Micro and Smart Systems

Prof. G. K. Ananthasuresh
Department of Mechanical Engineering
Indian Institute of Science – Bangalore

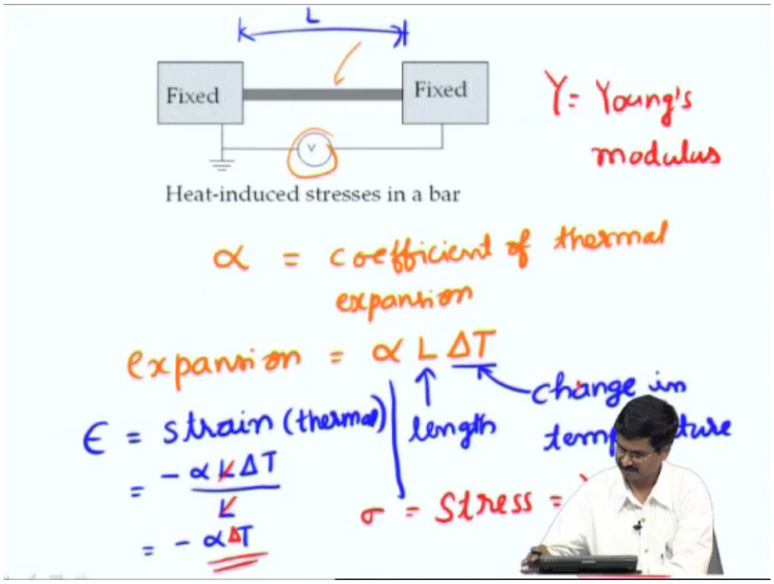
Lecture - 17
Residual Stress and Stress Gradients

Hello, as part of the micro and smart systems course, today we are going to touch up on another topic that is related to mechanical modeling of micro systems. The topic that we are going to discuss today is related to the Residual Stress and Stress-gradients. We discussed in the last lecture what is meant by stress and now there is a new term called stress gradient. Gradient usually refers to how a certain quantity varies with respect to spatial dimension.

The spatial dimension can be a thickness, width, length some other quantity in the spatial coordinates if you take x, y, z as your coordinate system how a certain quantity varies with x, y and z that is called the gradient. That is a derivative of the quantity with respect to that direction-- in one direction how that quantity varies. Stress and Stress gradients are very important parameters when it comes to the analysis of micro machine components.

They are important for Macrosystems as well but in Macrosystems you have stiffness that is very high compare to the stress and stress gradient that exist in them so we do not see the effect of that significantly, whereas in Microsystems you have components that have very low stiffness and you need to have appropriate adequate stiffness in order to counter for any bad affects that are cause by residual stress and stress gradients.

(Refer Slide Time: 02:01)



Let us take a very simple example that is shown here which is a very common example also in Microsystems not only in Microelectromechanical systems where there are deformable solids but also in Microelectronic Circuits where you need to have interconnects. Interconnects are the elements that connect to different devices. It can be to capacitors, to transistors, to diodes or diodes to transistors all kinds of connections they are like wires that we see around in electrical wiring we have this interconnect.

Let us say one of this interconnects that are shown here, this one, so this particular one is fixed at both ends as it is shown here and let us say we have applied a voltage potential between these two ends shown by this quantity. Now, when there is a voltage there will be certain current going through this and because of the current their will be joule heating of this interconnect or a beam if you will.

So it is suspended between two points where it is fixed at both ends and there is a current going through and there is a heat generated because of the current pass through it. It is a very common situation in Microelectronic circuitry as well as Microelectromechanical systems components. If you have current going through that because it is heating, and that heat naturally makes it temperature go up.

When some things, when the temperature of something goes up it usually will expand, so this bar here would want to expand that expansion is given by a quantity denoted by alpha which is coefficient of thermal expansion. This gives you the change in length or expansion of a certain material per unit length, per unit temperature raise; if this bar if this interconnect here raises by temperature V and its length is L the expansion is given by alpha L times delta T.

So here, the L is the length and delta T his changing temperature or temperature rise. So alpha of course is coefficient of thermal expansion. All material have certain alpha some have low thermal expansion others have very high thermal expansion coefficient, so these are expansion. So this interconnects wants to expand but it is fixed at both the ends, so it is not allowed to expand.

So what would happen, it would develop a stress because it is not allowed to move it will develop a stress and that stress we can call is a residual stress that is their because its heated, okay, as long it is hot the stress will remain there. How much is the stress? We have this expansion, I can write the strain epsilon which we have denoted in one of the last lectures also epsilon is a strain, in this case due to thermal load because the heating has taken place and it wants to expand but it is not allowed to expand.

So a thermal strain will get created with a negative sign. So if I heated, let us say delta T is positive and L is positive, L of course as we said is the length of this interconnect, L is positive and delta T is positive, alpha is of course positive for most material. There are some materials which can have negative thermal expansion coefficient but they are very rare, so alpha is usually positive, L is positive, delta T that is positive; then we have expansion which is positive that is actually expansion and not contraction.

Because it is not a load that expansion the stress created by that has to be opposed, so we will have a strain which is negative of this quantity, so strain as we know is change in length which is negative change in length, alpha L dT delta T divided by the length which is -alpha T. So this will be the strain created here which will a null or cancel the affect of the expansion. It wants to expand but it cannot so it develop stress in the opposite direction.

So expansion affects and residual stress affect both will cancel will remain there. When you cool it of course the stress will disappear then it is not residual stress anymore. For some reason if it remains there that can happen in processing due to varies reasons then we call it residual stress, right now this is simply thermal strain and if you want to write thermal stress we know the stress strain relation.

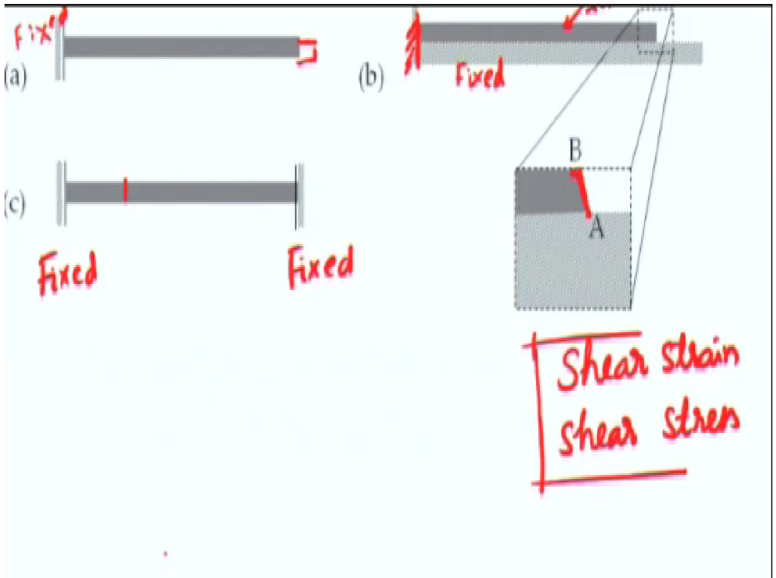
So in this case the thermal stress in this going to be Young’s modulus Y times epsilon that will be –Y epsilon, we have just written here as alpha times delta T so this should also be delta T, I forgot the delta so that is change in temperature. So we have only canceled L here that is alpha delta T, stress is –Young’s modulus times alpha times delta T.

Remember, that we are using Y for Young’s modules which is the proportionality constant in the stress strain relationship, stress sigma and strain epsilon related by Y. So we get the thermally induced stress, okay. Now when it is hot and let us say we lay down another material on top of this and if that new material and this material have different thermal expansion coefficients then the stress will remain in both the structures even after they are brought back to the original temperature.

So to see this let us say we have one layer which has certain expansion coefficient-- top of that another layer which is has a different (()) (09:20) expansion coefficient; we join them together, we first raise the temperatures join them together and cool it when they are cooled each of them will cool to different extents that means that one will contract to lower length than the other because they have different thermal expansion coefficients.

In those cases, when you bring it back to the original temperature both of them will retain certain residual stress. So there are several processing steps where we would have residual stress remaining in the structure, okay.

(Refer Slide Time: 10:03)



So let us take a few more examples. Let us say I take now a beam, so previously what we took is interconnect is also a beam, so this beam here is fixed at one end so this portion is fixed, okay this portion is free. If I heat this it will simply expand to by certain amount so there is nothing really interesting about this particular thing it will expand by whatever we derived in the case of interconnect where we said it will change in length will be related to this alpha times length of this times del T.

Another hand let us say we have still have it fixed at this end and we also have fixed on this entire length so this hatching indicates that we have fixed their also. Then what would happen is that it you cannot expand here, it cannot expand there, only the top three surfaces can expand.

So, you know that whenever we say that it expands the contracts it regards to heating, but let us say that the processing of this that is we have a substrate on top of that we have put another layer at a high temperature and now cooled it back to the room temperature the substrate would have and then the positive thin film, thin layer would have another alpha, so if the alpha is do not match one wants to contract more than the other when it is brought from high temperature to low temperature.

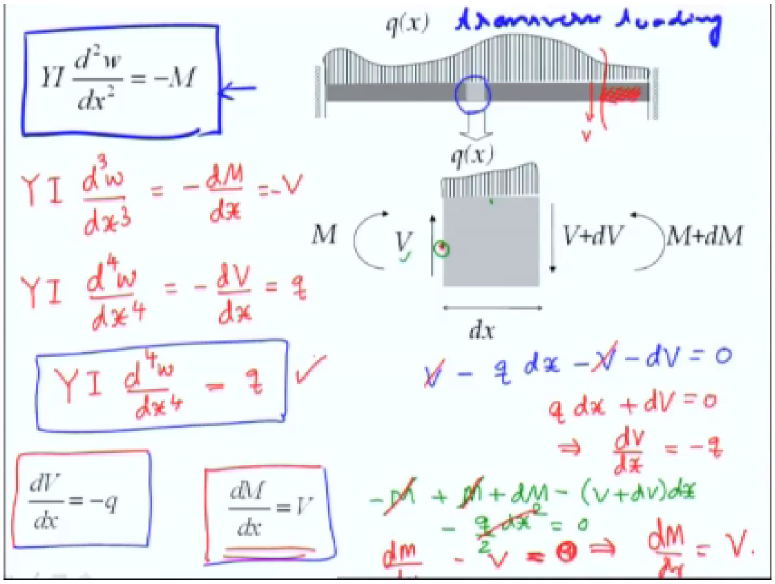
When that happens let us say that this material has developed a tensile residual stress that means that to counter that it has to contract and create a compressive residual and conciliated become stress free. So if you look at this portion the bottom one cannot because it is fixed to this substrate whereas the top one that we see here this top one that can—

So let us see this the mouse, okay all right I will just mark, so this is the top one and the bottom layer is fixed to the substrate so it cannot move whereas the top one can contract and that is what happened, if you look at the close of you view the top layer is -- the top edge of this thin film has moved to this point whereas the bottom one still remains there, so what you see here is that this surface is actually rotating.

When a surface rotates normally we call something that expands let us say you have something here and it expands that normal strain or normal stress when it contracts that is also normal strain and normal stress but things can also rotate as we see here that is called Shear strain and accompany Shear stress. So the shear strain and stress are different from the normal stress and normal strain.

They are perpendicular a normal to the surface whereas these are along the surface the surface itself, there is a force then it will shear like this, that is what happen in this structure. Let us look at the other quantity other figure where the beam is fixed at both ends. It is fixed at this end and it is fixed at this end. Now, when there is a residual stress here how that it affects the beam here this system and that is what we will discuss now. Okay.

(Refer Slide Time: 14:02)



So for what we will do is we will look at that system as shown here. We have certain loading in q(x), okay. And we are taking a small element. Why are we taking this q(x)? You know that from the beam analysis that discussed in one of the earlier lectures if they were to be a loading q(x)which is the transverse loading—it is transverse because it is perpendicular a transverse to the axis of the beam, transverse loading, this is a transverse.

When you have a transverse loading it create some stress. Now this q(x) is not in your load we just want to find equivalent transverse loading for the residual stress that maybe there in a beam such as this. If I have a beam and if there is a residual stress inside this if there is a residual stress, let us call that sigma r, how do we account for it? What we will do is let us find Equivalent transverse load qr, okay for sigma r because you want to see what happens to this beam.

Usually when this residual stress is depending on its sign when it is positive then this will be stiffer that is you have pulled it and kept it in some tension okay that is what a positive residual stress would do and that would make it stiffer, it is like if I take a wire and keep it taut by pulling on it then you know that it will be stiffer that is if it were to be slack with a little force it will bend sideways but if it is very tight very taut then if you pluck it, it will be much harder to pluck it.

And also the frequency of sound it generates also will be higher compare to that one is slack as oppose to one that is put it tension. So that is what happen when it is positive it become stiffer when it is negative it will become more flexible to the extent that if residual stress is very high it could simply buckle-- buckle meaning you just heated that develop contract tile residual stress or negative residual stress the beam itself might buckle like this.

Okay, this is thermal buckling, so the beam itself might just buckle like this. Okay. That is why some of the bridges also collapse when there this temperature mismatch. Now, when you go to this we want to find this q(x) or qr that you want to look at which is equivalent to our thermal residual stress or any other residual stress. In order to see that we want to get another small concept that we had discussed in one of the last lectures which is the governing equation of the beam, right.

If you recall Y was Young's modulus, I moment of inertia and w is a transverse displacement of this and M is the bending moment and the sign negative sign here because of the sign convention that you have taken were if there is a moment here that is a positive if it is on the left side that is positive and we also another quantity called V which does not appear in this equation right now but we will see how it comes about and that is positive on the left side, downward is positive side that is because sign convention you get a negative sign here.

Now, if I were to differentiate with respect to x, I would get; I want to differentiate this so I will get YI d cube w/dx cube = -dm/dx, okay that dm/dx = V as it shown over here, we will derive why that is the case so we will get V-V. So YI third derivative of the transverse displacement with respect to x= – of the V which is called the Shear force. So whenever you take a beam unless and loading, if you take a section there, there will be a force V as it shown here at the Shear force.

That force that acts in this direction in this direction that is V. If I take a cut here and consider this portion that would be my V over there as it shown. Now if I take another derivative I will have YI Young's modulus and moment of inertia d to the 4w/dx to the 4 = –dv/dx this turns out to be the load q that applied on this, this q(x) that we have taken that is what it turns out to be.

So in one sense what we have written YI d to the 4w/d x to the 4 = q is our differential equation for the deformation of the beam. This is the main equation. Okay, this is for general loading q(x) that is transverse. Now as we said we want to find an equivalent load q of r which is the affect of the residual stress that maybe in this thing. So going back to the previous discussion where we have this sigma r that we want to find what we are given, we want an equivalent loading qr.

Okay. So before that let us look at these two relationships where do they come from. For that let us look at actually this itself, okay this particular diagram. Now if I take a small element that is shown over here and blow it up as shown here then there will be a q(x) acting on it in the vertical direction; let us sum the forces there.

This V we will say positive is upwards then we have V – q dx, qfx is the force per unit length multiplied by length of this q dx and then this is downwards is –V - dV or –V of V+dV all of them have to sum to 0 for static equilibrium, so you can cancel the V and V okay. So what we get here is this relationship, so what we get is q dx+dv = 0 or dv/dx = -q so that is what we have here. Okay.

Similarly, let us say we take moment about this point and consider moment equilibrium then we get this relationship so we have here-- let us use a different color, okay. I am going to write now the moment equilibrium let us create some space to erase something. We want to derive this quantity by taking moment equilibrium okay.

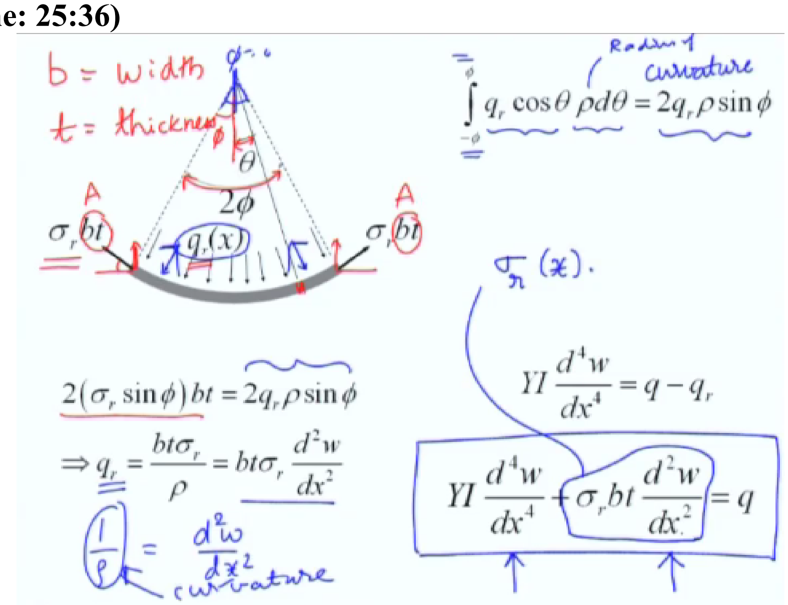
So let us say common to clockwise is positive clockwise is negative then this will be clockwise so we will have –M and then this is counter clockwise that will be +M dM and this force when we are taking moment about this point this will not contribute any moment whereas this would and that would +V+dv dx, in fact that will be clockwise because the force is downwards we need to make that change okay this will actually be negative that is –V+dv dx and so will be q.

So the moment due to that q the force is q dx and then we have to multiply by half of dx because it will act at the midpoint and this is the dx so it will be dx square/2 that should be equal 0. Now when you look at what can be canceled this M and M will get canceled, so observe what quantity is. So this M and M will get canceled we will have dM and then we will have Vd x and dv dx and the dx square.

When there is quantity dv multiply by dx, dx itself is small and the dV is also small product on the (()) (24:13) smaller so what we will have to do is we would say that the quantity that multiplies the dx and dx that will be very small we can neglect it so what will be left with would be dM by-- let us divide throughout by dx dm/dx – V dx dx goes and then we are dividing by dx here so then we will get –q.

This q itself is the force per unit length as we saw we neglect this quantity so that itself will be equal to 0 implies dm/dx = V, okay dm/dx = V. So we have neglected dx square quantity because that is very small and then we divided dm dm/dx and dv dx also we neglected. So we get this quantity to be dm/dx = V which is the-- this relationship. Okay. So having derived this general relationship now we want to find q related to this equivalent residual stress. Okay.

(Refer Slide Time: 25:36)



So for that let us take the – a little segment which we have denoted here which has bent because of residual stress, okay. Let us say that qr that is shown here so the qr that you see there is a little r over there that is acting on the beam in the transverse direction and we say that, that qr causes this residual stress sigma r, okay. Sigma r multiplied by b and t where b is the width of the beam let us say you assume rectangular section and t is the thickness or depth of the beam. Okay.

If you multiply them this becomes area. If you have some other cross-section you simply put area on both sides. Let us say the residual stress right now is uniform throughout the beam then we have this sigma r times A and the left side and right side but they will be in the tangential direction because this bend because of the residual stress , we are assuming that the residual stress will cause it to bend and we want to find the equivalent loading that causes the bending that is qr, okay.

Let us say this angle is two times phi this angle here two times fee and this other angle where you take some small element okay that angle is from the vertical that is this is a vertical this angle is theta. So now we want to see how force balance works here because there is qr acting on it if you take a small segment the total force acting on it okay and that has to be compensated by the residual stress times area that is the force and force here that has to balance only then this element will be in equilibrium.

First let us look at what happens to the components due to the sigma r. The way the direction is, so this sigma r if I were to this direction is like that, if I take a horizontal component, vertical component; horizontal vertical the two horizontal components get canceled whereas the vertical components do not get canceled they add up and shown up as this, the vertical component because this angle is 2 phi, this is going to be =2 phi.

And likewise if that is phi this is going to be phi and you are looking at the vertical component which is a sign phi so this is phi so this vertical component here will be sigma r sign phi times bt 2 of those that is the vertical forces this one plus that, that is this quantity, okay. Now what about-- how is that compensated and that is compensated with the qr that is the transverse loading, so let us use different color to make that.

So now we want to see the effect of this qr, that qr 2 when I take a force over here and the corresponding course by the same angle over there also the horizontal components get canceled, one goes that way one goes that way other goes this way they get canceled, only the vertical components will be there that vertical component is taken has is shown as qr times cosine theta, the theta is varying times rho d theta because qr as you know is a force per unit length, so multiply with a length.

Rho here is a radius of curvature, radius of curvature that is a length and force; the total force if integrate from –phi to +phi the central line is where you have phi = 0 this line and you have to go from here to here –phi to +phi that is what is shown here. Then if integrated we will get this that is what we have equated. So the vertical upward force should be equal to the downward force, when you do that then you get qr = sign phi, sign phi gets canceled you have bt sigma r/rho that is over here then we get qr = bt sigma r.

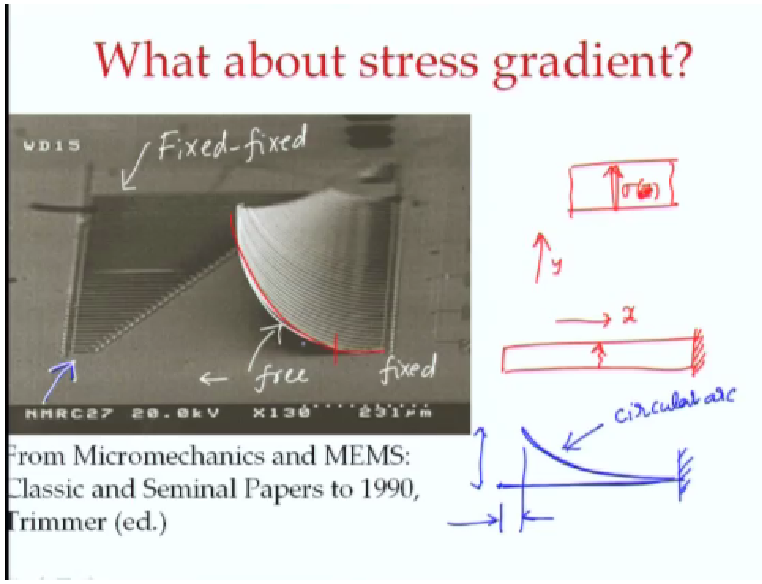
And then for rho that we have here we substitute it d square w/dx square because that is an approximation to 1/rho from geometry, so this is the curvature, so curvature 1/rho = t square w/dx square is what have used. Rho is radius of curvature. 1/rho is curvature. Okay. That is a curvature that is what we have here. So now what we have found is that qr which is an equivalent transverses load that causes a residual stress sigma r is given by this quantity.

So if we go back we have just derived that normal transverse load that maybe on the beam is given by this now we have qr and that qr if you say that residual stress sigma r is caused by transverse load it has to cancel that that is negative qr. So we can bring it back this side because qr is given by this quantity, okay so now we get a differential equation which is YI d to the 4w/dx to the 4+sigma r times bt * t square w/dx square.

Earlier we had only the fourth order derivative, now we also have a second order derivative where the sigma r bt is there. So if somebody says that the residual stress in it you have to put this in. And now you in our derivation we took a small element here so we did not make an assumption about stress being uniform throughout the beam. Now if it is were changing as well we can put this sigma r as a function of x.

So this sigma r here can also be a function of x residual stress can vary across the axis of the beam and you can take this into account, this is the and new governing equation that we need to solve compare to previous cases where this second term was now existent. Okay. That is affect of residual stress and whenever the residual stress that is positive then the beam will become stiffer when it is negative then it will become more flexible.

(Refer Slide Time: 32:38)

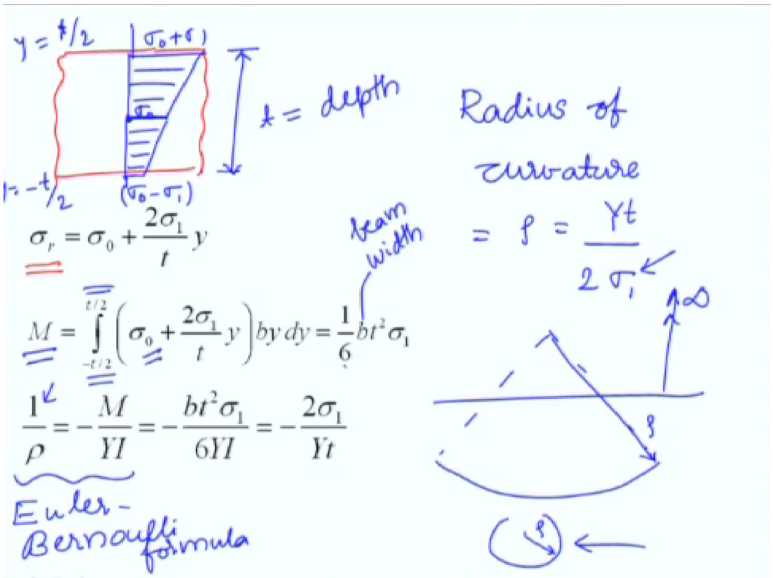


Okay, now what about the stress gradient. You see in this picture that is more than 20 years ago this is how the micro machine beams look like. On the left side over here we have-- let us choose a different color here we have some beams which seems to be fine (()) (33:04) flat. These are Fixed-Fixed beams so these beams are fixed at both ends, Fixed and Fixed. Whereas these beams are cantilevers so they are free on this side and they are fixed at this end.

These beams as you can see have curled up, right. Let us switch back to another visible color so all of them have curled up, why is that? Because they have stress gradient in the thickness direction, let us say I take a section of this beam, if I take that beam somewhere there, okay the stress in this direction varies if I call that let us say sigma z that sigma z is actually varying with the direction then that happens in the stress gradients it actually curled up like that.

The reason for that can be seen by looking at an example. Let us say that-- okay that direction instead of calling it the z let us call that direction the y direction okay, so when you have the beam which is in this figure it is fixed here and free there so this is the white direction and this is the x direction okay. And now there is a stress that is there inside this which is varying.

(Refer Slide Time: 34:44)



If I take that stress to be given as follows it varies to the y direction sigma r is equal to that is the residual stress, sigma naught +2 sigma 1/t times y. Okay. So sigma naught here refers to the stress that is there at some point and then when Y=t/2 half the thickness that will become sigma 0 + sigma 1. Or when the Y = -t/2 sigma naught – sigma 1 quantity, okay. So at one end it is sigma 0+sigma 1 other end it is sigma 0 – sigma 1.

We can take something like that so that is if I have the beam this is a beam if I start from here if I say the stress here is sigma naught and then it increases to a value sigma 0+sigma 1 because this whole thing is t/2 substitute that will become sigma 1 Y = t/2 and then negative it may go to another value which is much smaller, okay so this is sigma naught; this is sigma 0+sigma 1; this is sigma 0, sigma, sigma yeah that is sigma 0– sigma 1.

So, this sigma have to erase, okay. So we have stress vary in this manner. When you have that-- now what we do is we do the moment balance that is I take a section here where I know that there is certain thing acting. Now I take moment about certain point when I do that, that bending moment at that point will be there or causes a bending moment – the affect of stress gradient it is to bend and that is what we see here.

When there is a residual stress it actually makes this cantilever beam bend or curl as shown here and that bending moment equivalent bending moment can be calculated by take moment of this sigma naught+2 sigma 1/ty which you have shown here integrated from t/2 to+t/2 that is from here to there so this level is y = -t/2 and this level equal to y+t/2, okay then integrate you get this as 1 over 6 bt square sigma.

b is in this case we are using b to denote the width of the beam or a breath of the beam, okay that is the beam width and we have sigma that is given and this. Notice that sigma naught itself does not play a role only the-- the slope of that. If you look at the sigma r as a function of y this is the slope of that sigma r (()) (38:06) is intercept, that does not matter because that is there everywhere but the gradient is what matters that is given by this moment.

Now you know that 1 over rho is – of bending moment divided by Young's modulus times I, this is our Euler Bernoulli formula that we have used in one of the earlier lecture also. Now we substitute for M from here, when you do that and simplify it will turn out to be 1 over rho where rho is the radius of curvature is given by this quantity or another words radius of curvature rho due to the residual gradient, radius of curvature equal to rho is going to be equal to reciprocal of this quantity because you have reciprocal over here that will be Yt/2 sigma 1, okay.

The more the residual stress is gradient the smaller the radius of curvature whereas if the curvature is small that means it bends more. Note that, if there is a straight line its radius of curvature is infinite that means there is no curvature there is no bending of this thing. Whereas this one has certain radius of curvature so it has bent a little bit but if I take this in this manner radius of curvature is very small look at in each case this radius of curvature is infinite, okay.

Whereas this one maybe somewhere here so this is the radius of curvature somewhat larger and here it is very small. So when radius of curvature is small that means that curvature is more or bending is more which is clear. When residual stress gradient is very large than rho will be small meaning it bends more but if Y were to be larger Young's modulus is very large then even when residual stress is more overall quantity may still be large, in which case it will remain flat or bend only a little.

So you can see how it looks like and if thickness is more radius of curvature is more it residual stress is bending that is known from your common intuition that when there is a material whose Young's modulus is very high it resist bending or when the depth of the beam t here which we call depth of the beam that is the thickness or dimension of the beam in the direction of bending; when that is more also it will not deform much or it is a curvature is very large or if it is less then it is smaller it will bend like this. Okay.

So that is the radius of curvature; that is the affect of residual stress gradient. And this residual stress gradient is what is causing these beams to deform as it is shown here. Okay. And now let us see in my micro fabrication processes is what causes this residual stress there are number of them one we already discussed with very beginning of this lecture which is the thermal affects.

So most of the micro fabrication processes that happen at high temperature they develop certain thermal induced stress; when they brought back to the lower temperature this stress remains in them because different materials that are adhere to each other at high temperatures they are brought back to the lower temperature then we would have the residual stress remaining in them.

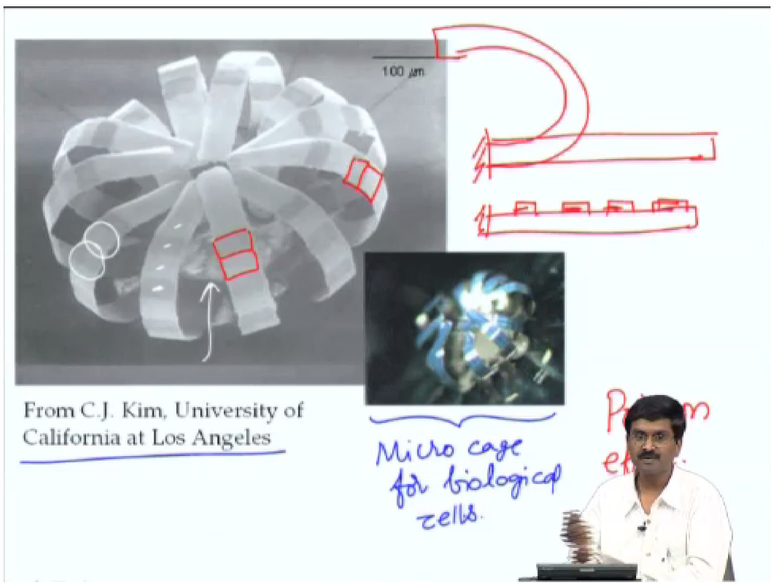
Other ways that residual stress can get created is because of oxidation, ironic plantation that is when you add certain foreign substances into a substrate or a thin film, then you have residual stress. Imagine how it works. Let us take-- we take a very live example of a bus, a city bus. When there is a pick hours there are people completely filled in it and more people get in and as a passenger in that bus would fill stress.

Similarly, when the atoms or molecules that are in the substance if we add more atoms and molecules into them obviously they will feel stress that is one way to have residual stress. Another is in certain processes such a epitaxial growth then here again making them grow in certain places but then lattice-- every atom will have a certain place in the lattice. But if there is a mismatch in the lattice then there will be a problem.

Again we go back to the as a city bus example there is a sit that can accommodate three people. Now a fourth person wants to squeeze in then all four people will feel stress due to their neighbors, so that is what happens. So many of the physical or chemical changes that happen in a material would cause residual stress, all of those now can be dealt with in the context of beams by adding this term. Okay.

In a general case, also similar term will come and that can be discussed when we come to the general form of elastic deformation equation, right now we have only talked about in the last one of the earlier lecture and today just discussing just the axial expansion contraction and bending of cylinder structure such as bars and beams. Okay.

(Refer Slide Time: 43:40)



So now we discussed the affect of residual stress and stress gradient. What are the applications of it? It is a bad affect in the sense that it will have some unknown stress which is quite difficult to measure. When you have in a lavatory process some micro machine structure assessing how much stress there is as a residual stress in it at the end of the processing when you package and ship there chips, it is difficult to assess.

There are methods, expensive equipment is needed in order to measure this residual stress but what can be done is to throwing a few cantilevers beams such as this and how much rotates. Let us say I have a cantilever beam of certain length and thickness and cross-section chip if I included there when I see that at the end of my processing it have curled up like that based on how much it has curled which we can measure by measuring this height.

And possibly this also maybe significant that is something like this here has become something like that which will be a circular arc, okay. By measuring these things, you can compute what this row is that is the radius of curvature. Note that, whenever bending moment is constant which is what we have here because bt and sigma 1 if it is constant in a process the stress gradient then it would cause a constant radius of curvature okay, that is 1 over rho = all known constant in this expression, so it will have to bend in the form of a circular arc.

If you measure the radius of it by measuring this height and this height this movement of this point in this direction then we can estimate what the residual stress is, so this becomes a sensor, so just add a cantilever into your layout of the micro machine structures based on whether it curves or not, if it does not curve you can be happy that there is no residual stress in that process that you have done otherwise by measuring it you can account for it.

There are other uses because if you do not account for a residual stress your calculation will be off, because residual stress thus changes stiffness of an elastic structure. You can also use this bad residual stress to your advantage as it is been done by a group in University of California at Los Angeles where they have made a cage for holding by logical cells, here is a picture of a real Micro cage-- this is micro cage for biological cells.

These days there is a lot of interest in studying mechanical or other prosperities of biological cells in order to single cell that are isolated from the tissues. So if you take a single cell you need to capture it. So in fact you can see that there is a small object which can be seen there, it is actually cell that is trapped and this one there all these fingers imagine your hand fingers these are all curled up like this and when you actuate from below by pressurizing.

It will open like the petals of the flower and the cell comes in and then you can close it. So this one here they are all these curled up beams each of my finger is a cantilever beam that earlier one had curled, you can also curl so all these things are like petals of a flower and if I pressurize from bottom I can make it go like this or open up okay by changing the direction of the pressure and when a cell comes in I can grab it and that is what you see here.

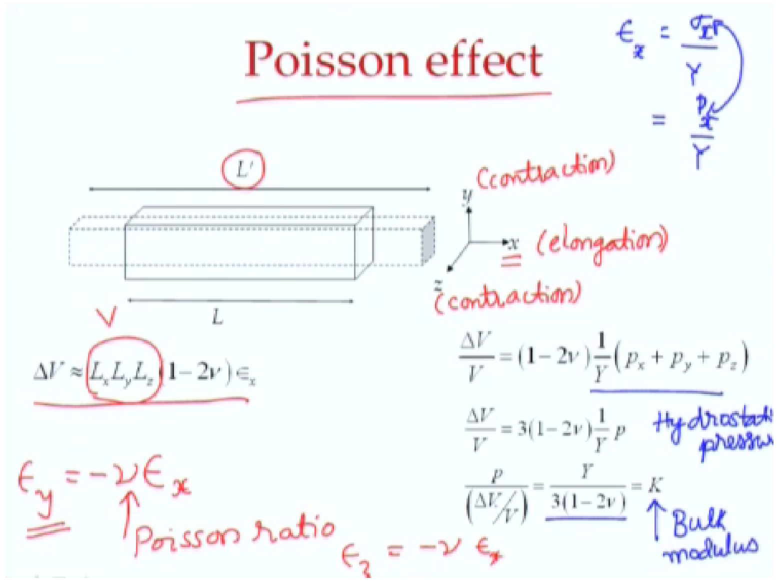
So there is a use for residual stress. We can actually make cell cages or one can even imagine putting drug medicines in it and you swallow it and inside you create the pressure open it and medicine can be released at the right time in the right amount there are lot of uses in drug delivery as it is called.

Now, if you notice in this figure there are some variations, if you look at all of them here, see this one looks a bit darker compare to this then there is a dark and bright. So what is that, why is that there? There is a reason for it, in fact if I look at any of these beams here that have curled up, so let us say I take beam that was originally a beam-- okay I have to change the color, so there is a beam which is fixed here and free here.

And the process will have induced the residual stress intentionally so that it can curl up and become like this, okay to become the cage. Now, what is done is if you were to just leave it, it can actually go and become completely close residual stress is more the bending will be more. But the reason for having this bright and dark portions is that what they have done is taken a beam and they have put this stiffness in some place okay.

So whatever it has curled up which has a stiffener is looking a little bit darker compare to the other one which is slightly brighter that they have added these ribs on top of it. Why did they add? For that we need to understand an affect that we have not talked about which you are probably familiar which earlier called Poisson effect. What is Poisson effect?

(Refer Slide Time: 49:51)



If I take bar, we always start with the simple example of a bar of length L and I have elongated new length L prime, okay. Then what will happen is that is the other dimension in the x direction we have elongated; in the y and z direction it gets contracted that is the Poisson effect, okay. In x direction we have elongation and that causes contraction in the z and y direction so y direction and z it causes contraction.

This is common knowledge if you take a rubber band and stretch it rubber band become thin that is the Poisson effect. And this has lot of implication especially in this particular example. What happens is that if there is a residual stress gradient causes it to curl like that perpendicular to this direction that is here it will curl the other way so when my finger let us say curls up this manner so observe that if it curls up in this manner.

If I want to look at this quantity that will curl up like this if I want to magnify this it will curl up like this, so one is going like this other would gather with, let us look at that in the form of the figure if the beam is bending as shown here in this direction in the cross-section direction it will go other way.

So let us look at the figure so where we have on the screen a little element that is shown where we have this bending in this manner this other one will bend in this manner creating a saddle surface like in a horse saddle and this is the affect of the residual strain and the Poisson affect.

Poisson effect is said that if there is elongation in one direction in the perpendicular direction there will be contraction okay.

So if there is a sigma x in the x direction in the other direction also there will be a sigma y which is negative of sigma x times a ratio called a Poisson ratio. Okay. Likewise, there will also be an epsilon z which is negative of this and then x. So if I want to have this effect and to prevent this from becoming a tube.

Now if you see this, this bends like this, other way it goes bends the other ways so what will happen is this will actually become if you were to imagine this like a tube and this tube bending is what you will see then it will not act like a cage each of my fingers that we saw earlier it become like curled tubes or thin wires with a hallow so it will not be effective as cage and that is why they have put this. Okay.

In order to model such as these and how to understand what it mean we need to look at the all mechanical effects and one of them is this Poisson effect. The Poisson effect while we are at it let us discuss another very short concept very small concept of how materials are compressible or not that is contain in this Poisson ration. If I look at this bar why do the change in volume of this. The change in volume is given by this quantity, how do we do that?

Now I have written the strain in the x direction and y direction and x direction and if I look at the total volume of the new one and the old one, if I take the difference approximately if I neglect the higher order effects then it will be the volume lx, ly, lz the length in x direction, the length in y direction, length in the z direction that is the original volume V. Now new volume is 1-2Nu epsilon x, that comes because this other strain that are created.

You need to calculate this delta V as the original volume the thin which is this – the Nu value of the dash line structured you get this. Now if you take the ratio of delta V/V that will be given by 1-2Nu*1 over Y okay. Here what we have done is we have taken the strain so now strain okay, we have the strain epsilon x = d stress that is sigma x/ Young's modules. The sigma S we can also replace by px/y okay that is why we have all these quantities.

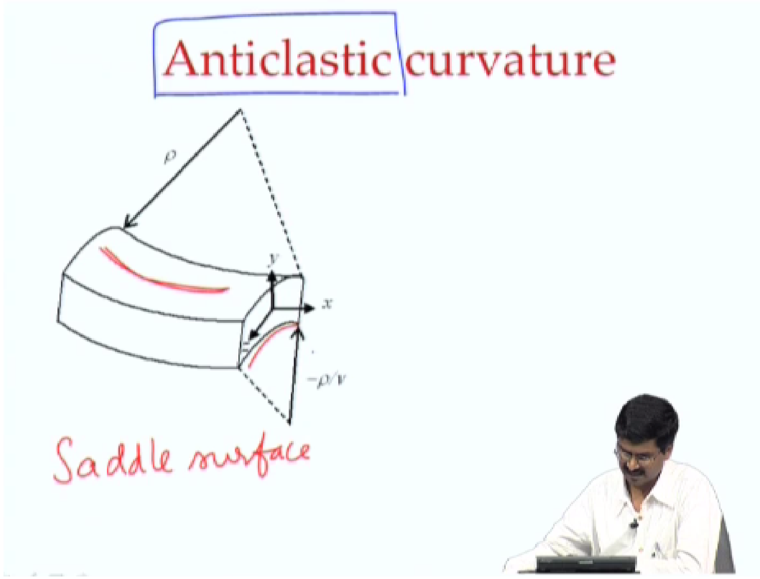
It is nothing but just epsilon x/y and the sigma is replaced with pressure because after all stress is (()) (55:10) and so is pressure that is what we have done here. So when you do this delta V/V is 1-2Nu * pressure in x, y, z directions because we have taken only x direction if it happens in the y and z directions we get this quantity and there are three of these pressures, if they are all equal which we call hydrostatic pressure.

So when you put something in water or a liquid for that matter the pressure in the x, y and z direction will be equal then we will get delta V/V three times of that pressure this. Now we can define a new quantity which will call the bulk modulus which gives you relative change in volume okay, for the applied pressure, so bulk modulus is how much pressure is needed for a unit relative change in the volume of the material.

And you can see that that is given by this quantity Young's modulus divided by three times 1-2Nu. Whenever Nu is equal to 0.5 this becomes 1, 1-1, 0 in which case K becomes infinite that means that such a material would have delta V/V because now that becomes infinite or this is 0 that cannot be compressed. So when Nu = 0.5 that will be incompressible. Whereas if it is more than 0.5 sorry < 0.5, 0.2, 0.3 which is for most metals and for silicon.

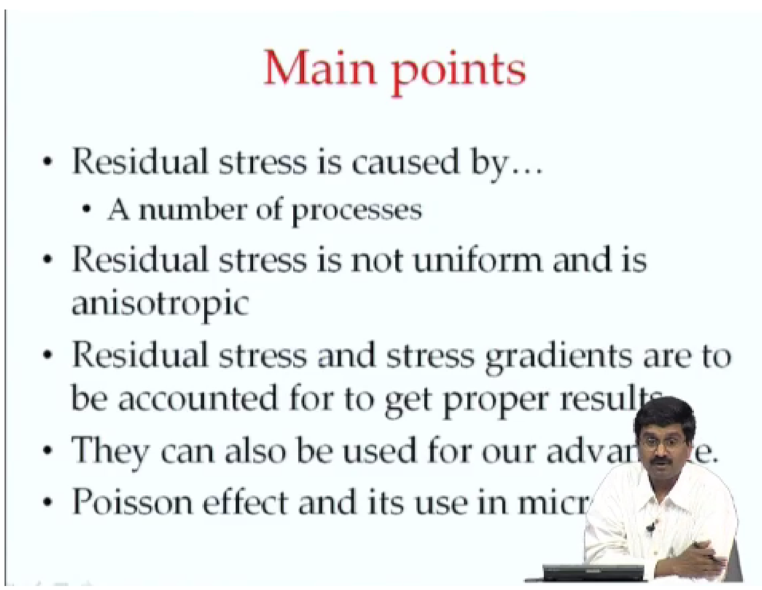
Also you can take it has 0.252, 0.3 then it will be slightly compressible. So we learnt another concept which is related to Poisson effect.

(Refer Slide Time: 57:13)



And we apply that Poisson to what is called Anticlastic curvature bend, beam bends in one direction. It will bend in the cross-section direction in the other way to create a saddle surface and that is also used to make the cages such as that one shown here. Okay. Let us summarize what we have discussed today.

(Refer Slide Time: 57:36)



We discussed how—what causes residual stress, thermal processes is being one but there are lot of other processes where extra atoms are introduced to the material and not only the stress, stress gradient is also important, so in fact we should note that the stress may not be uniform throughout the structure and it can even be different in different directions we cannot confer that using modeling whereas we have discussed only in this lecture in bars and beams which make up most of the micro machine structures.

And we have discussed the Poisson effect and how residual stress, residual stress gradient can be used for our advantage. In the next lecture, we will discuss some more concepts of modeling. Thank you.