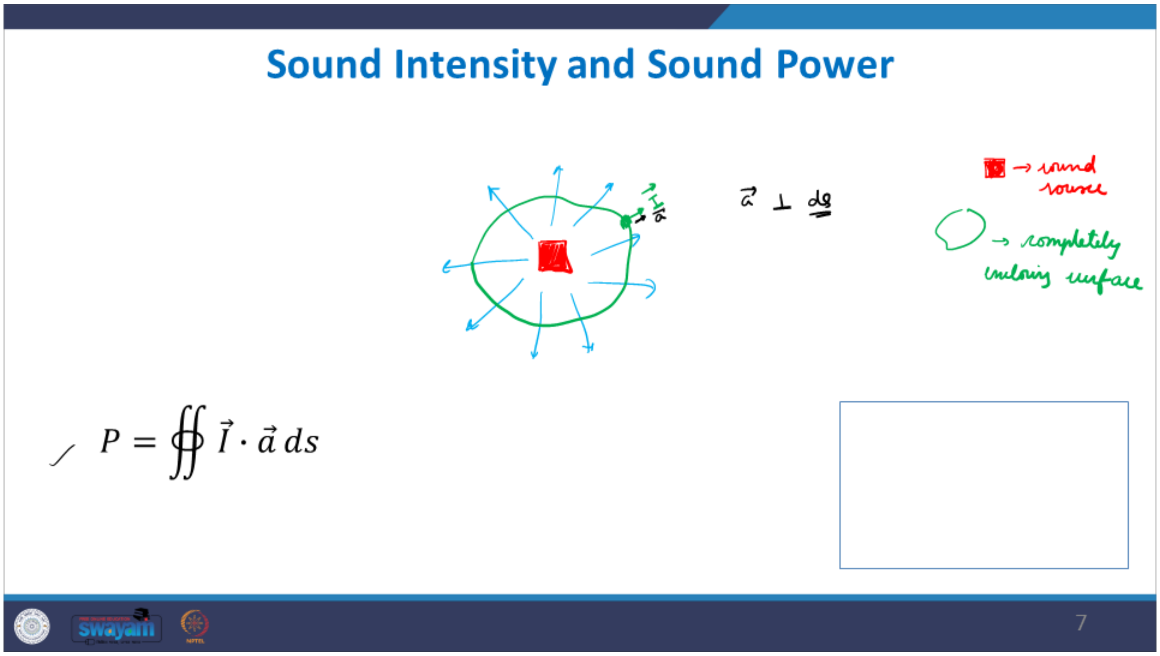
So, if you look at this particular equation here.

$$\mathbf{P} = \oiint \vec{\mathbf{I}} \cdot \vec{\mathbf{a}}\, \mathbf{ds}$$



Let us just clarify this. So, what does this equation mean? We have got, let us say, some sound source, which I am indicating by this red block. This is our source. Okay. This is the legend for this figure. This means this red block is the sound source. And then it is radiating away the sound energy, which I will indicate using these arrow-like lines, where each of the arrows indicates the direction of the wave propagation. So, the direction in which the energy is flowing outwards from the source.

Now, if we had some surface, which I will indicate using this green color, that completely encloses the source. So, no matter what the shape of that surface is, it has to completely enclose the source. So, this is a completely enclosing surface, and what we do is, let us say, we take an infinitely small element here. This is our infinitely small element, and there would be some intensity. The intensity vector should be in the same direction as the direction of the flow of energy. So, this is the intensity vector I. Then we have got some area vector, which should be normal to the surface area of this infinitely small element. So, this area vector A is what it is. Normal to the elemental surface area. We do the dot product, which means we are simply multiplying them, and then we integrate it over the entire surface; then we should be able to get the complete power of the source. So, that is what this equation indicates.



So, what should be the unit of sound intensity? Just like power is watts, sound intensity is power per unit area. So, it should be watts per meter squared. There is another way to, you know, define sound intensity because here, if you remember, this is the sound wave that is propagating, and how is the energy being stored here? It is in the form of the kinetic energy of these oscillating particles in the medium. As the wave propagates, we have these medium particles; they strike one another and keep propagating the energy from one layer to the adjacent layer and so on, and the wave propagates in space. So, it can also be defined as the rate at which this work is being done by the medium particle on its adjacent particle. So, this will be given by

$$\mathbf{I(t) = pv}$$

How are we getting this? Let us say we have got some medium particles. It is oscillating back and forth, a layer of medium particles, and they are transferring their energy to the next layer here. While striking and so on and so forth, the energy is propagating. So, the intensity is the power per unit area, and power is work done per unit time. So, what is the work? It should be the intensity here should be work done per unit time per unit area.

So, let us derive this formula. So, the instantaneous intensity at any point would be what? It could be the force being applied by that layer of fluid particles. So, and force multiplied by the distance, and because it is a longitudinal wave. So, here the direction in which they are applying the force is the same direction in which the movement is taking place.

So, although we do the dot product for the F vector and the dx vector, ultimately it becomes a dot because they are both in the same direction, and we end up with multiplying their magnitudes.

$$\mathbf{I(t) = F.\,dx}$$

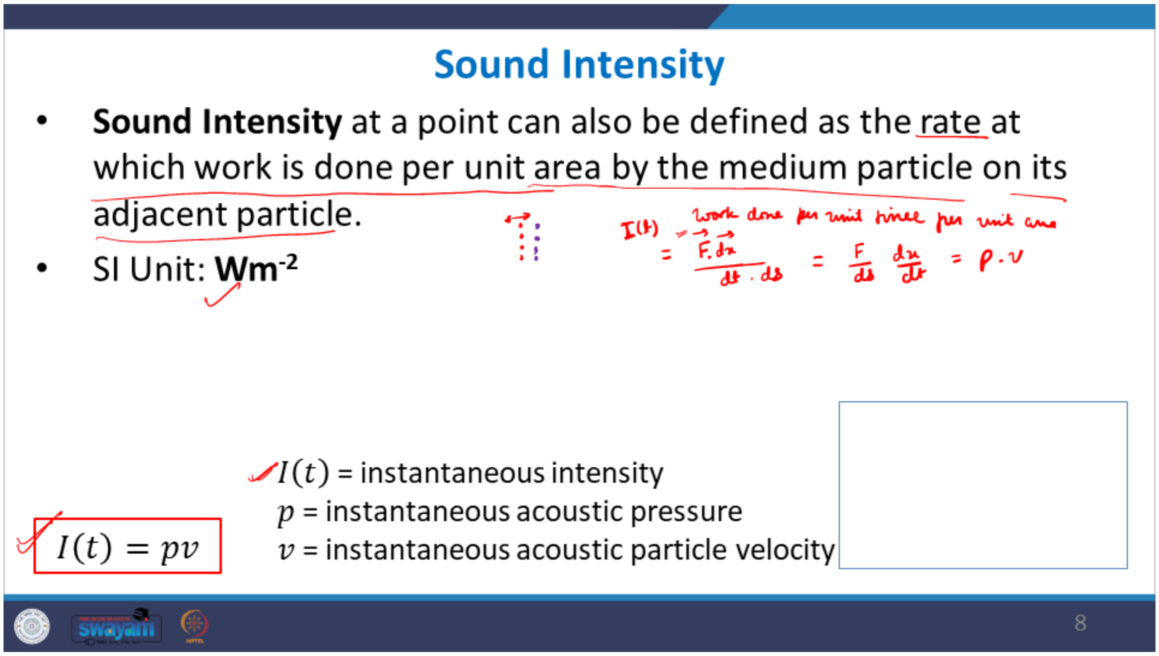
So, let us do that. So, that is the work done then per unit time by dt and per unit area by ds. The surface area over which the sound wave is propagating.

$$\mathbf{I(t) = \frac{F.dx}{dt\ ds}}$$

So, this becomes now F by ds would simply become, because it is force per unit area. So, it should become pressure, and then dx by dt we are doing. So, we have pressure multiplied by the velocity. Okay. So, that is how we are getting this formula P into V. This is the instantaneous intensity, and it is what? It is the product of the instantaneous acoustic pressure and the instantaneous acoustic particle velocity.

$$\mathbf{I(t) = pv}$$

Because it is a sound wave, so both the pressure, which is a scalar, as we have already seen here, initially it was a dot product of two vectors, because they are in the same direction, it ended up being the multiplication of the magnitudes of the two vectors, and P being a scalar quantity, we simply multiply it with the instantaneous acoustic particle velocity.



So, this becomes the very first equation,

$$\mathbf{I(t) = pv}$$

As you know, what is acoustic pressure? It is a fluctuating wave. So, the pressure P is changing over time. So, which means that at some point the intensity would be high, at some point it will come to a low, and at some point, it will become 0. So, the instantaneous intensity should have some kind of fluctuating waveform. It should change, which would not make sense. So, what we do in general is take one-time period of the wave, and in one-time period, we try to do an averaging to find out how much intensity we are getting in one complete wave cycle. That gives us a more sensible term compared to the instantaneous intensity. So, we get this time-averaged intensity. In most of the cases, equations, and in most of the cases, when we discuss or describe the intensity of a sound wave, we generally mean the time-averaged intensity, not the instantaneous intensity, because that is a fluctuating quantity. So, this time-averaged intensity I is what you get by doing the time averaging of pv over the time period T. It can also be written in the form of an integral like this.

$$\mathbf{I} = \frac{\mathbf{1}}{\mathbf{T}}\int_{\mathbf{0}}^{\mathbf{T}} \mathbf{p} \cdot \mathbf{v}\ \ \mathbf{dt}$$

Now, if you see here, the magnitude of

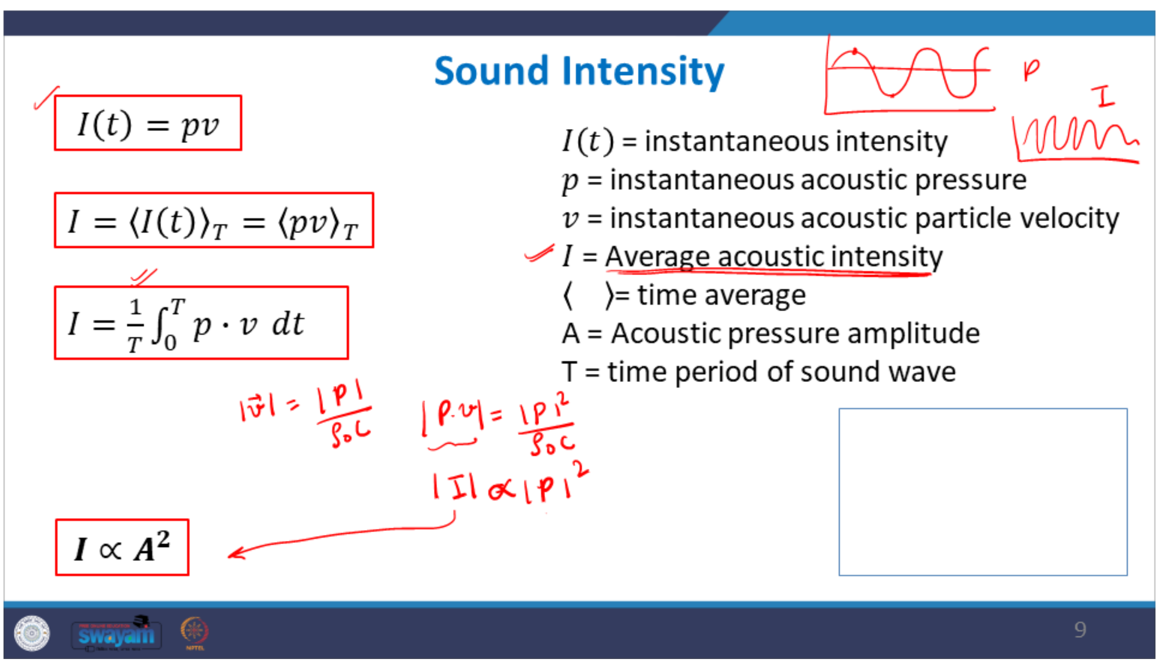
$$\mathbf{v} = \frac{\mathbf{p}}{\boldsymbol{\rho}\mathbf{0c}}$$

So, p.v the magnitude of this should be what?

$$\mathbf{p.v} = \frac{\mathbf{p2}}{\boldsymbol{\rho}\mathbf{0c}}$$

Right, so that means that this is what this becomes: the magnitude of the intensity. So, the intensity is directly proportional to the pressure square, the magnitude of the intensity. And this p is what it has got a certain amplitude, so we can directly come to the conclusion that the intensity would be directly proportional to the amplitude square of an acoustic wave.

$$\mathbf{I} \propto \mathbf{A}^{\mathbf{2}}$$



Now, let's see another wave front, which is called the spherical waves, and then we'll derive the equation for sound power and intensity for the harmonic plane waves, spherical waves, and the cylindrical waves. So, for the harmonic waves, how were they generated? They were generated by some kind of reciprocating flat surface inside a long tube, and the wave front, which is the surface of constant phase and amplitude, was perpendicular to the direction of the wave propagation, and the amplitude did not change over time.

So, there was no attenuation. Now, let us imagine we have a point source, and this point source is radiating uniformly the sound energy in all directions. Now, what do you mean by a point source? Obviously, you know, in real life, we do not have anything that is a point. So, what do you mean by a point source? In acoustics, we will call anything a point source if the source's largest dimension is much smaller than the wavelength under consideration. So, let us say we are dealing with low-frequency sound, where we are dealing with some phenomenon involving low frequencies. So, the lower the frequency, we know that

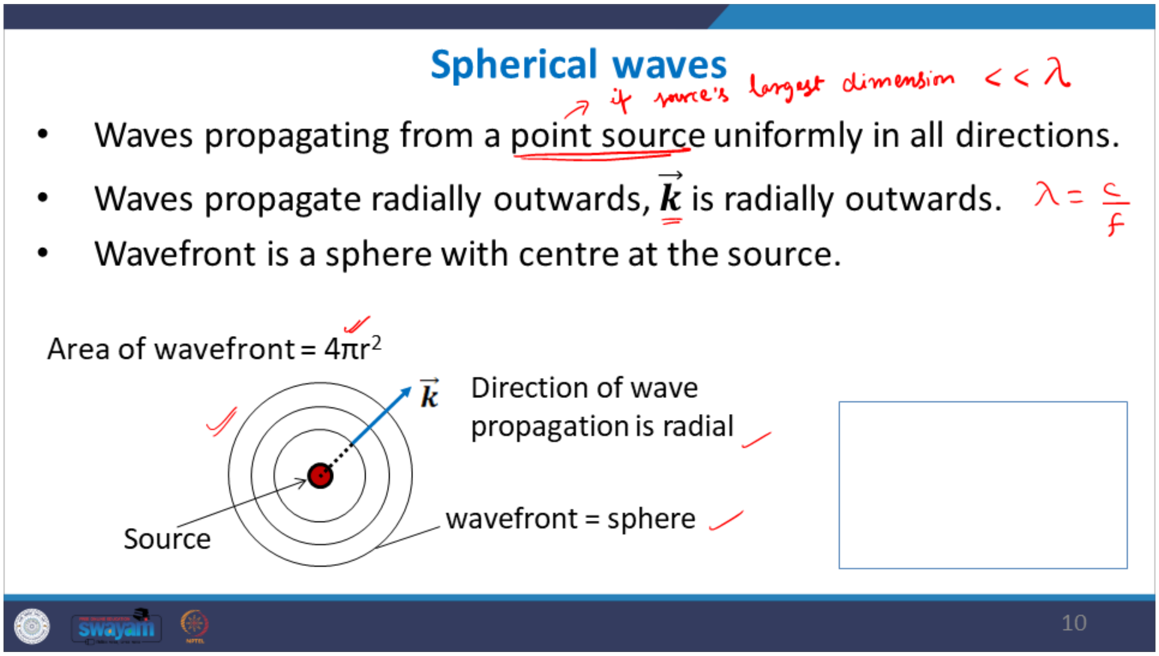
$$\boldsymbol{\lambda} = \frac{\boldsymbol{c}}{\boldsymbol{f}}$$

So, the lower the frequency, the longer the wavelength.

So, if you are dealing with the low-frequency region, this λ becomes large, and most of the ordinary small objects behave like a point source. Whereas, if you are dealing with a very high-frequency regime, then that same lambda will become small, and the ordinary objects in centimeters or whatever will not behave like a point source. So, we have a point source, and it is radiating the waves uniformly in all directions, and here the k vector becomes radially outwards. And the wave front that we are getting, if you see this figure, is a spherical wave front.

Instead of harmonic plane waves, we are getting a spherical wave front. If you see at any point r, what is the area of the wave front? It is the area of that spherical shell that is surrounding the source. So, the area of that spherical shell is given by

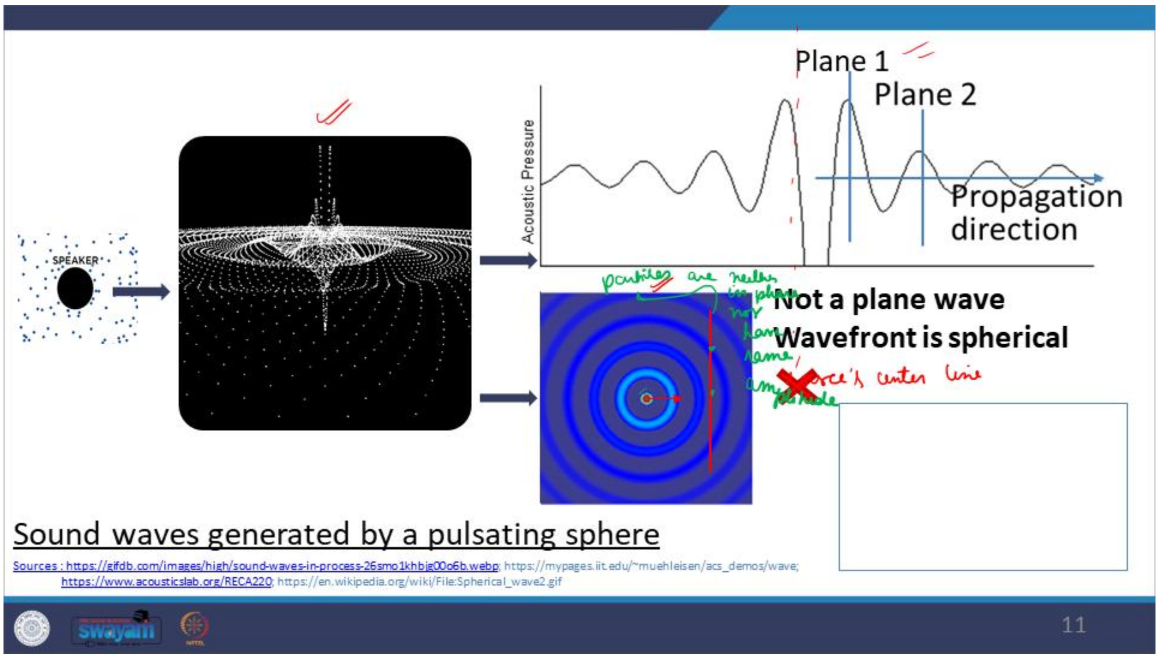
$$A = 4\pi r^2$$

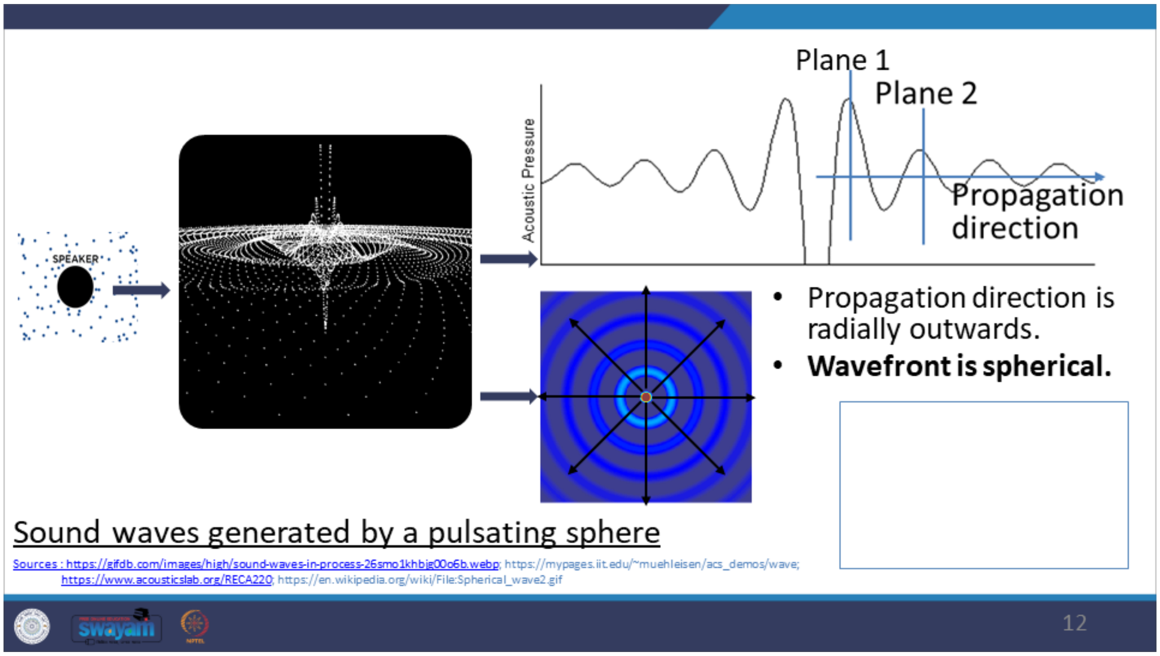


So, let us see a typical spherical wave. Let us say we have got a small ball, and that is pulsating, which means it is expanding in volume, contracting, expanding, and contracting. And by means of expansion and contraction, it is sort of driving the surrounding medium particles back and forth and is able to create waves that are radiating uniformly in all directions.

This is the 3D view of this, and this is the kind of wave if you measure the wave away from the source. What you see here is that it is still a sinusoidal wave as it is going away from the source. So, let us say the source is located. This is the source's line or source's center line. So, as you are moving away from the center line the wave fronts are still sinusoidal in nature, but their amplitude decreases, and this is the contour plot of the acoustic waves. Let us take a snapshot just like we did for harmonic wave. So, we want to observe at a particular instant in time; we took a snapshot. Now, let us say we draw a plane surface perpendicular to the direction of wave propagation.

So, if we have this. So, do we observe that all the particles are in the same phase and have the same amplitude? So, no. So, here the particles are neither in phase nor have the same amplitude. Okay. The amplitude is varying because you can see here the wave front as it changes over distance. So, the amplitude here would be higher, and the amplitude here would be lower, and the phase as well is different. While one is experiencing a trough, the other one is experiencing a peak. So, this is not a plane wave front; it is a spherical wave front, okay. So, the direction is radially outward, and the wave front is spherical.



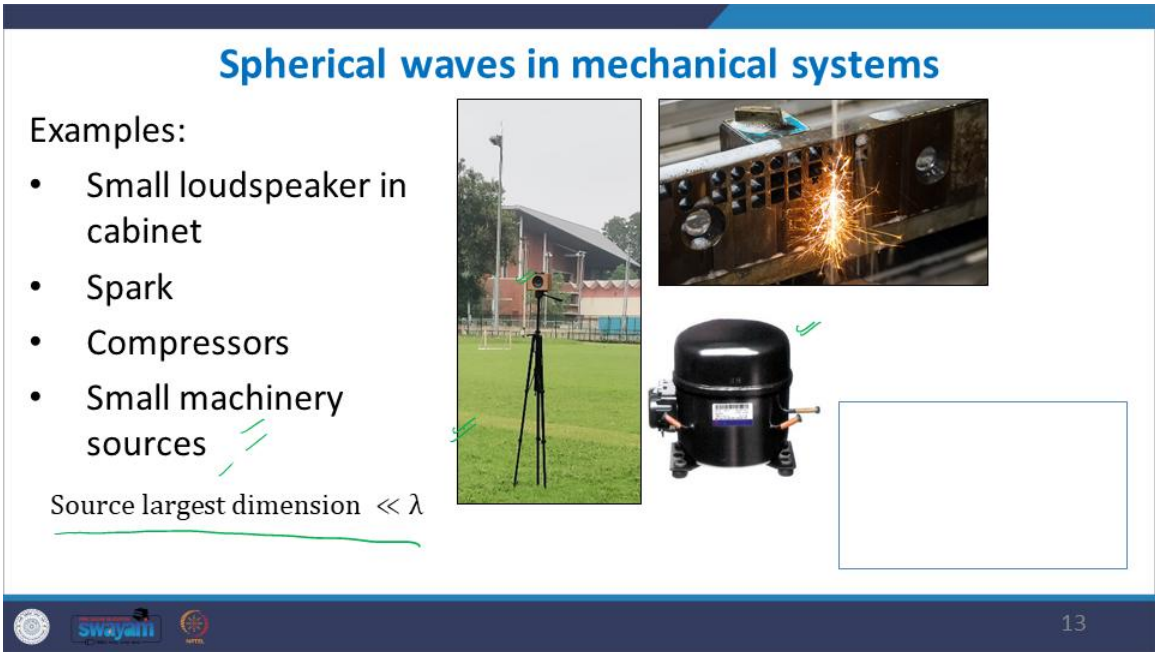


Now, what are some spherical waves in mechanical systems? So, because this course is about noise control in mechanical systems, let us see some common sources. So, at the very first, I gave you the definition that if we have a point source, which is sort of vibrating uniformly all across its surface and it is radiating outward, then it shows or gives away spherical waves. Now, for example, let us say we have a boxed speaker, a speaker inside a cabinet.

Here, the diaphragm of the speaker is moving back and forth, and it is generating a spherical wave. This is a figure from one of the experiments that I did with my doctoral student. In order to get a spherical wave front, how did we create it in real life? We had a small boxed speaker. We played it in a low-frequency regime, less than 500 hertz.

So, in that case, the dimensions were extremely small compared to the wavelength. And as the diaphragm was pulsating, the waves were coming out, and it was kept more than a meter and a half away from the ground to avoid the reflections, and we were getting a good spherical wave front. In the same way, you know, there could be various such small sources, such as a compressor. So, inside the compressor, you have a lot of mechanisms, but the outside surface of this closed compressor starts to vibrate and pulse. It behaves like a pulsating sphere and generates spherical waves. In the same way, you have sparks or

Even you know, me speaking here, the voice itself behaves like a point source, the vocals of a human. So, then we have any kind of small machine resources; all of them will behave as long as this condition is satisfied.



So, let us solve and find out what should be the solution for a spherical wave. This is our equation to begin with. It holds true for every kind of wave front in a homogeneous medium. So, this is the linear acoustic wave equation.

$$\nabla^2 p - \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} = 0$$

Now, a spherical wave is typically propagating in the radially outward direction.

It is not propagating like in the particular x-direction, y-direction, or z-direction. Let us say we have a source somewhere at the origin, and we sort of plot the sound waves. We can think of them as waves that are propagating along some radial direction. So, it makes sense. to then derive the equation in the spherical coordinate system.

So, this is our spherical coordinate system, okay.

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r} + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial^2}{\partial\varphi^2}$$

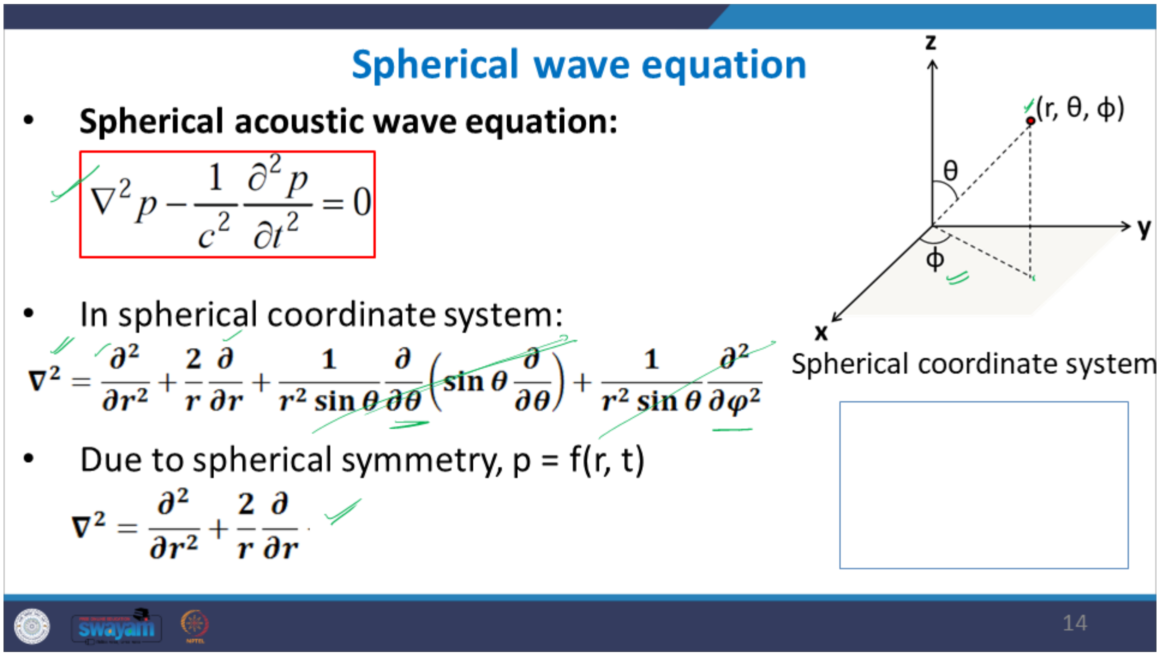
r, Θ, and Φ, r being the radial vector where we are trying to make the measurement of the acoustic pressure. Φ is the azimuthal angle, which means that if you project this point onto the xy plane, the angle that it makes with the positive x-axis becomes your Φ. And Θ is like the elevation angle. So, how it is measured: you try to draw this radial vector and see what is the angle it is making with the positive z-axis. So, you get these, and this is how you represent them in the spherical coordinate system. So, the same equation in the spherical coordinate, and this kind of complicated term having Θ and Φ all around.

But the waves are spherically symmetrical. See, all of this, they should be spherically symmetrical, which means that if, suppose we have the sound wave, it is radiating, so it is spherically symmetrical. So, what is here in one point, it should be the same in the other point. It should be independent of Θ and Φ. It should only depend upon the radial distance from the source.

So, for that uniform spherical wave front, which is symmetrical, So, this should mean that $\partial/\partial\Theta$ and $\partial/\partial\Phi$ should be 0 because the pressure does not change with Θ and Φ. So, hence these terms are automatically neglected, and we end up with

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r}$$

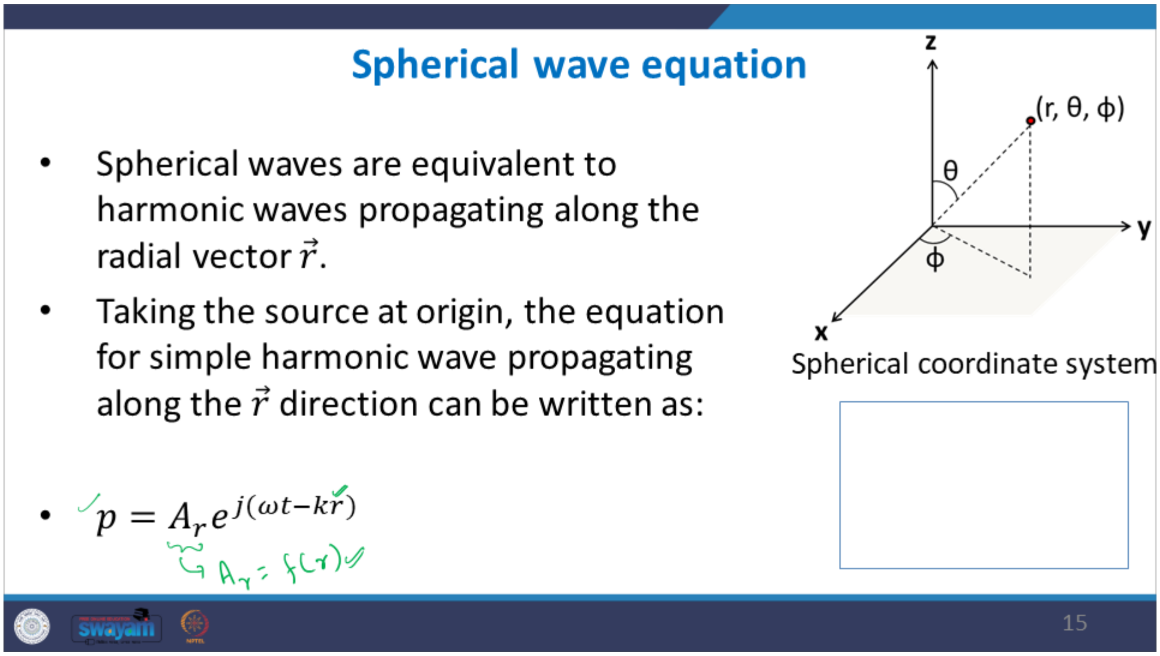
So, they behave like, as I said, some kind of sinusoidal waves that are propagating along the radial direction.



So, why don't we start with and represent the pressure in the form of whatever? So, I told you in the very beginning, when we were studying harmonic plane waves, why we study them because Once we know this basic solution, we can solve complex waves and various kinds of complex waveforms as the superposition of harmonic waves or changing harmonic waves. So here also, we will begin with the solution as a harmonic plane wave.

We will think that it is some kind of a harmonic plane wave, but because the amplitude is changing with distance, so here this amplitude, instead of being a constant term, is A_r, which is a function of the radial distance r. So, we represent it and the direction of wave propagation is this r vector:

$$\mathbf{p} = \mathbf{A_r e^{j(\omega t - kr)}}$$



So, now let us see what this A_r is. Suppose we have a source at the origin, and we are measuring its power or intensity at any point r. We know that the intensity at r should be powered by the surface area over which it is being measured. Here, we have taken a sphere, so the surface area becomes $4\pi r^2$.

So, this intensity is inversely proportional to r^2

$$\mathbf{I_r} = \frac{\boldsymbol{p}}{\mathbf{4\pi r^2}} = \mathbf{I_r} \propto \frac{\mathbf{1}}{\mathbf{r^2}}$$

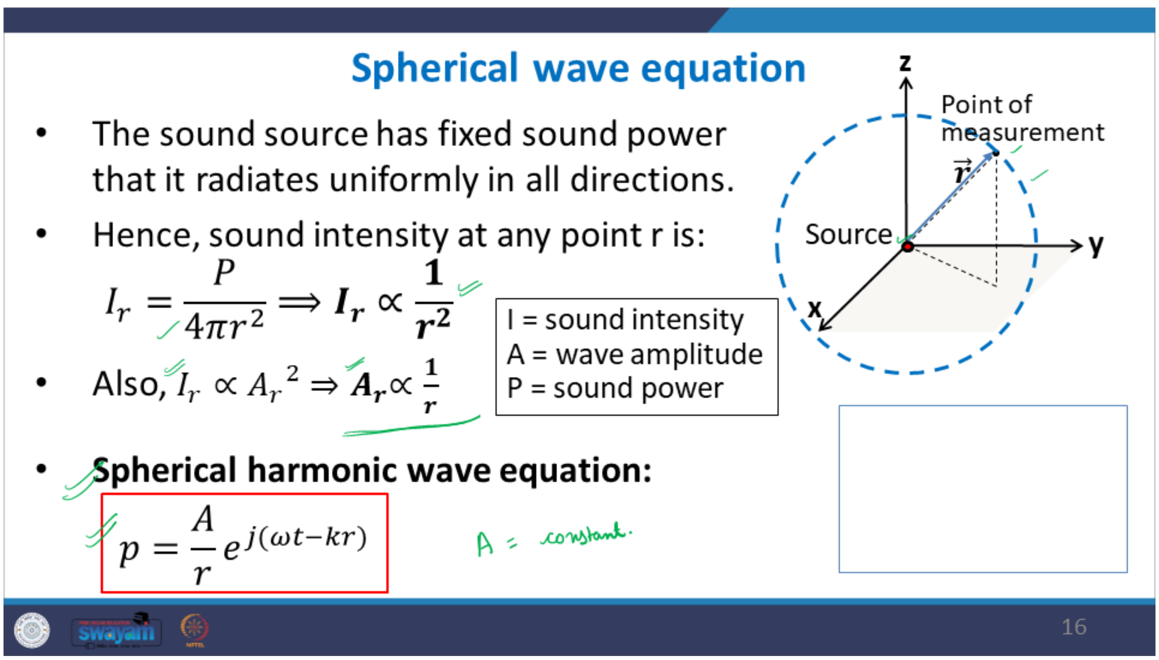
and because the intensity is directly proportional to the amplitude square, the amplitude becomes inversely proportional to the radius.

$$\mathbf{I_r} \propto \mathbf{A_r}^{\mathbf{2}} = \mathbf{A_r} \propto \frac{\mathbf{1}}{\mathbf{r}}$$

So, now we have come to know that this is a term which is inversely proportional to r. So, this A_r, which was a function of the radial distance, is now,

$$\mathbf{p = \frac{A}{r} e^{j(\omega t - kr)}}$$

where a becomes some constant term which is determined by the boundary conditions or the problem at hand. So, this becomes our final expression of a spherical harmonic wave equation, fine.

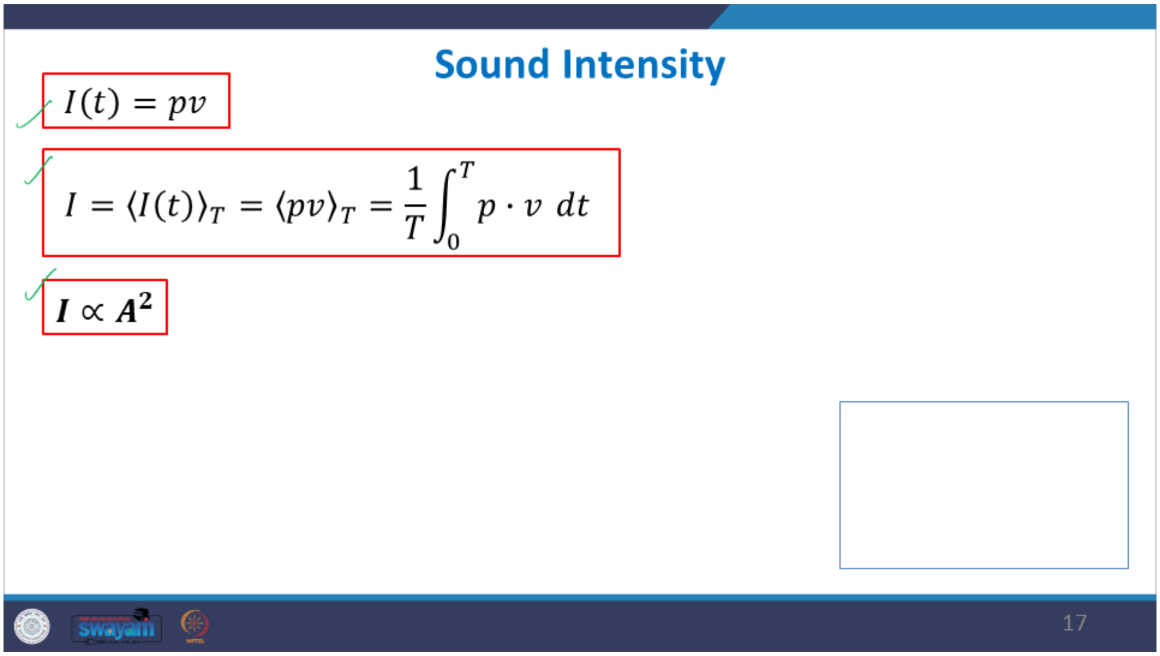


So, the sound intensity let us derive that. These are the equations we have already seen; using these equations, we are going to come at sound intensity.

$$\mathbf{I(t) = pv}$$

$$\mathbf{I = \langle I(t) \rangle_T = \langle pv \rangle_T = \frac{1}{T}\int_0^T p \cdot v \ dt}$$

$$\mathbf{I \propto A^2}$$



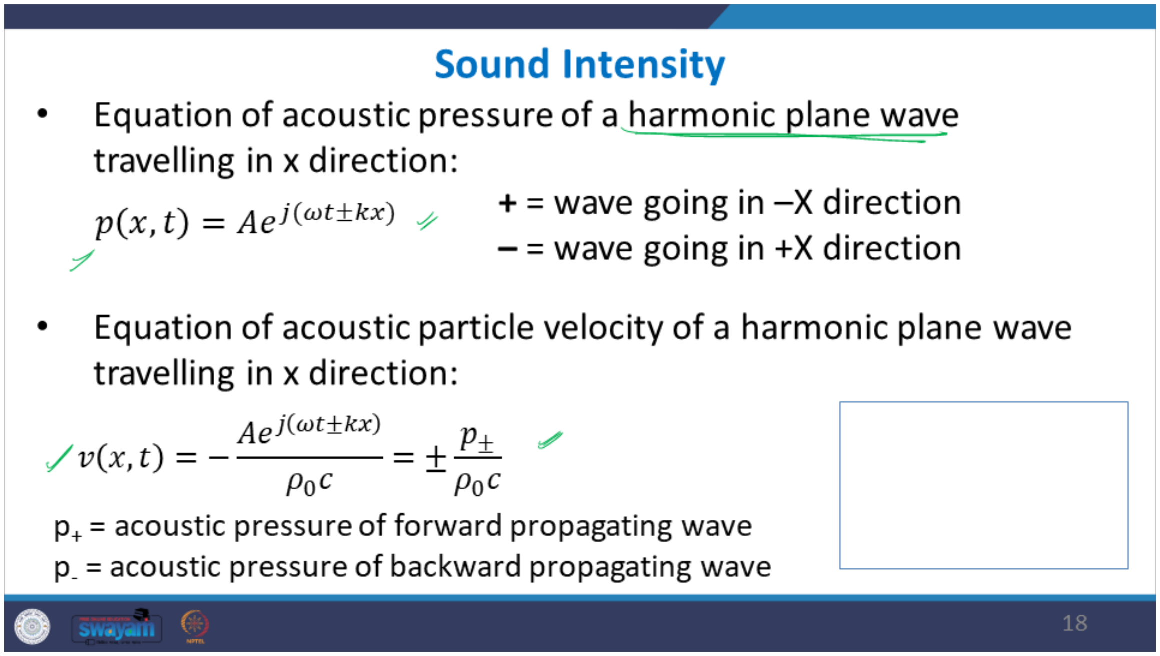
So, let us start with the sound intensity of a harmonic plane wave first. Let us consider a plane wave traveling in the x direction. If you see, this becomes the equation:

$$\mathbf{p}(\mathbf{x},\ \mathbf{t}) = \mathbf{A}\mathbf{e}^{\mathbf{j}(\boldsymbol{\omega}\mathbf{t} \pm \mathbf{k}\mathbf{x})}$$

This becomes the velocity equation, which is represented by

$$\mathbf{v}(\mathbf{x},\ \mathbf{t}) = -\frac{\mathbf{A}\mathbf{e}^{\mathbf{j}(\boldsymbol{\omega}\mathbf{t} \pm \mathbf{k}\mathbf{x})}}{\boldsymbol{\rho}_0\mathbf{c}} = \pm\frac{\mathbf{p}_{\pm}}{\boldsymbol{\rho}_0\mathbf{c}}$$

Then we add the plus or minus signs based on whatever is the direction of wave propagation.



So, if you multiply the two things together, you end up with this term from here and here. So, which means that the intensity is simply some, you know, it's the magnitude of the intensity. This is how it's represented, so it depends on the properties of the cosine waves. We will not go into detail of this:

$$\textbf{Instantaneous:} \quad I_{plane} = pv = \pm \frac{{p_\pm}^2}{\rho_0 c}$$

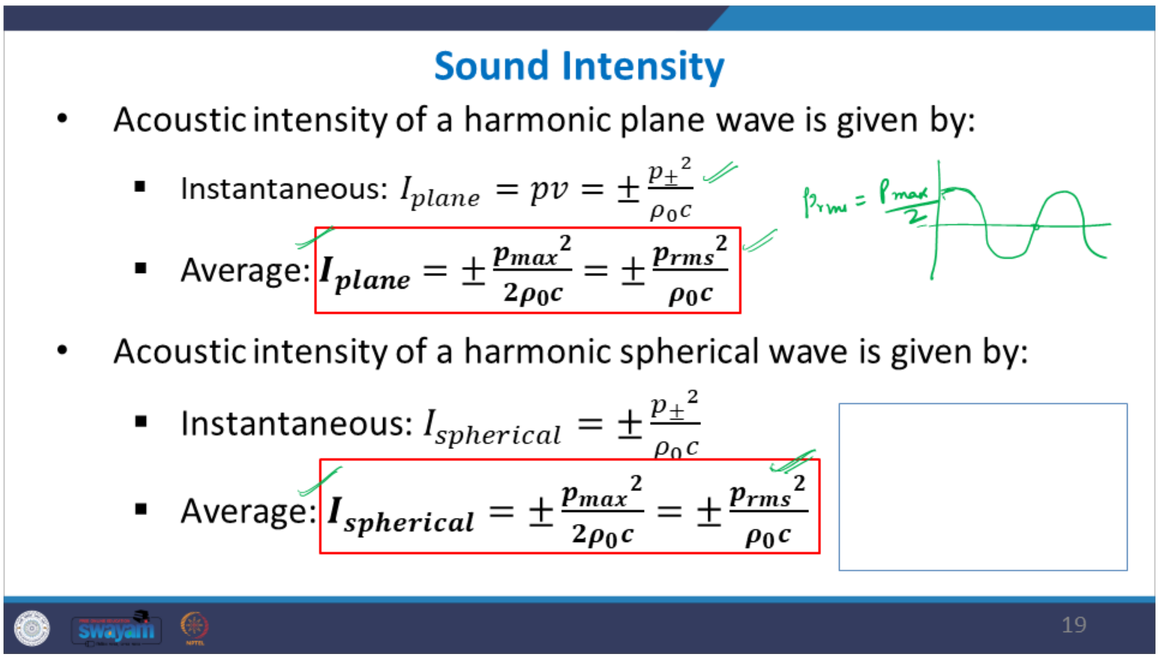
$$\textbf{Average:} \; I_{plane} = \pm \frac{{p_{max}}^2}{2\rho_0 c} = \pm \frac{{p_{rms}}^2}{\rho_0 c}$$

And for a spherical wave, also, the same equation holds true, and we end up with the same equation, which

$$\textbf{Instantaneous:} \quad I_{plane} = pv = \pm \frac{{p_\pm}^2}{\rho_0 c}$$

$$\textbf{Average: } \mathbf{I_{plane}} = \pm\frac{\mathbf{p_{max}}^2}{\mathbf{2\rho_0 c}} = \pm\frac{\mathbf{p_{rms}}^2}{\mathbf{\rho_0 c}}$$

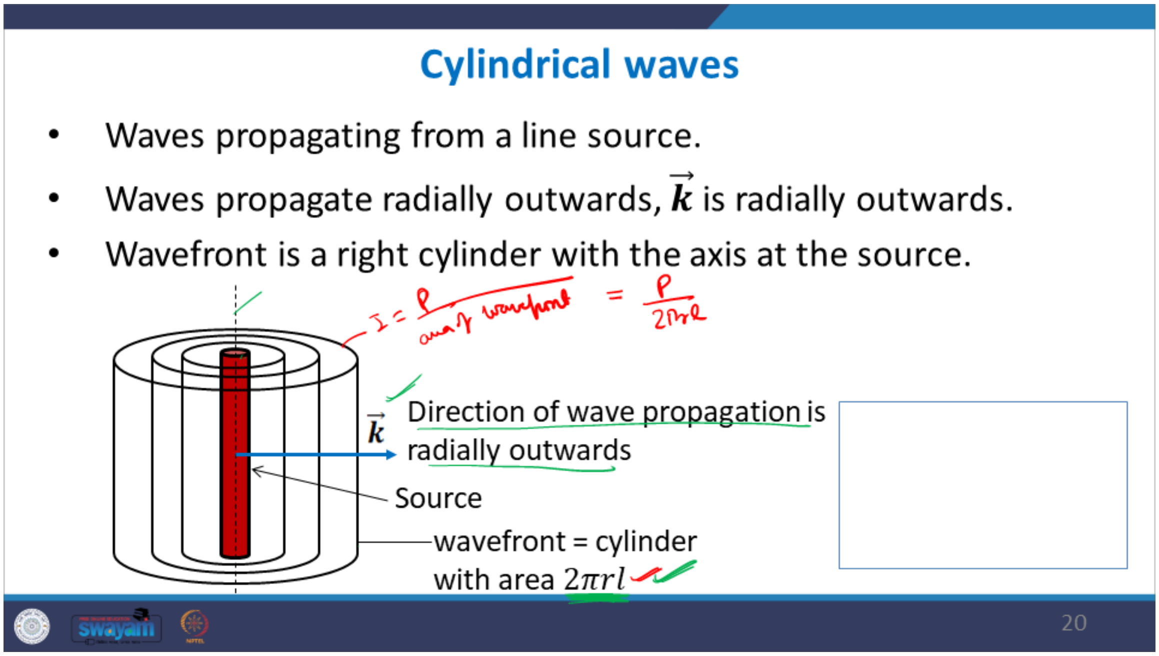
Now, these equations you have to just remember if you want to be in the field of acoustics or noise control, because they are going to be used in a lot of ways.



Now, the last kind of waves which we will discuss in this course is the cylindrical waves, and depending on the wave front that is being generated, it depends on what source we are sort of taking into consideration.

Like, for example, with the spherical waves, it was a point source that is uniformly radiating throughout in all the directions. So, which means that the vibration velocity stays constant over the surface of that point source. Suppose you have got some kind of line source, and it is vibrating uniformly; then you generate a cylindrical wave front. So, this is a line source. Okay.

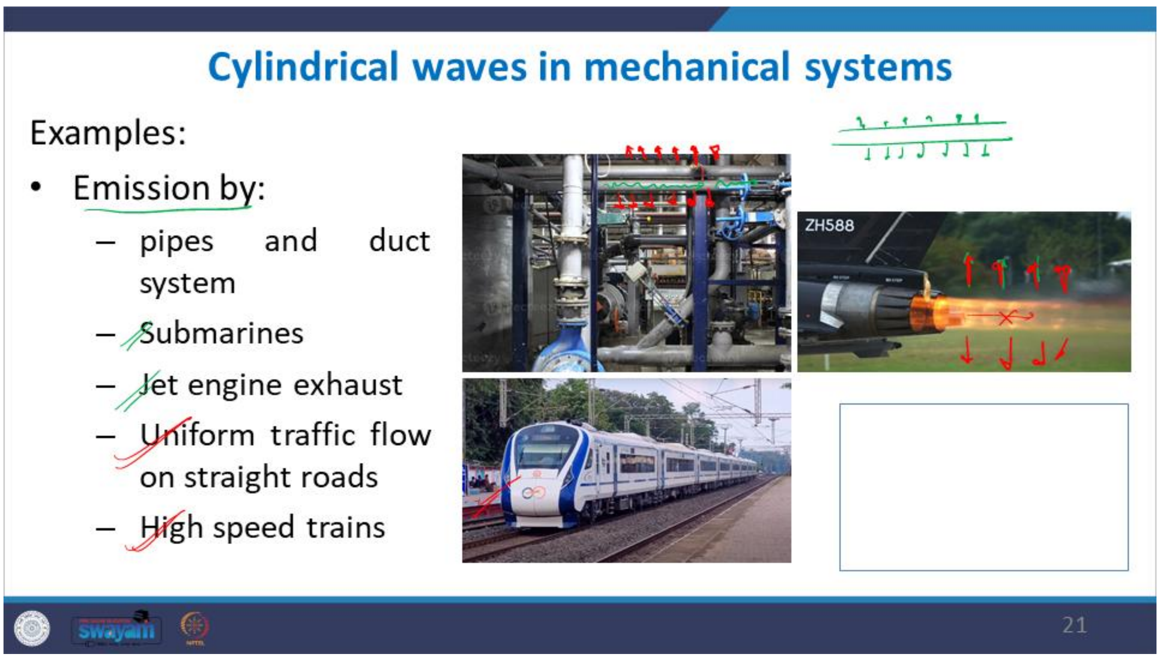
And the wave front at any distance would then become a right cylinder or a right circular cylinder, and its axis would be the same as the axis of the source. So, again, the direction of the wave propagation is radially outwards from that source line, and the area of the wave front at any particular distance becomes the surface area of this cylinder, which is the circumference of the circle multiplied by the length of the source. So, it is 2πrl.



Some of the examples of cylindrical waves in mechanical systems are pipe and duct systems, you know. So, here it is the emission by, it is not the emission, the noise inside. So, that is the distinction I want to make because when we were discussing harmonic plane waves, we saw that if inside the pipes, we have some reciprocating motion that is generating sound waves. For example, inside the tailpipe of a vehicle, we had the piston of the engine that is pushing the, you know, sound waves through the tailpipe. Then that becomes, inside the pipe, it behaves like a harmonic plane wave, and as it emits outside into an infinite baffle or atmosphere, it becomes a point source, so it emits a spherical wave. Okay, so this is not that. It is the waves that are created So, suppose some kind of fluid was flowing through this pipe here, and because of the flow of the fluid inside these pipes and ducts, the surface of the pipe starts to vibrate.

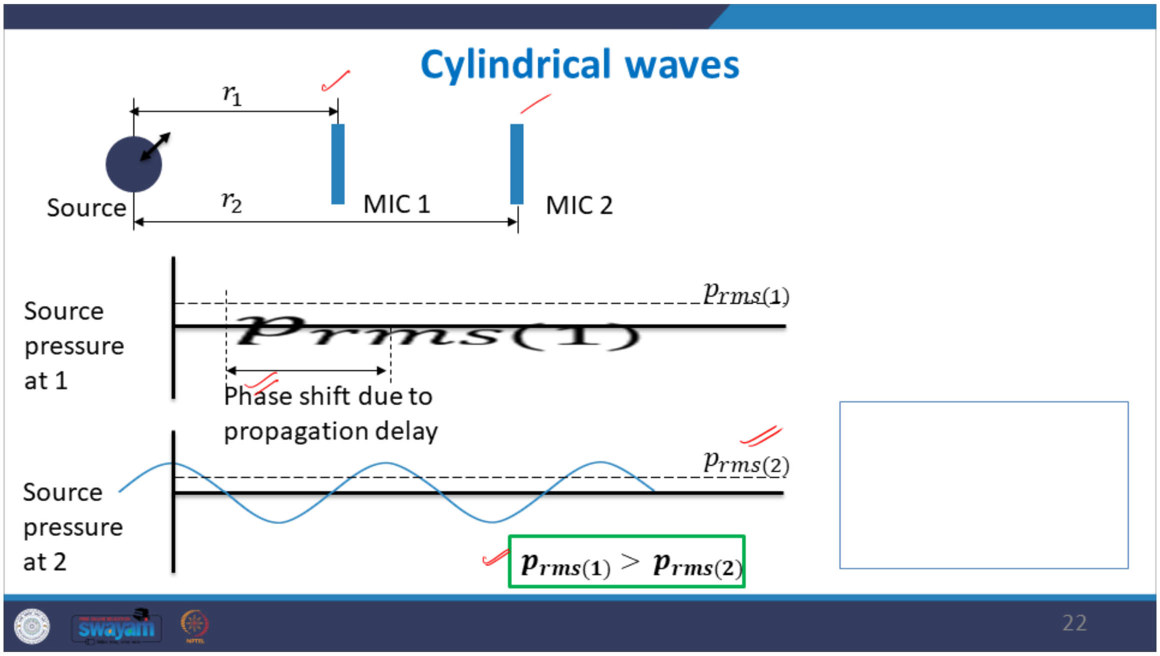
So, it is not the waves flowing inside it, but rather because of the flow inside, the surface of the pipe is vibrating, and that pipe surface, when it starts to vibrate back and forth, it behaves like a line source, which creates the cylindrical waves. In the same way, you know, submarines, jet engine exhaust by the means of, you know, the way they are sort of having the fluid flow go over here, they are creating these harmonic waves. To make it more clear, this is not harmonic. This is they are creating the harmonic waves in the surrounding atmosphere where they are behaving like a line source, okay.

So, I hope this makes the distinction clear between, you know, the confusion of whether the pipe or a duct has a harmonic plane wave front or a cylindrical wave front inside. If it is a reciprocating motion that is pushing the fluid inside, then it becomes a harmonic wave front. But by the virtue of the flow, if it is able to generate the structural vibrations in the pipe itself, then outside it radiates a cylindrical wave front. Similarly, the uniform traffic flow on a straight road or high-speed trains generate cylindrical waves.



So, what happens?

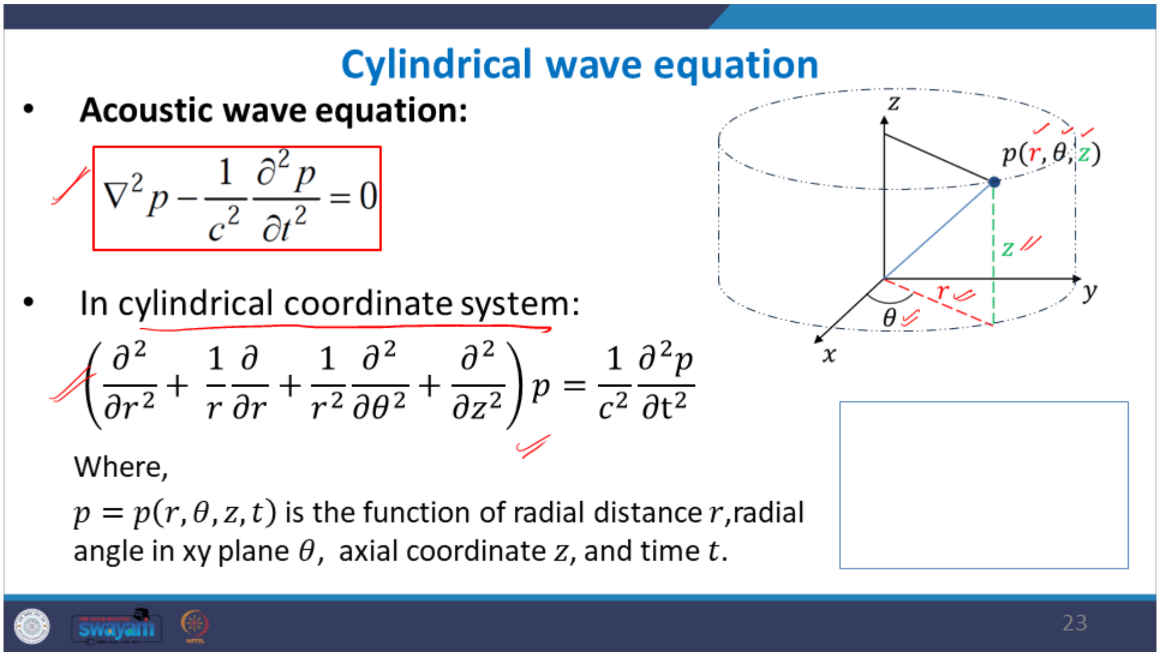
So, suppose there is some line source, and we are giving a side view of it, and we take two microphones at two separate distances and measure the pressure. So, there should not just be a phase shift, but there should also be a change in the amplitude. And this P_{rms}, just like a spherical wave front, here the intensity is decreasing, and hence the amplitude is also decreasing with time. Okay.



To obtain the cylindrical wave equation, we are applying this acoustic wave equation in the cylindrical coordinate system, where any point P, where we want to measure the sound pressure, is given in terms of r, Ө, and z. r is where, if you project this point onto the xy plane, the distance in the xy plane becomes r. The angle that this projection makes with the x-axis becomes Ө, and z becomes the z coordinate. So, that is how you represent it.

$$\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial z^2}\right) p = \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2}$$

Now, if you solve it, which is not within the scope of this course because it is still a beginner's course in noise control and mechanical systems. So, we will not go into the derivation, but we directly come to the solution of this particular equation.



It is slightly more complicated; we get some amplitude which is a function of the radius. But how is it a function of the radius? It is the Hankel's function of second order multiplied by some constant into k times r, $e^{j\omega t}$ which is a time-varying sinusoidal signal.

$$\mathbf{p(r,t) = AH_0^{(2)}(kr)e^{j\omega t}}$$

Now, let us see how the pressure varies with the radius. I told you here that as you move your microphone away, the amplitude should change. How does it change? Let us see the sort of function. So, at any point P, suppose we measure it. What should the wavelength be at any point P? This should be

$$\mathbf{I = \frac{P}{Area\ of\ wave\ front}}$$

So, the sound power divided by the surface area. So, this should give you

$$I = \frac{\mathbf{P}}{\mathbf{2\pi rl}}$$

which is the area of the wave front. So, ultimately it comes out to be here

$$\mathbf{I_r = \frac{P}{2\pi rl}}$$

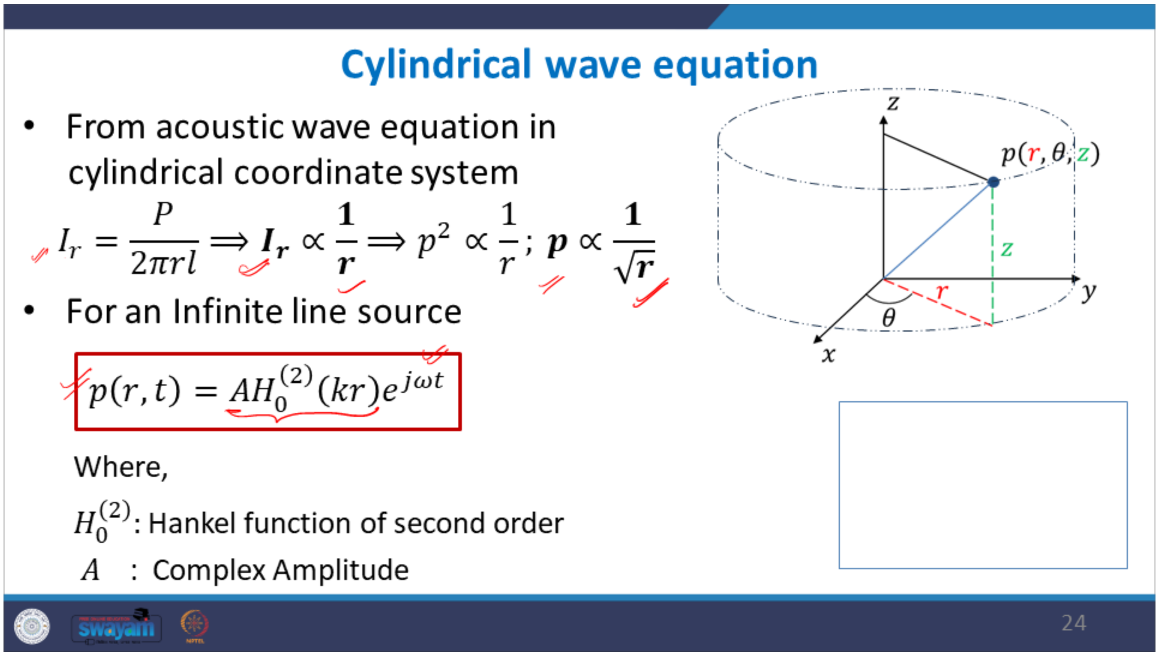
So, which means the intensity is inversely proportional to r.

$$\mathbf{I_r \propto \frac{1}{r}}$$

In spherical waves, it was inversely proportional to r^2. We also call it the inverse square law for spherical waves. In cylindrical waves, it is just inversely proportional to r. So, the pressure is actually inversely proportional to the square root of r.

$$\mathbf{p \propto \frac{1}{\sqrt{r}}}$$

That is how the variation happens.



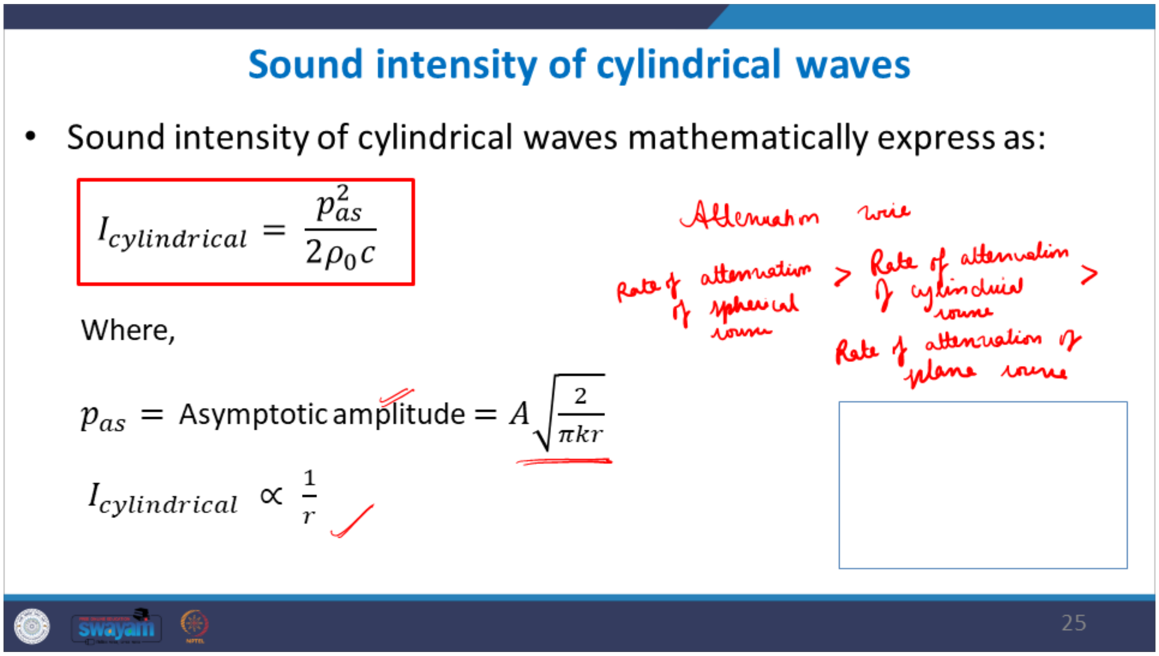
And the intensity of the cylindrical wave is given by

$$\mathbf{I_{cylindrical}} = \frac{\mathbf{p_{as}^2}}{\mathbf{2\rho_0 c}}$$

Here P_{as} is the asymptotic amplitude, which is this constant here.

$$\mathbf{p_{as}} = \textbf{Asymptotic amplitude} = \mathbf{A}\sqrt{\frac{\mathbf{2}}{\mathbf{\pi kr}}}$$

Okay. Fine.



So, you know before we end this lecture, just a small conclusion of all this exercise is that See, suppose you have a source which is emitting spherical wave front, that means that with the distance, the attenuation is automatically happening. It is attenuating, the pressure is attenuating at the rate of inversely proportional to r^2. If you have a cylindrical wave, it is inversely proportional to r, and if it is a harmonic wave, there is no attenuation at all. So, in some ideal situation in a machinery setup, we have some source that is spherical, another source that is cylindrical, and then another source that is a harmonic plane wave or harmonic plane source. The harmonic plane source is more dangerous because there is no attenuation no matter how far away you are.

That takes priority over cylindrical and spherical sources. A spherical source will automatically attenuate over space or distance, and the same goes for the cylindrical. So, attenuation-wise, you can say the rate of attenuation of spherical is larger than the rate of attenuation of the cylindrical, spherical source, cylindrical source, which is larger than the rate of attenuation of a plane source

So, a plane source takes priority because there is no attenuation at all, whereas with the spherical, you automatically get attenuation. One concept that can be used in various machineries is, by some means, if you can convert a plane source into a spherical source, then automatically you get attenuation. Okay with this, I would like to end this lecture and thank you for listening.