NOISE CONTROL IN MECHANICAL SYSTEMS

Prof. Sneha Singh

Department of Mechanical and Industrial Engineering

IIT Roorkee

Week:2

Lecture:8

Lecture 08: Propagation Through Medium Boundaries: Numerical

Hello and welcome to Lecture 8 in the series on noise control in mechanical systems. I am Professor Sneha Singh from IIT Roorkee.

So, in today's lecture, we will solve some numerical problems related to the topic on propagation through medium boundaries. A quick review of the previous lecture: we had been able to derive the pressure reflection coefficient as,

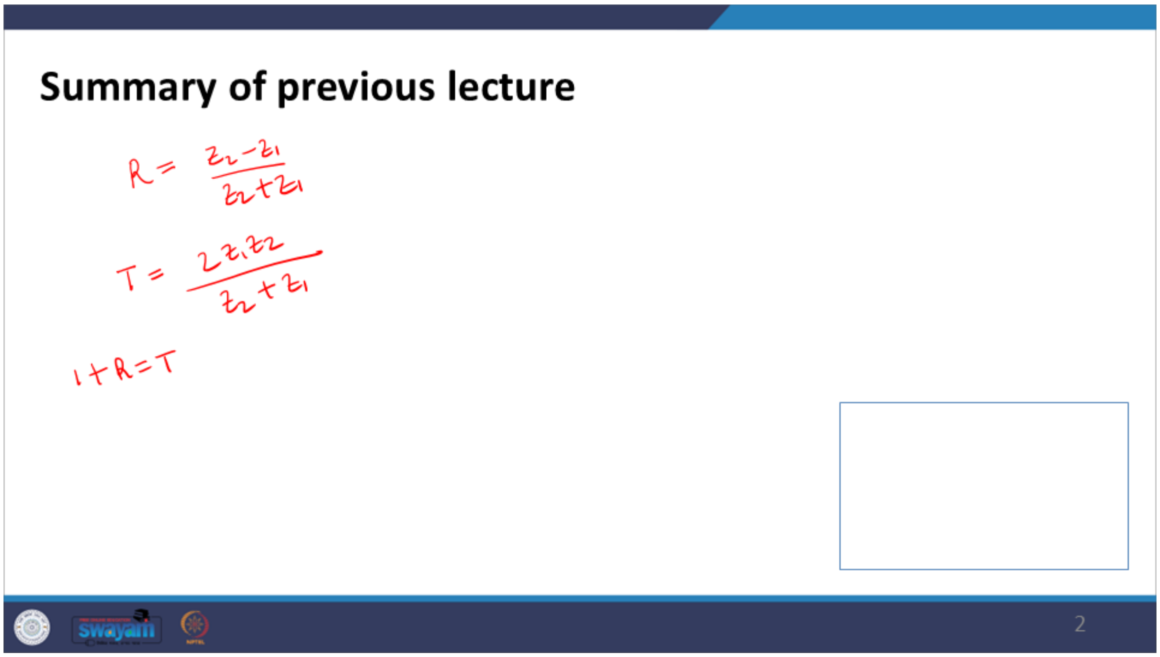
$$R = \frac{z_2 - z_1}{z_2 + z_1}$$

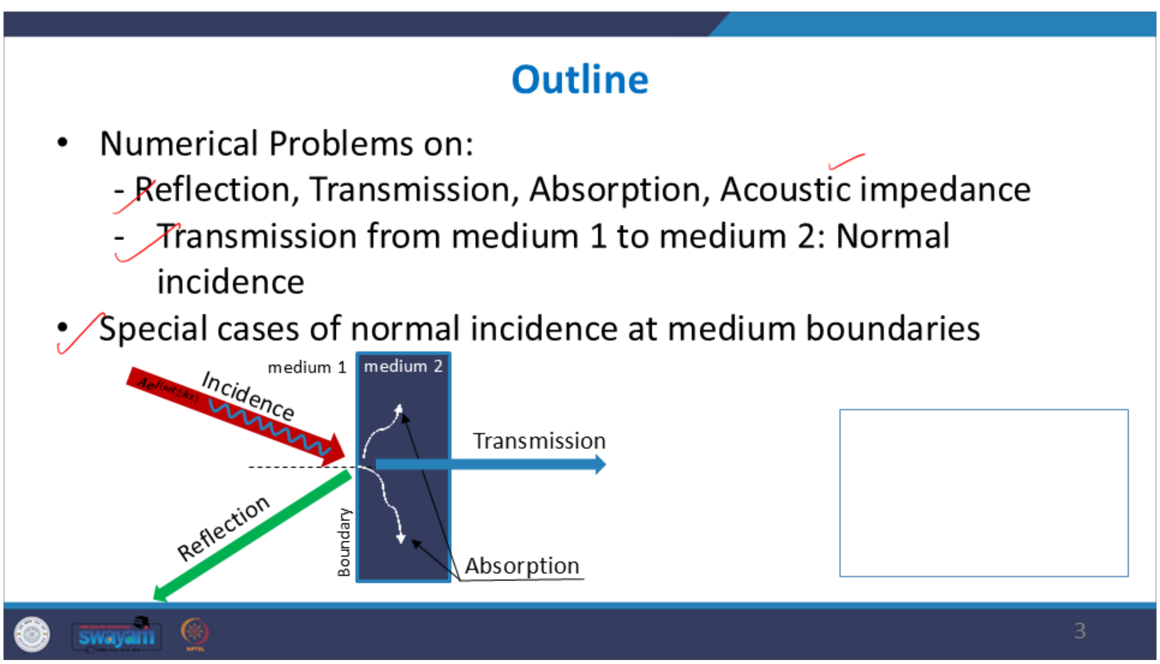
And the transmission coefficient, which is the pressure transmission coefficient, as

$$T = \frac{2z_2}{z_2 + z_1}$$

And another relationship was,

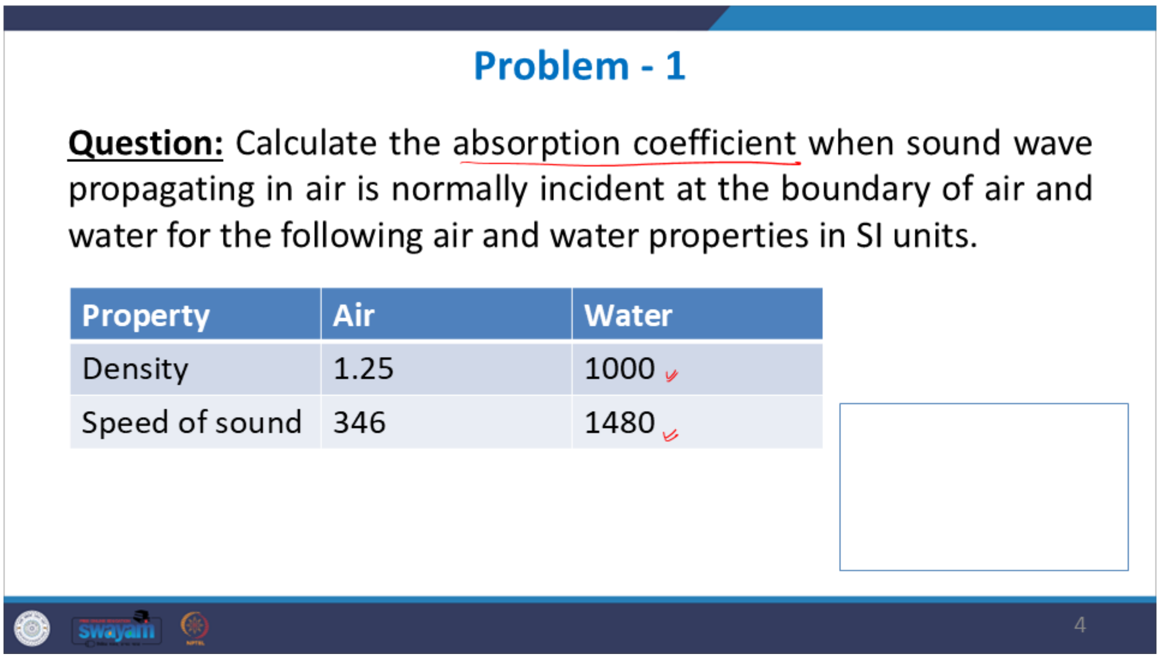
$$1 + R = T$$





All these relationships were derived when the sound wave is incident normal to the planar boundary of the next medium. Today, let's solve some numericals related to that.

The outline is that we will solve the numericals related to these concepts: reflection, transmission, absorption, acoustic impedance, and the transmission from medium 1 to medium 2 at normal incidence. We will also see some special cases of normal incidence at the medium boundary. So, let us see the first problem here.



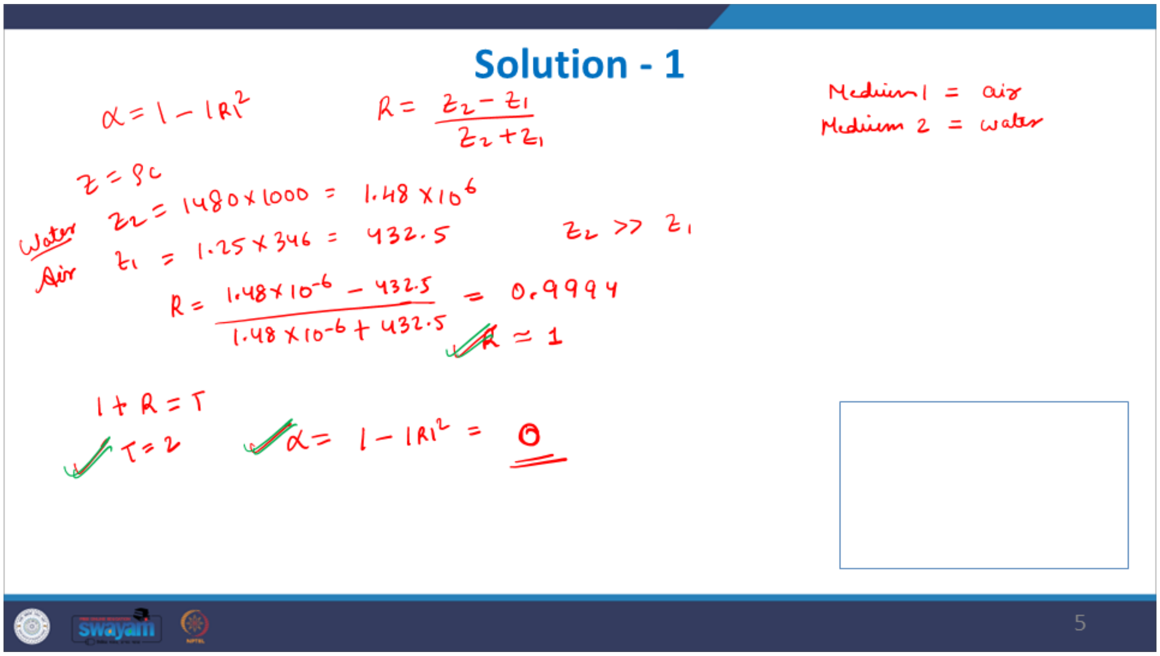
Calculate the absorption coefficient when the sound wave propagating in air is normally incident at the boundary of air and water for the following properties in SI units.

So, here we have, from air it is traveling to water, and typically, the speed of sound in water is much higher, and the density is also much higher. So, this is what is given to us. Let us calculate. So, to find out (α). This is the relationship.

$$\alpha = 1 - |R|^2$$

So, we need to know the pressure reflection coefficient.

$$R = \frac{z_2 - z_1}{z_2 + z_1}$$



Here, our medium 1 is air because the sound is traveling in air, and medium 2 is water. So, what happens? Z is,

$$z = \rho c$$

which is the characteristic impedance.

So, Z2 would be,

$$z_2 = 1480 \times 1000$$

$$= 1.48 \times 10^6$$

So, this is for water, And the Z1 is going to be, if we see here the values, which is 1.25 and 346.

$$z_1 = 1.25 \times 346$$

$$= 432.5$$

this is for air. both in the same SI unit. So, here you observe that Z2 is of 10 to the power of 6, whereas the Z1 is 10 to the power of 2. So, the Z2 is here 4 times higher in the order of magnitude. So, Z2 is much higher in magnitude than Z1. If you try to calculate, what overall you see is something like this.

If you solve it, it will come very close to 1

$$R = \frac{1.48 \times 10^6 - 432.5}{1.48 \times 10^6 + 432.5}$$

$$= 0.9994$$

$$R \cong 1$$

So, this is a special case which we will be discussing very soon. So here, what happens is that when a sound is coming in contact and it is encountering the boundary of a medium which has a much higher impedance, if you find out the reflection coefficient, it is almost equal to 1, which means that whatever pressure is being incident, that same magnitude pressure is getting reflected back, and if you have this,

$$1 + R = T$$

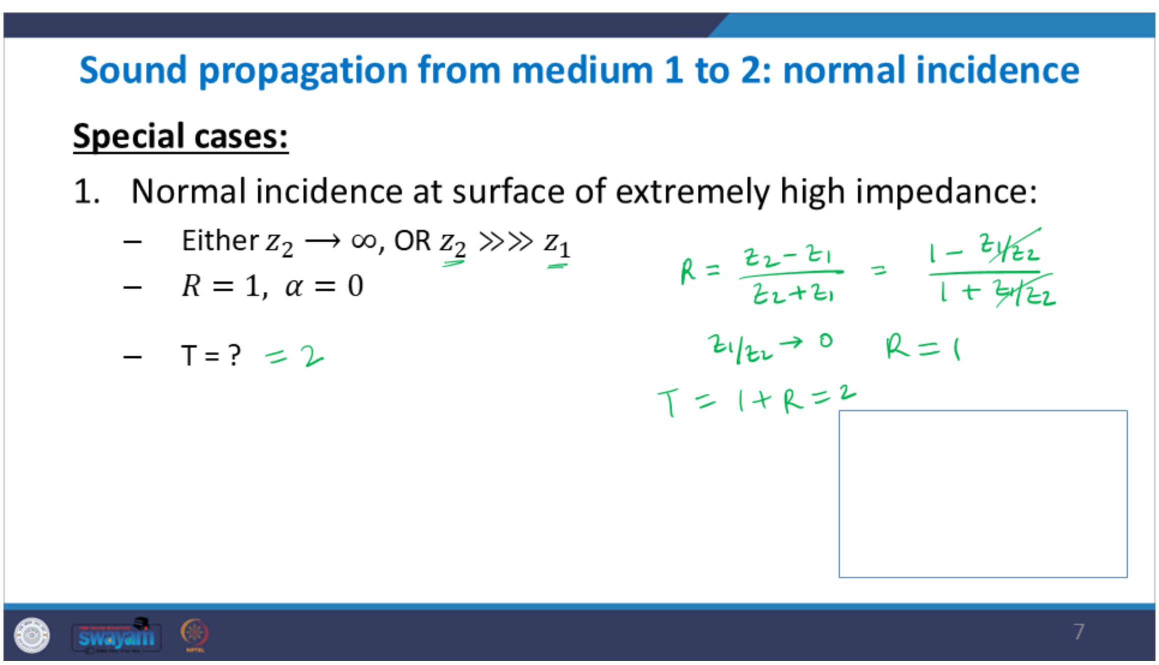
$$T = 2$$

a very peculiar case which you will see, and if you try to solve for alpha, which is given in this question here,

$$\alpha = 1 - |R|^2$$

$$\alpha = 0$$

that you have to find out the absorption coefficient for this boundary. So, let us discuss about this special case $R \cong 1, T = 2,$ and $\alpha = 0$ these three quantities over here.



So, here this is the special case that we have encountered with this problem. Here, what is happening is these are the two special cases which have similar results: either the impedance of the next medium or medium 2 is much higher in magnitude compared to the impedance of medium 1, or the impedance of medium 2 is infinity, which is an extremely high impedance surface we are encountering. Then we see that r,

$$R = \frac{z_2 - z_1}{z_2 + z_1}$$

if you solve it, let us divide the entire thing by Z2, we get this expression,

$$R = \frac{1 - \frac{z_1}{z_2}}{1 + \frac{z_1}{z_2}}$$

and since Z2 is much higher than Z1 or Z2 tends to infinity in both these cases, this quantity here

$$\frac{z_1}{z_2} \to 0$$

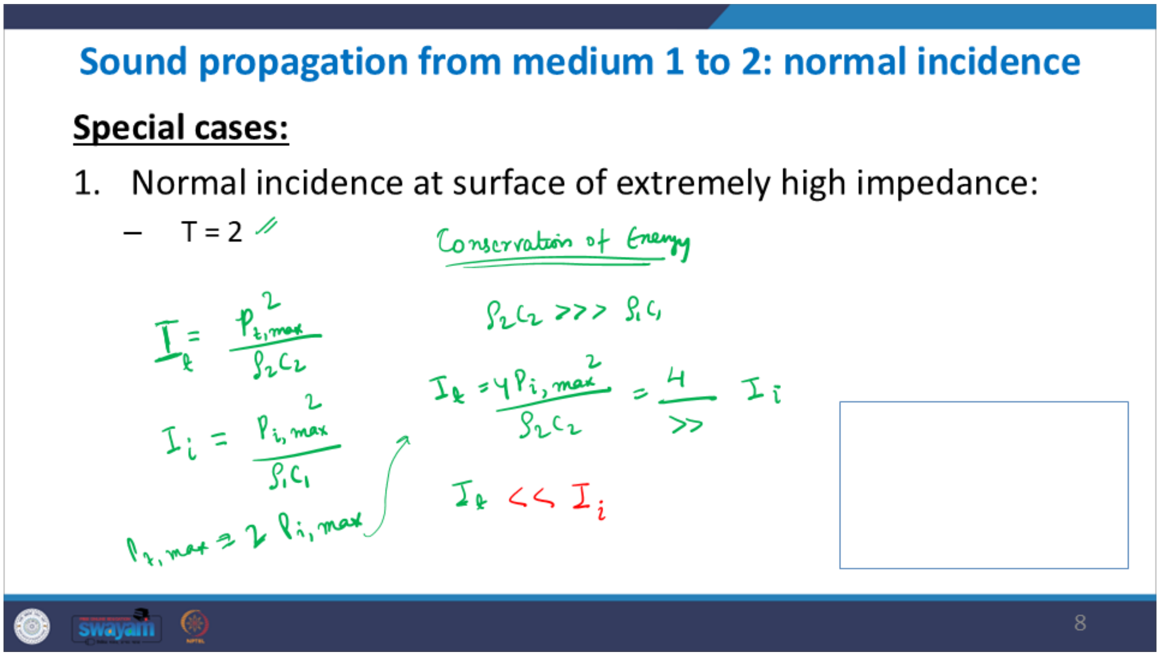
because the denominator is much higher compared to the numerator, it can be neglected. So, if we simply neglect this quantity here compared to 1 over here, we have neglected this. So, what we are getting is that

$$R = \frac{1}{1} = 1$$

$$T = 1 + R = 2$$

Now, it seems quite counterintuitive that the sound wave is being incident on a very hard or high impedance surface, and whatever the pressure wave is incident on, that same magnitude pressure wave or that same amplitude pressure wave is being reflected back, but at the same time, you know, intuitively, we'll think that okay, whatever sound is being incident, it's getting reflected back.

So, there should not be any transmission, but that's not the case. The pressure of the transmitted wave or the amplitude of the pressure of the transmitted wave is almost twice that of the incident wave. So, it is doubling in magnitude, very peculiar. So, does that mean that you know suddenly we are creating new sound energy or suddenly something weird is happening? That is not the case. What happens here, Let us see and compare in terms of the sound energy and the intensities in the two mediums.



So, by the principle of conservation of energy, even at this particular interaction, the energy should be conserved. We are not suddenly generating new energy, okay. So, that means that the total energy at the left side of the wall should be the same as the energy at the right side of the wall. Let us find out the intensity for these respective waves. So, the intensity of the transmitted wave is what? Let us find the intensity of the transmitted wave. It comes out to be,

$$I_t = \frac{{p_{t,max}}^2}{\rho_2 c_2}$$

And the intensity of the incident wave comes out to be,

$$I_i = \frac{{p_{i,max}}^2}{\rho_1 c_1}$$

because we are using this relationship that 'I' is equal to rho C. So, if you see here, rho2 C2 in this case is much higher in magnitude than rho1 C1.

$$\rho_2 c_2 \ggg \rho_1 c_1$$

So, overall, even if this, Now, we know that Pt max has almost doubled in magnitude compared to the Pi max,

$$P_{t,max} = 2P_{i,max}$$

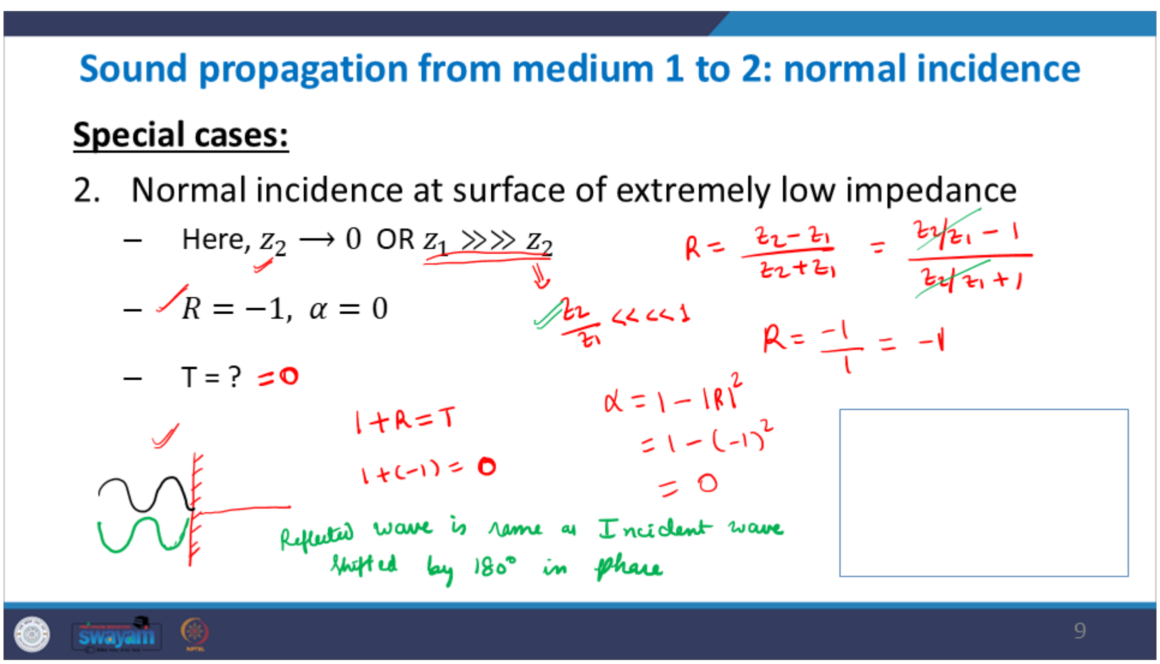
because T is coming out to be 2, right.

So, if we equate this, then what do we get?

$$I_t = \frac{4\,P_{i,max}^2}{\rho_2 c_2}$$

which means 4 by some very large quantity, I am just going to write it as a very large quantity, Into I_i. So, 4 divided by a very large quantity. So, ultimately what we get is that the intensity of the wave or the energy density, you can say for the transmitted wave, is actually much lower in magnitude. It is much lower in magnitude than the intensity of the incident wave. So, now this interaction makes more sense that if we have a high impedance medium, although the amplitude of the pressure wave is increasing, but the intensity of sound actually decreases quite a bit in the high impedance medium.

Let us see another special case which is Normal incidence at a surface of extremely low impedance. So, now it is just the reverse case of the previous one.



So, from a high impedance suddenly we are going to an extremely low impedance medium. So, either z_2 is approximately 0 or z_1 is much higher in magnitude compared to z_2. So, in both these cases if you solve for r which is

$$R = \frac{z_2 - z_1}{z_2 + z_1}$$

and if you solve it this way it becomes,

$$R = \frac{\frac{z_2}{z_1} - 1}{\frac{z_2}{z_1} + 1}$$

Since z_1 is much higher in magnitude compared to z_2, so what are we getting here?

$$\frac{z_2}{z_1} \lll 1$$

So, essentially this term here, can be neglected. Since we are neglecting this term, so what happens? r becomes,

$$R = \frac{-1}{1} = -1$$

And alpha (α) is,

$$\alpha = 1 - |R|^2$$

$$= 1 - |-1|^2$$

$$= 0$$

And what do you think the transmission coefficient is going to be in this particular interaction? Once again, you can use this relationship.

$$T = 1 + R$$

$$T = 1 + (-1)$$

$$T = 0$$

So, 0 transmission.

Okay. So, essentially what is happening is that when the wave is incident normally and traveling from a high impedance to a very low impedance medium, there is a reflection, but transmission coefficient is 0. So, physically, what happens at this boundary? Let us say this is the boundary we are talking about. Let us say this is our incident wave. That is an incident wave. Now, what do you mean by minus 1? Which means that the magnitude or the amplitude of the reflected wave is the same as the amplitude of the transmitted wave.

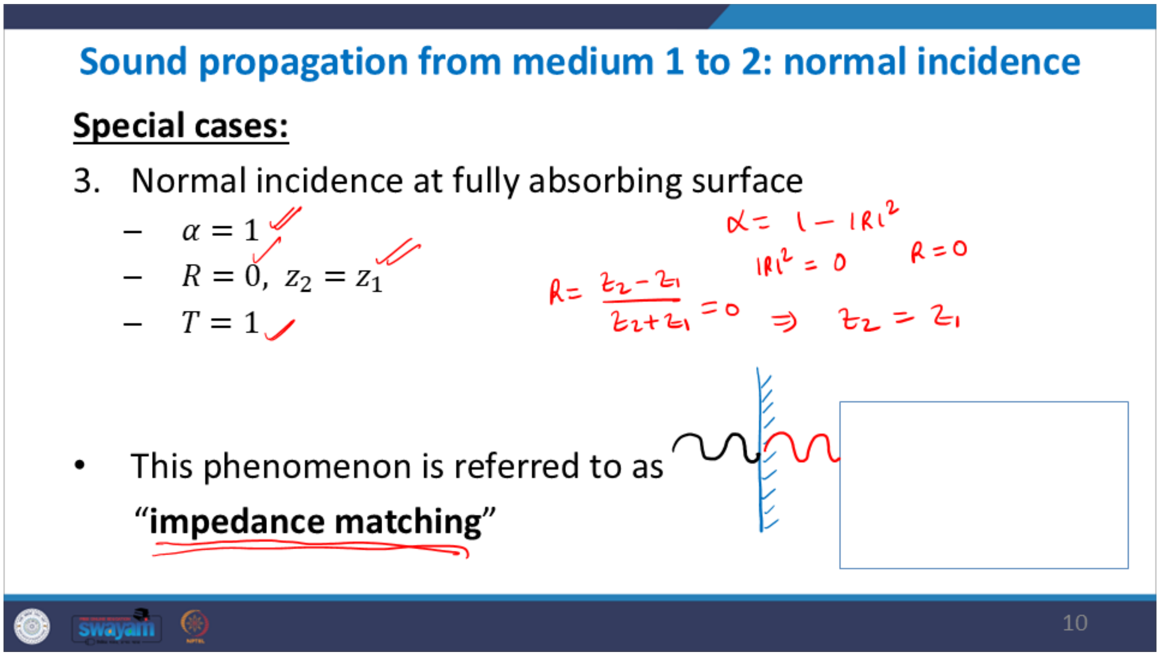
But minus 1 indicates that wherever we have a peak of the incident wave, we are actually experiencing just the negative of the pressure, which means we are experiencing a trough in the reflected wave. So, the reflected wave should be something like this, just the negative of the incident wave. So, it should be like this (refer slide above). So, basically what is happening is that the reflected wave is the same as the incident wave shifted by 180 degrees in phase. So, because of the 180-degree shift, it is just the negative or the opposite of the transmitted wave in the waveform where the amplitude remains the same.

Overall, the pressure is canceling each other out, as you can see the incident wave and the reflected wave together, when they are added or superimposed on each other at every point in space, they are canceling each other at the boundary. So the pressure essentially at the boundary is becoming 0, and the pressure at the left-hand side is 0, and in the same way, the pressure at the right-hand side becomes 0. So, by the continuity of pressure, this wave is flat; no wave at all. So,

$$T = 0$$

So, this is a very favorable case. What is happening is that suppose we are traveling from a high impedance to a low impedance medium in terms of noise control in mechanical systems. If some such kind of interaction is encountered, what happens is, we are able to bring down the transmission to 0 when this kind of, In an ideal situation, if this interaction is encountered, the transmission through that boundary becomes 0. So, the wave stops and does not penetrate into the new material, and at the surface, the pressure also becomes 0, because the reflected wave cancels the incident wave. So, at the surface and near the surface in such interaction, the acoustic pressure becomes 0. Near such boundary surfaces, which is a very favorable condition for noise control.

Now, the third and the last case is that, suppose the sound waves are incident at a fully absorbing surface, that is when the absorption coefficient of that surface is equal to 1.



So, in that case, alpha (α) is,

$$\alpha = 1 - |R|^2$$

and alpha (α) itself is 1, which means that,

$$|R|^2 = 0$$

So, in that case, R becomes 0, and if R is becoming 0, what does it mean for the impedances of these interactions? If you look at this relationship,

$$R = \frac{z_2 - z_1}{z_2 + z_1} = 0$$

if this is 0, this means that Z2 is the same as Z1. This phenomenon is also called impedance matching, which means that the impedance of medium 1 and medium 2 completely match, and in that case, T,

$$T = 1 + R = 1$$

So, essentially what is happening here, if we again represent it through what actually happens at the boundary, like we did in the very first case and the second case.

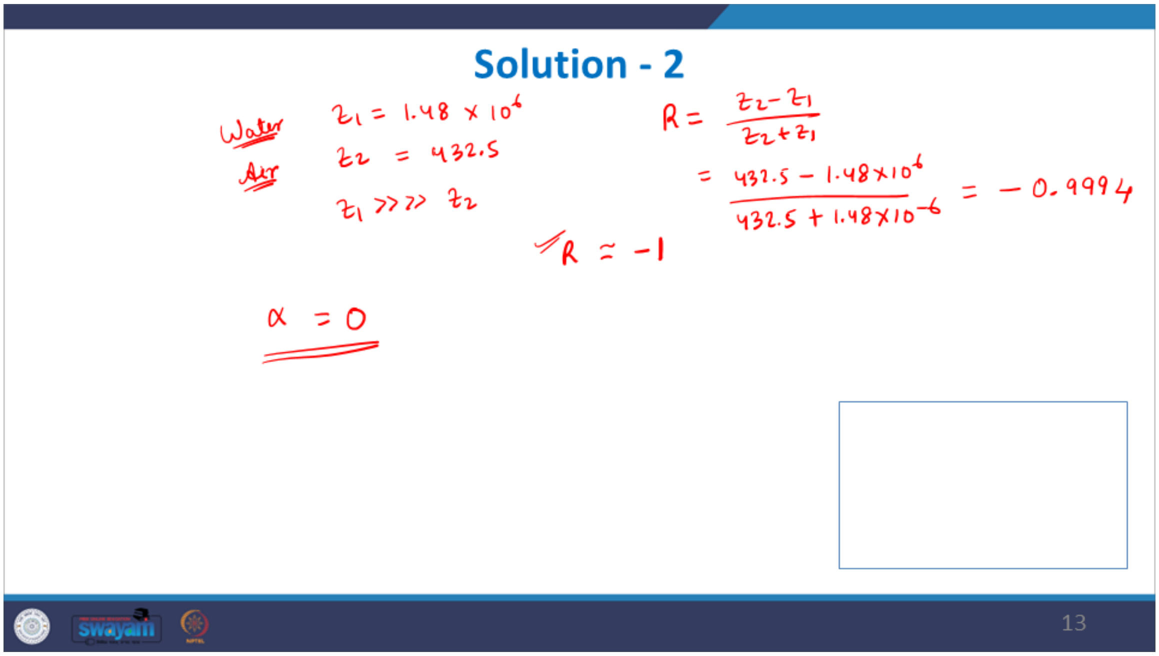
So, what we see here is that let us say this is our boundary. Let us represent each thing with a different color, and we have an incident wave that is coming in at a certain amplitude. There is no reflected wave because $R = 0$, and the transmitted wave has the same magnitude. and the same phase as the incident wave. So, essentially, what you see is it behaves as if there is no medium change at all. This is because, the characteristic impedance of the two media, is the same, and we discussed this in the previous class that, the characteristic impedance ρc. It sort of quantifies how resistant that medium is to sound wave propagation. And in this case, because the characteristic impedance of the two media is the same, it means that they both are acoustically are same. They might be different in other parameters. We might have the medium which has different densities. They might have different bulk modulus, but overall, the impedance becomes the same. So, acoustically, for a sound wave, it behaves as if the medium is continuous in nature and there is no difference at all. Both are offering the same resistance, and hence the sound wave propagates through as if there is no change in the medium at all.

Problem - 2

Question: Calculate the absorption coefficient when sound wave propagating in water is normally incident at the boundary of water and air for the following air and water properties in SI units.

Property	Air	Water
Density	1.25	1000
Speed of sound	346	1480

So, this is the impedance matching case. Now, let us solve problem 2. So, the same as problem 1, in problem 1 what happened was that the sound wave was propagating from air and it was incident on the air-water boundary. and the water had much higher impedance than air. Now, we reverse the case; it is propagating in water and then it is incident on the air. What is happening here?



$$z_1 = 1.48 \times 10^6$$

So, z_1 here is the water because it is traveling in water and then it has to emerge out into air. So, air it is again. The same way as we solved for a previous case, it is,

$$z_2 = 432.5$$

So, here the reflection coefficient, so now this represents, you can see here itself that z_1 has become much greater in magnitude than z_2.

$$z_1 \ggg z_2$$

So, this represents our second special case, where R which is,

$$R = \frac{z_2 - z_1}{z_2 + z_1}$$

$$= \frac{432.5 - 1.48 \times 10^6}{432.5 + 1.48 \times 10^6}$$

$$= -0.9994$$

this quantity here. as we had found for the previous case with a negative sign. So, approximately you can say.

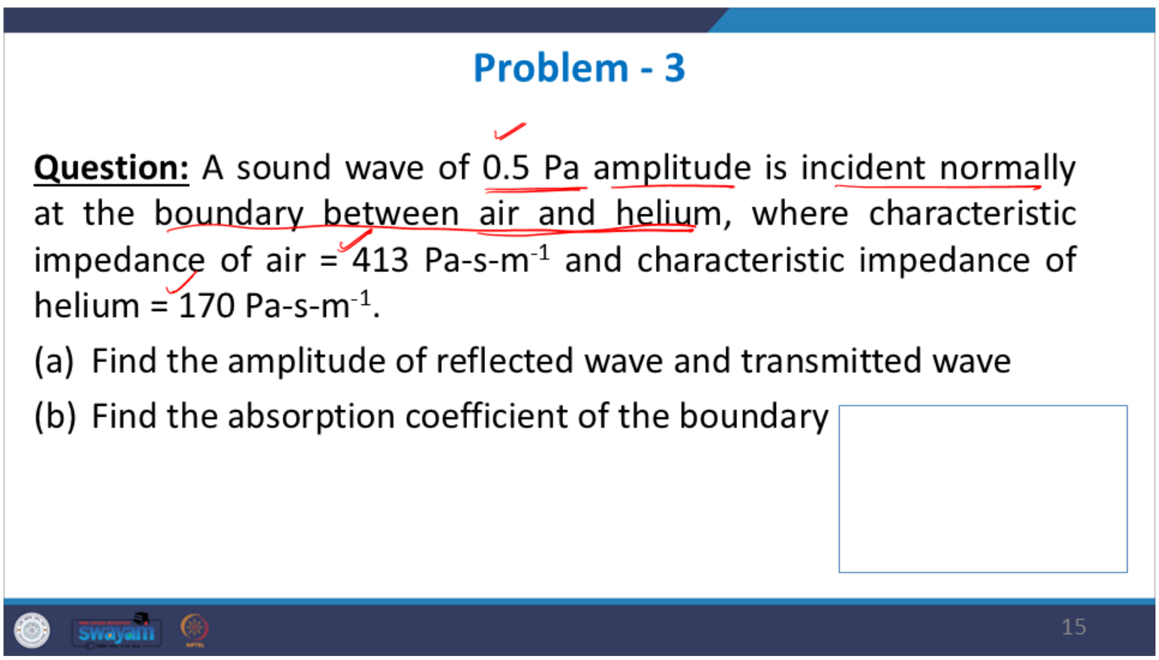
$$R = -1$$

And what we have to find here is the absorption coefficient, so alpha (α) once you know these special cases, you don't even need to solve them because once you know that, One impedance is much higher in magnitude than the other, you see which special case is this, and you can directly come to the conclusion that the R is either minus 1 or it is plus 1. so here in this case. If you know the special case, you can directly solve and say r is equal to minus 1 and alpha, which is,

$$\alpha = 1 - |R|^2$$

is becoming 0. That is our answer.

So, for both these problems, absorption is 0, but the nature of interaction is pretty different. Okay.



Now, let us see a third problem in this situation. Here, we have a sound wave of 0.5 pascals amplitude. Normal incidence at the boundary between air and helium. Now, here it is not given whether it is propagating in air or in helium, but suppose it is given that the boundary is between air and helium, and air is coming first. So, we assume that air is medium 1 and helium is medium 2. And the characteristic impedance of the first medium is given to you, and the characteristic impedance of the second medium is given; you have to find out what is the amplitude of the reflected wave and transmitted wave, and also the absorption coefficient of the boundary.

Let us first solve the reflection coefficient; here,

$$R = \frac{z_2 - z_1}{z_2 + z_1}$$

medium 1 is air, medium 2 is helium. So, z_1 which has already been given as 413. And the z_2, which is in helium, is given to us as 170. So, this R can be which we solve as.

$$R = \frac{170 - 413}{170 + 413} = -0.417$$

So, let us solve for the first one; you have to find out the amplitude of the reflected wave and transmitted wave. So, the amplitude of the reflected wave uses the relationship of R,

$$P_{r,max} = R\ P_{i,max} = (-0.417) \times 0.5$$

this comes out to be,

$$= -0.2084\ Pa$$

what do you observe here is that, the pressure, the amplitude of the reflected wave, has reduced, and at the same time, it has become negative. So, it is suffering from a 180-degree phase shift. The reflected wave is shifted 180 degrees in phase from the incident wave, and its amplitude has also decreased. So, even in this case, when you add up the acoustic pressures in the interaction. So, at the very left-hand side of the boundary, the reflected and the transmitted wave are coming together, and the reflected wave is sort of subtracting or nullifying the incident wave, but there is imperfect cancellation. So, an imperfect cancellation is happening. So, the pressure is going down, but it is not coming to 0.

So, definitely, there is some transmitted wave, which we will find now. So, in the same way, the amplitude of the transmitted wave,

$$P_{t,max} = T\ P_{i,max}$$

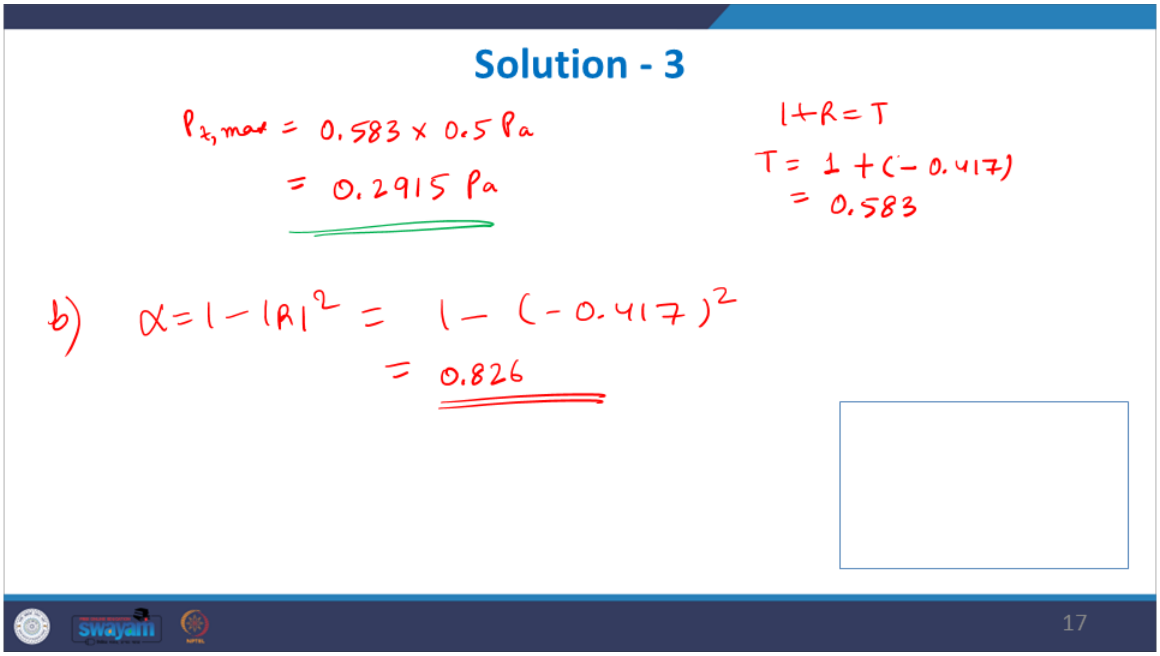
So, this p t max is, so t in this case, let us find out T first:

$$T = 1 + R$$

T becomes,

$$T = 1 + (-0.417) = 0.583$$

So, Pt max should be, $0.583 \times 0.5\ Pa$



So, overall, you get 0.2915 pascals. Now,

let us solve the last part of this question: the absorption coefficient of this boundary. So, alpha (α) is

$$\alpha = 1 - |R|^2$$

$$\alpha = 1 - (0.417\ )$$

If you solve it, it comes out to be 0.826, which is a pretty high number. So, it is a highly absorbing surface. The closer the alpha is to 1, the greater the absorption.

So, with this, I would like to conclude this lecture on the numerical problems for sound wave propagation at normal incidence. Thank you.