Micro and Smart Systems
Prof. G. K. Ananthasuresh
Department of Mechanical Engineering
Indian Institute of Science – Bangalore

Lecture - 19
Vibrations of Microsystems Devices: Part - 1

Hello, as part of the Micro and Smart Systems course today, we will discuss vibrations, the basics of vibrations has it relates to micro systems.

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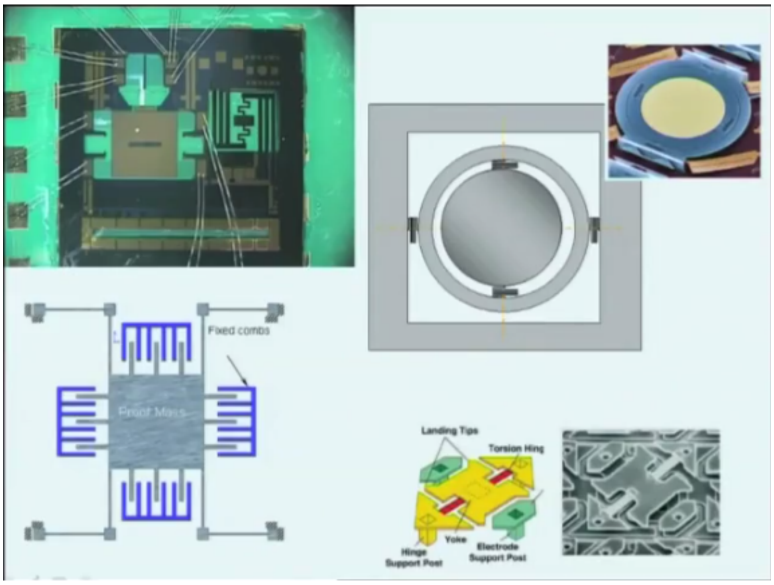

Let us consider micro systems that are shown here. The one on the top left corner is an accelerometer chip. You see a mass where it is suspended by certain beams which we have analyzed for static deflection in our earlier lectures. Now, let look at this as a dynamic device because whenever there is acceleration it is going to move which we called vibration. Any motion that is periodic in nature can be called a vibration.

Does not need to be exactly periodic but it is a reciprocating motion. Something goes back and forth like what a pendulum does. When you have such a system we want to ask a number of questions about those systems, at what frequency do they vibrate? And what is their frequency response? What is their time constant? And how much time do they take to reach steady state and if it is a sensor such as the accelerometer, what is the bandwidth of that sensor?

What is the response time of that sensor? And many such questions. Below here we have a proof mask suspended by again beams here 4 sets of L shaped beams and this one acts like gyroscope which we will discuss in the next lecture but here again we have a vibrating mask which is connected to these beams which are characterized by springs and in the lower left corner this one, we have this torsion mirror which twists about this beam.

Which is this red colored theme on either side, this yellow part can twist the H shaped structure can twist about this axis that also is like a vibratory motion because something will be tilting back and forth. And what we show in the top right corner is another kind of mirror. It is a real device that we discussed in one of our earlier lectures where this is a mirror that can tilt about this axis as well as this axis. The 2 axis mirror.

All these are examples of devices that undergo vibrations or small motions about a certain configuration that is going back and forth oscillations. That is what we are going to discuss today. So having seen these devices they look quite complicated because there are lot of elements here as well as here and they twisting of the non-circular shaft and then here a detail that you cannot see. But it is a serpentine beam.

This little one is serpentine beam which makes certain rotation stiffness about this axis to give this mirror 2 axis rotation capability. They all look complex but when we analyze them we try to abstract then as simple as possible and then consider the basics of vibration and then we will come back to these devices and then see how we can analyze them.

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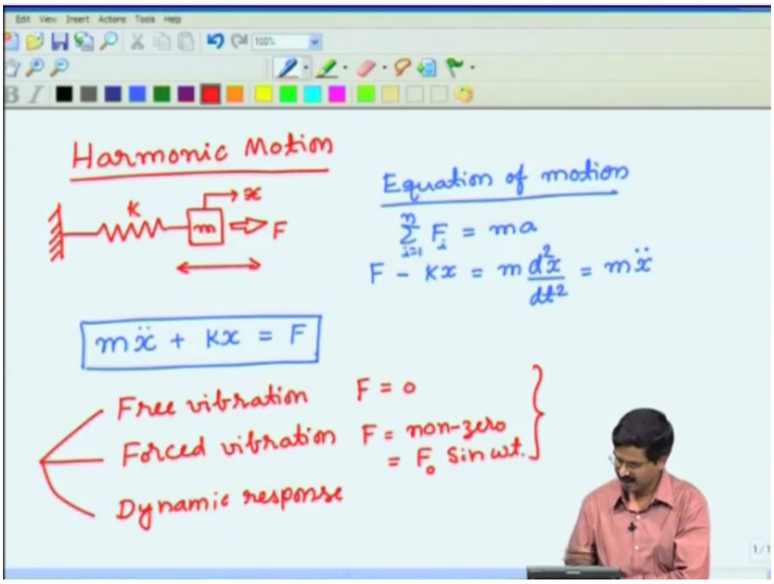

So let us start with some basics of vibration and there we have to begin with this concept of harmonic motion. Harmonic motion that is our starting point for vibration. What do we mean by harmonic motion or simple harmonic motion, SHM as we call it. Does not it be always simple but we will start with this basic harmonic motion where we start with a spring which is attach to a mass and let say this can vibrate on along this line here.

We will measure the displacement of this with x, okay and let say that there is a force acting on this mass that is called that F. Now, when we write the equation of motion for this one we have to invoke Newton's second law. Let us write equation of motion for this simple spring and mass. Let us say spring constant is K and mass is M. So if you want to write the equation of motion for this device we use summation of forces = mass times acceleration.

That is newton's second law. So what are the force acting on this? There is of course the force F that we have apply, these are all the forces FI, I = 1 to N, here we have F here and also this spring force kx. That is in the negative direction because when I pull the mass to the right the spring will extent and it wants to go back to its original state and that is why it will apply a force in the negative direction, negative x direction. If I call this as a x direction.

So that force magnitude is kx that is - = m times acceleration. Acceleration as we know is the second derivative of the position x with respect to time. D square x/dt square which in short form will denote as x. x double dot, x dot is velocity x double dot is acceleration. Now the equation for rewrite we will have F is equal to or that is actually write it as first on the left hand side. We will write mx double dot+ kx = F.

So this is the equation of motion for a simple spring mass system, okay. Now depending on what this F is we will get different types of vibrations or motion that we describe. The first one what we call free vibration is, free vibrations that is when this F = 0. That is there is no force and yet it will vibrate if there is a disturbance. If you leave spring and mass just like that it is not going to move.

But if you displace the mass a little bit and leave no force after that will oscillate or vibrate that is called free vibration. The absence of the force external force whatever vibration we get is called free vibration. And the next one is called Forced vibration where F will be non 0. F =

some non 0 function, okay. Normally, we keep this as some F0 some force with a magnitude time's sin omega t where omega is a frequency of that force.

And the third one is general dynamic response. It is not a vibration per say being that if I have a block there and have this spring if I apply the force if the force is large it will keep on going and especially when k = 0 the body would start moving that is the dynamic response of the system where we take an object or if draw a projectile, take a stone and throw it the motion of that stone is determined by an equation.

Again that equation will be similar to this in fact it will be the same except that you put that force that is over there and it will not result is oscillations not vibrations, okay. So today we are going to look at these 2. The free vibration and forced vibration, okay.

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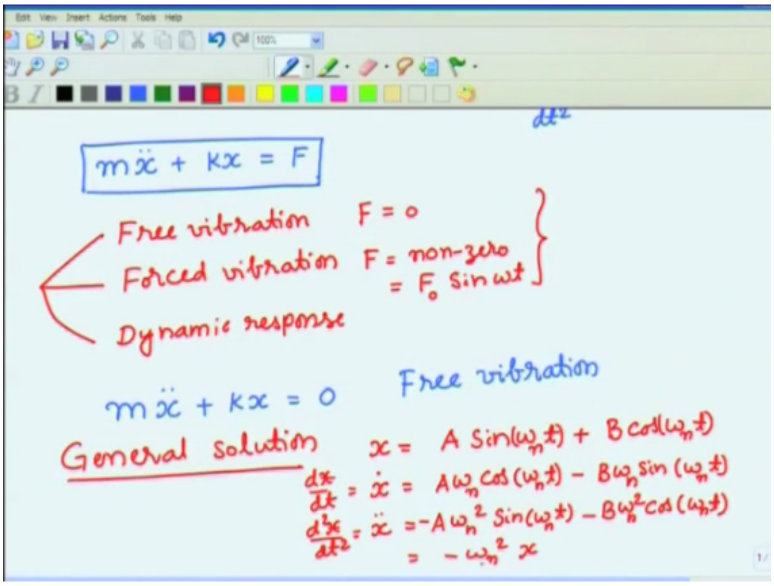

So, let us look at this free vibration. So we go back to the equation and then say mx double dot+kx = 0. We are considering the case which we called free vibration where there is no force. If you look at this equation, it immediately strikes to us that the solutions of sin and cosine are going to satisfy this equation. So the solutions of this equation has a general form of having sin or cosine.

Because if I take sin, if I take one derivative it will become cosine if I take another derivative, second derivative it will become – sin. So that is what we have here. We have mx double dot+kx. If x and x double dot have opposite sins we can satisfy this equation. So we write a general solution of this equation is x = A sin omega nt B cosine omega nt. So sin omega nt cosine omega nt. You could have just put sin t and cos t.

But then we know that if it is periodic there is a omega or any other constant it will still satisfy this equation. That is why you call it general solution. In order to see what is omega n is let us take derivative of this x that is I write x. which is dx/dt. It will become A omega n cosine omega nt - B omega n sin omega nt. That because derivative of sin is simply cosine where is derivative of cosine is – sin.

And then we are taking derivative in spite of omega nt the omega n will also get multiplied. Now if I take another derivative x double dot that is d square x/dt square is A now because cosine we are doing it will become negative omega n square sin omega nt – B now sin becomes cosine with – n remains omega n square cosine omega nt. So what we got is – omega n square times x itself.

Because that we started out both are minus if we take – omega n square out we get that back.

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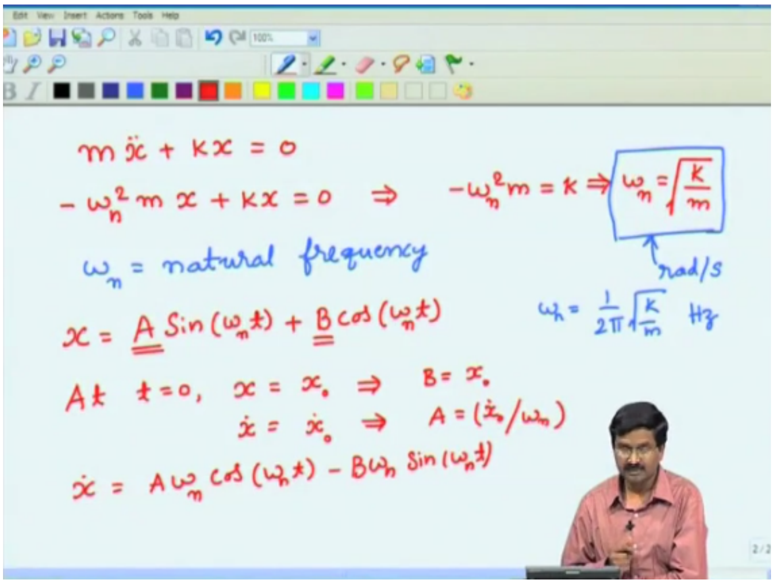

So what we have now is if we go back to our original equation mx double dot+kx = 0. If we substitute what we just cut that is instead of x double dot, we substitute – omega n square and of course m will be there x+kx = 0 and x is not 0 in general. So, what we will get is that – omega n square m = k that means that omega n = square root of k over m, okay. And this omega n is called natural frequency of this spring mass model.

There is only one natural frequency here because this mass has only 1 degree of freedom, just moving back and forth. If I take 2 masses there will be 2 frequencies and so forth. What we are considered is a 1 degree of freedom vibrating system which has natural frequency given by square root of k over m. The units of this are going to be in radiance per second. If you want it in hertz we have to divide by 1 over 2 pi.

So that is will become omega n = 1 over 2 pi square root of k over m hertz. That is cycles per second, so many cycles per second. Okay, now we have the natural frequency define that is the fundamental characteristic of a vibrating system that is if there is a system if you pert a little bit it will vibrate at a certain frequency that frequency is called natural frequency. These are free vibration.

Now, if you want determine this constants that we had if you recall we had an x = A sin not cosine we had used A sin omega nt+B cosine omega nt. Now we want to determine this constants A and B for which we need the initial conditions of this system. At t = 0, wherever you start as a reference time if we know what x is and what x. is then you can determine 2 constants with these 2 conditions.

Let us say x at t0 is x0 and that at velocity at time t = 0 is x.0 then we will see immediately that if I say t = 0 this will vanish this will give us that B = x0 because cosine when t = 0 is 1. So if B is x0 that will tally and similarly A will be given by. Now I have to take derivative of this that will become x =, so let us just write it x. is A omega n sin omega nt+actually it is – because it is a cosine – B omega n sin omega n, sorry.

This Will become that is correct but this is the one that will become cosine since we are taking derivative sin becomes cosine this will be there and t = 0 this term will go to 0 and this will remain that will have A omega n so A is going to be x.0 divided by omega n. So we can evaluate the constants. Then we get general solution of this free vibrating system.

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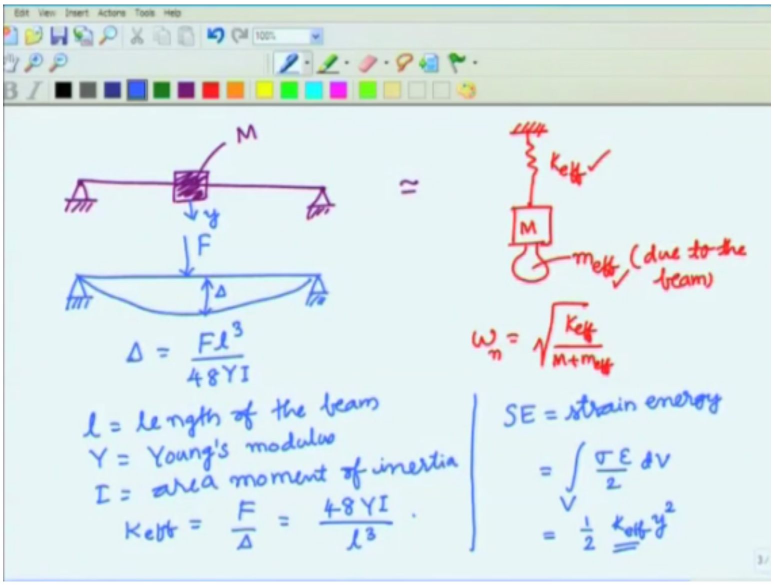

Now, let us look at what happens the case when we have a more complicated system that is a simple spring mass system instead let us say we have actually a beam. Let us say we have a beam that is hinged the 2 points but in the center let us say there is a mass, okay, this as a mass M. This is a typical situation in micro system devices where you have a proof mass and then we have suspension beams. How do I know the natural frequency of this system?

If we were to be just 1-degree type of system meaning that if we were to be able to approximate this as what we have done earlier that we have this mass which when is restrained by a spring if I can get this k effective and also the mass M we have but then there will be an additional mass which we can say something like that added to it which we will say m effective. That is this m effective is due to the beam.

The beam also has a mass this is due to the beam and m is the mass that we have already. Now both of them will contribute towards the mass of the system or a general we call it inertia. Inertia is something that an object has because of its mass and that is moving. So, if we can get this m and m effective and Keff we can again write the natural frequency of this in radiance per second as square root of K effective divided by M+m effective.

So we need to find this and this. We already discussed how to find this k effective. That is we look at the for example if you take the beam theory, it is a beam here if there is a force in the middle how much does this beam deflect. Once we know the deflection we can get the K effective. That is let us write it here, if I were to have this beam that this we have already discussed we are doing it again.

If there is a hinged-hinged beam with a force F acting here whatever that force is when it deflects like this, this deflection if I call that delta, the delta is given by this F times l cube divided by 48EI. Where l is the length of the beam and E is actually we have been using Y so that there is no confusion with electrical field that we will encounter later in this course. So we are using Y for young's modulus and I is the area moment of inertia, okay that is the thing.

Now, if I want to know k effective is simply F/Delta. If I do that I get this as 48 YI divided by l cube. Now, how do you do M effective? Before that let us actually see what is the basis for getting the k effective this way? And the basis really is if I look at this strain energy, a concept that we had described in one of our earlier lectures in the context of applying Castigliano's theorems and other energy theorems.

If I have this strain energy which is defined as integral of stress time strain half of that integrate over the entire volume of the elastic body. In this case the beam. If I do this and equate the strain energy to half this k effective times motion of just this point. Now let us call this motion of this mass as Y, okay, then I simply say Y square. As the spring is moving as pre-extension is Y, okay.

That is how we obtain this K effective that we have shown here. In other words, half k effective Y square is = the strain energy stored in the continuous beam. Similarly, when you want to get the effective mass we would try to get effective mass such that half m effective times Y. square that is the kinetic energy. If I have a mass that is move with velocity V its kinetic energy is half mv square.

Here that velocity is time derivative of this Y, okay. So if I know that y half m effective y. square should equal to kinetic energy of the whole system. So whatever we have done now for the strain energy let us do this for the kinetic energy or so.

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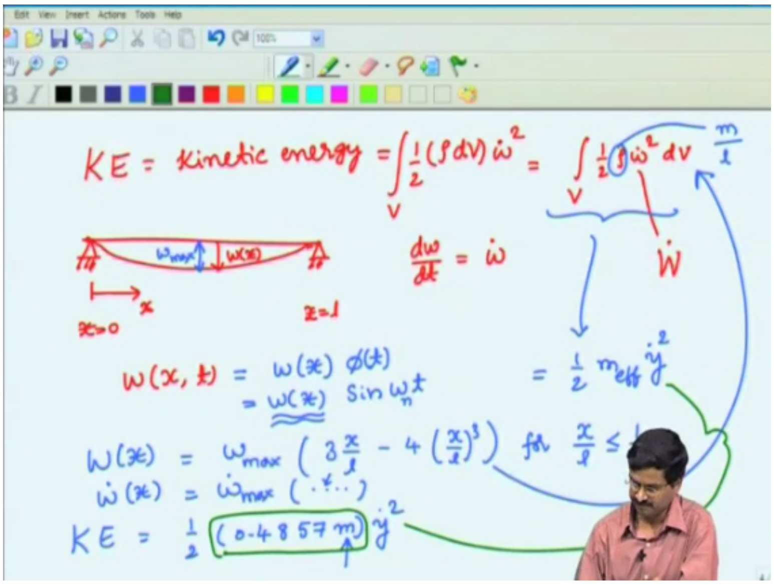

So the kinetic energy which we denote by KE kinetic energy =, if I take a small particle with volume dv and density row that is its total mass that mass multiplied by y. square, okay where this Y could be the extension. So let us write it generally, okay if I take the, go back to the beam, we have this beam which is hinged here and here. Now when its starts vibrating that way this way whichever way it wants to vibrate.

When you have that let us denote this as wx where x goes from here x = 0 here, x = l here then wx is displacement and dw/dt, w is the function of x and t. So, w is a function of x and t because every point is moving with respect to time and also it varies along x. So we can take derivative with respect to x as well as t. dw/dt is or we can call it wt is the velocity.

So if I take the kinetic energy for this I will write w. square and half will be there we have to integrate over the volume. You already have dv integrate the volume. In other words, what we have is half row w. square dv, okay. Not that this is not omega this is actually w. So this one is w. okay, that is the velocity of each point in this beam. Now, this kinetic energy that we have here this kinetic energy we want to equate it to half m effective times y. square.

Y. if you recall is simply the motion of this mass and of course the beam at that mid-point, okay. So that is what we would like to have. So how do we get this w of x? Here if take that and I integrate this whatever I get if I can put in this form where m effective can be obtained. So now this w of x that we are talking about here can be written, I am not writing the time dependents, just writing the x dependents.

Because we can write this w of x as something that depends only on x and then something that depends only on time. What depends on time usually will be in the case of vibrations a simple harmonic motion or harmonic motion sin omega t. Omega can be anything. Omega n it can be omega or any other frequency. So we only look at the x dependents because that is what need here in order to integrate.

So w of x here will have this form which is w max which in this case happens to get the mid-point that is your w max times 3 x/l – 4 x/l whole cube for x/l <= to half, okay. So we can put this w of x into this integrant so this one goes into this integral then we get that KE here which turns out to be when you do the integration. By the way this w max is nothing but what we called y that we have here, okay.

So here we need to put w. then we take this wx when we take the derivative of this, this depend only on x or w max keeps changing so w. of x which we need will be w max. that is dw max/dt times at same thing. Whatever you have here the same thing will come down. So w. here is w max. times something depends on x we substitute over there and notice that w max. is nothing but what we called y. here.

Because if you go back again you see that that is the motion of that mid-point. So now what we get this integration will be 2, we get it as half 0.4857 m*y. square. Where does this m come from? This m comes from this row that is a mass per unit volume this quantity row is the mass of this per unit length because we do the area also is taken. If I say this row is m/l*A this A and this a will get cancelled.

That is A that comes over here and that A that is there in dv both if I cancel I will just have m/l. So that m is coming because of that. Now, if you take this KE and see that it is actually should be equal to this then what we see is this, this quantity here becomes our m effective. So m effective is 0.4857 times the mass of the beam.

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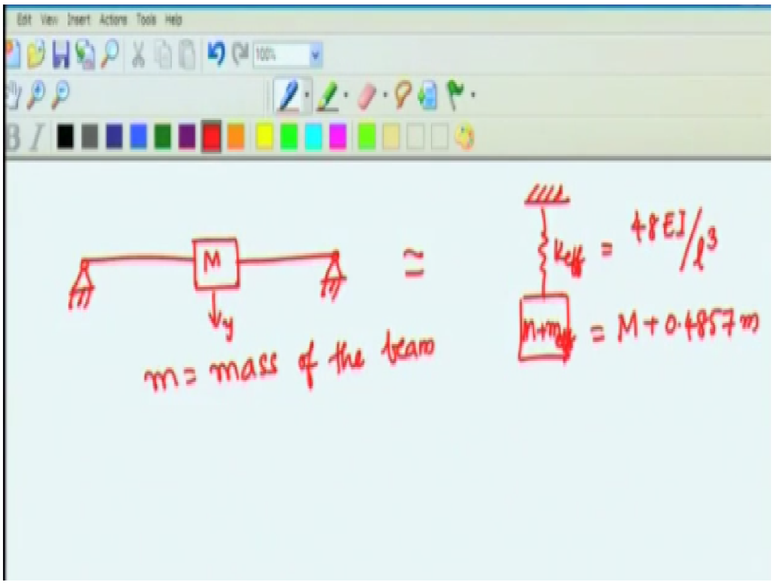

So if we go back to our primary question if I have a beam with some mass attached to it that mass is capital M that we have like a proof mass and accelerometer or gyroscope or any other main structure if it is pin here and pin here then this becomes equal to A spring with K effective and 2 things here M+m effective this is k effective which you have already got in it as 48EI/l cube and this now we get M+0.4857m, where m is the mass of the beam.

So not all of the mass is moving as much as this Y. that is motion we said Y, okay Y. is dy/dt. This mass here is not moving at all this mass is moving little bit and this is moving a lot at the centre that is why only less than half of that mass of the beam becomes effective mass. So, this is a simple technique that we can reduce any vibrating system however complicated it is.

If you want to reduce it to 1 degree of freedom that is just 1x then you have to find the equivalent mass. This we can do for all the complicated things that we see here. So, if I take let us say this gyroscope in fact will have 2 degrees of freedom meaning that it can move in the X direction are Y direction. So, you have to go back. So let me get a pen here, okay.

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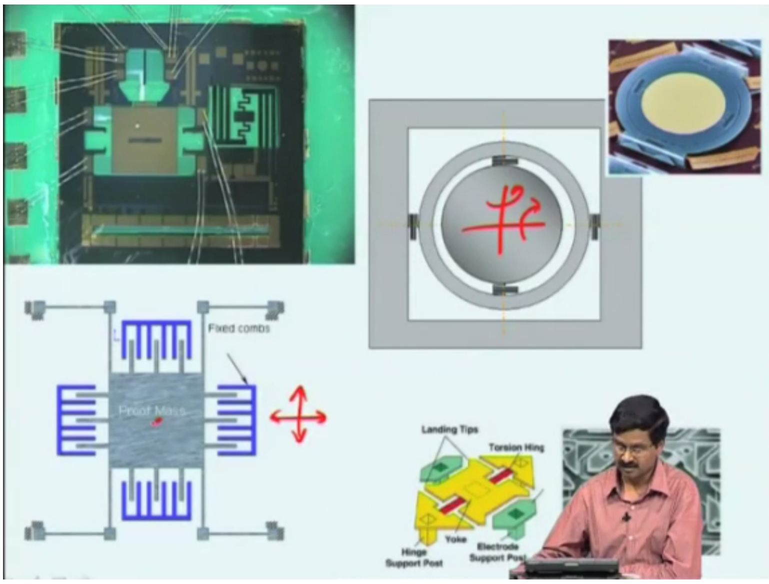

If it vibrates in this direction this profile in this direction are it vibrates in this direction, then we can reduce it to something like this.

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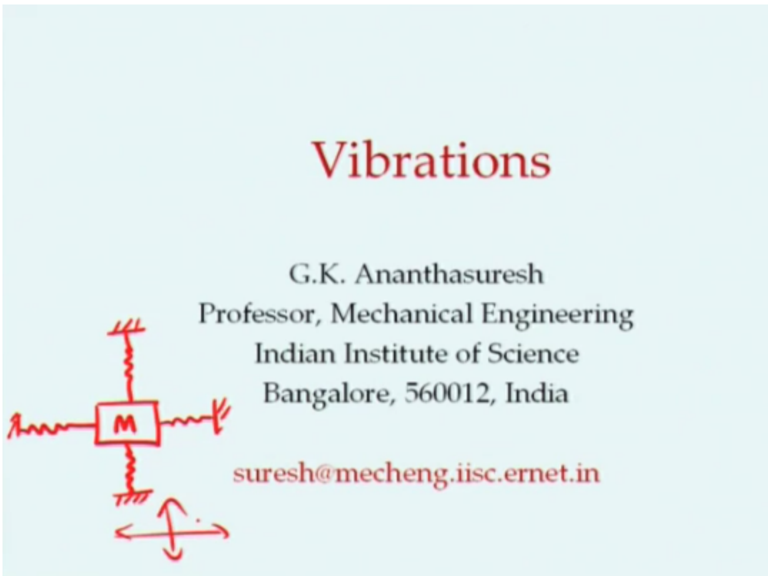

We can reduce that to, here is the mass M with a spring here and a spring there and a spring here and a spring there, okay. Now this can move in both X directions like this or like this, okay. Similarly, if we take this that can reduce single degree of freedom this can reduce 2 degree because this can oscillate about that axis as well this axis. Either way there is a resistance for it is a spring torsion spring which we discussed in the last lecture.

And that can be taken into account again using this method. The idea is we have to find m effective such that half times m effective times the velocity square of the point that we are considering in this case could be centre of the proof mass that should be equal to total kinetic energy of this system and k effective we takes strain energy and get the equivalent lumped constants, okay.

So, now let us go back to our discussion where we want to find the natural frequency of this. So, now natural frequency omega n is simply is going to be equal to k or m meaning effective k effective divide by M+m effective because we have the additional mass of the centre that m goes in there and that is how we get the natural frequency of this system like wise we can do this for any other system that we have noticed here, okay.

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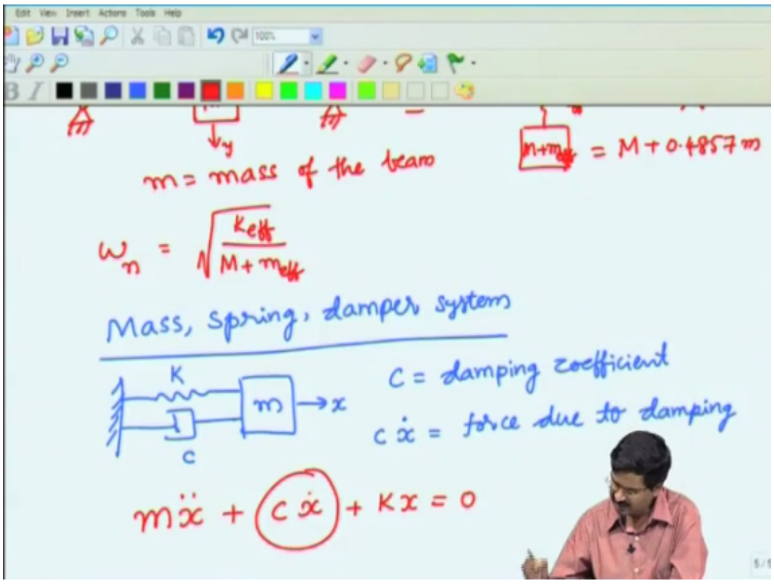

Now, let us move to a second level where in dynamics systems we have inertia, the mass and we also have the stiffness K that is what we are considered Spring Mass System. Now, we moved to another level where we say mass, spring the 2 things that we have already discussed damper system. All of us are familiar with the concept of damping which is something that tries to oppose the motion or retards the motion.

That is the better word, retards the motion. If there is friction if I am moving it will resist my movement. Similarly, if I have a pendulum hanging when I set into oscillation if you come back after one hour and see oscillation that you have setup has disappeared. It has disappeared because it has damped out this oscillation what has damped out in the case of pendulum it is the viscous damping that is the air damping around.

Similarly, if I put a ball in water if I push it will keep on going up and down like this after a while it would have stabilized. Again here the water viscous damping that is acting on the ball would have reduced it. So, now let us consider those systems where we have a spring restraining our mass but also we have another restraint which is the damper. So, spring constant K the damping coefficient we put it as C.

So, C is damping coefficient. The case stiffness m is inertia or mass c is the damping coefficient. Now what happens if I take free vibration of this that is I have taken this mass moved it by certain amount and left it then what happens. So, for that the equation of motion now will involve the effect of damping and this damping the force due to damping is cx dot force due to damping.

So, if I again take this as x here it is moving c times dx/dt is the damping. If that is the force, we can write the equation here whatever we had earlier mx double dot+cx dot that is the force due to damping +Kx = 0, okay. When you want to move this here we know that inertia force when I move in this direction the force that will act in the other direction will be m times x double dot that is acceleration ma = f, right that mx double dot and the damping is also put in this way kx also put it this way.

These are the 3 terms that try to balance on the force that is applied on the mass. That is true in general. Inertia force, damping force, spring force all of these are going to try to pull you back from the direction in which you are applying the force. Now how do you solve this equation? When this term was not there we just saw that sin and cosine where the solutions but if they are there now we need to find the solution.

For this the easiest way is to take the Laplace transform this equation.

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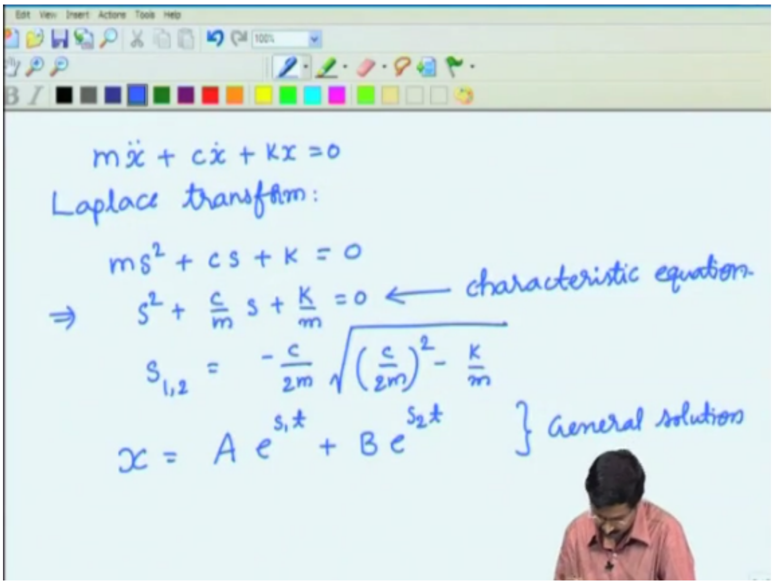

Let us write the equation again, mx double dot+ cx dot +kx = 0 if I take Laplace transform. I hope you are familiar with the concept of Laplace transform. If I take Laplace transform then I will get this as ms square+cs+k = 0 or I will try to make this just s square dividing by m = 0. It is a quadratic equation. Then there are 2 roots for it, S 1, 2 which will be - c/2m square root c/2m square – k over m.

Now, general solutions of this is going to be A times e raise to s1t+B times e raise to s2t. That is the general solution of this differential equation. If we substitute this back into this equation by taking x dot here and x double dot we will see that it will match up and give you the solution where s1, s2, are given by solution of this characteristic equation that we get when we take Laplace transform and of this equation. This is the characteristic equation, okay.

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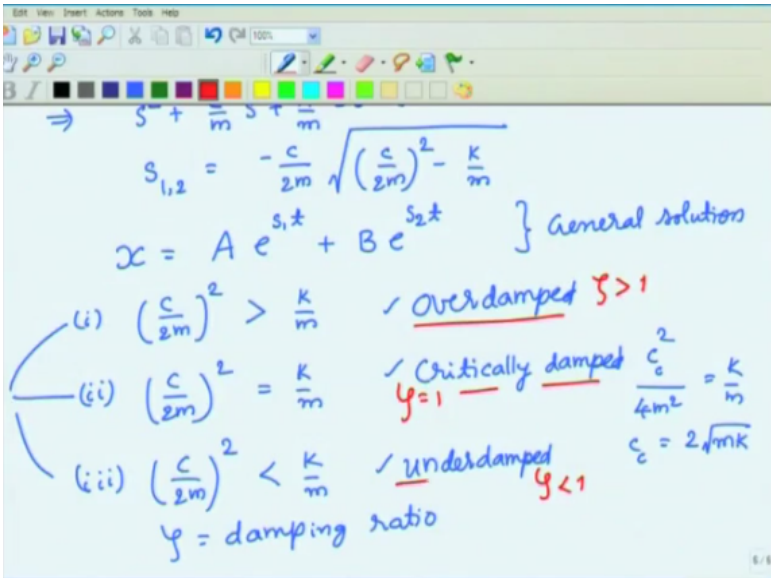

Now, if these are general solution there are certain things that we can say based on different conditions of damping. The first one is when c/2m square is > k over m we call it over damped, over damped oscillator. There is lot of damping over damped. Second one, is when c/2m square = km we call it critically damped state, okay. In fact in that case we can define a Cc which is Cc square/4m square, I am just squaring here that is = km.

So, Cc is going to be = 2 times square root of mk because one m another m this cancels and there will be mk and 4 with a square root will be this. The third case is where you can guess what the third case is. Simply that c/2m square is less than km which we say c is less that will be under damped, okay. We have 3 cases over damped, critically damped, under damped and for that we define something called a damping ration which we do not with this zeta.

So, here that zeta = 1. Here it will be < 1, here it will be > 1, okay. This damping ratio is an indicator of how much damping is there if there is lot of damping we call it over damped. If there is a just sufficient damping, sufficient for what that we will see something like critical in damped state. Other one if we have less damping we got it under damped state.

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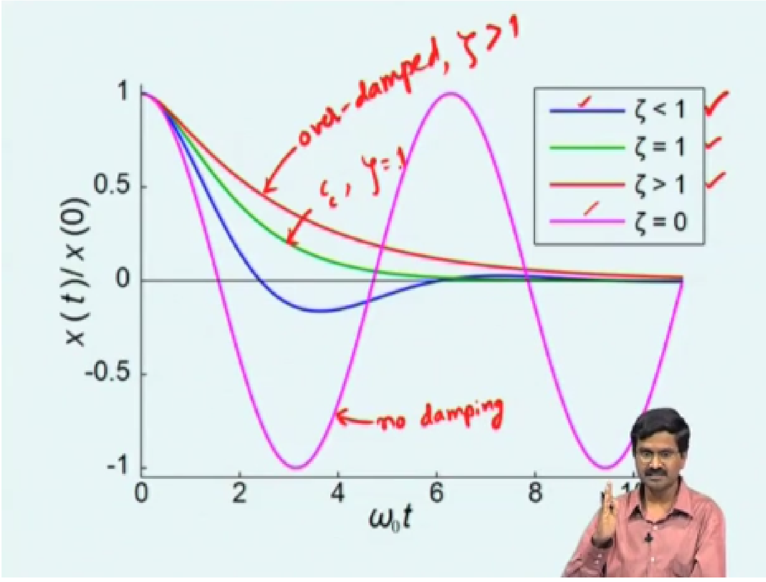

What does it mean? Let us look at this here, what we see here is that the first case when this zeta is = 0, we have this curve, this is no damping. So, this means that if I start out we are showing here time just multiply omega 0 does not matter just the time and here we have the ratio of xt at any time t divide by what it was at the beginning just normalized. So, if you start here it will just keep on oscillating.

That is if I have a pendulum you will keep on moving with the same amplitude that you set out with at the beginning if there is no damping it will just go on. Now, if take this case that is the blue curve it will go like this and then oscillate after a while die out. That is what have with the real pendulum. It you set in a large motion after a while it will reduced and reduced and finally it will just come to a rest.

Now, if you look at the case of critically damped you just comes down like this if it is over damped that is > 1 it comes down this way, okay. So, each of these as a different behaviour. So, the critical damp is the transition behaviour where the oscillations do not see. This is over damped where over zeta is > 1 that just goes without any oscillation. Whereas when it is < 1 that you see here for this blue curve and this magenta curve there will be oscillations.

Okay, when it is 0 which is this curve it will not reduce at all. Oscillation amplitude keeps this remains the same between over damped no oscillations to oscillation case that is our critical damping where that cc are we said zeta = 1. And that is what it means, if there is damping it will quickly stabilized if there is no damping it will keep on oscillating by the same amplitude that we set it out with.

So, now let us go back here and then try to see the significance of this damping in the dynamics systems that we encounter in micro systems, okay.

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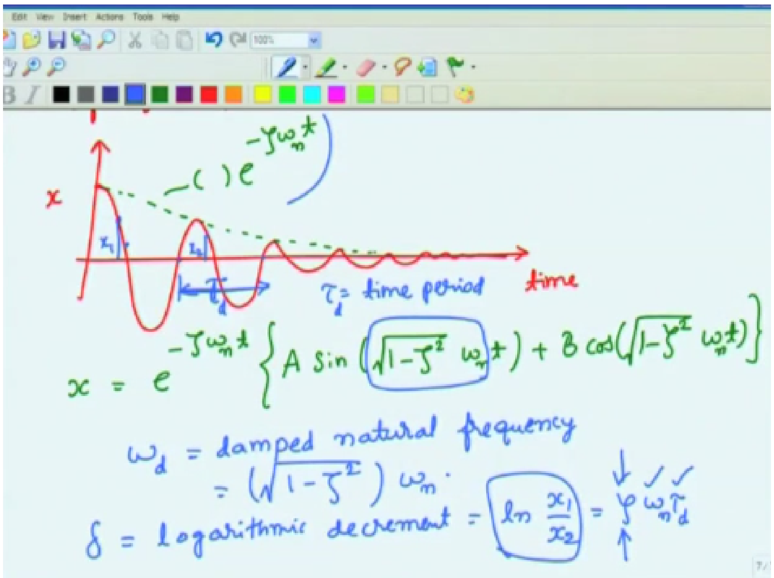

Now particularly, if we have this zeta > 1 if you say that zeta actually > 1 is not so intense with oscillations let us take the case where there is less than 1 that is the under damped case. Now as we saw in that figure if I start out this is x, this is time, if I start that x with some this time that is pulled it and left it then it will go and then it will go back. Now, if we look at this envelop here where the amplitude of this oscillate thing is decreasing, okay.

And that curve will have some constant times e raised to - zeta omega nt. We only talked about what omega n is natural frequency that will be exponentially it is decreasing the amplitude from here to here to here to here it is decreasing. That is what we see. So, now if I have to look at the solution, we have this s1 and s2 that we have derived here s1 and s2. If we substitute them back into this general solution that we have written which is over here we get something like this.

We get this x to be e raised to - zeta omega nt times A sin square root of 1 – zeta square times omega nt+B cosine square root of 1 – zeta square omega nt. Now previously we had only omega n but now we have omega n times this square root of 1 – zeta square this thing is called damped natural frequency. We have defined the concept natural frequency when there was no damper.

Now, we have a damper we call it damped natural frequency which is square root of 1 – damping coefficient square times omega n, okay. When this is 0 there is no damping it reduced to omega n. So, this is a general one. Now if you want to know what this damping coefficient is for a real system let us have a built in accelerometer and you want to know what the damping is. We will discuss that in one of the future lectures.

But now we will just see that you do not know how to model it but you want to know what it is. You can determine that from an experiment. If you take a micro system device you said the proof mass into some motion and if you can capture that motion you can measure the motion of something in various ways. The most direct way is to have a vibro-meter for example, will give you the motion as function of time.

If that function of time looks something like this then if you can measure at what rate the amplitude is decaying that is if you can measure this quantity then we can estimate what the damping coefficient is because we would know what omega n is natural frequency that is k effective m effective if we do square root of that ratio if you take that you will get omega n. Now we will want to know what the damping is.

For that what we need to know is or omega n experimentally you can just see what this time period is. So, if you take one oscillation let us say this 0 here again it is 0 here this one if we can measure that is your time period tau. Tau is the time period here inverse of tat we will give us omega n, okay. And now if you want to see actually, sorry this will be time period under damping.

So, we will say tau d because there is damping here otherwise it have been jut normal time period. Now it is tau d and we can take that tau d and try to determine what will be the omega d here and what will be this zeta. So, for that there is a technique where we call it delta which is logarithmic decrement because something that is going exponential has a logarithmic nature that is decrement is simply the natural logarithm of x1 and x2 that is x at 2 instance.

So, I take x there and x here. I take the ratio of these 2, take the logarithm that can be shown to be = again if you have x here valuated at some point and then you evaluate not immediately afterwards actually. If you take it here you have take a similar one over there.

So, if you take this is x1 you take here x2, okay. That is after one period. If you take then that can be shown to be = zeta times omega n times tau d. okay.

Now we know this because you can measure it and this logarithmic decrement we can measure it, right. All of this things we know then we can compute this zeta or damping coefficient. These are very useful thing because you have a vibrating system. If you can measure its free response when it is damped if you can get the time period which you can measure once you have a wave form like this and you can compute its natural frequency.

And if you can compute this logarithmic decrement that is x1 and at certain point and one cycle later if you can get x2 the difference of the ratio of this logarithm that if equated to these 2 this unknown thing becomes your damping coefficient you can compute. It is very useful in experiments.

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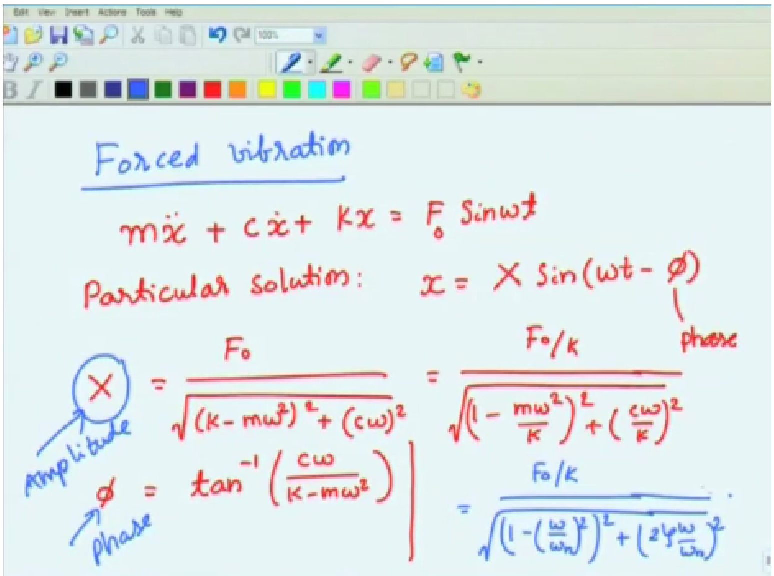

Now let us discuss the force vibration. So, we have forced vibration where we had the equation mx double dot+cx .+kx = F0 we will say =- sin omega t that is a periodic force. If we have that the general solution will be same as what we had earlier, okay. Now we can also write a particular solution for this force that we have x then will have whatever solution that we had earlier.

In addition to that we will get a particular solution which will be of this form where x is just some constant times sin omega t the same frequency - some phase phi. So, phi is called the phase difference. That is if there is a system that naturally vibrates at one frequency, if we have a force that is being applied at another frequency of solution will have that frequency natural frequency as well as a new one.

That is why there is a phase lag between the 2 that is the phase. The solution of this can be written in this fashion if I give you X and phi we are done because omega is already given. So, X here the amplitude is given by F0 divided by square root of k – m omega square square+c omega square and this phi which is the phase can be written as that or tan inverse c omega/k – m omega square, okay.

Now this part can be rewritten in a convenient way F0 divided by k square root of 1 – m omega square over k square+c omega/k square. We have basically divided by k both the numerator and denominator. The reason we do this is to see a pattern in this. F0 over k divided by square root of k omega n square, so we get 1 – omega/omega n square square+this portion turns out to be 2*zeta omega/omega n square.

So, this gives us a way of visualising this amplitude X as omega varies. This is the amplitude and phi amplitude and this is the phase. We are interested in the amplitude if we plot that using this expression here we get something like this.

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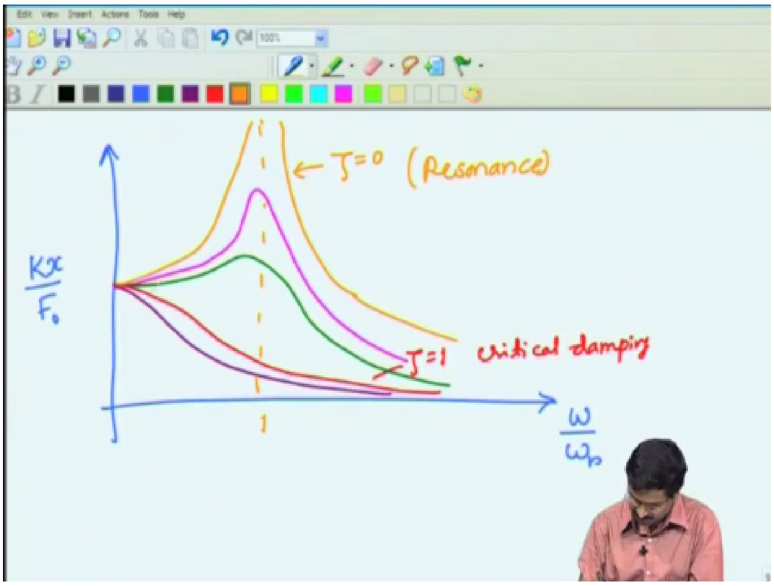

Let us say I plot here omega/omega n and here I have kx divided by F0, okay because that is what we are interested in. If I plot this kx/F0, if I take this that is if I go back this F0 will go to denominator k will go numerator. That is kx divided by F0 the one over this quantity that is the denominator of this quantity. If we plot that it is going to look like this for various values of damping coefficient.

When damping coefficient is critically damped one if I start here let us say that it is = some value that we have started out with it will go like this. Now if I take it to be slightly larger lesser this is the, this = 1 that is a critically damped case. Critical damping it is going to look like that and if it is more than critical damping then it would go like this. If it is less than critical damping then we will see a behaviour like this.

If it is further less it will be like this and if it is 0 it will be go like this. Okay, this is the case where zeta = to 0 no damping all of these peaks occur at omega = to omega n are in this part it will just be 1, okay. That is at the natural frequency, when the applied frequency is = natural frequency we see something like this. That is it will go to resonance. That is the condition of resonance which we tried to avoid or in some micro system devices we also take advantage of it.

Now if I were to zoom in this portion, okay then we would see that is look at this in a plot, okay.

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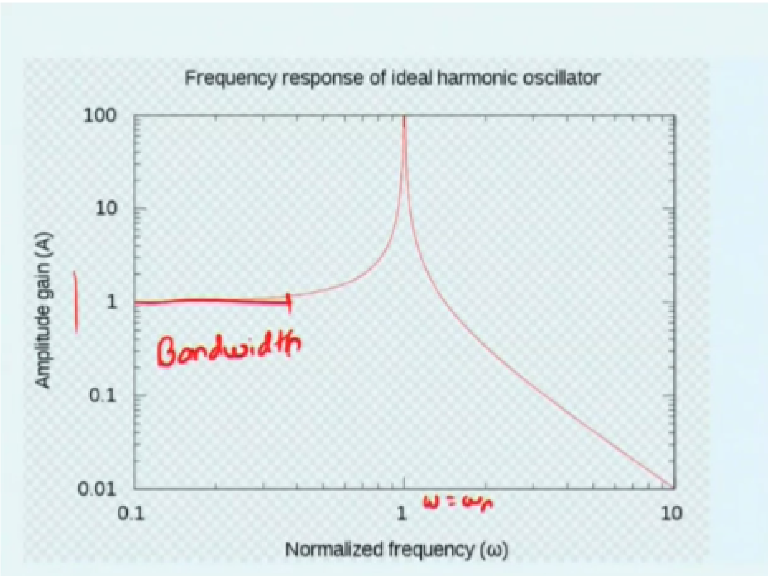

Now, we are looking at the frequency in amplitude on this axis what we see here is that at this natural frequency that is here is where omega = omega n we have a peak and this is linear and also noticed that this is on a log plot, okay this is log plot. 0.01, 0.1, 1, 10 and 100 over a region it is constant. That means that the amplitude here is independent of the frequency and that is what we call it as bandwidth.

So, whenever you have an oscillator spring mass oscillator that is the bandwidth. So, we will discuss this bandwidth and how it relates to sensors in the next lecture. Thank you.