NOISE CONTROL IN MECHANICAL SYSTEMS

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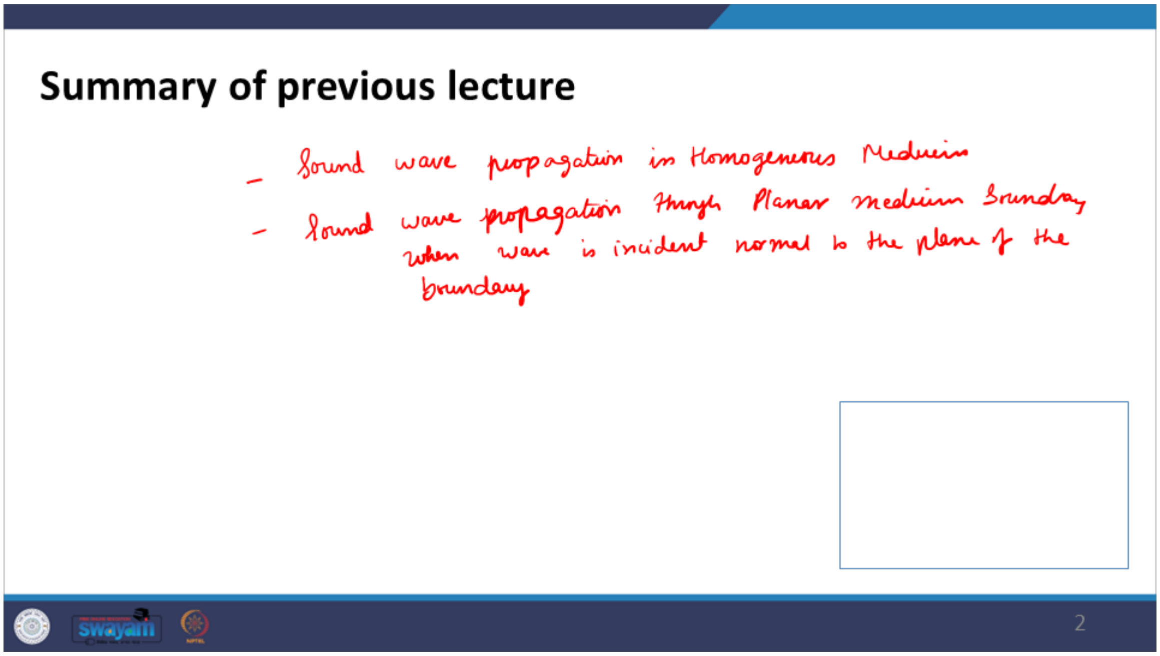
Week:2

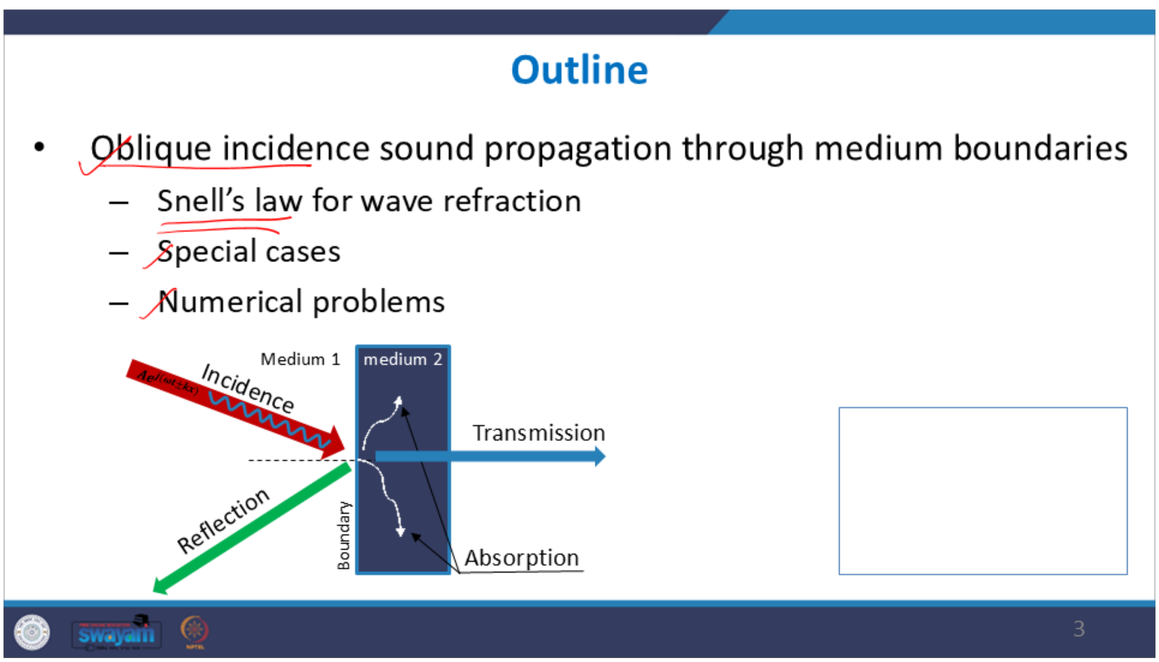
Lecture:9

Lecture 09: Propagation Through Medium Boundaries: Oblique Incidence

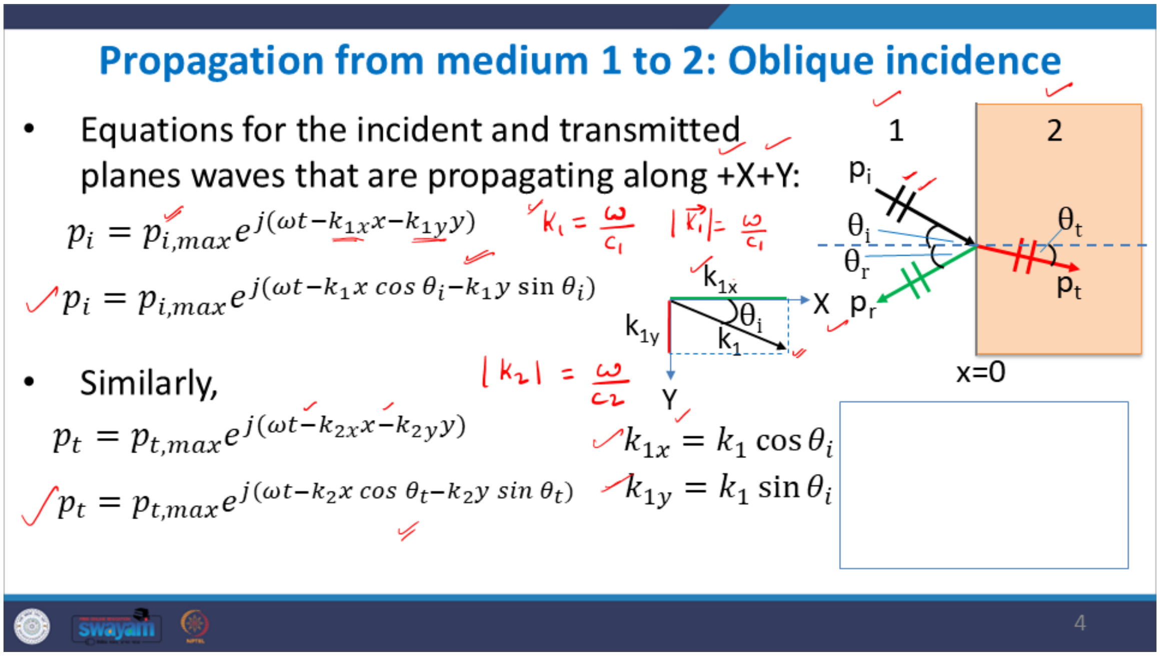
Hello and welcome to Lecture 9 in the series on noise control in mechanical systems. I am Professor Sneha Singh from IIT Roorkee. So, to sum up till now, we have studied sound wave propagation in a homogeneous medium, where we studied the different wave fronts. And then we have also studied sound wave propagation through a planar medium boundary, when the wave is incident normal to the plane of the boundary. And we have been able to solve some numericals based on this interaction, but in real life, sound waves can come from various directions depending on the location of the source and may be incident at varying angles at different planar boundaries. So, we need to solve and see what happens when a sound wave is incident obliquely, or at a random angle to the plane of the



boundary, which we will cover in this particular class. And while we discuss the phenomenon, we will come to know about Snell's law of wave refraction. We will do some special cases and then solve some numerical problems.



So, let us see, this is our medium 1, this is our medium 2, and some sound wave. Again, for solving all these interactions, we have chosen that the wave is a harmonic plane wave.



But the relations derived, in general, hold true for spherical and cylindrical waves as well. Because they can be decomposed or thought of as a special case of a harmonic wave front. So, here I am representing this using this arrow. The arrow represents the direction of propagation of the harmonic wave, and this indicates the wave front from the top view. So, this schematic you can look at it in my lecture on harmonic plane waves, where I have discussed what this arrow and the perpendicular things mean. This notation, or what is the meaning of this symbol, you can see in that particular lecture.

Here we see an incident wave at some angle θ_i with the normal to the surface, and then it is getting reflected and transmitted. So, let us represent. So, here our incident wave. Let us take, first of all, our x and y axis.

This is our x-direction and y-direction (Refer slide 4). So, if this is the axis we are choosing, then in that case, the instant wave is propagating in the forward x and y axis or the plus x and plus y direction. It is propagating in that. So, forward in x and forward in y. So, accordingly for a harmonic plane wave, we can represent it as

$$p_i = p_{i,max} e^{j(\omega t - k_{1x} x - k_{1y} y)}$$

Now, let us see this is the x-component of the wave propagation vector, and this is the y-component of the wave propagation vector. And what is the propagation vector? It has the magnitude of the wave number, which is

$$k_1 = \frac{\omega}{c_1}$$

In our case, we have two mediums, so we can represent it as omega by c1 because in these interactions, omega, which is the frequency, remains the same. So, both f and omega remain the same for such interactions. It is only the speed of the sound that is changing from medium to medium. So, this is our overall k1, and the k1 vector is something with the magnitude as omega by c1. and the k1 vector direction is along the wave propagation direction. So, this is our k1 vector (Refer slide 4). And as you can see, the wave is propagating with an angle of θ_i with the x-axis, so this becomes the direction of our k1 vector, and k_{1x} is the x-component, and k_{1y} is the y-component.

We are using k1 because it is the k of medium 1. So, what is the relationship between the two? As you can see from the figure and simple trigonometry,

$$k_{1x} = k_1 \cos\theta_i$$

$$k_{1y} = k_1 sin\,\theta_i$$

So, let us replace this in the above equation. So, this is, after replacing, the equation we are getting.

$$p_i = p_{i,max} e^{j(\omega t - k_1 x\, cos\,\theta_i - k_1 y \sin\theta_i)}$$

In the same way, we can do for the transmitted wave. The transmitted wave itself is also traveling forward in the x-axis and the y-axis. So, here, that is why we have the negative sign and the symbol k_2 instead of k_1 because it is in a different medium. So, here,

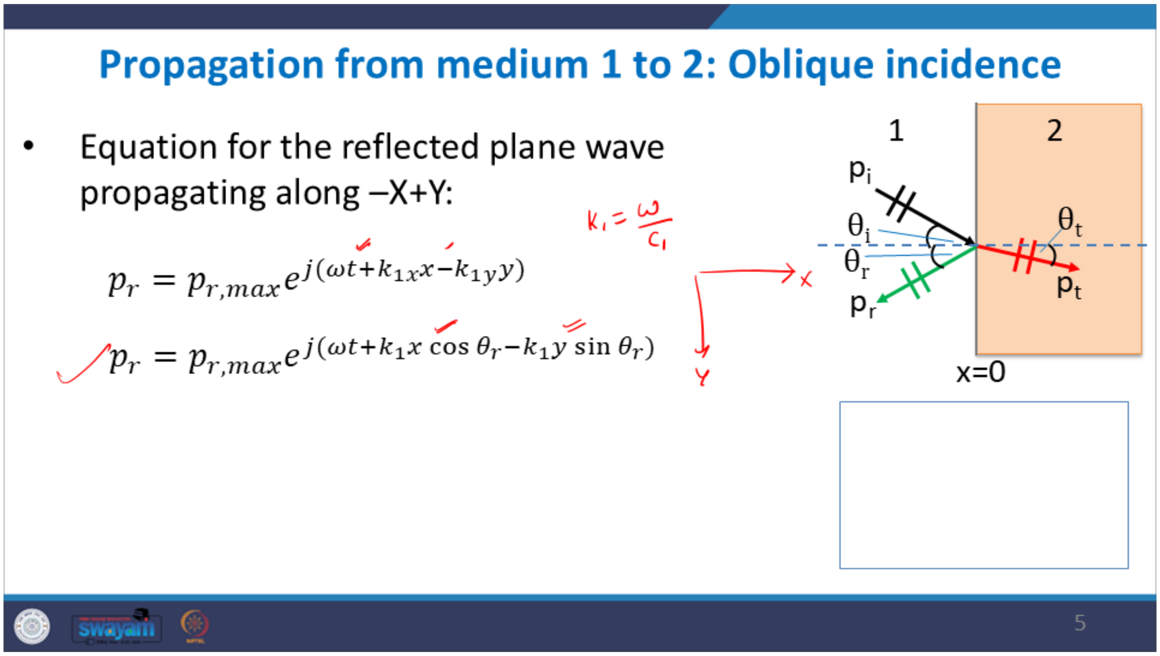
$$k_2 = \frac{\omega}{c_2}$$

Just like how we found the relationship between k_{1x} and k_{1y} in terms of the overall k_1, in the same way, we solve it and come to this format where it is

$$p_t = p_{t,max} e^{j(\omega t - k_{2x} x - k_{2y} y)}$$

$$p_t = p_{t,max} e^{j(\omega t - k_2 x\, cos\,\theta_t - k_2 y\, sin\,\theta_t)}$$

where theta t is the angle between the k_2 vector and the x-axis. In the same way, let us derive the equation for the reflected wave. Now, as you can observe, the reflected wave is traveling sort of forward in the positive y direction because this is our x and y coordinate, which I will draw here as well. This is our x, and this is our y, and what is happening to the



reflected wave? It is traveling in the positive y but the negative x direction. So, that is why we have a plus sign in the x component and a minus sign in the y. If you again resolve this and write it in terms of k1, and here, because the reflected and incident waves are in the same medium, they both have the same wave number k1, which is

$$k_1 = \frac{\omega}{c_1}$$

So, you resolve it, and the wave vector of the reflected wave is making an angle of θ_r with the x-axis. So, ultimately, this is the relationship we get

$$p_r = p_{r,max} e^{j(\omega t + k_1 x \cos\theta_r - k_1 y \sin\theta_r)}$$

when we resolve its x and y components. Now, let us see the same continuity of normal pressure on the boundary as we have been doing for the case of normal incidence. So, at x equals to 0, We add up the pressure just on the left-hand side of the boundary and the right-hand side of the boundary, and the pressure is continuous.

There is continuity of the pressure at that point. Hence, P at the limit of x tends to 0 negative should be the same as P at the limit of x tends to 0 positive.

$$P_{x\to 0^-} = P_{x\to 0^+}$$

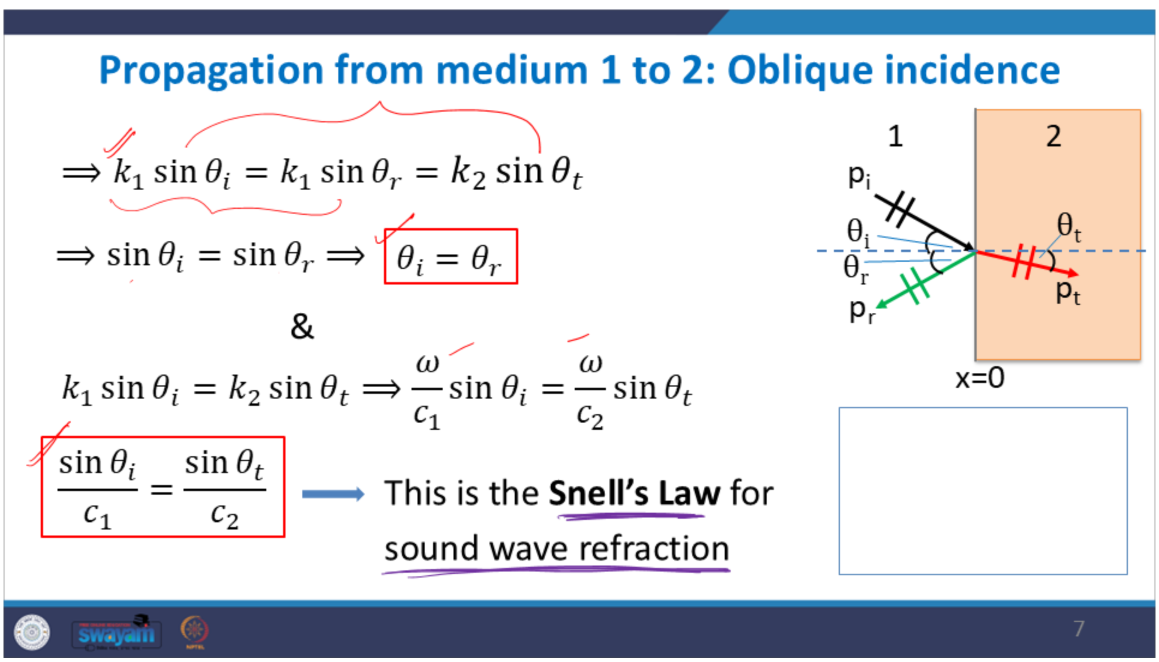
We are not suddenly creating new pressure at this point. So, when you solve it and put x as 0 in these above equations that we have found (Refer slide 4), You will see that we arrive at this particular format.

$$p_i(0, y, t) + p_r(0, y, t) = p_t(0, y, t)$$

Now, in order for this equality to hold true, because here what we have got certain numbers, and then we have certain exponential terms, and they are being equated. So, for that, all exponents may need to be equal so that this equality can hold true. So, we have already stricken off this e to the power j omega t from everywhere, and this is what we are arriving at.

$$p_{i,max} e^{-jk_1 y \sin\theta_i} + p_{r,max} e^{-jk_1 y \sin\theta_r}$$

So, we equate these exponents. So, all of this has to be equal to each other. So, this thing here, this new equation that we get by equating the exponents on the left-hand side and the right-hand side.



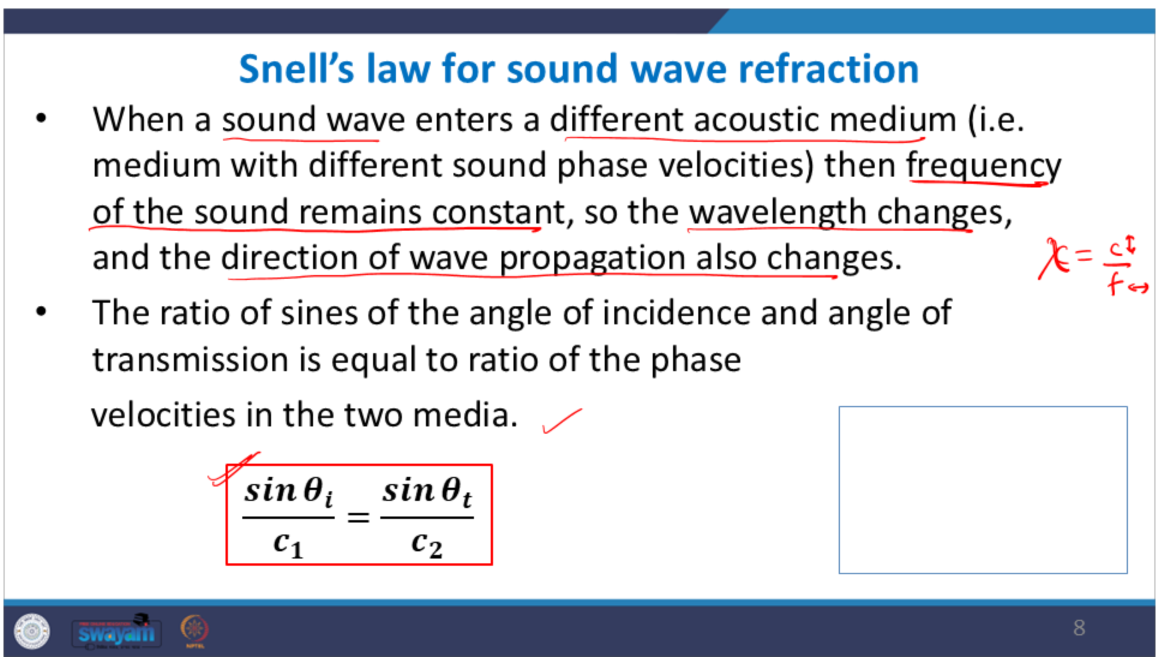
So, every term's exponent has to be the same for the equality to hold true. So, with this, what we get from the very first two equations, is

$$\sin\theta_i = \sin\theta_r$$

Given that we are dealing with the period between 0 to pi, both θ_i and θ_r will be the same because their signs are coming out to be the same. So, what does it mean? That the reflected wave and the incident wave, just like in the case of light waves where they are reflecting from a mirror, they both have the same angle with the normal. When you solve these two equalities, so you write it like this and see over here, this is the final equation we are getting:

$$\frac{\sin\theta_i}{c_1} = \frac{\sin\theta_t}{c_2}$$

Now, this is very important because this is what we call Snell's law for sound wave refraction. Now, there is a Snell's law for light waves, there is a Snell's law which is applied to an electromagnetic wave in general, but because the context of our lecture series is the acoustic waves or the sound waves, we are referring to this as Snell's law for the case of sound waves. So, let us state Snell's law.



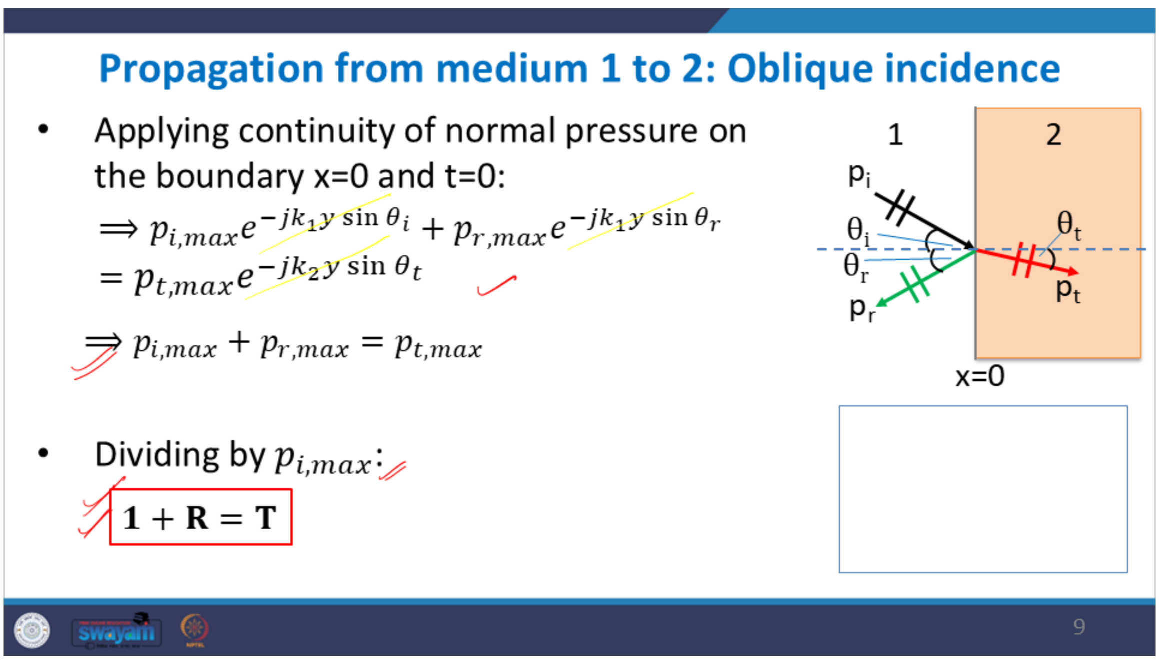
So, verbatim, what it says is that 'when a sound wave enters a different acoustic medium, the medium having different sound phase velocities, then the frequency remains the same'. So, always remember that the frequency of the sound wave is independent of the medium it is traveling in.

And the various kinds of boundaries and the various kinds of elements it encounters on the way. It's independent; it remains the same. The frequency only depends on the source that's creating that sound wave. So, till the source remains the same, it doesn't matter how it propagates and what path it takes, the frequency remains the same. So, the frequency remains the same, but in this interaction, the speed is changing,

$$\lambda = \frac{c}{f}$$

This is changing, and this stays the same (Refer slide 8). So, lambda changes. So, the wavelength changes, and at the same time, the direction of the wave propagation is also changing. And what is the manner in which it is changing? It is changing so that this relationship is satisfied.

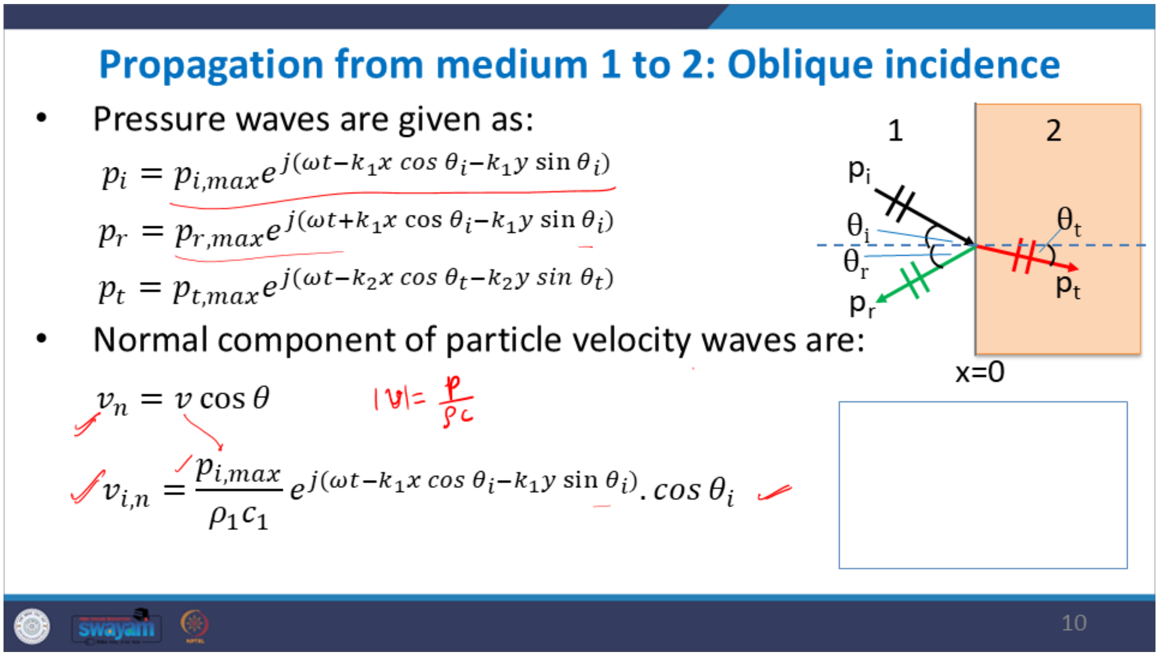
So, the sines of the angle of incidence and angle of transmission are equal to the ratio of the phase velocities in the two media.



Now we have found this relationship between θ_i , θ_r and θ_t . Let us now again apply the continuity of normal pressure, but now we equate it at one time instance, which is t equals to 0 at the boundary. So, once again, this is our equation, okay? And in this equation, because we have already established that, for this equality to hold true, all these exponents must be the same. So, let us just strike them off. This is a common term we are removing from the equation. So, we end up with this particular equation here.

$$p_{i,max} + p_{r,max} = p_{t,max}$$

So, this is the equation of the respective pressure amplitudes. If you divide it throughout by the incident pressure, you again get the same relation. As you got for the normal incidence.



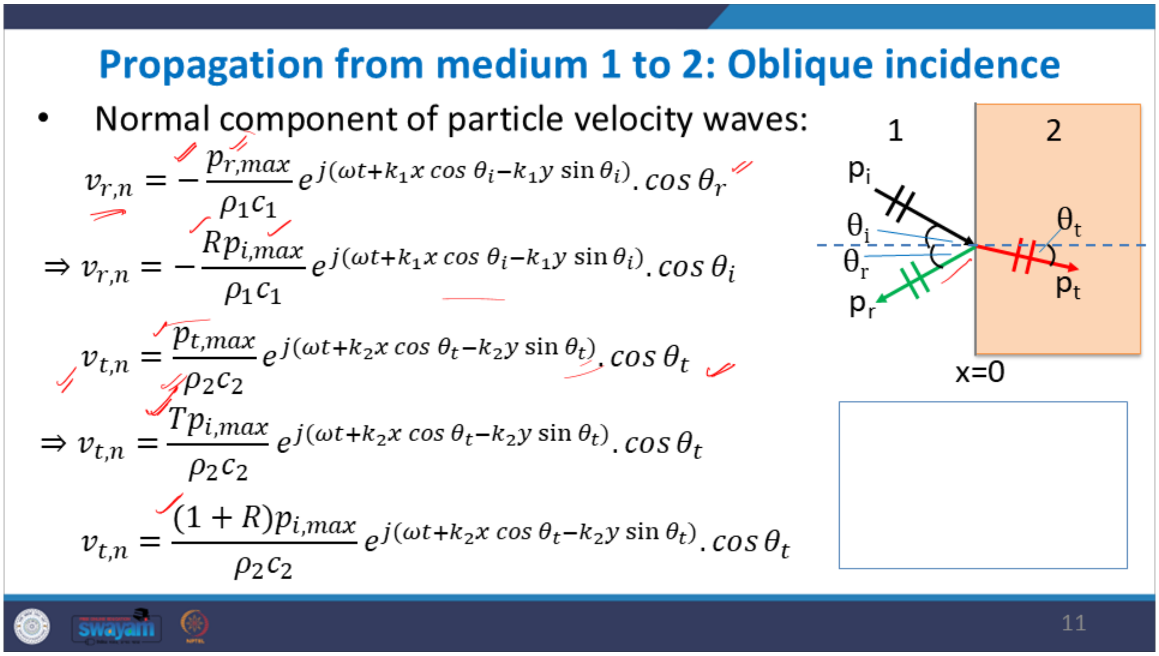
Now, once again, have a look at these equations. We have these equations for pressure. Now, let us find out the equations for the normal component of the particle velocity at the medium.

Because here, in the previous case, when it was normal incidence. So, in normal incidence, Vn was the same as V, and we did not have to use any *cos* term in that, and we directly derived everything. But over here, V n is not equal to V. So, the normal component of the velocity in terms of the V would be *cos* times that angle. It is the cos component. So, velocity is in a particular direction. We want to decompose it along the x-axis. So, that is what we get.

So, for the incidence angle, the normal velocity is given by. So, also, we know that

$$|v| = \frac{P}{\rho c}$$

So here, for the incident wave, we write the format of p here. So, this long expression for p_i we are writing it here (Refer slide 10) and then divide it by $\rho_1 c_1$ for medium 1. Because here, the incident wave is traveling forward in space, the normal component is along the positive x-axis, so we have a plus sign, and then this times $\cos\theta_i$.



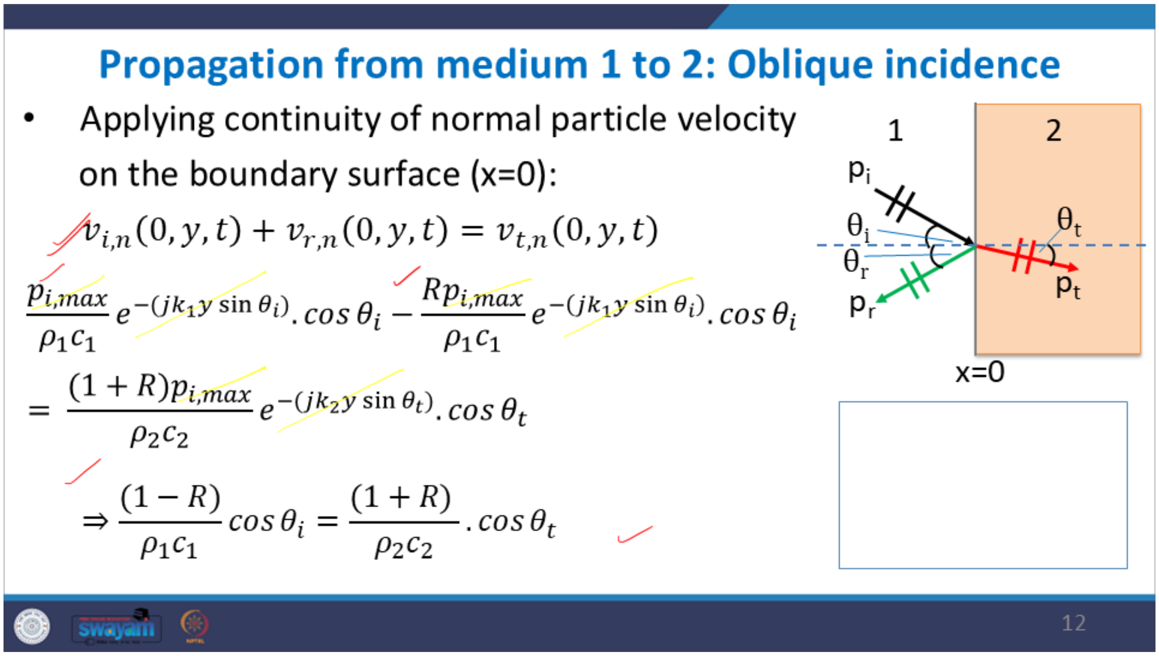
In the same way, we can find out the normal component of the reflected wave. So, once again, we use the equation of the reflected wave. We divide it by $\rho_1 c_1$ for that medium and then multiply by $\cos\theta_r$, and then we place a minus sign here because the wave is propagating like this. So, if you resolve and find out the normal component along the x-axis, the wave is actually propagating along the negative x-axis. So, the normal component is opposite in direction.

So, we put a minus sign here. And how can we write this? So, in the same way, just like how we did the derivation for normal incidence, we will do it the same way. So, this can be written as $Rp_{i,max}$. In the same way, the normal component of the transmitted wave velocity, once again, we write the equation of the transmitted wave, this one here, divided by $\rho_2 c_2$ because it is in medium 2 (Refer Slide 11), multiplied by $\cos\theta_t$, and we have a positive sign because the normal component of it is along the positive x-axis.

So, this can be written as $Tp_{i,max}$. You see here; what we have already found is this equation

$$1 + R = T$$

So, here we can replace this T as $1+R$.



Now, applying the continuity of normal particle velocity at the boundary, let us just equate that. So, we are adding up these velocities. Here we add up the magnitudes. So, plus minus like that, and when we did the same thing for pressure, we did not. Use a minus sign because pressure is a scalar quantity; it anyways gets added, but in velocity, we consider the direction as well, ok. Now, let us remove all the common terms; if you remove that, all of this is the common term. We have already found that these exponents are equal; we remove the common term.

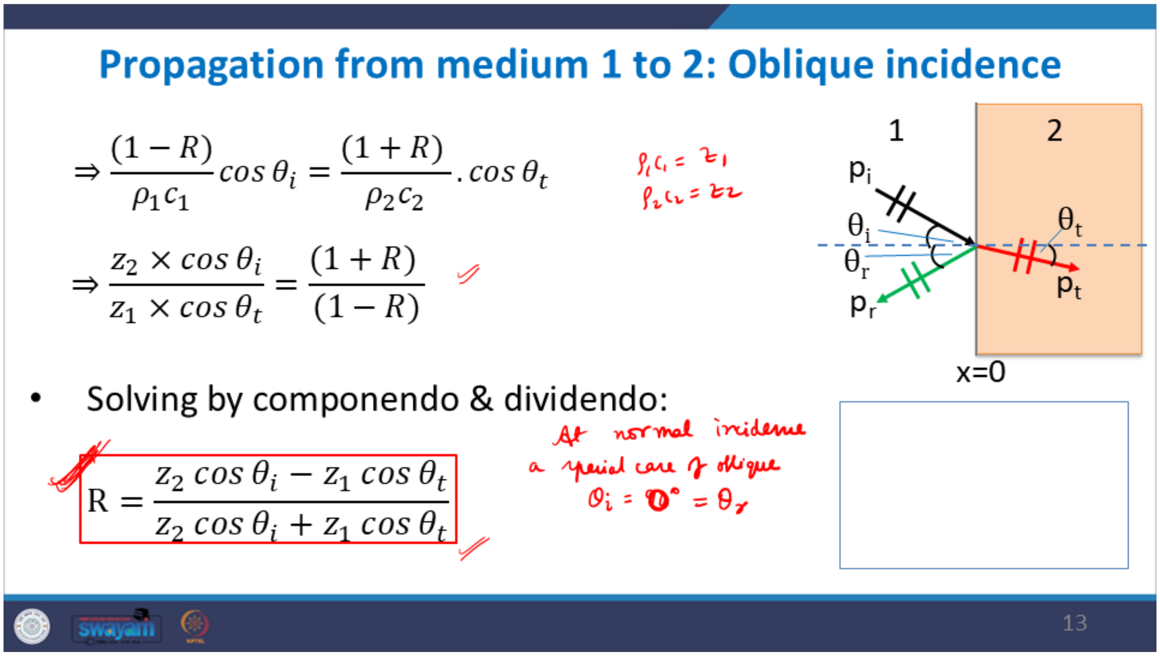
And this is another common term we are removing. So, after removing these common terms, this is the equation we are ending up with, okay: $1-R$ times this quantity here and $1+R$. times this quantity (Refer slide 12). Now, $\rho_1 c_1$ is what?

$$\rho_1 c_1 = z_1$$

$$\rho_2 c_2 = z_2$$

which are the characteristic impedances. So, if you replace this, ultimately, this is what you are getting (refer slide 13). And if you solve by componendo and dividendo, finally, this is the result of your reflection coefficient, which is

$$R = \frac{z_2 \cos\theta_i - z_1 \cos\theta_t}{z_2 \cos\theta_i + z_1 \cos\theta_t}$$



If you take that this is a generic equation, and normal incidence becomes a special case of this generic equation. So, at normal incidence, which is one particular normal incidence, a special case of oblique incidence. So, for this generic case,

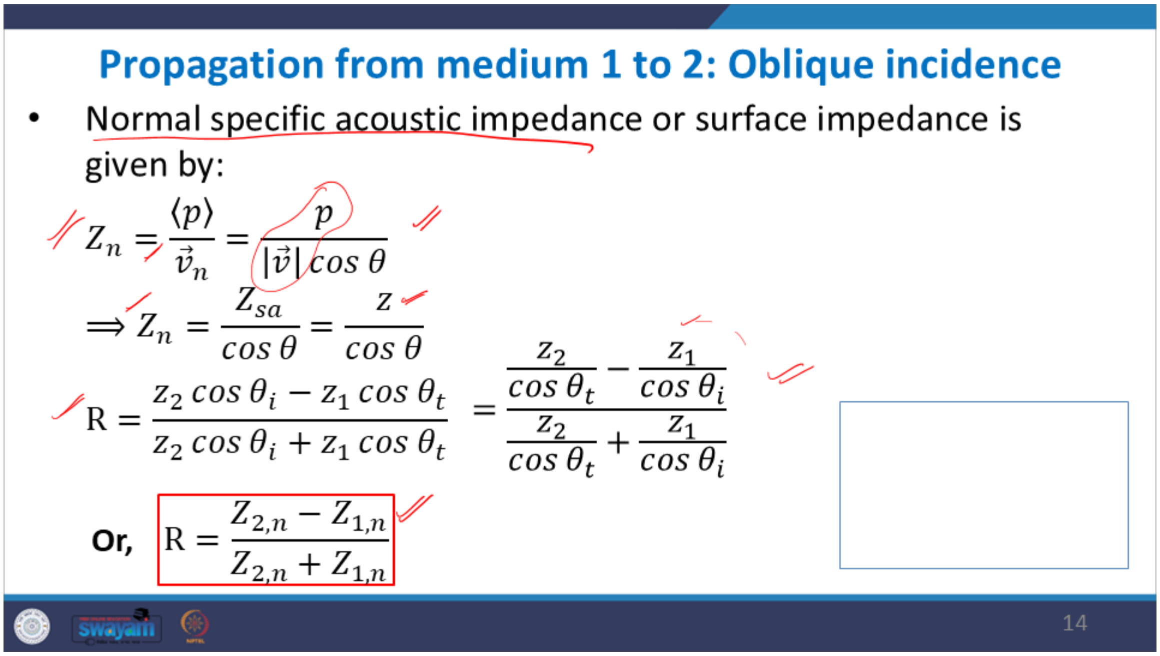
Normal incidence now becomes a special case where θ_i is equal to 0 degrees, and this is the same as θ_r. So, if you equate this and if you solve using Snell's law, Then ultimately you would end up with,

$$R = \frac{z_2 - z_1}{z_2 + z_1}$$

This is for you to solve and see for yourself that normal incidence can be thought of as a special case of oblique incidence.

Fine, so now, because it is oblique incidence, the specific acoustic impedance, it is Z, the normal specific acoustic impedance is not the same as the characteristic impedance. This is the normal specific acoustic impedance. By definition,

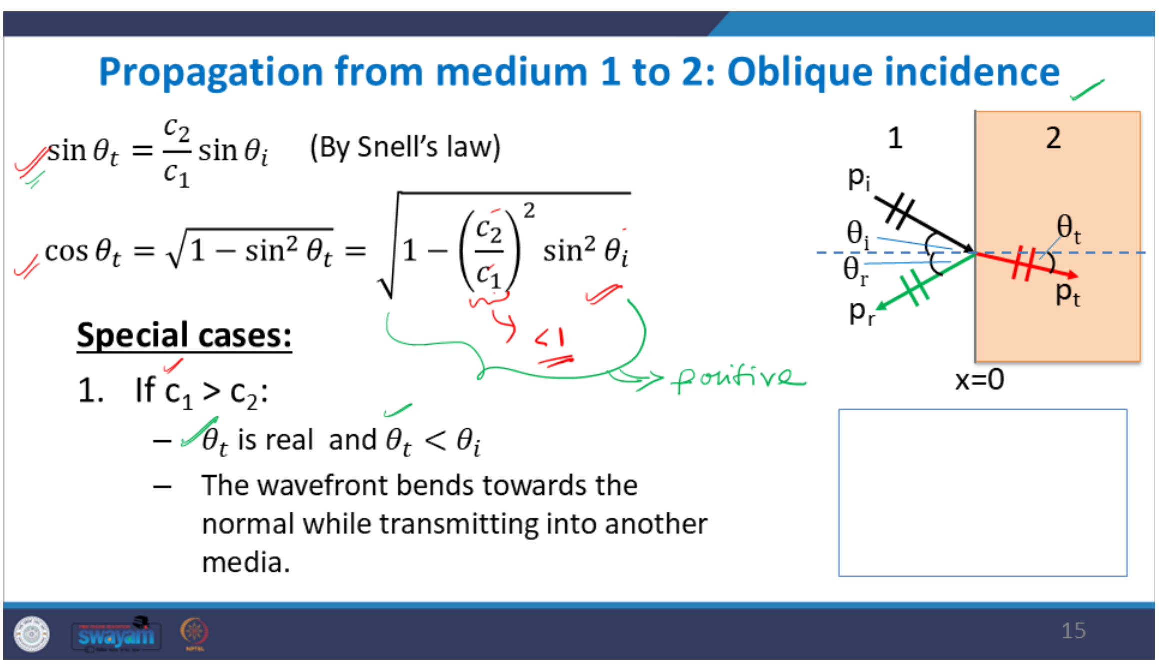
$$Z_n = \frac{p}{|\vec{v}| \cos\theta}$$



So, you can say that this quantity here is your Zsa or small z. So, the Zn becomes,

$$Z_n = \frac{z}{\cos\theta}$$

So, the same equation that we had found if we divide by $\cos\theta_i$, $\cos\theta_t$ in the numerator and denominator, this is the expression we end up with (refer slide 14). This is the normal specific acoustic impedance of the medium 2, and this is the normal specific acoustic impedance of the medium 1. So, ultimately, you know you get the same format. The normal specific acoustic impedance follows the same format as in the case of normal incidence, and this is in terms of the characteristic impedance, this particular format here.



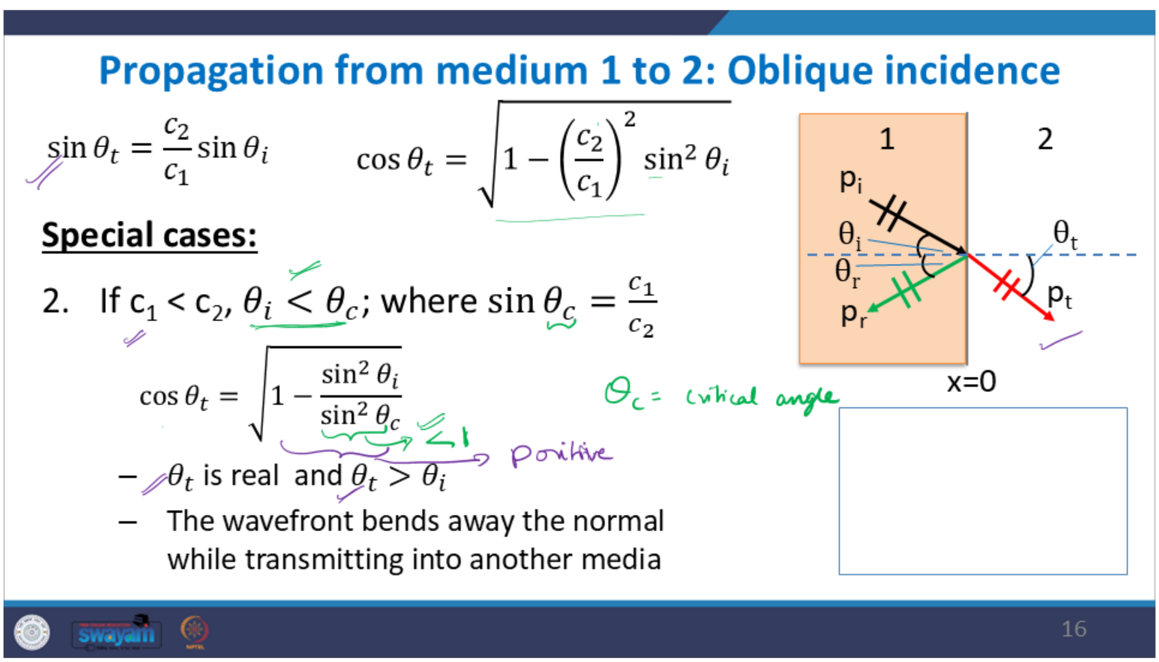
Now, let us do some special cases of oblique incidence. So, by Snell's law, this is what we are getting. So, $\sin\theta_t$ we obtain in terms of $\sin\theta_i$. And

$$\cos\theta_t = \sqrt{1-\sin^2\theta_t}$$

$$= \sqrt{1-\left(\frac{c_2}{c_1}\right)^2 \sin^2\theta_i}$$

So, this is the expression of $\cos\theta_t$ in terms of c1, c2, and $\sin\theta_i$. So, if these quantities are known, let us consider the first case when c1 is greater than c2. So, as you see here, if c1 is greater than c2 (refer slide 15), this term here would always be less than 1.

So, in that case, the variable inside the expression, that is inside this square root, would always be positive. So, a square root of some positive numbers means that in this case, θ_t is real and by this relationship, θ_t is smaller than θ_i. So, what is happening here? We are getting a real transmitted wave, and because θ_t is smaller than θ_i, you can see this particular case here. So, the wave front is bending towards the normal while transmitting into the other media.



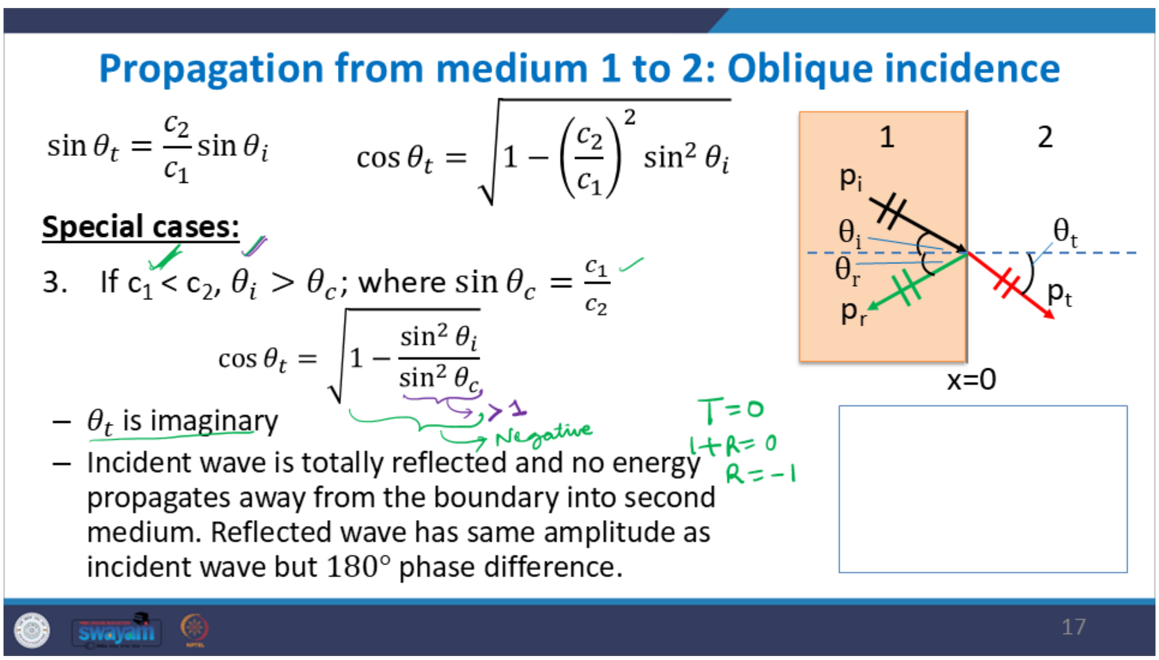
Now, let us see the second case, which is slightly more peculiar. Here, c1 is smaller than c2. and this is our expression (refer slide 16). Let us say we have one critical angle. So, theta c is the critical angle in our situation, and this is defined as

$$\sin \theta_c = \frac{c_1}{c_2}$$

So, if you replace this expression here, in terms of the critical angle, what you get is

$$\cos \theta_t = \sqrt{1 - \frac{\sin^2 \theta_i}{\sin^2 \theta_c}}$$

And in the first case, θ_i, is smaller than θ_c. So, overall, this term here would remain smaller than 1, and hence this entire term inside the square root would be positive, and hence we will get a real transmitted wave. θ_t would be real in nature, and by Snell's law, because c1 is smaller than c2. θ_t becomes greater than θ_i. So, here, what happens in this case? As the wave is incident, it transmits, or it is transmitted into the other medium while bending away from the normal, as represented here.

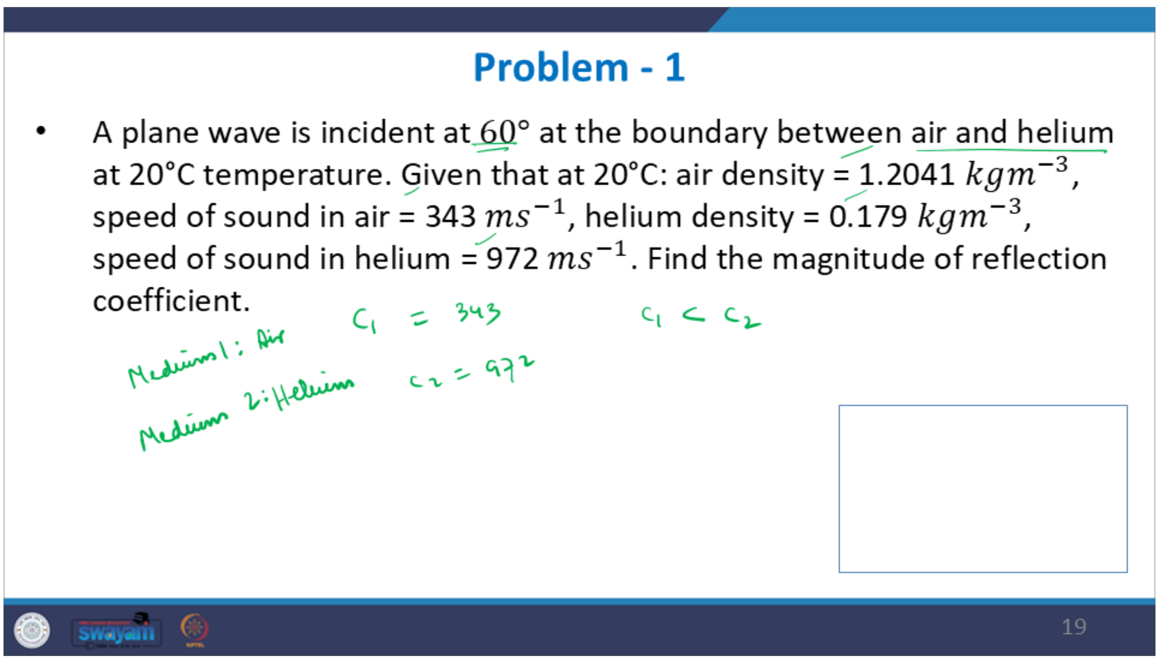


Now, this is the third case, which is the most peculiar case. What happens here is that now we have, once again, c1 smaller than c2, but now θ_i, is becoming greater than θ_c. So, in this expression, now, what happens to this particular thing here? θ_i, is smaller than θ_c. So, this would become greater than 1. So, then the entire thing inside the square root becomes negative.

So, this means thatcos θ_t becomes some kind of imaginary quantity. How do we interpret this? We interpret that the incident wave is now getting totally reflected, and we are getting an imaginary transmitted wave, which means that, in reality, there is no pressure wave that is transmitting into the next medium, and at the same time, what is happening? Because there is no transmitted wave. So, T is becoming 0.

So, 1 + R = 0. So, R is coming out to be minus 1. So, once again, that same case we had discussed when R is minus 1, what does it mean? It means that the wave is having the reflected wave is having the same amplitude as the incident wave, but now it is shifted by pi wherever we are getting a peak in the incident wave, we are getting a trough in the reflected wave and vice versa.

So, they cancel each other out. So, this is what we obtain.

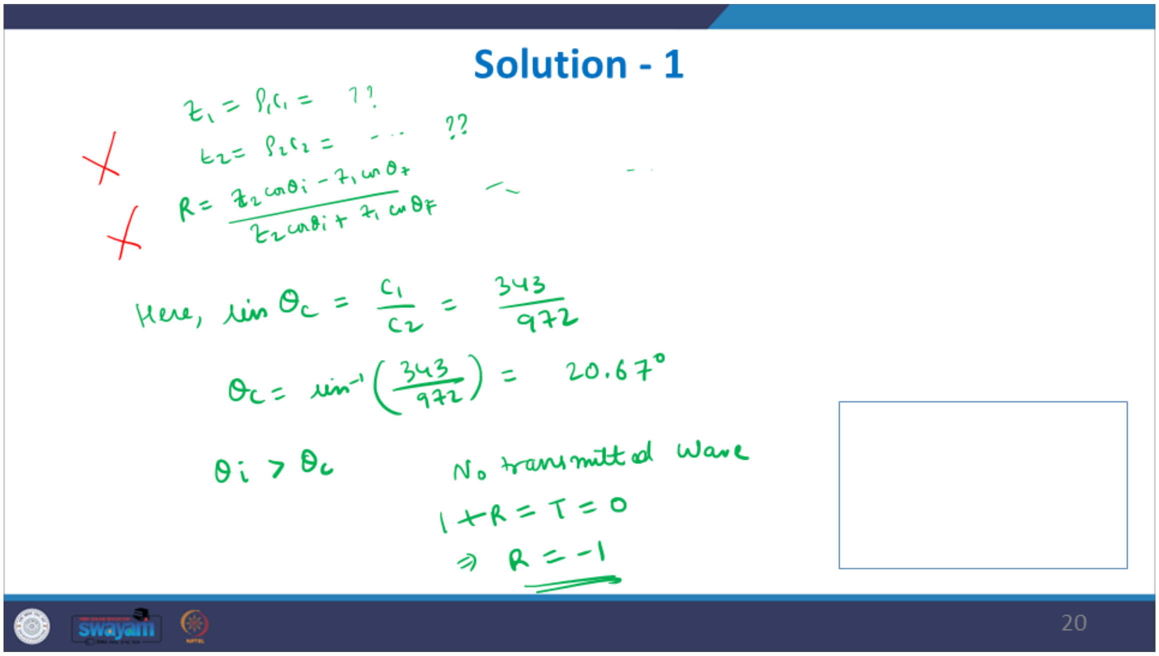


Now, with this understanding, let us solve one problem on oblique incidence before we close this lecture. Let us say a plane wave is incident at 60 degrees on the boundary between the same air and helium, our same old friends. So, here the density and the respective speed of sound for the two mediums are given to us.

Now, we want to find out the magnitude of the reflection coefficient. Now, if it is some student, without thinking, they would just go ahead and start solving because it is an oblique incidence case. They will first of all find out what is Z1, $\rho_1 c_1$, they will find out a certain value. Z2, $\rho_2 c_2$, and then again, they will find out a certain value, then R they would find as

$$R = \frac{z_2 \cos\theta_i - z_1 \cos\theta_t}{z_2 \cos\theta_i + z_1 \cos\theta_t}$$

like that, and all these calculations they will do. But I have already taught the special cases of incidence.



So, before you do all of these manual calculations, you have to see whether it falls under a special case or not. So, here, and so whenever the speed of sound in medium one is larger than the speed of sound in the second medium, you can stay cool, and you can just go ahead and solve the case because it will always be the same way; the wave there will be some reflected wave and transmitted wave, and the transmitted wave will be bending towards the normal, right? But here it is the other way round; here c1 because our medium 1 is air, medium 2 is this So, c1 here is 343, c2 is 972.

So, what is happening here? c1 is coming out to be smaller than c2. So, it is following the special case either 2 or 3. So, in special case 1, there is nothing to worry about, but when it is special case 3, then you have to see. So, you have to first ensure that

You know, it is whenever c1 is smaller than c2, you have to first ensure that the incidence angle is below the critical angle. So, let us find out what the critical angle is for this particular interaction.

So, the critical angle is defined here as

$$\sin\theta_c = \frac{c_1}{c_2}$$

So, let us see this. So, first of all, before doing all of this, we begin like this.

$$\theta_c = sin^{-1}\frac{343}{972}$$

,sine theta c. So, theta c is the sine inverse of this quantity here. So, if you solve it, you get the answer as 20.67 degrees, which is way smaller than θ_i. So, what you are getting is that θ_i is greater than θ_c. Let us see what is happening here. So, if you see, this is the case we are talking about: c1 smaller than c2 and θ_i is greater than θ_c. So, now our angle of incidence has reached that critical angle zone where the transition takes place, and beyond that, there is no reflected wave.

So, you do not need to solve anything; you already found that here this is a special case where c1 is smaller than c2, and we have reached the critical angle and are beyond that. So, no transmitted wave. So, what is happening here? Since there is no transmitted wave, what is the magnitude of the reflection coefficient?

R is coming out to be minus 1. So, in that way, you can solve this particular problem. And with this, I would like to close this lecture. So, thank you for listening.