Micro and Smart Systems

Prof. G. K. Ananthasuresh
Department of Mechanical Engineering
Indian Institute of Science – Bangalore

Lecture - 15
Deformation Strains and Stresses

Hello, as part of the Micro and Smart Systems Course today we will start a new topic and that is the modeling of solids. What makes Micro Systems different from Microelectronic Circuits is the movement of the solids and the flow of the fluids. So we have solids and fluids which move and give the functionality to Micro Systems. Here in this series of lectures we will look at how we can model the moving solids.

The solids can move in two fundamentally different ways and they are one is the rigid body motion by that what we mean is that if I have an object that can only translate in 2 directions, but also can rotate that we call it a rigid body motion, but if you want to define rigid body motion a good definition would be that if you take any 2 points in the solid then as the body moves the distance between those two points will not change.

Let us look at the picture that is shown over here which has a body with two dots shown on it. These two dots let us magnified a little bit. Let us go to the page width that is not enough.

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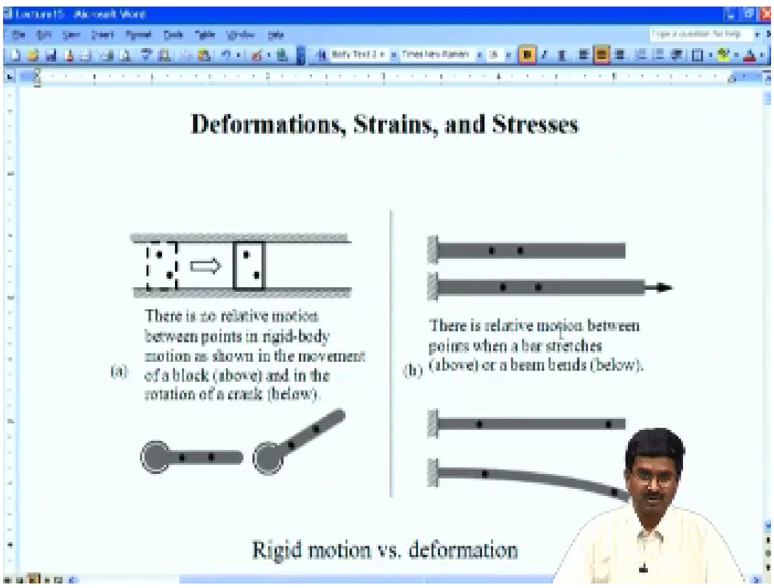

So if you look at this body here which is shown in dash lines and it has been moved to another location where it is shown in solid lines. If I look at these two points before it moved and after it moved the distance between these two points will remain the same if the movement of the body is called a rigid motion. Likewise, if I have a rotating body as shown in the diagram over here.

We have again taken two points on this body and this has rotated by certain angle and we look at the same 2 points in the rotated position. When the distance between these 2 points remains the same under the rotation also we have rigid body motion. There are lot of examples around us which indicate this type of rigid body motion. If you look around I am sure you will find an example of a rigid body motion around you.

For example, if I just take my eye glasses the hinge motion that is there in this eye glasses this pair of eye glasses is a rigid body motion because if I take any two points here the distance remains the same as the body is moved that is one way solids deform and another way is where this distance does not remain the same and that example is shown over here. If you look at this picture, we have a rod and we are applying a force and in the process we have stretched it.

If I want to take 2 points in the original position when it is stretched because the entire body is stretching distance between any 2 points will also increase here. So it is not rigid body movement there is what we call elastic deformation of the body that is taking place in this example. Likewise, if I take the same body I indicate 2 points on it and I apply a force in this direction.

When I do that then this bar is going to actually bend we call such a thing a beam under the transverse force that is the force acting in the direction perpendicular to the axis of the beam then the beam will bend when it bents the distance between these two points would have changed so that is also not rigid body motion. So we note that solids can deform in 2 different ways. One is rigid motion other is deformation.

Rigid motion are the example shown here where as you rotate or translate the distance between any pair of point does not change that is the definition of rigid body and whereas in the deformation things will be different that is any pair of points you take a distance need not be equal and such a thing we call an elastic body. We all familiar with lot of elastic bodies if you take a piece of paper you can fold the paper that is also a deformation.

If the paper comes back to original state, we call it an elastic body. There are lot of elastic bodies around us. So these are the 2 things that we need to keep in mind as we proceed to work towards the modeling of solids.

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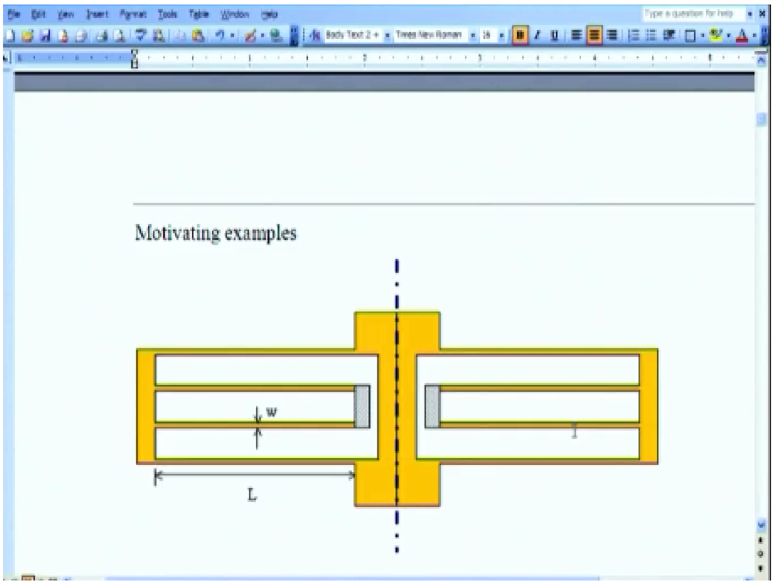

Let us look at some examples of micro system devices in order to get motivated, to study the deformation of solids. The rigid body motion of solids is important and there are a few micro system devices where you need to study the rigid body motion of solids, but many other micro system devices undergo elastic deformation and for that purpose if you want to model them we need to look at elastic deformation of the bodies. So here is an example which is a folded beam suspension.

So we have 2 points here this portion not point this portion and this portion when we anchor them, when we fix them this entire thing can move relative to these 2 points. That movement will happen because the beams that are here. There are 4 beams here and another 4 beams there only because of the deformation of these beams the central I shaped mass or a solid portion can move only when these beams bend.

So there is one example of elastic deformation that gives the motion and if we ask that question if I apply certain force here how much displacement do I get here. Okay coming back to this example when we have a mass with 8 beams connected in this fashion whenever there is a force applied we would like to know how much displacement this body undergoes. So for that we need to study the elastic deformation of these beam like structures.

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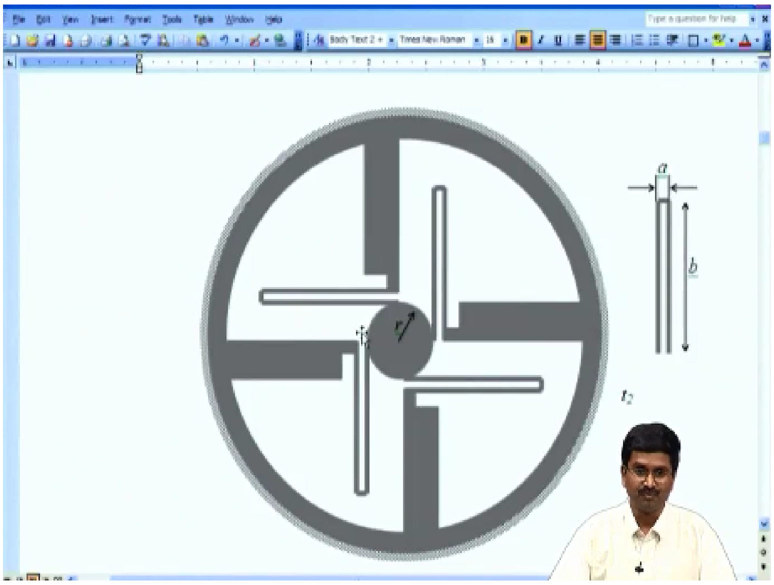

Another example if I take a suspension which is a different kind here we have a central platform, circular in shape here and we have a outer platform and we have beam like structure connected to the central platform to the outside one. In this particular case I can use it to raise the central platform up and down relative to these outer platform so that I can have it like a X, Y or Z stage I can move it in the X direction, Y direction as well as up and down.

So If I again ask that question if I want to apply some force in a vertical direction how much does it move we need the study the deformation of these beam like elements.

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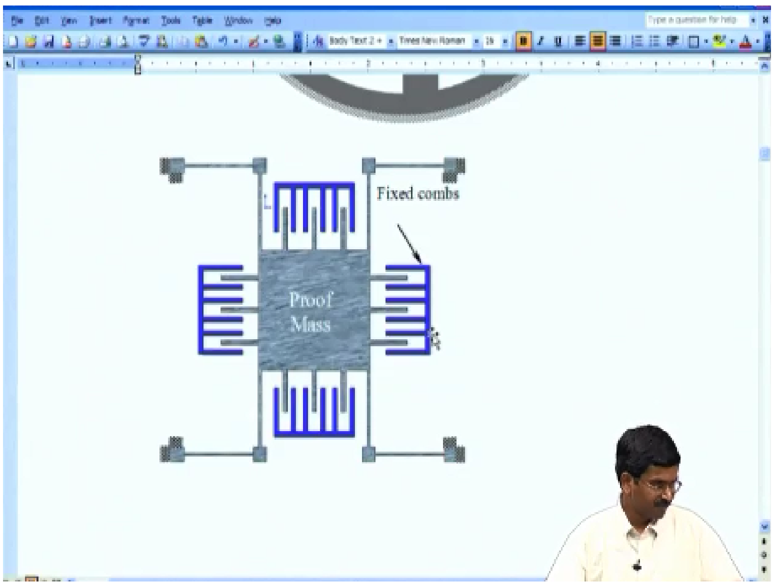

Let us look at one more example where we have a suspension which is a different kind. We call it suspension because the central mass which we call proof mass that we will be able to move in the X direction, Y direction and that is given by the suspension. These beams 1,2, 3, 4, 5,6, 7, 8 that is 4 pairs of beams suspend this mass in such a way that it can freely move in X and Y direction, but will have difficulty to rotate.

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Such a platform I have it here which I will show if you see here in this particular device we have a suspension if I move it in this vertical direction like this you can see it moving and you can also see some other beams observe here for example these beams are deforming. They are not just rigid body movement their motion is not just rigid body motion, but actually deforming as you can see.

Similarly, if I apply force in this direction there are these beams that are deforming. So this has perfectly decoupled motion in the X direction and Y direction. What we have here on the screen is also a suspension similar to that. So we want to know again if I apply certain force in the X direction how much does this proof mass move. Again we have to study the deformation of beams.

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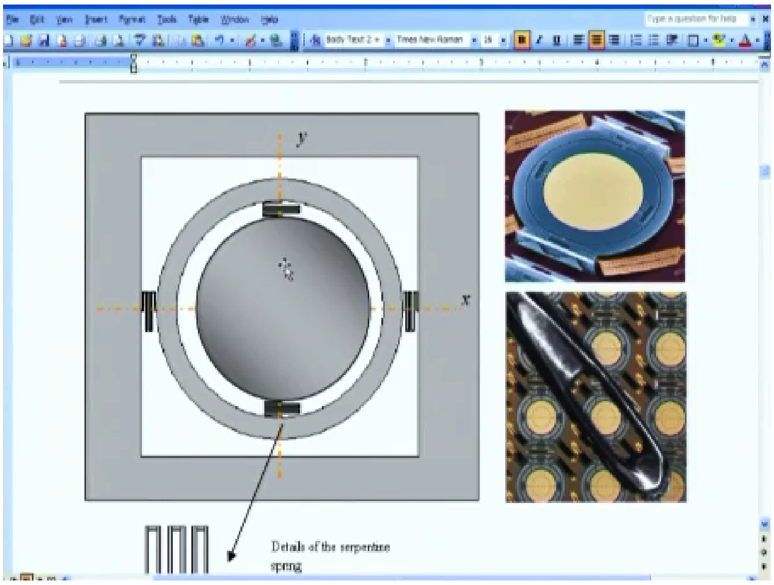

Let us consider one more example before we begin to study this. So here you see an eye of the needle and within that we have lot of these micro mirrors. One of them is shown in a closer view. This particular mirror is schematically shown on the left side and here we have the serpentine beams as shown here each one of those 1, 2, 3, 4. Four of these have this serpentine beams.

And they provide the central mirror and ability to rotate about 2 axis about X axis as well as Y axis. If I call this X axis you can rotate about that we call this Y axis we can rotate about that as well. Again we need to study how these beams deform in order to solve problems of this kind.

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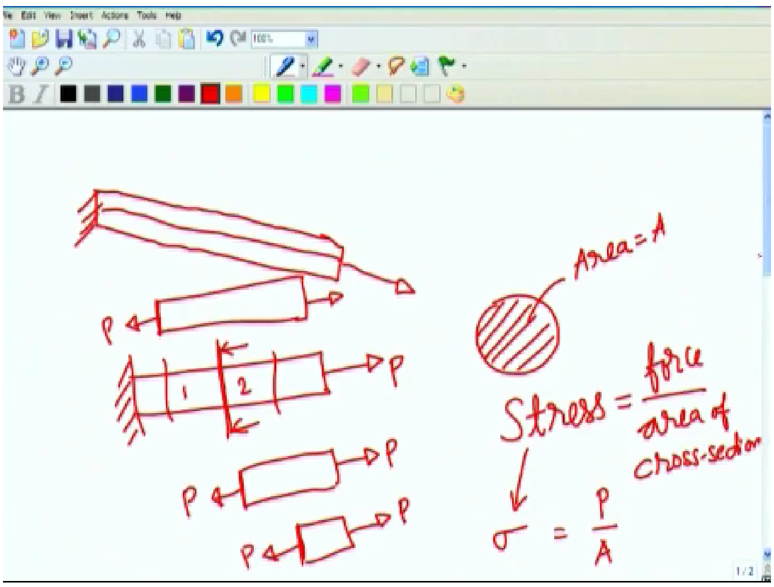

With this motivation let us begin with something very simple. Let us take a bar like that. Let us say that it is fixed at this end and let say this is the axis of the bar and we apply a force in this direction. Let us draw a straight bar. When I apply a force here let us call that force P and if I cut here I will see if I look in this direction I will see certain cross section. Let call it a circular cross section of area A.

So I have taken a bar of circular cross section applied a force P. When I take a section here that is how it is drawn up here. Now what will be the force in this section because this section before it was cut it was holding together the 2 pieces. So the piece here one and 2 if I call it. Let us draw this piece 2 imagine that we have the bar and we have made a cut over here and separated it we have a force P on this side.

We should have a force P on this side also only then the bar that is cut is going to be in equilibrium. This is sum of forces in the X direction has to be 0. So if I cut let us say over here and take a small piece there. Again we will have the force P and force P here. So the force at this section will also be P at this section it will be P or if I cut there we have to take a longer length of the bar. Again for that we have this P and we have this P/

And on this also it will be again force is P. So anywhere I take along this part if I cut it my internal force is going to be P. We now define a concept of stress because a bar that is under some loading is going to experience some stress. That stress is defined as force per area of cross-section. So if I indicate with what we have the symbols the force we applied is P and the area of cross section is A and the symbol that we use for stress is sigma.

So sigma is a stress which is force per unit area. If I take force divided by area of cross section, I get the stress. So stress is something a quantity that we define to understand the effect of a force on an elastic body. You cannot see stress. Stress is a concept we have to imagine, understand that whenever we take an elastic body applies forces on it, it going to experience some stress.

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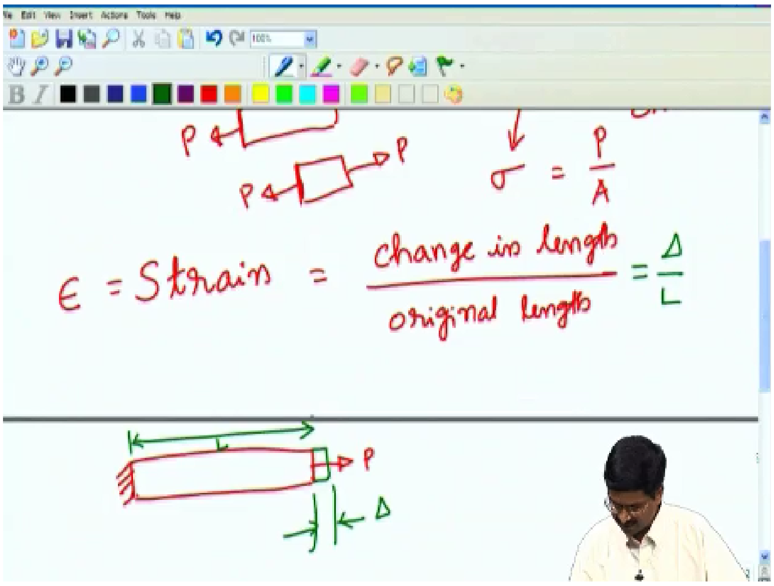

But what we can see is what we can call strain. Strain is defined as the change in length/original length and we use this symbol epsilon to indicate strain. So epsilon is a strain. It is defined as change in length/original length. If we go back to our bar let us draw it again we are fixing the bar at one end and applying a force P because of this force the bar is going to expand a little bit.

Let us say the bar has moved here. So this quantity let us call this delta. This expansion of the bar when you pull on the bar it is going to change its length it is going to increase that length by an amount delta. So if we go back to our strain definition I would put that as delta/original length of the bar. Let us denote b L so that is L here. So the strain is going to be delta/L.

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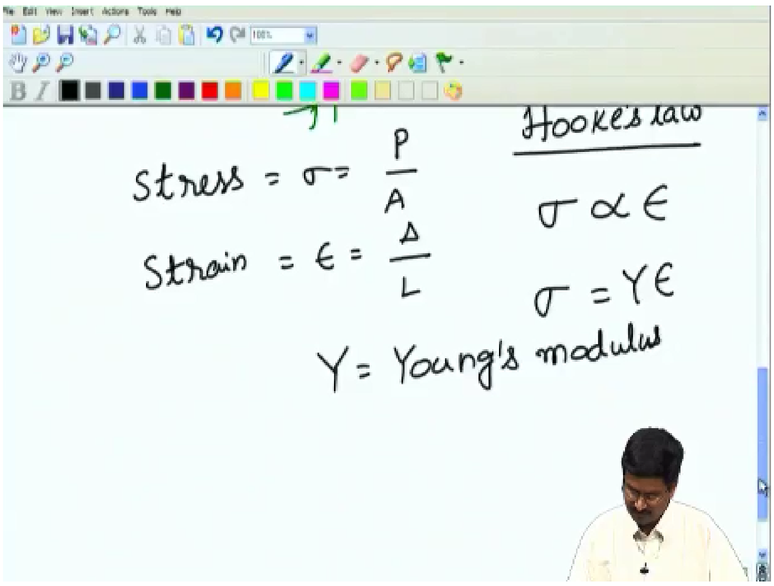

Now we have defined 2 quantities. One is stress which we denote by sigma which is P/A force by area of cross section and then we have defined another quantity strain which is delta/L. These two are related because of the nature of the material and that goes under the name of Hooke's Law. Robert Hooke who propose this law. He found out that the stress and strain are proportional to each other.

So stress here is proportional to strain and the constant of proportionality is called Young's Modules. So Y that we have used here is called Young's Modules which is the property of a material.

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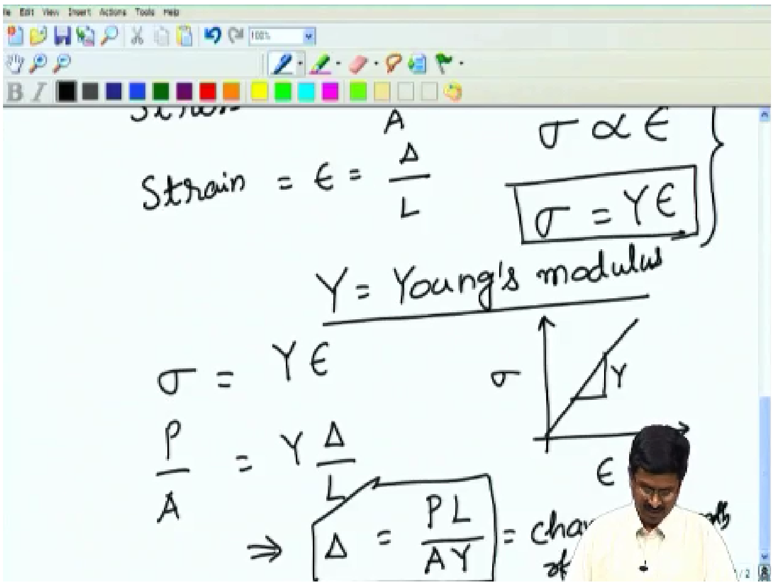

So if I want to draw this stress versus strain the beginning of this curve is going to be a straight line and the slope of this is Young's Modules Y. Now let us relate using this Hooke's Law which is this by substituting for stress and strain in this bar. So if I do that sigma let us rewrite it sigma Y times epsilon sigma is P/A and Y epsilon is delta/L. So what we find here is that delta can be written as P times L over AY.

So the elongation or contraction depending on the sign of P the change in length of the bar that we have taken is given by this formula, but note that this is not a formula that we need to memorize. It can be derived if we go back by considering equilibrium of the segment that we cut at every point so that we will know internal force acting at each point one should use internal force you use the definition of stress force/area of cross section.

And the definition of strain which is change in the original length by using these and Hooke's Law which tell you how the material is going to vary its stress and strain based on the property that we call Young's Modules we can derive this formula rather easily. As we will see as we consider more complicated elastic bodies the bar by the way is a simplest elastic body that one can imagine. We can only actually stretch and contract.

If we take more complicated elastic bodies our principle is still going to be the same. We use static equilibrium so that all forces are balanced and later on we will also have to balance what are called moments. After that we use our definition of stress and strain and then relate stress and strain based on the property of the material which is Young's Modules and we can compute the elastic deflection as shown here.

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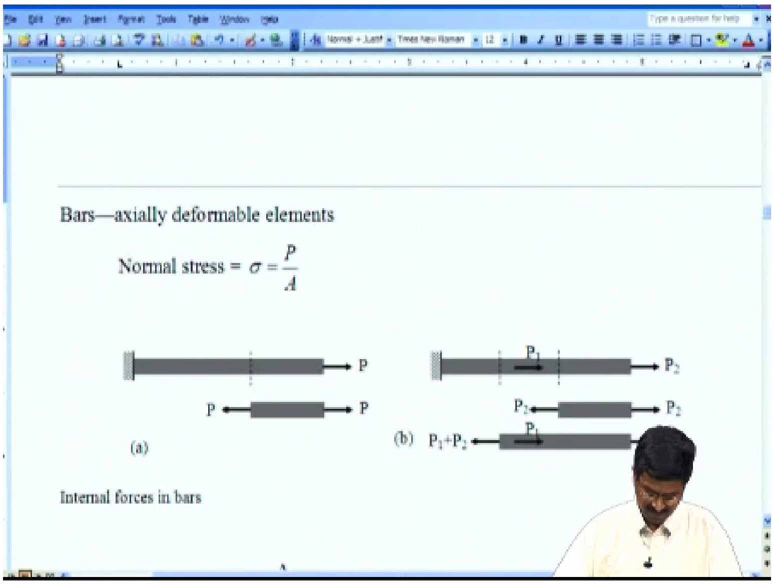

Let us look at a case where we have the same bar where again we have the normal stress by the way this is called normal stress because this stress is going to act normal to the surface of the body. So we have the body where we have cut and it is going to act normal to that that is why it is called a normal stress. If there were to be 2 forces as shown there is a P1 here and there is a P2 here as shown in the bar over here.

When you have such a situation the internal force is going to change.

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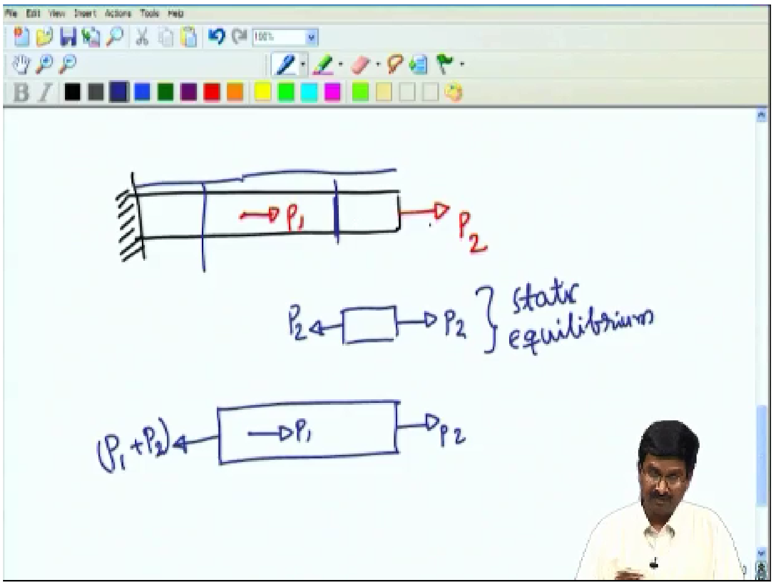

Let us go back to the pad and then see what is going to happen? Now I am going to take a bar again let us consider the fact that it is fixed at one end. Now I will say that there is one force here let call this P2 and then somewhere else at this point have a force P1. Now let us take sections of this bar if I want to cut over here and separate or do segment the internal force that is going to act at this section is going to be just = P2.

Because only then this bar will be in static equilibrium whereas if I cut somewhere here if I draw that piece of the bar so I have P2 here, but somewhere here I also have P1. So this force that is going to balance the forces P1 and P2 has to be P1+P2. So in this case it is going to be from here to here it is going to be P1 and then it is going to be P1+P2 from there to there. So the stress need not be same in elastic bar when there are different forces or if it has different areas of cross section.

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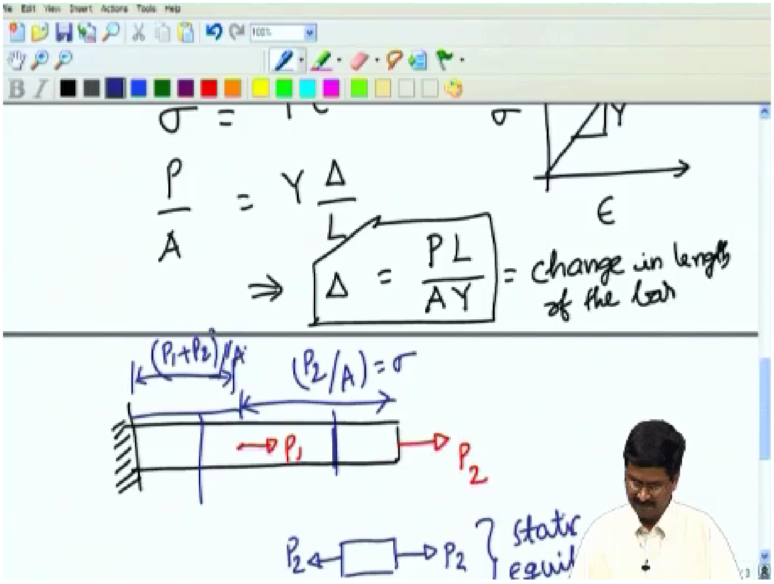

So here if I go back for this portion the stress is going to be P2/area of cross section this is the stress whereas for this portion the stress is going to be P1+P2/area of cross section. So because of this we have to keep in mind that whenever there is an elastic body when there are different forces acting at different points our stress is going to be different because the internal force might be different.

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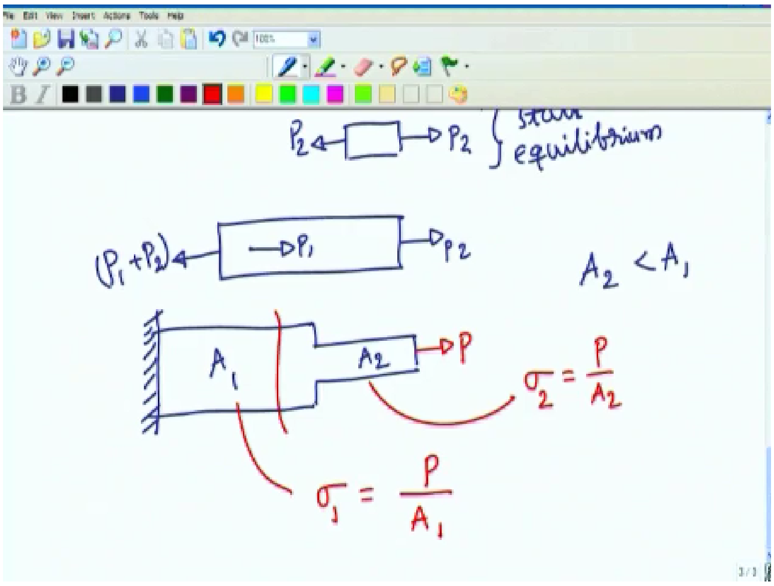

We can also have a situation where I can have a bar with change in area of cross section. So we have here a bar which it is fix at this end area of cross section here is A1, area of cross section here is A2 these are smaller cross section A2 is <A1 if we have that. In this portion anywhere let say we have a force P acting on it. In this portion the stress sigma normal stress sigma is going to be P/A2.

Whereas in this portion the stress is going to be the same force P because if I take a section here it will be the same force acting on that, but area of cross section is A1. If we call this sigma 2 we will call this sigma 1. So when there is change in area of cross section again the stress will be different at different place in the bar. There can be an additional force can vary, internal force may vary or area of cross section may vary in order to change the stress. So these about the bars.

And the bars happen to be in micro structure in many, many different places.

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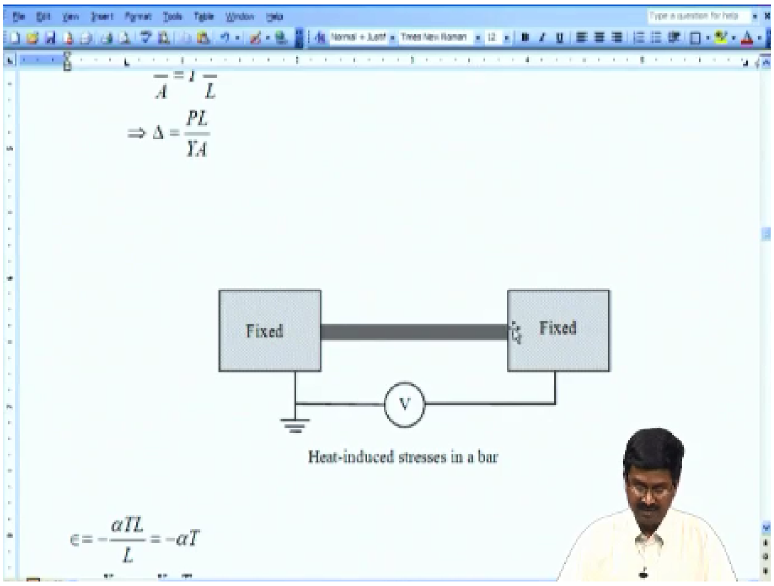

Let us take for example a resistor which is shown in a figure here. If I take a resistor like this. Resistors are very common. There are conductors, resistors all over the integrated circuit chip as well as micro system chips when you have something which is fixed at both ends. So far we can start bar which were free at one end where we can apply a force and other end it was fixed whereas here we are fixed at both ends.

And here we actually applied a voltage potential so that current can pass through that. When current passes it is going to get heated because the heating most of the materials will expand. When they expand they want to change their length, they want to increase their length, but it is fixed at both ends so it cannot. In that case the material inside will develop a stress and that stress in this case will have to be compensated by the stress created by this bar.

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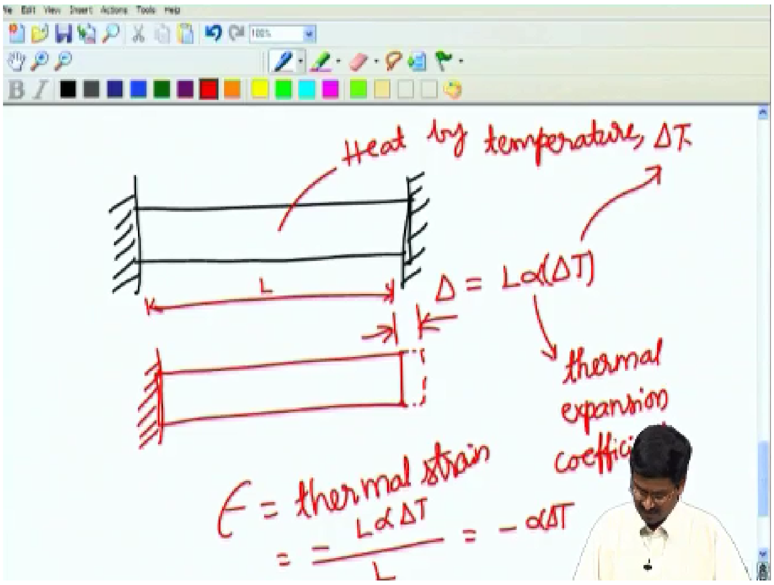

If I take a normal bar if I take let us say the same resistor which is a bar which is fixed now at both the ends. Now I heat it by a temperature heat/temperature delta T. When I do that this bar wants to expand how much will it expand if it were to be free. Let us say I take this to be fixed at one end. It would have expanded by an amount which we can call delta that will be given by the length of this bar L times alpha times delta T. Delta T is the raising temperature that is what we have indicated over here.

This alpha is a material property called thermal expansion coefficient. So if you have a bar of length L if we heat it/temperature T if it is going to be fixed at one end it would have expanded by this much. The thermal expansion coefficient is the elongation of a material per unit length per unit temperature rise. You want the total elongation we will multiply by the length and the change in temperature.

But now since it is fixed over here it cannot expand. So it will have to develop a strain to compensate this. That strain which we will call thermal strain. In this case has to be negative L alpha delta T that is a change in length/the original length which is L which will be –alpha delta T. Alpha is a thermal expansion coefficient multiplied by the change in temperature this is the thermal strain.

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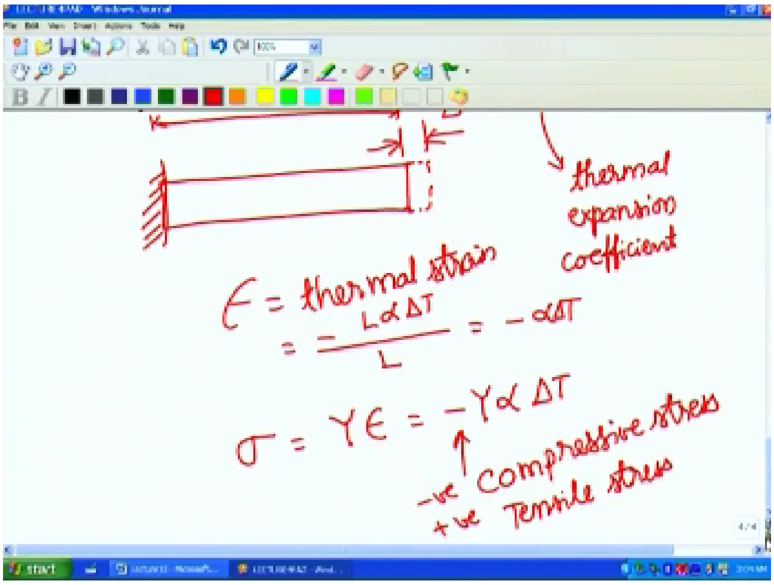

Thermal stress will be what we have written earlier Young's Modulus times the strain which in this case is going to be –Y alpha delta T that will be the thermal stress. Whenever the stress has a negative sign we call it compressive stress. If it is positive, we call it this is negative if it were to be positive we call it tensile stress. So we have compressive stress here and tensile stress. So a bar can also have loading not just thermal loading, but also we can have the mechanical load.

It can come up active materials can have different stress and strain. Whatever it is we can relate it if we know that it is linearly proportional we can relate it with Young's Modules, but sometimes this relation maybe even non linear. The stress strain relationship had been linear and non linear then also we can deal with it that is what we can say about bars. Now let us look at the next case where instead of having a bar just one bar.

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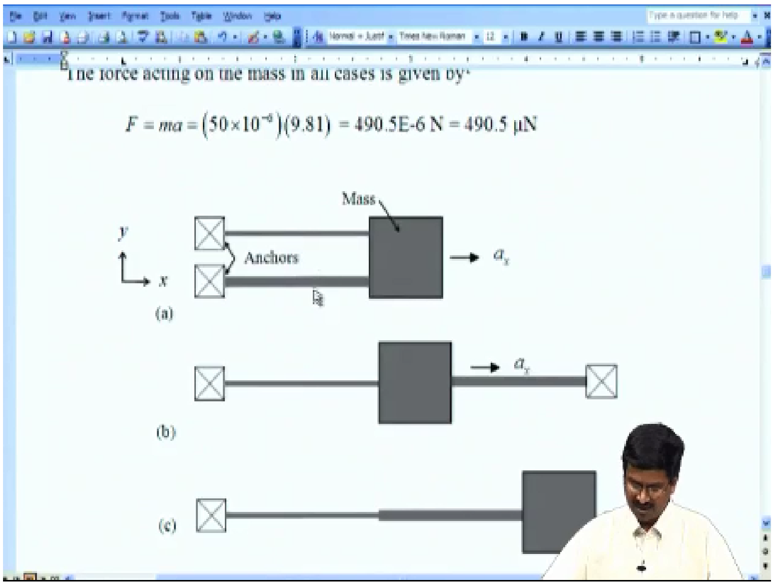

Let us take a mass as shown here connected with 2 bars. So we can get stiffness of this bar and this bar as shown here if I say. The first one we have a stiffness indicated as K which is given by area of cross section Y times L. How did we get this?

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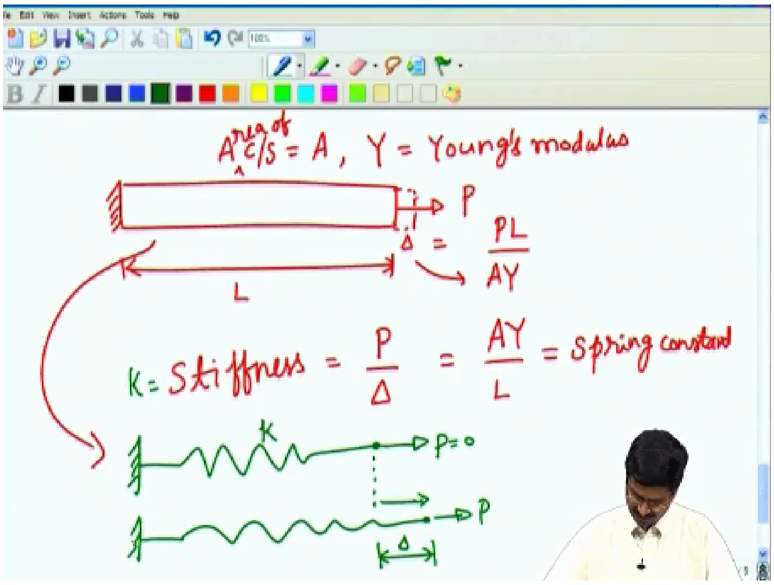

Let us define what stiffness of the bar is. So if I have a bar and I fix here what do you mean by stiffness if I apply a force P let us say the length here is L and the area of cross section is A this is area of cross section A. If it is made of a material whose Young's Modules is Y. We just derived above that the expansion of this bar which we can call delta that was given by P times L/AY that is what we derive a bar.

Now what do we mean by stiffness? Stiffness is the force which in this case is P/elongation delta. What is P/delta? In this case it will be A times Y/L. So P I bring delta this side and AY goes there, L goes here. We get P/delta =AY/L which is also called the spring constant of the bar. If we were to represent this bar as a spring. So if I take this bar I can say that this bar is like a spring and I have a force P then it would deform by an amount delta with this force P.

The P initially = 0 and P goes to some value from what the bar was here we will move to this point. So here we denote this as K. The stiffness of the spring that same thing as stiffness of the bar. So entire bar we can represent as one single spring and the stiffness as spring is given by AY/L. So when we go back to the example that we consider here if I have a mass where there is certain acceleration A of F acting on it. It will have a force which is mass times acceleration in this particular case the mass.

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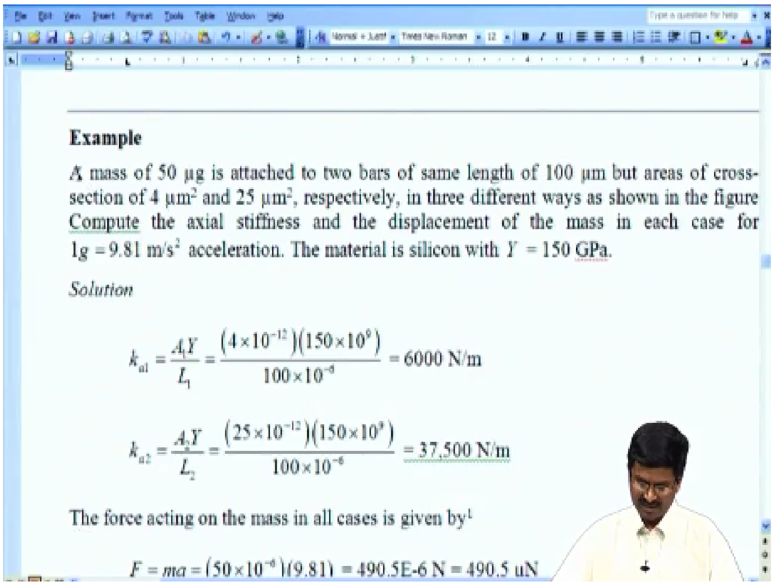

Here is the example we want to take a mass of 50 microgram acting on a body the body here is some square mass it has 50 micrograms and that is 15, 10 power -6 kilogram and it is subtended with 2 bars of lengths 100 microns, but different areas of cross section. One is 4 micron square other is 25-micron square. The smaller one is 4 micron square larger one is 25-micron square. If I have this, then I can represent this particular thing as a mass suspended with 2 springs.

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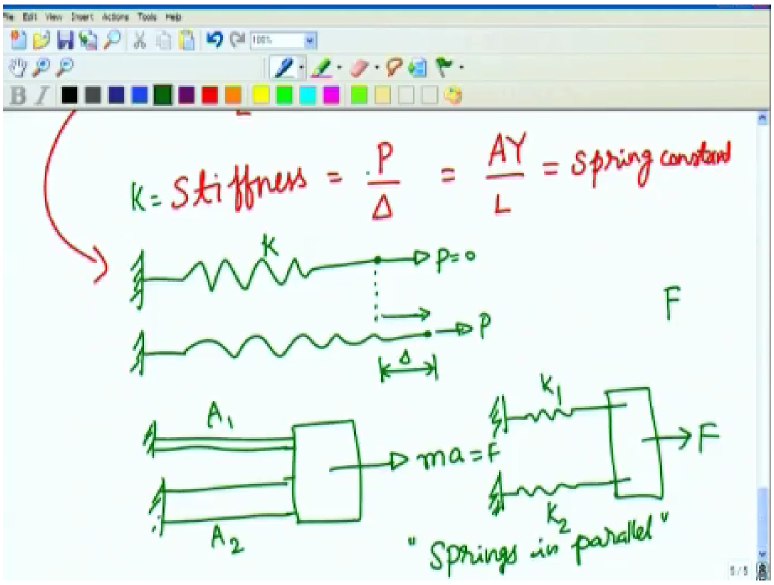

So just as we have done for the previous one now we will have a small bar and a large cross section bar and each of these are connected to a big mass. So this is fixed here this is fixed here area of cross section A1 area of cross section A2 we have the mass on which there is a force the M times acceleration. So this can be represented as 2 springs and there is a mass here. So if I fix the spring I can have spring constant K1 for that K2 for this.

These 2 springs area said to be in parallel so these springs are in parallel. Springs in parallel that means that they share the same displacement that is they have the same displacement whatever the spring elongate the spring has to elongate by the same amount and they will share the force applied on this. This MA is the force here that force is shared by these 2. That F that we have.

So we saw earlier that the spring constant times the delta is going to be = force.

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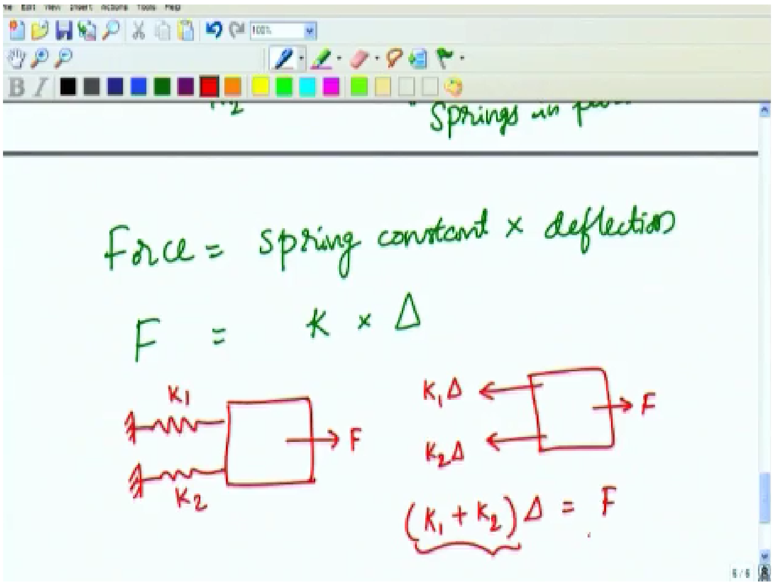

So let us write that so we have the force is = spring constant times deflection of the spring. This is the force F spring constant K times the deflection delta. So now when we have 2 springs like this there will be force acting this way which is K1 times delta if we denote the movement of this mass as delta there will be K1 delta and K2 delta which we will do it here now again.

Let us draw this mass and show this spring of constant K1 another spring of constant K2 which are both fixed here and there is a force F acting here if I want to break away these springs then I can put the mass and there is a force acting on it and there will be a force which is K1 times delta another one K2 times delta. So that means that K1+K2 times delta = force or the new spring constant which is F over delta.

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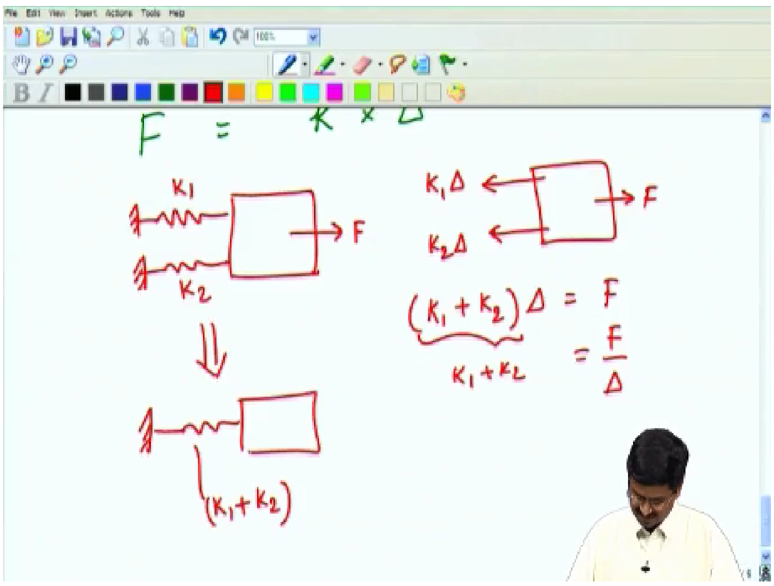

The new spring delta which is F over delta =K1+K2. In other words, this thing I can show it as a single spring which has a stiffness K1+K2. So springs in parallel we will have their stiffness added in order to get the equivalent spring constant.

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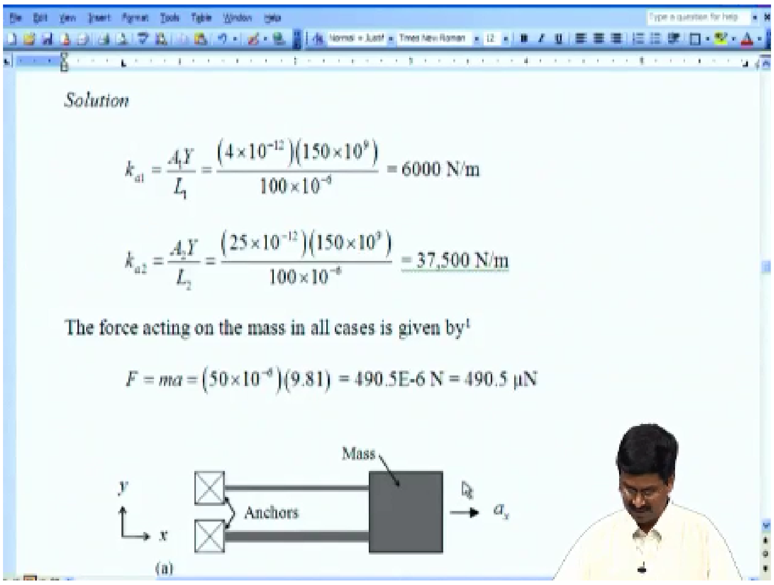

Now if we go back to this example. We have 2 springs of spring constant K A1, K A2 because of the area of cross section different for these 2 parts. One turns out to be because area of cross section 4 micron square times length 150-micron square which is specified in the problem and Young's Modules 150 gigapascals that is Y is given as 150 10 power 9 pascal gigapascal is 10 power 9 that is Y

The length is 100 microns then you get 6000 Newton per meter stiffness and another one 25 microns square cross section 150 gigapascal for Young's Modules/100 micron length I will get 37500 Newton per meter stiffness. So when we know the total force acting on it we can divide this force by K1+K2 and get the deflection for this mass.

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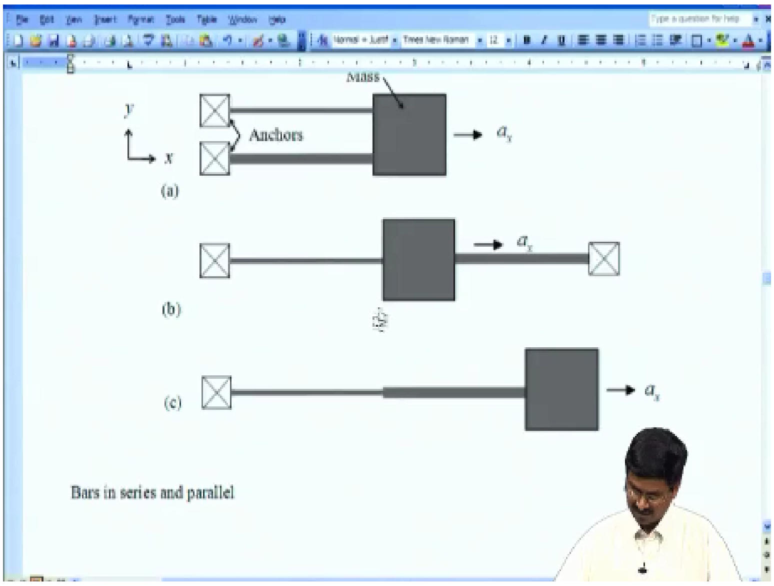

If we take a different scenario we put a narrow bar over here and a wide bar over there even then the 2 springs have the same force acting on the because the same force will act through this bar, same force will act through this bar again these are in parallel. Whereas if I were to attach the springs into end I have shown here the narrow spring and wide spring. I am calling it a spring even if it is a bar.

Because the bar can be represented like a linear spring and attach the force here then the force that we see in this bar as well as in this bar the force is going to be the same whereas the deflection is not. In these cases, the deflection is the same here and here because in this case both are elongation both are contraction. Here one will be let say if the force is applied in this fashion.

We will have the compression of this bar elongation this bar both magnet will be equal then we call them springs in parallel whereas if they have the same force then we call them in series in which case displacement will be different. This portion will deflect something this will deflect something the mass will be total of these 2 deflection in that case we call them spring in series.

In that case what we will have is a situation that is slightly different from springs in parallel.

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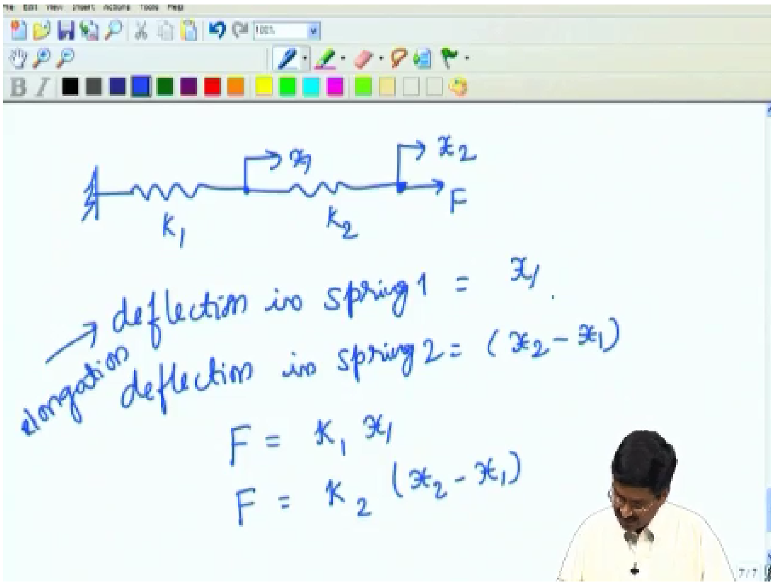

So if I have 2 springs as shown in that picture. Let us call this K1 and another spring this is K2 and if I have a force applied over here this will deflect let us say an amount by displacement X2 this is X1 the deflection in spring 1 this is the deflection or elongation in spring 1 =X because that is what this point is going to move and the deflection in spring 2 is going to be X2-X1. Okay.

This elongation when we say deflection we actually mean it is elongation how much the spring is going to move or increase by X1, X2-X1 they are not the same. So a spring in series, but the force is the same. So we will have the force F = K1 times X1 and it is also = K2 times X2-X1. The force is the same, but deflections are different.

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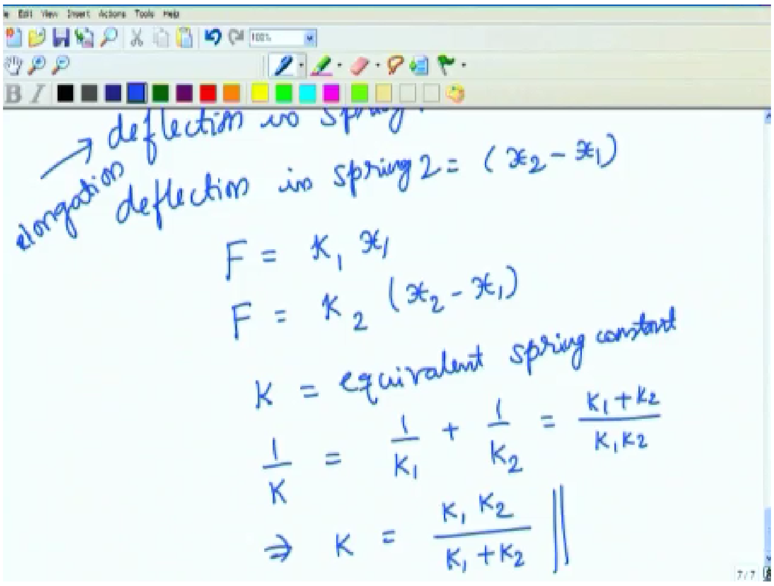

In this particular case what we get is that the equivalent spring constant if we solve these 2 things we get that the K equivalent spring constant will turn out to be harmonic mean of the 2 springs. One over K1 one over K2. In other words, the K = K1, K2/K1+K2 one over K here we will have K1+K2/K1, K2. If I take reciprocal I will have this. This is what you get for springs in series. So this is something we need to remember when they are elastic body if we can break it into little pieces I have shown here we can get equivalent springs and combine them together to get the total spring constant.

Now let us move on to the next level of detail which is bending of beams. In the case of bending of beams.

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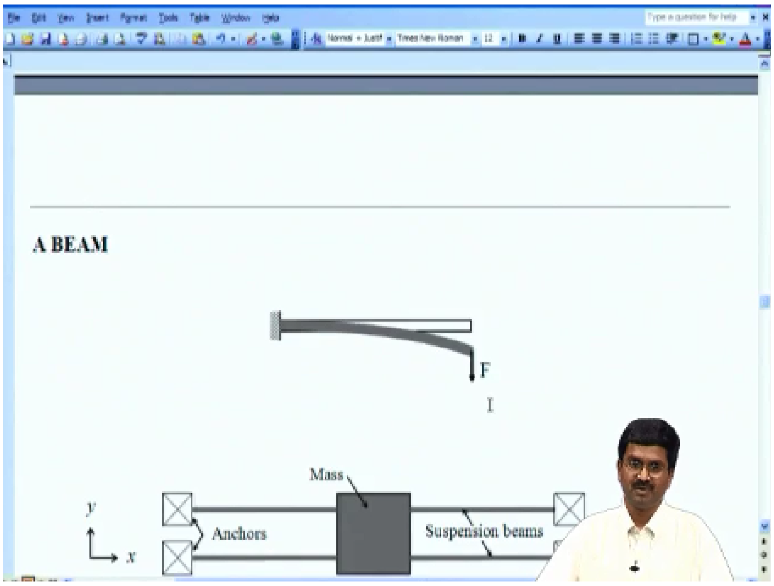

Now let us move on to beam where a beam is going to bend as opposed to just stretching or contracting like the bar did. In the case of beams, we need to define a few more quantities and what kind of stresses will act when a beam bends.

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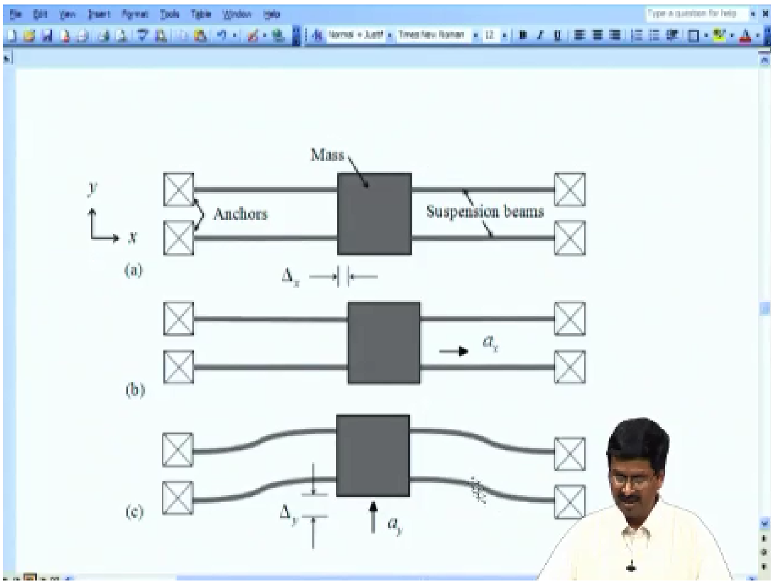

So we have a different kind of suspension we have a mass here and a beams. When there is acceleration in the X direction these beams will only experience axial stresses like bars we just did, but when there is a force or acceleration in the vertical direction then these will deform as shown here. In order to look at those things, we need to again take the beam and show the forces. Now the forces are going to be perpendicular to the beam. So if I take a beam.

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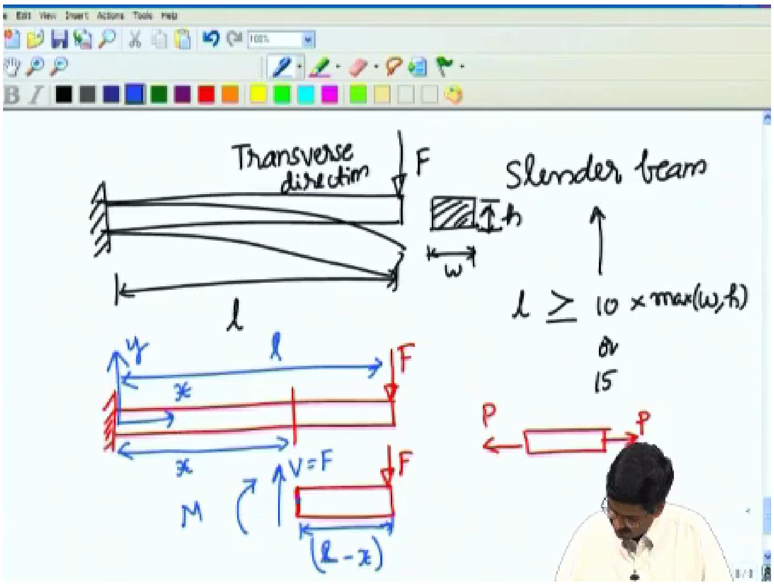

Let say that I show a beam what should be what we call a slender beam. In the case of bars as well as beams we want it to be very long compared to its area of cross section or the dimensions that define the area of cross section. In the case of beam if you want to apply the theory that we are going to develop we want the length of the beam L and let say we take a certain cross section.

Let say this has a cross section that is rectangular. If there is width W and the height H we want the length to be more than at least 10 times or 15 times 10 or 15 times the maximum of W and H. If this is true we call it a slender beam. In such a slender beam if we have a force F acting in the vertical direction which we call transverse direction what is going to happen? this beam is going to bend.

So in order to analyze that bending what we will do is we will take a piece of the beam. Let us draw that slender beam one more time with a force acting. I take a section here like we did for the bar, but in the case of bars the force was axial in this direction. So we are able to just say that is a force. If recall if I take axial force if there were to be force in this direction whether that is the force, we had P but now we had vertical force which were denoted by F transverse force.

In this case if I were to take this segment of the beam. There is a force F that is applied what will happen here and that will have a force that is in the vertical direction which we denote as shown in the picture here which is a vertical shear force because effect of this is to shear this material. So this is our vertical shear force which happens to be =F in magnitude because this is up this is down this is an equilibrium as far as the force is in the Y section that is Y direction here.

So now we need to denote all the things this is our Y direction this is our X direction. The force in the Y direction this has to be F to compensate for that one, but if I take moment about let say this point this vertical shear force does not contribute any moment whereas this will contribute. Let us take this length to be okay let us denote where we have cut this distance as X.

So this is going to be L–X. So this is the distance which is L–X because the total is from here to here is X the total length we take it as L here so there is L–X. If I take moment of this force this 4 times L–X will act like this.

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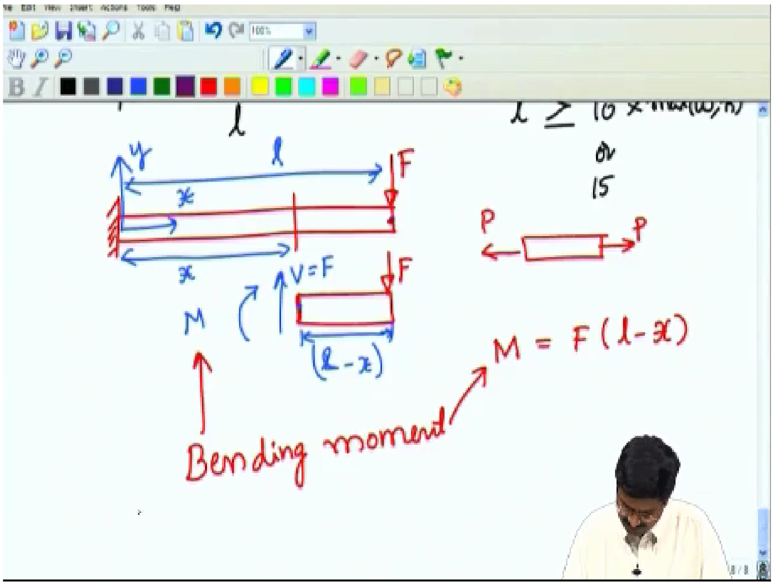

And this we call the bending moment. It is this moment that tries to bend the beam this moment. So that in this particular case will be = this F times L-X that is a bending moment. As we move from this end to that end when we go here L-X. X =L at this point. Let us say this point X = L. So there will be L–X0 no bending moment. As we move towards this bending moment is going to increase as X becomes smaller and smaller as you go from here to here the bending moment is going to increase.

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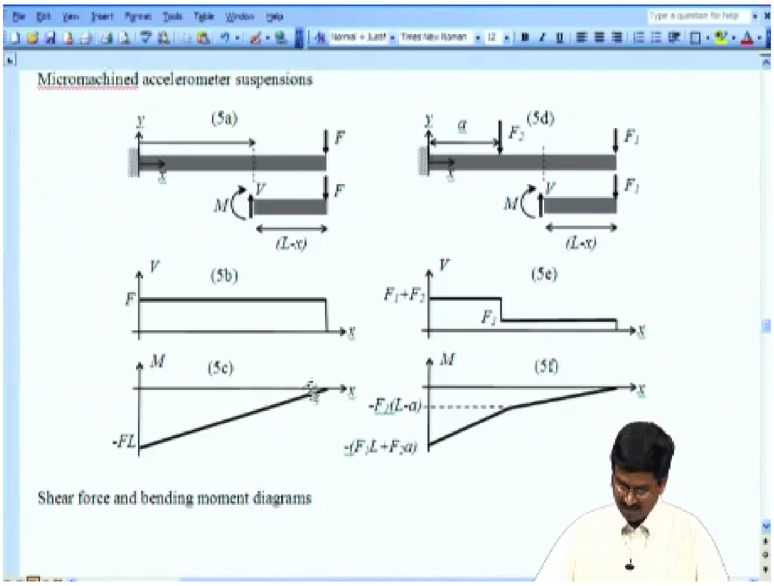

In fact, we can draw this bending moment diagram for this and also shear force diagram. So which is shown in this picture over here the left side picture as I take sections at different points I will see that this vertical shear force V remains constant that turns out to be just V all through from X = 0 to L whereas bending moment it will be 0 here this L–X. This is 0 there and then as we go it is going to be –F or L.

Because our sign here whenever there is moment this way if we go back and look at the sign of the moment here this is going F times L in the clockwise direction. This has to be counter clockwise direction we see it at that bending moment is negative and that is indicated over here that is our sign convention. Bending moment is negative for something like this then apply a force it is going to bend like this in a convex fashion and that is the negative bending moment affect.

Like we had in the bar if there were to be 2 forces then there will be a change in the vertical shear force just as in a bar there is change in the axial force or internal force. If I have F1 here and F2 up to this point there will be just F1 vertical shear force V is going to going to be just F1 and after that is going to be F1+F2 if I cut anywhere here it has to compensate this force and this force.

And I will get this vertical shear force in this fashion and likewise the bending moment 2 when I cut from here to here anywhere the bending moment Is going to be the F1 times L-X/ So that will have certain slope here it will reach a value –F1 times L-A if I indicate this distance as A. I am coming from X=L back to X=A decreasing X value because I am moving towards this X=0 at this end and after that I am going to have a bending moment which is F1 times L-X and then F2 times A-X.

So together from this point onwards the moment due to this force and this force which this bending moment has to compensate it goes like this. And it reaches a most negative value –F1 times L+F2 times A that will be bending moment. So given any beam we can draw the shear force diagram and the bending moment diagram and try to get the first the affect of shear and then affect of bending on the bar as X varies from 0 to L how they change.

And we have to consider what is the affect of bending moment as we just said affect of bending moment is to bend the beam and shear force is to shear the beam. We will consider shear affects later, but let us look at what is the affect of the bending moment.

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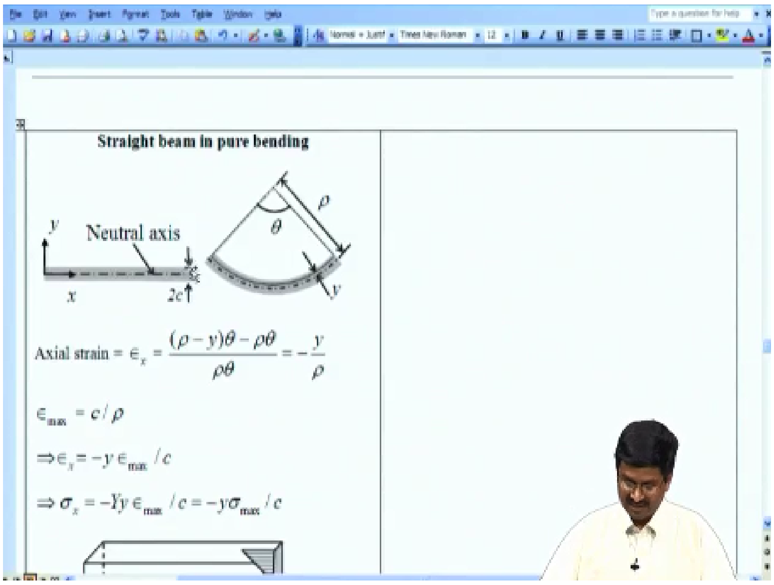

For this let us consider a small straight segment which is under what is called pure bending that is there is a bar and we are applying moment that tries to bend this straight bar into a shape that is shown here. If that bend bar has a radius of curvature rho because it will bend into a circular arch shape the straight one and become circular arch shape that has a radius of curvature rho and an angle theta the sector that is shown here.

If I take radius of curvature from here to here and here to here this is the center of curvature septangle subtended by this bend bar is theta, then we look at this cross section and we define what is called a neutral axis which is shown as a dash dot line in this figure also. Neutral axis is a line within this bend beam where the beam does not extend does not contract. In other words, it remains in the same length and the ones above the neutral axis will have to decrease in length.

The ones below neutral axis have to increase in length as the beam bends. So let us look at that here.

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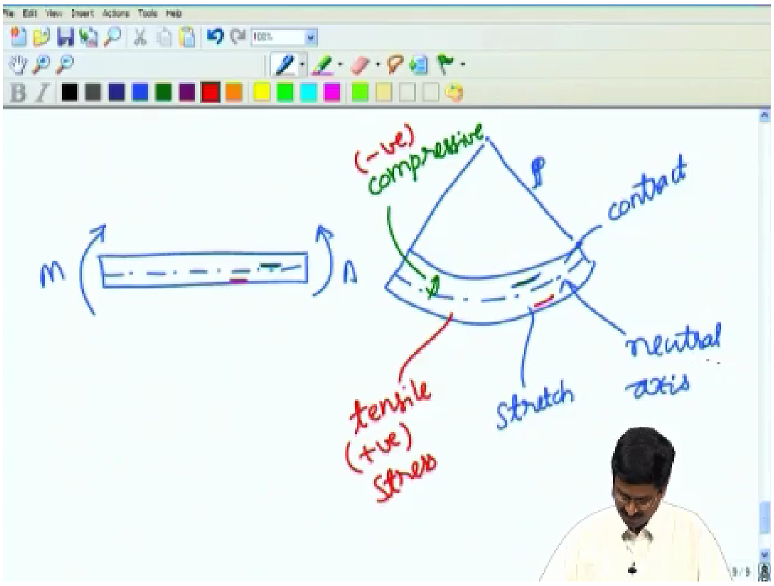

So we have taken a straight beam and we have rotated by applying moment bending moment for this by applying bending moment to rotate it into a circular arch where there is center of curvature and rho which is the radius of curvature. We are saying that there is going to be an axis which will neither stretch nor contract which will call neutral axis. If we have a neutral axis any fiber if you will if you take that and look at it before the ones above have to contract.

So this portion will have to contract and this portion has to stretch okay if I take a small fiber here if I take the same one it would have stretched whereas if I take a small one over there that would have contracted. So the one above this portion that will be under compressive stress where the normal stress has a negative sign whereas this portion will be what we call tensile of positive stress and the beam bends in this manner.

Whereas this will be negative stress. So for the case of beams we need to discuss a little bit more as to how to get this stress that is above which is compressive below the neutral axis which is tensile and we have to relate it to the forces that are applied because if we go back to our problem we have the beams which have transverse force acting. So this is the kind of thing that we are looking at and how do we determine this bending which we will discuss in the next lecture just to summarize what we have discussed today.

We first discussed what is meant by the motion of solid bodies it can be a rigid body motion or elastic deformation and then we looked at the motivating examples which will revisit in the next lecture as well and we discussed at length about how bars actually deform and then we discussed the concept of normal stress, a normal strain and Hooke's Law with which we can determine the deflection acting in a bar that is subject to axial loading which is given by load times length of the bar/Young's Modules times area of cross section.

We also discussed how thermal loading will cause stresses inside bars and we considered bars in parallel and series which are like springs in parallel in series where we have shown how a bar can be treated like a simple spring and we moved on to beams and defined the concept of the vertical shear force that we see whenever it intersection or bending moment in order to keep the broken piece in equilibrium.

It has to be in force equilibrium as well as moment equilibrium. We will consider how the beams bend in the next lecture.