Micro and Smart Systems
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Lecture - 18
Torsion and Twist

Hello, as part of the micro and smart systems course today we are going to study at another modeling topic which is Torsion and Twist. We know that elastic structures deform in many different ways, one that we have studied the simplest case is Axial stretching and then we have studied bending, today we are going to study, a discuss Torsion.

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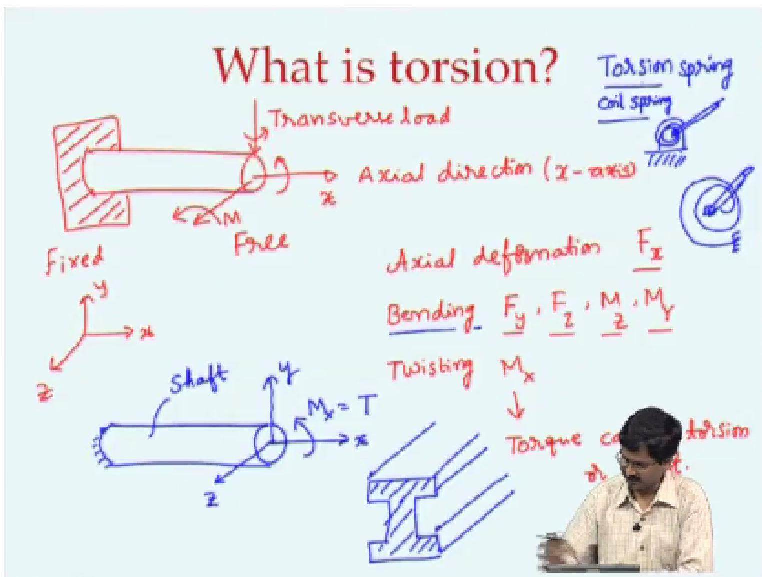

First, let us discuss what torsion is. All of us know if not torsion the word but twisting is something that we all are familiar with. Twisting is you take something it can be a stick or a piece of string or a wire or a plate, a shell anything that you take you apply a certain motion which makes it deform in a way that is different from bending that we have studied. So let us look at that. Let us take the kind of structure that we have already studied.

Let us say I take something that looks like this, let us say it is fixed into the wall on the left side, this side it is fixed, fixed into the wall. And this other side is free. Okay. When I apply a load in this direction all it will do is, deform in the axial direction so this for is this axial direction. So if I apply a force in this direction it is going to stretch that we have already discussed.

Now if I apply a force in this direction which we call transverse direction, transverse load or transverse direction so this is axial direction or axial load. If you apply there it is going to stretch or if it other direction will contract. If it is this way or if it is this way okay, if I put x, y, z here; if this is x, this is y, this is z okay, x, y plain is x-axis this way which I – let us say this is x-axis axial direction and the y or z axis if I apply forces this going to bend.

So this is one way of deformation which is axial deformation, okay. This is x-axis that is axial deformation. And if it is loading about the y-axis or z-axis in those directions we have bending, this beam length structure is going to bend. Now we also have to look at the moments. If there were to be a moment about let us say the z-axis, okay if I-- so this is the force in the x-axis bending will be caused by force in the y-axis or force in the z-axis.

And if there were to be a moment which is whose affect is to turn something. If there is a moment here, let us say that is about the z-axis we have studied that moment as well, whose affect is also bending and if there were to be a moment about the y-axis that is-- this is our y-axis, if there were to be a moment about that the beam will bend into the plain or out of the plain. So moment about y-axis also will cause bending, that leave one more thing, we covered affects Fy, Fz and moment about y-axis and z-axis.

What if there is moment about x-axis that is like that. Okay. The affect of that is to twist beam that the twisting and this loading we call is Torsion or Torque, okay. Torque causes torsion or twist, okay. So this is familiar to all of us that if we take a shaft, okay something like this; it is a beam basically and if we fixed one end and apply a moment that moment is about this x-axis, this is what we are calling it x-axis here up is y-axis and going that way perpendicular to the axis of the beam in this way is that.

If there were to be a moment about x-axis that is called a torque, we use capital T to denote the torque. Okay. Now the affect of this is to twist it. How much is it twist? So if I take something and hold one end fixed and twist the other end how much does it twist and what kind of stress does it create in this beam which we normally because shafts are the ones that hold this torque load we usually call it a shaft were it is a beam.

It does not need to be only of circular cross-section it can be any cross-section I can twist them. So I can have I beam cross-section which we see in civil engineering structures, but some of the microstructures also have this-- I can have something like this a beam what we call an, I beam because it has I type of section here. Okay. So when you have this, this a section this also can twist.

Sometimes whenever there is a building collapse or something if you go and see look at the beams you will see them bent and twisted in different shapes and that twisting is bending – bending or very common features of the deformation of elastic bodies in general. There is one more thing that we should not get confused at this time were what we use is called a Torsion Spring, okay.

This torsion spring is actually a bending spring in the sense that if there were to be a pin joint and we have a rigid body going rigid body something where we neglect the deformation, if I connect something like a coil spring, if it has that coil spring—spring actually does bend it does not twist, twist is something like this. We take something and then apply a torque on it.

To see this if this is a beam bending is apply the load this way or that way, apply force that way or this way that causes bending but if I-- or moment about the this axis or that axis. If I apply moment about this axis this way that is torque, right. If I take something that is very flexible such as a straw, then I do this-- you can actually see the twist and that is what we are talking about.

Whereas this coil spring that we are talking about here which is simply a spring that is wound in the form of a coil like—I am just exaggerating it, so if I have a pin joint here and attach something here one is attach to this other is attached to a fixed frame. Now as I rotate this is going to bend and that will also going to provide a torque about this axis so people call it a torsion spring but that torsion is not torsion actually it is bending, okay.

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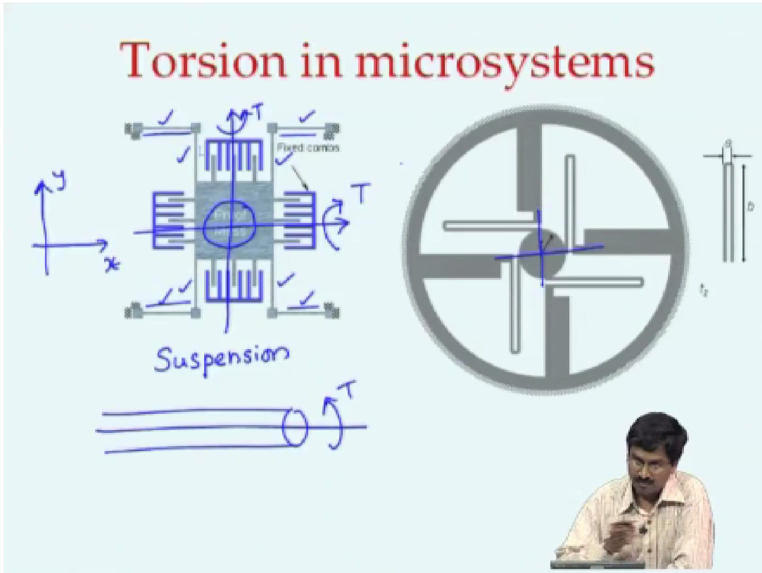

Having understood what torsion is, let us see where we find it in Microsystems. In fact, we find it in many places and Microsystems is not an exception. Let us say we take this suspension that is shown over here this is a suspension if you recall for a gyroscope or it is a suspension for a 2 axis accelerometer, so this Proof Mass the center what we have shown here.

This one can move in the horizontal and the vertical direction that is this is horizontal direction and this is a vertical direction or I can call it x and y with equal stiffness it is a symmetric it has that. But what if there were to be a moment like a titling of the pieces over which this 2 accelerometers is placed and there can be a moment or twisting about let us say this axis. What if there is a torque applied on this?

Then these beams that you see that is this beam, this beam, and this beam and this beam will start twisting or if there is a moment about this axis here then the beams which are vertical here they also start twisting, so we need to account for when there is a torque here they will twist, when there is a torque here this other beams also will twist.

So you have to see the affect of torsion on these so that we can see whether the Proof Mass and the capacitance that these beams will change or not. Okay. Similarly, here these other beams also depending where the torque is applied on this central platform this also can also experience twisting. So whenever we have the axis this way if there is torque T acting that will cause this body whose axis is this to twist, okay.

So when apply torque this way we have to see which beams will experience because the beams that are horizontal like this they are experience twist; when it is here there axis aligns like this they will start twisting, okay. So in these cases we probably do not want the beam to twist but they will, so we have to account for those unintended affects to be taken into account of the design so that any of these other motions would not affect our accelerometer.

But there are also a few devices which need this twisting or torque and we look at one example over here.

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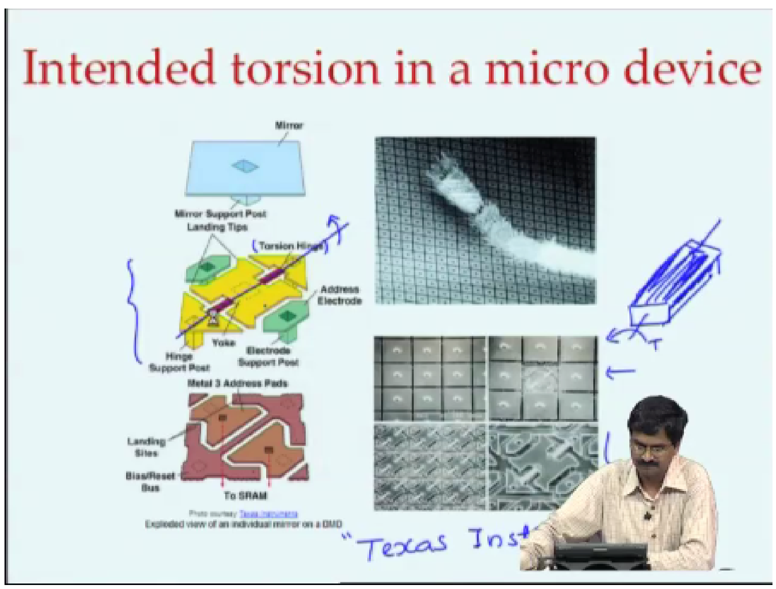

So this is Texas Instruments, this is a product from Texas Instruments company who have done extensive research on this Microsystems device which is an array of micro mirrors which are shown here with the leg of an ant to compare how small these mirrors are, we can see a little bit of close up of view here under just one mirror is shown here. Okay.

And if you look at the model of this which is colored for a clarity we can and also blur into pieces, so here is the central portion. We have the yellow structure and there is the red ones which are called here Torsion Hinge, torsion hinge because that-- they make this H shape structure to tilt this way or that way and that is done because these torsion hinges can twist.

And the force application is done through this electrocutes are underneath which will exist an electrostatic force to tilt this H shaped one this way or that way. In other words, this is shaped structure will twist about this one to make this one move this way or that way and there is a mirror attached and the mirror will tilt this way or that way to shine the leg beams in different direction as desired.

This is an array of micro mirrors which is used for projecting images such as what we connect to the laptop some of the laptop project the systems have this array of micro mirrors. So this is what we are talking about. Here is a beam, the red colored one. If I were to just write it, draw it here, okay this is our-- the red colored system, now we have a torque on it. Again know that, the axis of this red colored beam has a rectangular cross-section and there will be a torque load.

Because if I apply a load here the moment about this axis which we call torque and that torsion of this is what makes this mirrors tilt one way by the other. So there are a number of examples in Microsystems where torsion is important. Let us take a closer look for this Texas instruments mirror. So this is where the torsion happening, okay we want to model that. And you can see the real structure scanning electro micro mirror supplied by Texas instruments where this micro mirror is there underneath that you can see the electrodes.

And these are the critical portions as far as the mechanical behavior is concerned and that is the crucial part. This mirrors undergo go many millions of cycles probably when billions in their lifetime because if they use in the projector systems they have to twist many number of times. Okay.

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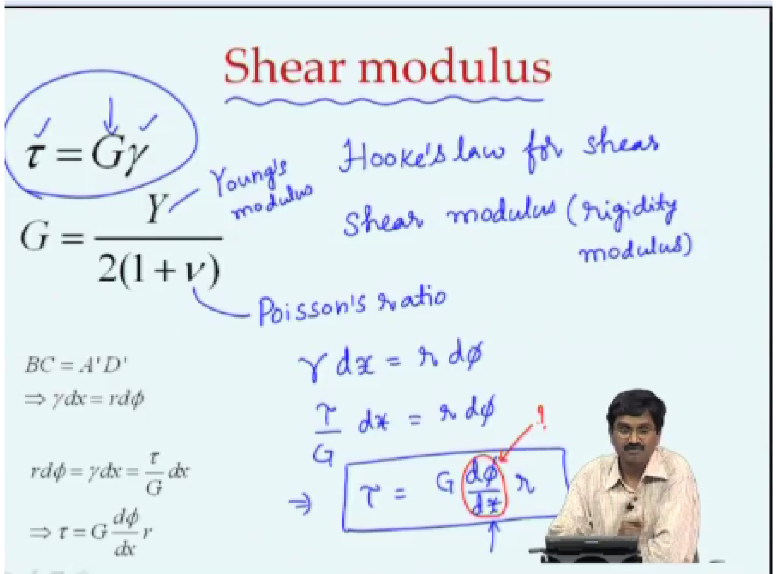

Let us discuss this with the simplest case which is the Torsion of a shaft. So shaft as I said is just a rod of circular cross-section and it is fixed at one end and free at the other, does not need to be free to apply torque or torsion but we just take a simple case where we have one end free other end is fixed and now we apply a torque that is shown here this is torque T, okay then what happens to it.

Then imagine that this and this fixed that is completely fixed, this is completely free; in between if you take something if I take this disk that is shown so the disk should have the same size as this one so we just take, if taken in inside of certain radius r rather than taking the-- this is small r whereas the radius of the big one is capital R, that is this diameter of this one is 2R but we have taken a disk, there is only a little bit and then see what happens to it.

So you can imagine that when I take this one and try to twist it the whole thing is going to twist, it will be zero twist here at the fixed end and at the free end it is going to have a maximum twist, that is if you imagine a straw and do it you will see that one end that you are holding fixed is not going to twist much other free end will twist a lot.

Now if you take that disk somewhere in between and look at what is happening to that thing, so here we are taking the disk the same disk that we have shown here, okay this disk will have blown up to large size to see what happens. If I take some of these letters are move let us not worry about it, we will redraw them. So I am going to take this piece a very small piece on that the disk, this is A, this is B this is C and this is D, okay.

Now this is a center of that circle, so when it twists A will move to another point, okay. Let us say A has moved to A prime and D has moved to D prime and B and C we assume they are fixed because want to see the relative deflection, so relative deflection will relative twist, so let us say the left end of this disk I keep it fixed or imagine it to be fixed related to that at a distance dx how much does it twist.

So I have taken on the periphery of this, let us draw it for clarity again. If I take that disk okay, I assume that this is fixed so that I can talk about relative deflection. So now if I have a rectangle here, we take it to be very small so we take an angle so that this portion can be approximated to a rectangle so here we have A, B, C and D; this is a center, when this what was there here. Now A goes to A prime somewhere here D goes D prime.

Since these are fixed we will leave them as it is and now we will see what happens to A, B, C, D it actually becomes A prime, B, C and D prime, okay. So the affect of torsion is actually to cause what we call shear in this rectangle that we have taken. To see that here we have shown in the dotted line, okay so I have taken a square, I have shown here. Now I have made this point move here, move point here just like what has happen here. Okay.

So if I say this is my A, B, C and D over here this has moved to A prime that is this point, this has moved to D prime. So this has been turned this way that is I have applied a force here to make it shear and that is what happens to torsion because when the surface rotates little to the other one it actually causes a shear that is something that we have not discussed at length but we did talk about normal stress and normal strain a lot.

A normal stress, a normal strain you have a force applied normal to the surface that changes the volume of the body, if there is a stress here it will go this way; if there were to be stress here it will go that way, right original one has become this rectangle, if there were to be a stress there this can become a rectangle like that, that we talked about normal stress. The shear stress is different from it.

Here we apply force parallel to the surface; here it is perpendicular or normal; here it is parallel. So that causes what we call shear strain, so we have shear stress which is denoted by Tau and shear strain which is denoted by gamma. How are these related? There is a property, material property which is called Shear modulus. This is similar to what we had Young's modulus which relates normal stress with normal strain and that relationship we had called Hooke's Law.

Similarly, this is a Hooke's Law for shear. If there is a shear stress, there is a shear strain will have a modulus which we call shear modulus because its affect is to shear parallel to the plain and that is related to Y Young's modulus and Nu Poisson's ratio. Poisson's ratio is something that we have already discussed in— another previous lecture. This Poisson's ratio and this Young's modulus give the shear modulus sometime this is also called rigidity modulus.

So having defined Hooke's Law, now go back to see how we can define strain here which is shear strain, if you go back to the previous slide we said this rectangle that we have shown is going to shear to other triangle which you have shown here. Okay. And this angle is our shear strain according to the way it is defined. So that is what we show here.

If I look at look at that BC okay that is this length from here to here will be still equal to this length so if I say the BC here is equal to AD which is also equal to A prime D prime and if you look at this BC that we have and if we say this length of it from here to here it is a going to be the same which we can write gamma times dx and r times d phi because we have taken the length of this if you notice we have taken that to be dx, okay.

This r how much it has moved, okay there is a this is little distance that the R times d phi because what was here has turned by that angle or this is turn by this angle that small angle is d phi, r times d phi = gamma the shear strain times dx. So we have this here. So gamma times dx = r times d phi, okay. And we will replace this gamma with Tau by G from here. Okay, Tau/gamma = Tau/ G = gamma. So gamma will replace by Tau/G*dx = r d phi r.

Or we can Tau = G*d phi/ dx*r which is what we have written here. Shear stress is equal to shear modulus times rate of twist this is the quantity; rate of twist of the shaft along the x-axis along the axis times the radius at different radii remember that we have taken this disk of radius r which is < the capital R, at any point we will be able to know what the shear stress is, provided we know what this is.

We know G and at any point R would like to find the shear stress; we have to know what that is, okay. How do we find that? Here you wrote is that he introduces 2 things, we introduce the Shear Stress Tau and then Shear Strain, we know one point (()) (25:26) that relates both of them but that is the Hooke's Law but in order to determine statics we need Hooke's Law and we also need equation equilibrium, so that is what we will next go to in order to find this rate of twist along the axis of the shaft.

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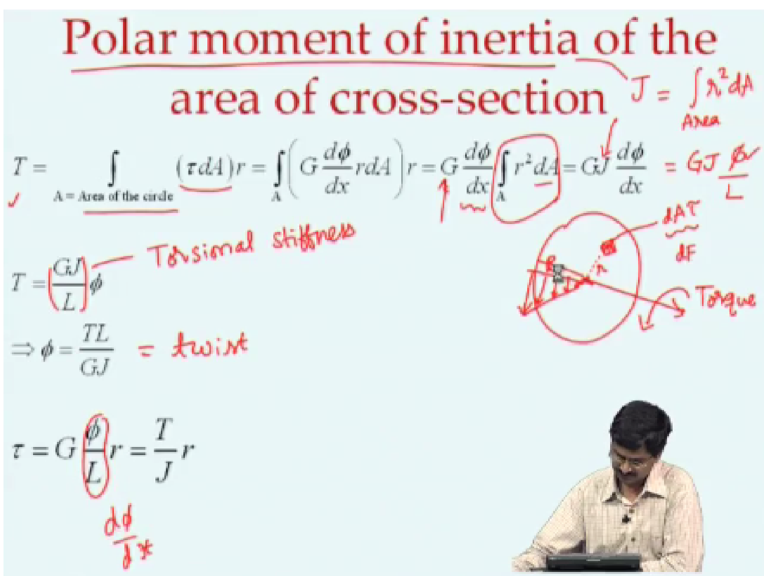

And that leads us to the equilibrium equation which is if I take that torque multiply by area of cross-section I get a force that is acting parallel, so I have this shaft surface and our shear stress everywhere and multiplying by shear stress with the area and then integrate over the entire area by taking moment. So I take a point at the center and I look at the all the area and try to do that, so let us sketch that.

If I have cross-section area like that, this a center I take a small piece here, okay that has in area let us say dA, we multiply that with the shear stress, stress is of course per unit area, that will be the force which is acting parallel to that surface, okay. And then I multiply that force with that radius where it is located and then integrate over the entire area of the circle, okay.

So we take a piece anywhere and try to fill it that is what integration means to get the total torque due to the force, this is the dF that acts parallel to that surface integrated over the entire thing and when we do that we substitute for Tau which we just derived G times D phi/dx times r and then we have (()) (27:17) integrate, you notice that we make an assumption here that d phi/dx does not change on this area of cross-section.

So when you take area of cross-section everywhere it is the same; that is an assumption that is valid for circular cross-sections, because all the twisting happens in the same place by the same amount it does not change within the area of cross-section; that is not true for other cross-sections as we will mention later. And G of course being a property of material that can be taken out of integration because that also does not change on this area dA over which you are doing integration.

Now this quantity which is integral of r square dA done over the entire area is denoted with letter J which is called Polar moment of inertia of the area of cross-section, okay, that is a J. Okay. That as you can see is defined as integral r square dA done over the entire area of cross-section. So we have derived the-- because I take this force and take the moment about this axis if I take moment about this axis right what I get will be the torque that I am applying which we discussed earlier today the torque.

So at torque = this total thing that = G j times d phi/dx. Okay. Where we assume that d phi/dx does not change over the cross-sectional area. Another thing we also say is that this d phi/dx can simply be put as phi times L, if you are interested in the total torque somewhere d phi/dx; if you say that is constant from the fixed end to the free end of the shaft we can say if phi is the total twist at a free end then you can say that is phi that is over a length L instead of dx we put L that becomes our twist and torque relationship.

We had Hooke's Law to relate shear stress and shear strain, now we have got relationship by using static equilibrium sometimes relates torque with the twist. Okay, that is T GJ/L. And this we can say is Torsional stiffness, torsional stiffness, just like we had a bending stiffness, axial stiffness now we have torsional stiffness. This is a torque which have, a unit of Newton meter and twist will have radians or degrees and then torsional stiffness will have Newton meter per radian.

Okay, so we have that T over phi and phi as we just called is the twist of the shaft. Now if any r if you want we go back to the relationship we had T = G times d phi/dx instead of d phi/dx we have put phi/L, okay which we had discussed over here, what you have done then we can say T/Jr because now you look at this relationship between T and phi, so we can get shear stress as T/J times r.

Now these 3 quantities that is phi/L times G, we can write T/J from here phi/ L T divide by GJ over here, so if I take that relationship I get T/Jr, this I can get that shear stress. When r=0 there is 0 shear stress that is clear. When I have twisting it this point will not experience inertia the point that is away will get a twisted little bit and little bit and so forth. When I go to the edge then this will be the largest.

In other words, if I take any line here it will has 0 shear stress; it will be maximum it will go like this. And r=0 that will have 0 shear stress, and r = capital R, okay this is our capital R from here to here that will have the maximum shear stress, that is what shear stress varies here. Okay.

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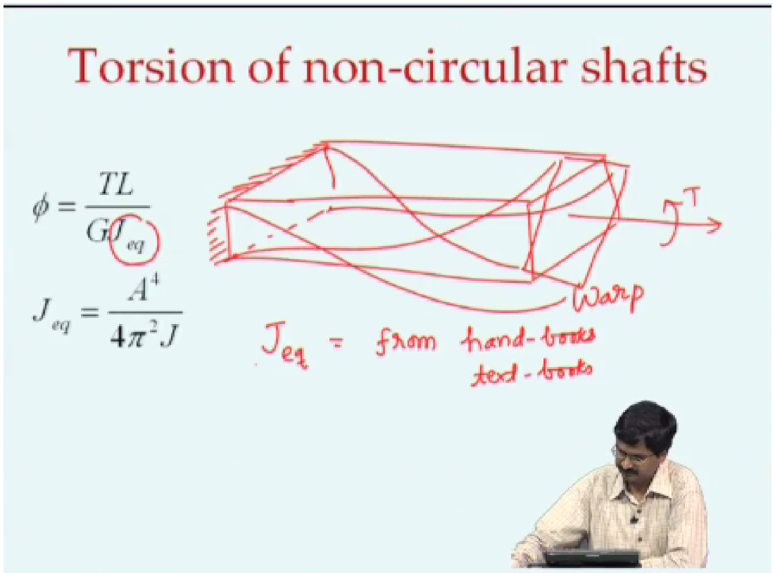

Now, what happens when you have non-circular shaft. Let us say I have like the TI mirror if I have a rectangular beam which they called Torsion Hinge, let us I fix it over here and about this axis I apply a torque, okay. Can we apply that formula that we just derived? Actually we cannot because some of the assumption that we have made here the d phi/dx is constant over the area.

And also it is a same or it is linear as a function affects that is d phi/dx if you just put it as phi/L the twist is 0 at the fixed end that is here and then its maximum at the end that will also be valid rather cross-section. Because if you imagine this rectangular box are rectangular beam if we take it and twist it, it will warp, okay. So we have defined something called a Warping function, and here this warping.

Because this is rectangle that will be remain like this, let us say this gets rotated because that is the affect of twisting, if it becomes let us say at an angle. From here to here the whole thing because what was here now let us as gone here so it has warp like this, this has gone to the other side and so forth everything as turned, okay so the weight has gone. So this warping is rather difficult to imagine.

But if you can imagine that whatever assumptions we made earlier for circular cross-sections will not be valid here. For thus, there are some approximations where we can define a J equivalent, okay that is a polar moment of inertia equivalent and still you use the formulae we have derived for twist of the circular shaft, this J equivalent will be given in handbooks for different non-circular cross-sections.

If you look at any strength of materials books handbooks or textbooks you can find J equivalent for different cross-section and that is what we need to use if you want to analyze the Texas instruments torsion micro mirror.

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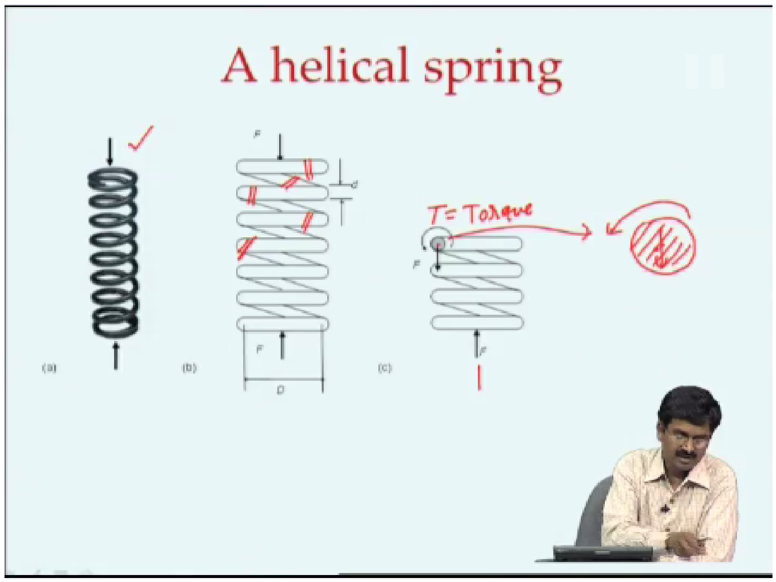

Now, let us look at an application of this torsion for something that is very simple and what we are familiar with, if we take any ballpoint pen all of them have a spring in it such as the one shown here which you call a helical spring, if it is compression spring you apply a force it is going to decrease in length, what happens how do we analyze?

It turns out that the kind of stress that this torsional the helical spring experience is actually torsion, so that diagram is shown here, if I cut somewhere like I have shown if I just cut the spring over here I would see that there is a force here since its upwards over here there should be a downward force on the cross-section for equilibrium, okay and if I take moment about that point that we call torque.

This torque is actually moment about the axis coming out of the screen, this is the torque. Okay. Likewise, if I cut anywhere on this anywhere we only see torsion and there is a force parallel, if I were to take this a (()) (35:55) cross-section and blowing it up, okay there is one force that is acting over here that is parallel so that is called the shear and also there is a torque about and axis perpendicular to the cross-section area that also cause a shear.

So this is a case of structure that is deforming where there is shear due to this vertical shear force a concept we had discussed in the context of bending of the beams and also we have the torque now which also called as shear and that shear effect is what we need to take in order to analyze this. If I surprised that something where you have taken a coil spring and you are just compressing it and that is causing torsion like affect.

Let us take a piece of string that is show here, so I just hold it hole one end and start twisting the other end. So I am holding this end fixed, and start twisting this other end. Okay. So do it as many times as you wish the more you do it the better will be the effect and after that just like this helical spring just bring these 2 ends together and what you see is that it twists, right. If I undo it and let us say all these torsion is gone.

If I take this and bring it closer it just bending nothing is happening to it, the moment I take this keep it taut and do this twisting that you, you can see what I am doing, just turning that is the torque that is the twisting and after that when I bring this becomes this. So here this torsion spring that we are talking about is actually undergoing this torsion affect and that is why it is coil and when you do it gets more coil and the torsion if you see.

And DNA in our bodies also has this kind of a feature where there is a helices that turns around what are called histones and that makes it collapse into a loop such as the one that you just saw with the this piece of string. Let us analyze this using what we have just discussed.

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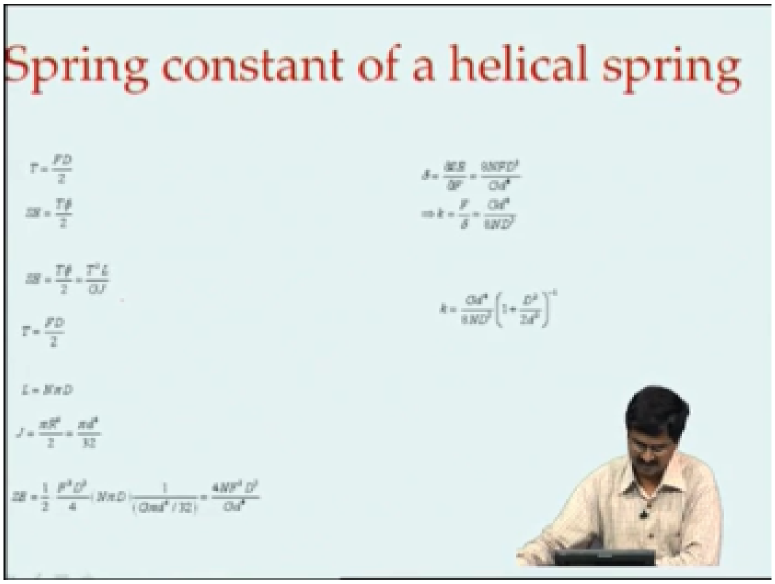

First the torque the torque we have seen so we saw that there was this cross-section and there was a force f here and there was a torque and that torque is f times the diameter of the helical spring divided by 2 so if the D here, this is the quantity. Okay. So diameter of the coil of the helical spring is D half of that because we have to take moment about this right, so this force acting here, F times D by 2 that is the radius of this one, half that thing. That will be the torque acting on this on this section. Okay.

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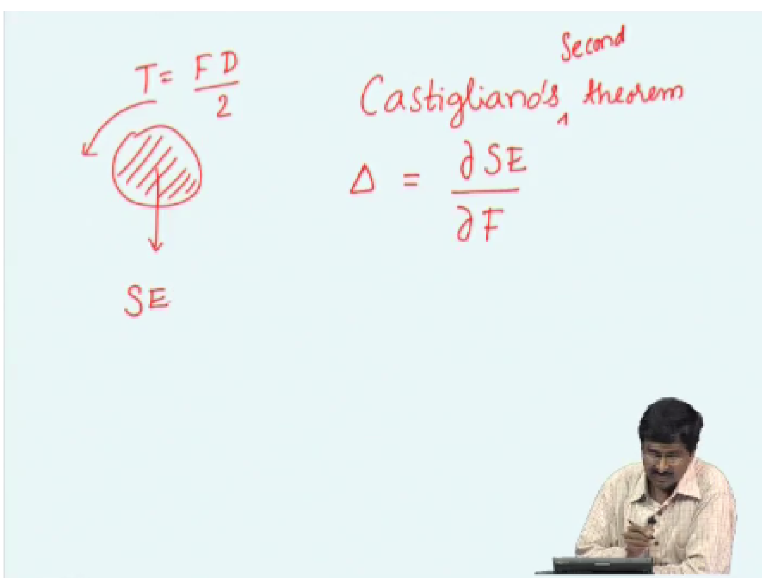

And now we will use this occasion to discuss the application of Castigliano's theorem which we had discussed in one of the earlier classes, Castigliano's theorem which says that due to this torque if I want to know what that twist is we have derived the formula just now, but if I have to look at this entire spring for a particular force that is applied on this if I want to know how much it is going to compress.

Let us say I keep this end fixed apply a force this will move down here, if I want to know this deflection we will write the strain energy of this whole spring and take the derivative of so if I want to find that delta of the spring I will take partial derivative of the strain energy of the spring with respect to the force, this is Castigliano's second theorem if you remember from one of the earlier lectures, that deflection delta is simply (()) (39:58) with respect to F.

So you have to write strain energy of this structure of the entire spring, strain energy of the spring. Let us do that.

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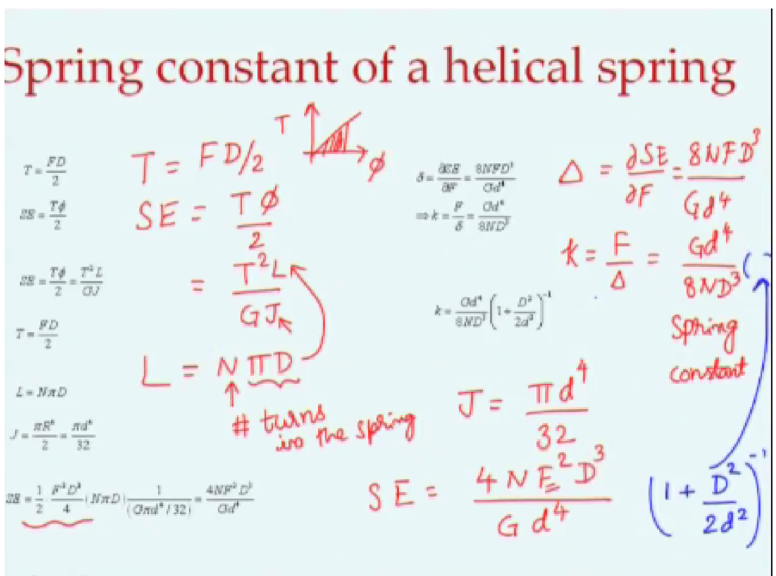

So first we have here this torque = FD/2 and the strain energy which is the area under the torque and the twist diagram that is if I were to plot the torque here and twist here the area under this is a strain energy just as for a spring half Kx square, half Kx is the force, x is displacement so we have torque here multiplied by F that we will have units of energy that is the strain energy and we substitute here for torque what we have just derived that was.

Phi L/GJ, if you go back in what we have just discussed the torque here is equal to phi GJ/L. So if we substitute that or here then we will get this stain energy = T square L/GJ, okay that gives us the strain energy and the T here is if FD/2 then we have we can substitute for T FD and also the L here is N times pi D, how does that come about? pi D is, if I take each turn of this spring here this is pi D 1 coil and I have N turns like that in a coil, there is a total length of this.

If you go back to this string experiment I want to take the entire length of the spring, if I take this spring and uncoil it I get a shaft length L = number of turns times the periphery of each of the loops okay length of each loop of the thing.

So N here is number of turns in the spring turns in the spring and we can substitute for L over here and we need to know the G, the G for circular cross-section, okay the J sorry the polar moment of (()) (42:32) cross-section is given by pi wire diameter of the spring, the spring there is a wire and wire is wound in a perfect coil to make the spring that is D, D to the 4/32 substitute all of that we can get the energy which is shown over here let us write it little bit bigger.

The strain energy turns out to be 4N substituting all of these F square capital D coil, diameter cube/G and D raise to the 4. Okay. So we have this total strain energy. If you take (()) (43:10) F then we focus on this the 2, 2 will become 4 and 2 8 the delta that we wanted which is dou SE/dou F turns out to be 8 and FD cube/Gd to the 4 okay that is the deflection.

So how do we define spring constant K? FD divided by this delta which will give us G, F goes-- this is F here F/delta this way it will go Gd to the 4 divided by 8 and d cube, that is the spring constant, we have used this concept of spring concept many times when we discuss the lumped modeling of the suspension of the Microsystems.

And if you have a real spring such as the one shown here we can get a spring constant in terms of its parameters as shown here where we have used Castigliano's theorem and the fact that the compression of a helical spring is equivalent to twisting its wire as we just saw the string experiment and that spring constant we have derived.

And if you also take into account the shear over this area because of the force F, if you write the strain energy to that also and again use Castigliano's theorem we will get an additional component which will give this K in addition to this there will be another term a correction term if you will that will be given by-- so there will be a correction term here which is 1+capital D square/2 small d square reciprocal of that 1 over that, that will be the affect 1+ this going to be correction.

Whenever we have this d/d ratio beyond certain points only then this correction factor becomes important, otherwise it is not going to be an important contribution to the spring constant. So that comes because of the vertical shear force due to the force F.

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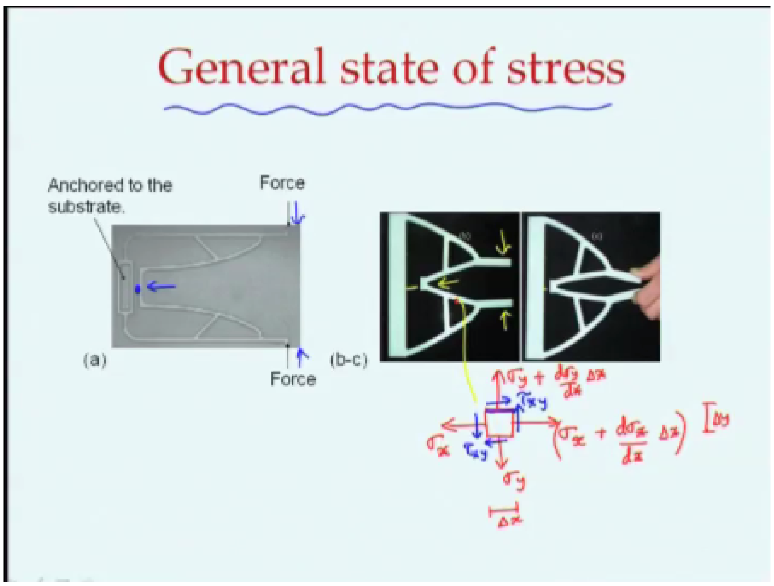

We have illustrated the affect of torsion and twist in Microsystems and also did a generic example of a helical compression spring that we see all around us including our ballpoint pen; we have derived an expression and revisiting Castigliano's theorem to apply it and find a formula. Now we have also introduced a process a concept of shear stress.

We have talked about normal stress quite a lot, today we have introduced concept of shear stress whose affect is only change the shape of a body to shear it where there is a force parallel to the surface is oppose normal stress where the force normal to the surface. Having discussed all of that today we can make an attempt to consider a general state of stress, instead of thinking about bodies let us look at only few dimensional bodies.

A Microsystems griper is shown here when you apply force here and here this portion will move, there is an object here a biological cell for example which can apply force on it, today number of people are looking at making this micro machine gripers where you can hold single a biological cells, cell such as a red blood cell, white blood cell or any number of cells that are there we can hold them and test them mechanical properties, understand the role of mechanical behavior or response in the assessment of the biological state of a cell.

So when you look at those devices let us say we have a macro version of that a large scale of version that is shown when I apply a force over let change the color of the ink, if we have apply a force over here, this portion is going to move as you can see this gap that you see has reduced to a small one (()) (47:37) force, how do you analyze this, what happens any of the structure.

If I take an element over here, okay an element of such a structure what will be the state of stress? I have a taken a small segment just imagine a point where there is a small square what kind of stress will be there, there will be a normal stress, okay in a x direction as well as y direction; there will be a sigma x here and there will be a sigma x plus a little bit more because stress if you assume that it is not constant which is true for a structure like this.

We have taken a small one at one side if it is sigma x other side you say that there is a little bit of variation which we say d sigma/dx times delta x; where we say the size of the square is delta x that way and similarly, the size of this in that direction is a delta y, okay that will be the stress on this side and let us say if I say this is sigma y and this will be sigma y+d sigma y/dx times delta x.

In addition to this 2 normal stresses what we have shown, there will also be a shear stress, a shear stress that will act on it like this and also like this that we denote by Tau xy, Tau xy means that its x the force acts in the x direction that is this direction and it happens to be on this surface whose normal is given by the y-axis. We have Tau xy the same Tau xy here. So that is the state of general stress a general state of stress acting in 2 dimensions.

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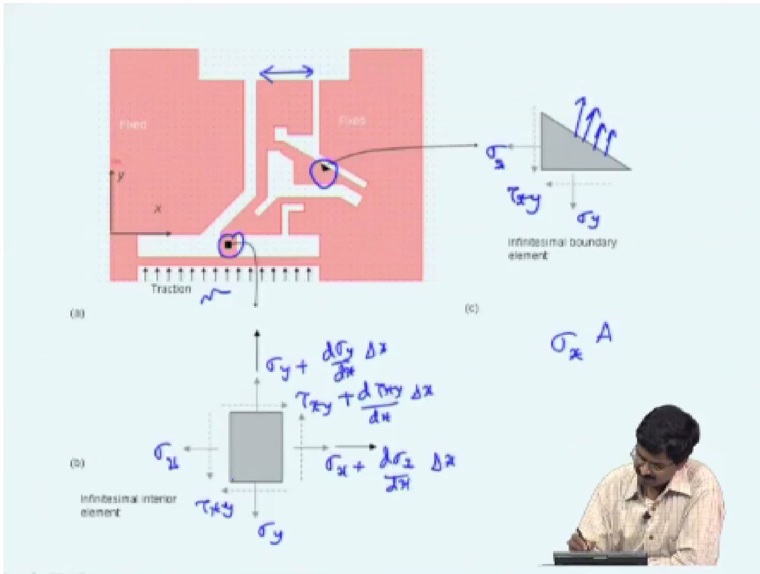

So if I take another structure, another micromechanical structure where is there is some force over here and its going to make this platform move this way or that way depending on force direction is this way or the other way. And if I take a small particle like this there will be sigma x and little bit more than sigma x+d sigma x/dx delta x and there will be sigma y as we just said sigma y+d sigma y/dx delta x and there will be Tau xy over here and this corner and this sides.

The other side we have moved in x direction and y direction that will be d Tau xy/dx delta x that one as well as this will be Tau xy/d Tau x/dy time delta y that is this one, okay. If you take all that and do the force balance because these are stress we can multiply by area, so sigma x I multiply by area and the other side I multiply sigma x+this by area and if I sum the force in x direction and y direction and do the moment, okay.

Then I get the equations which become our equations of equilibrium which we have taken this element, okay and one more thing is that what if there is an element on a boundary that become. This will have external torque, external force not torque applied on it and this side we will have sigma x, we will have sigma y and then have Tau xy, okay.

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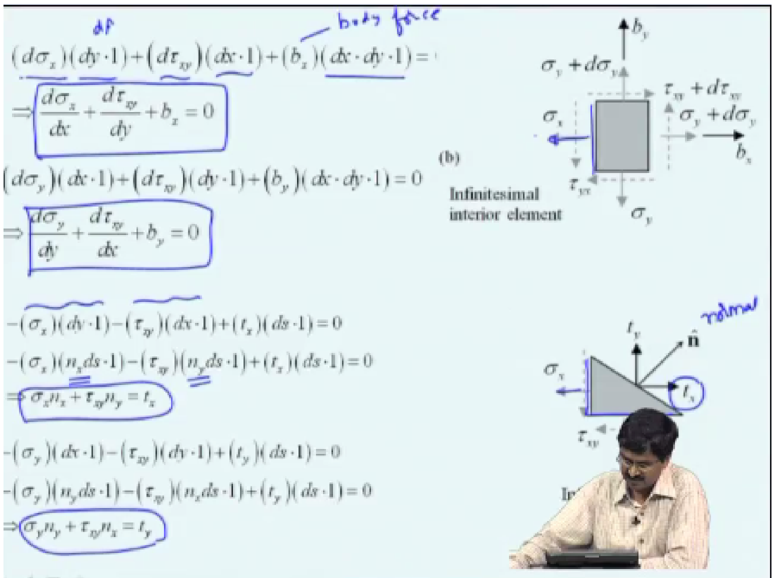

For both of those that is interior element and the boundary element if we sum up all these forces that is as we just said d sigma x/area of that area of this sigma x is acting over this over here, this height is divide, and if you assume unit thickness into the thickness of this sheet or a 2-dimensional structures that becomes dA, so force times stress is equal to sorry—

Force, this is the area times stress = force similarly, stress times area = force in x direction, if you sum up all that; if you also assume that there is a body force this BF is Body Force-- Body Force is like gravity or centrifugal force and forces like that which act at every point inside that body that is why it is called body force, that will be if I say force per unit area or unit volume we take dx dy and unit thickness that gives us the dx.

If you do that we get an equation of motion for the x-axis. Similarly, if you do that for the y-axis you get that, we will do further wedge element so you take the force sigma x here sigma x times dy times 1unit thickness that is the force due to that and there is also a shear stress which is active over dx time it is one that is dx for us that is this that is acting this way.

Let us also – here, - here and then this external force that is acting which we denoted by t the component dx that is shown here that acts over a surface ds that is what we indicated this is dx with Castigliano's theorem that will be square root of dx square+dy square and if you introduce this direction cosines nx, ny; nx is the cosine of the angle between the x-axis and this dx that is normal to the surface over there.

And this is the cosine of the angle between y-axis a normal to this is a normal to the wedge at the surface point, okay if you do that we get this third equation which is the boundary condition similarly, if you do same thing y-axis you will get further boundary another, if you put together all of them we get the equations of static equilibrium for general state of stress. These are equations of static equilibrium in 2 dimensions.

Note that these 2 we got by writing the force balance the x and y directions for an element which is interior and these are the force balance equation we wrote for element that is on the boundary like a wedge element; this for the interior; this for the boundary. These are the equations that we solve in order to analyze a structure such as this or a structure such as this; a structure such as this.

Any stress as we imagine we can solve them but of course we have to use numerical methods such as finite element analysis or boundary element method for solving these things; whereas the bars, beams and twisting of the shaft we can do analytically by writing down formulae.

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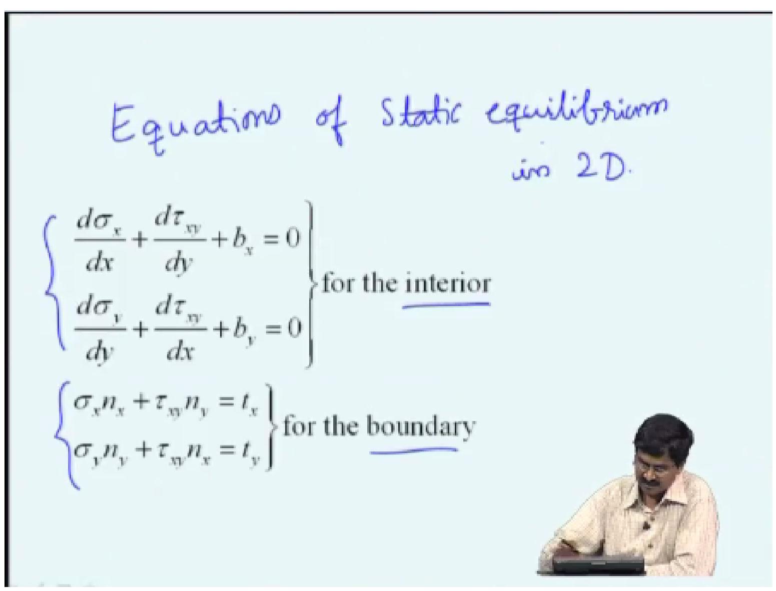

And let us also discuss, now that we have discussed static equilibrium that came from the balancing of the forces let us see the other concept which we have always said we need only 2 things when it comes to static of elastic bodies one static equilibrium balancing of forces and the other is Hooke's Law. What is equivalent of Hooke's Law for the general state of stress?

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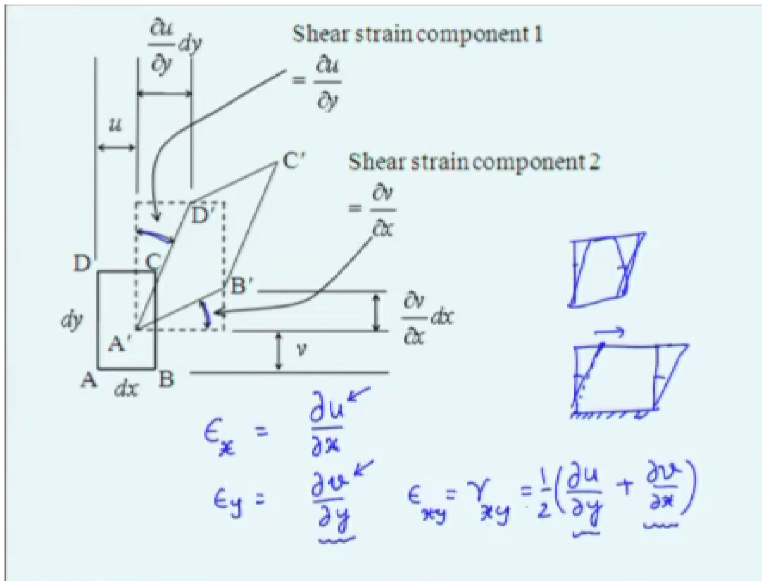

So for that we also need to define strains that we have already visited. We have the strain for the x-axis and y-axis which are normal strain and there also a shear strain where the element such as this gets sheared to that one then we say this angles are the shear let us draw it properly, if I have that let us I hold this fixed and apply a force as we have done today if this point moves here this point moves here, okay this angle and this angle are the shear strain, now we are divided into this portion and this portion.

So we can actually write the definition of this strains here, so epsilon x that is the normal strain in the x-axis is given by dou u/dou x and normal strain in the y-axis is dou v/dou y, u is the displacement in the x-axis, v the displace in the y-axis then the shear strain which you denote by gamma xy or some people write as epsilon xy also that is given by dou u/dou y+dou v/dou x and half of that.

See that this is dou u/dou x, here it is dou u/dou y it is dou v/dou y, here it is dou v/dou x half of it the shear strain string.

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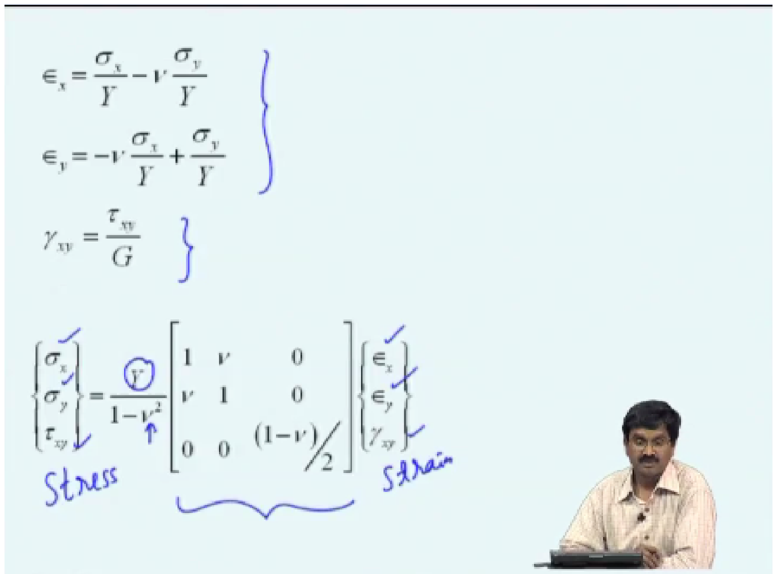

And what relates this stress and strain the Hooke's Law we have, we had already seen this in the context of defining Poisson's ratio and today we have defined this Hooke's Law for shear. When we put these things together we get relationship where there is general state of stress into the general state of strain that is normal stress, normal stress, normal strain, normal strain, shear stress, shear strain this becomes the equivalent of Young's modulus.

There is Young's modulus Y and Poisson's ratio Nu for a isotropic material. And similarly, we can do it for the 3-dimensions also. But of course the equations equilibrium that we just saw over here these have to be solved numerically, so we have to go for numerical solution when the geometry is complex for that finite element analysis is absolutely necessary.

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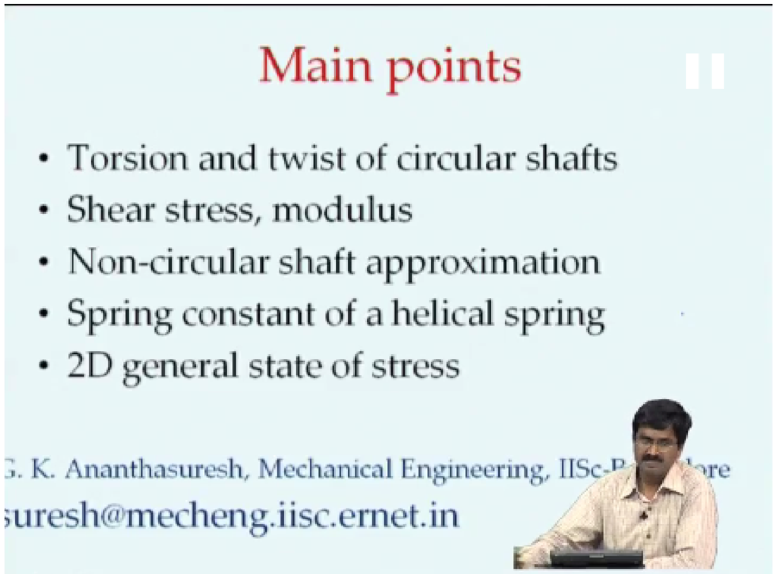

Let us just capture the main points of what we have discussed today. We have discussed Torsion and Twist of circular shafts and introduced the concept of shear stress and shear modulus, and we said people use an approximation for non-circular shaft and we also did an example to derive this spring constant of a helical spring in the concept of torsion we have learnt, and we also finally discussed the general state of stress for 2 dimensional bodies.

If you have any questions, you can send me an email at suresh@mecheng.iisc.ernet.in. Thank you.