Micro and Smart Systems
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Lecture - 20
Vibrations of Microsystems Devices: Part -2 Micromachined Gyroscopes: Part - 1

Hello, in the last lecture we were talking about vibrations as it applies to micro systems because this is part of the micro and smart systems course. We were looking at vibrations which are everywhere and now we want to see their relevance in micro systems.

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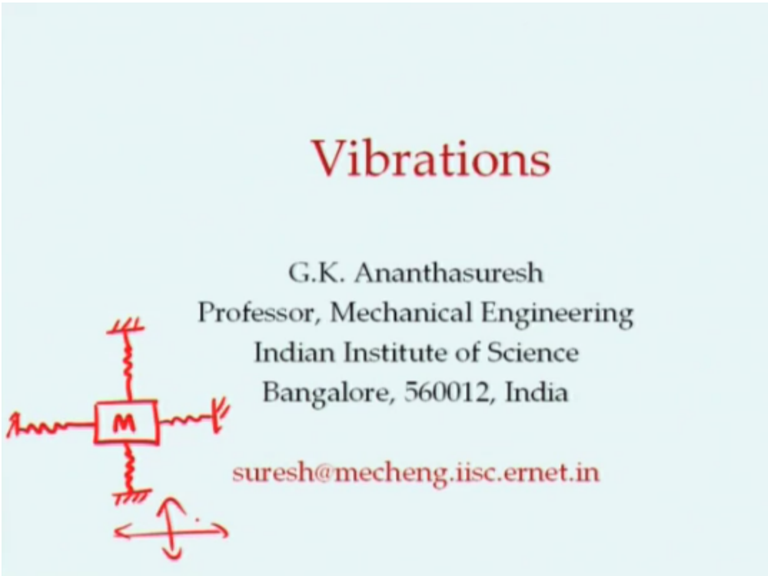

We had discussed a number of things related to vibrations and we will just briefly review what we had discussed in the last lecture and finish up one small concept of bandwidth that we are discussing towards end of that last lecture and then continue today with another topic related to vibrations called the micro machine gyroscopes. Let us quickly review the vibrations that we had discussed last time.

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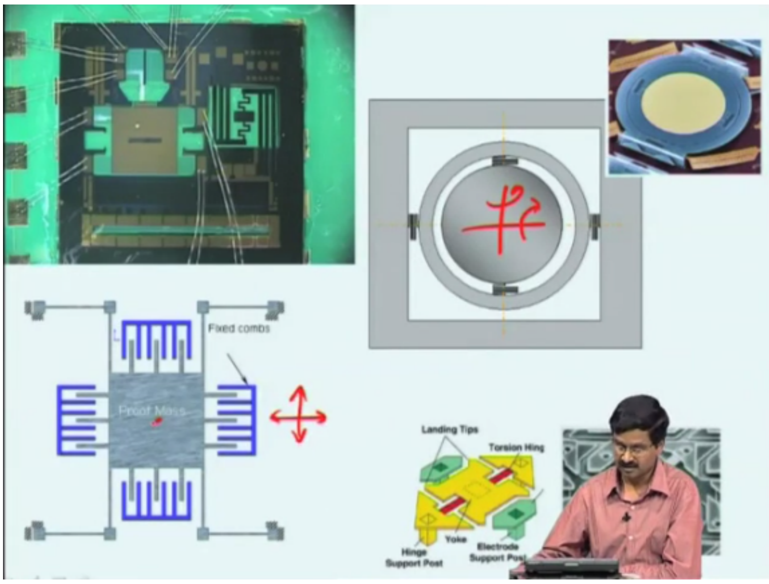

And first we had discussed the motivation for studying vibrations in the context of micro systems. So, this is an accelerometer, a chip and this is a gyroscope that we will talk about today. And here is a mirror that tilts about 2 axis that is as shown here rotation about this axis that is the Y axis if we call vertical axis and horizontal axis and there is a mirror about a single axis this can tilt. So, that is another one.

This is a 2 axis mirror, the single axis. All these are dynamic systems they undergoes small motions which we call vibrations.

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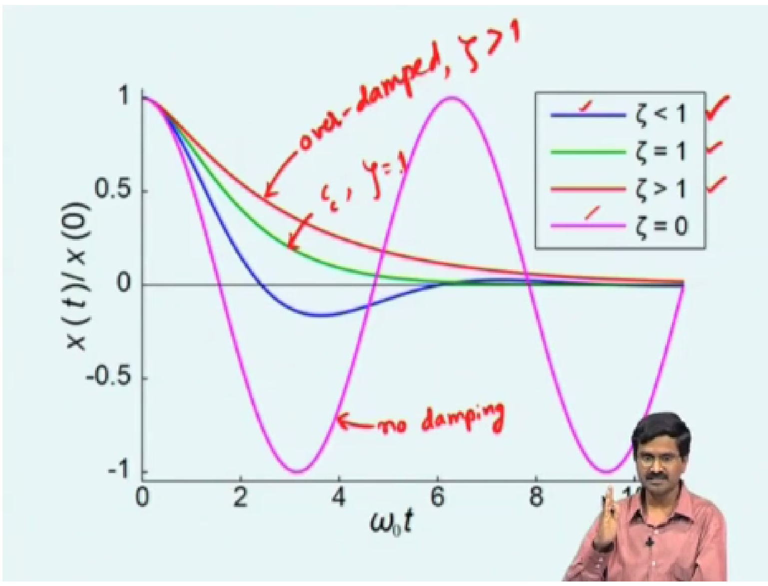

So, towards that we discussed if we just quickly go through the concepts that we had discussed in the context of last lecture. Okay, let us see let us bring up the last lecture here, okay.

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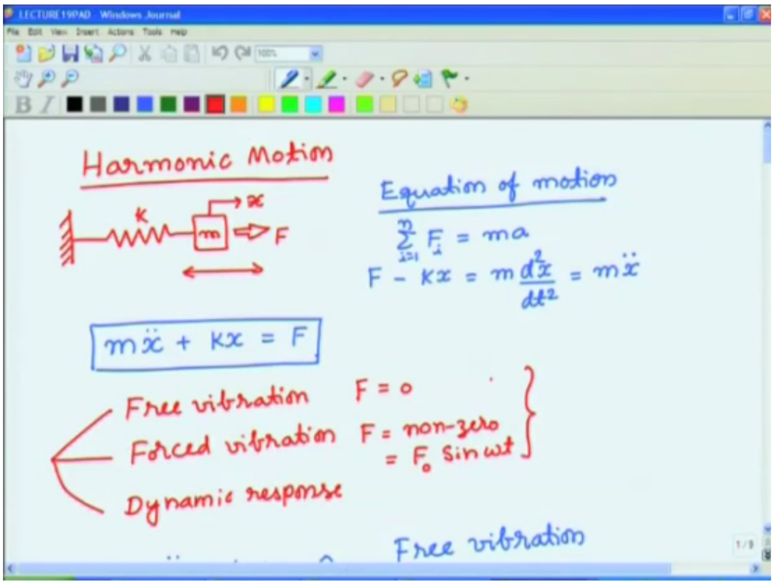

We started with the concept of Harmonic motion which is simply there is a spring and a mass and the equation of motion for that is as shown here, okay. mx double dot kx = force that is applied. We said that when the force is = 0 we call it free vibration, it is the free vibration and then when there is force then we call it force vibration. When there is arbitrary forces we generally call it a dynamic response.

So, free and force vibrations are special cases of what can be called dynamic response.

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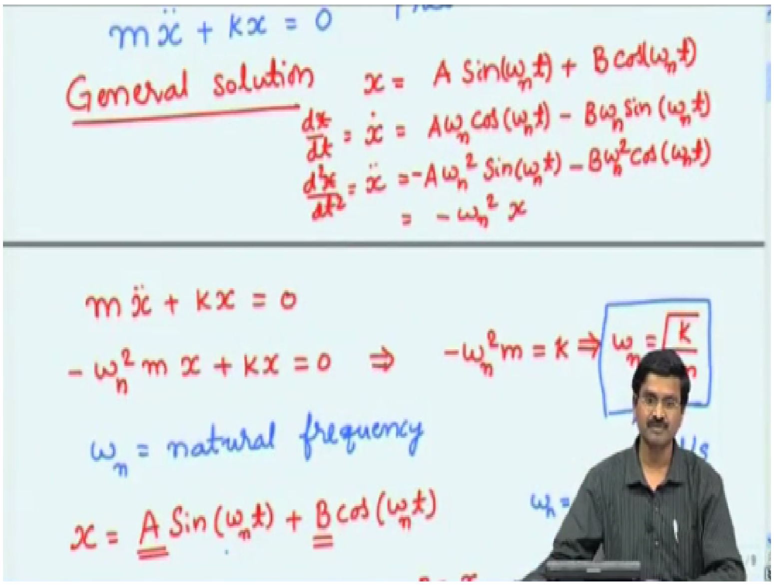

In the case of free vibration, we saw a general solution for it which is composed of sin and cosine terms and we had defined the concept of natural frequency that is left itself this spring mass system will be vibrating at a certain frequency going back and forth that is time period.

That is one-time period inverse of that is the frequency. So, formula for that is square root of K over m where K is stiffness and m is the inertia.

We discussed how we can obtain the stiffness for a given complicated system. However, complicated it is it may be we can always lump it to a single degree of freedom if it is possible. Then there is just one spring and one mass, the mass counts as inertia K transfer to be stiffness and we said that we do that in a way that enables us to do this.

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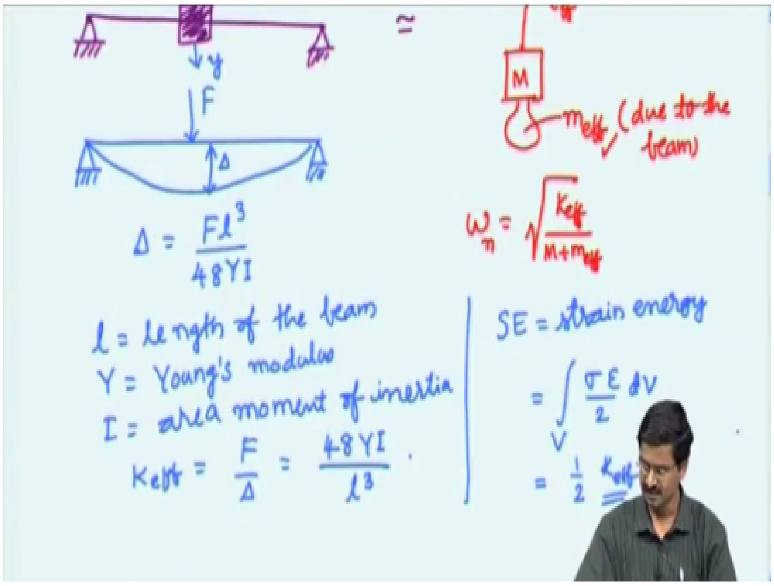

That is, when do we say the stiffness is K for a system if half K times displacement square is = the strain energy stored in that complex elastic system. Then that is the lumped stiffness which we can call K effective. Okay, just recalling what we had discussed in the last lecture. If the strain energy stored in an elastic system if that can be expressed as half K effective times displacement square, then that is the K effective.

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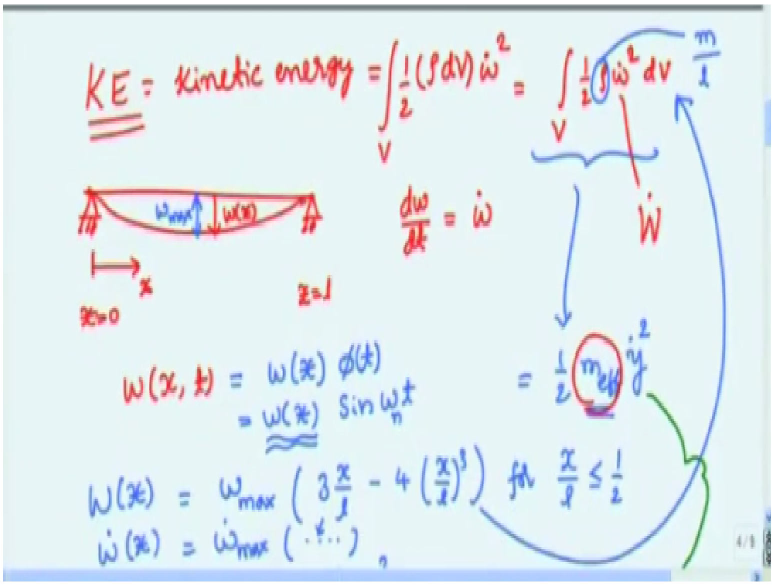

Similarly, for inertia we use kinetic energy. So, we have this KE, kinetic energy and we say that when we have a system which we can express as half m effective y dot square if we do that, okay. This is the effective inertia and y dot is the velocity of the degree of freedom that we are looking at in the lumped model. If we can do that then that is the m effective. In order to do that we had to do this integration by taking mass per unit length in the case of a beam.

If it is a general 3D structure mass per unit volume times the velocity square of that one and then integrate over entire volume get the numerical value. You know y. then you can get m effective. Once you know K effective on m effective you can compute the natural frequency for free vibration.

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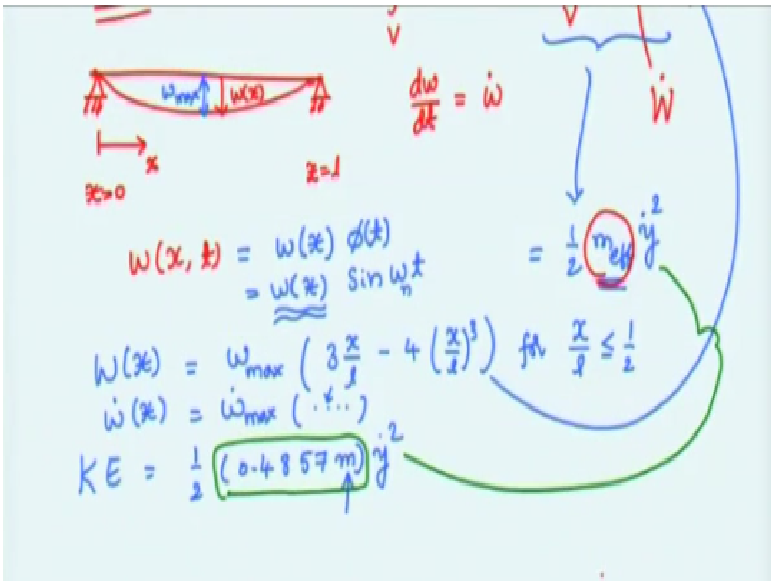

We had done a small calculation for a beam such as this if the mass of the beam is m only 48% of it participates in the vibration because all of the beam is not vibrating by the same amount where this point is fixed, this point is fixed, right. So, this point is what that is going to have lot of displacement and other points have less displacement. So, overall it amounts to only less than 50% of the mass actually moves.

If we take the maximum displacement as were y. A reference displacement in the lumped model. That is what we had discussed and then we moved on to adding a damper the system.

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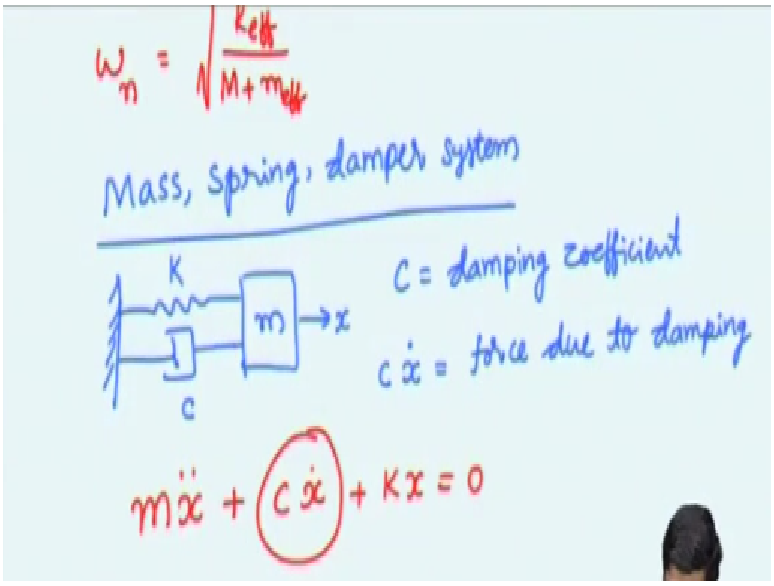

Now this is the new element, damper. We talked about how to get K effective m effective and then we have this damping coefficient c. In a future lecture we will discuss ways to compute c effective, okay. But today after reviewing this we are going to start on a vibration related topic which is micro machine gyroscopes. So, this cx. term we had considered, earlier we had a solution without this term.

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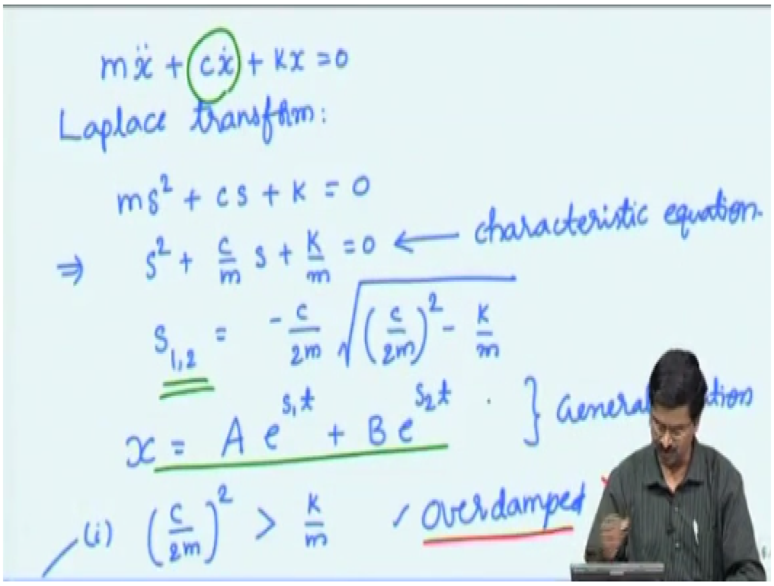

Now we wrote a solution after introducing this term for that we use Laplace transform technique to come up with the solution. This is the general solution of the system and then we also classified it in 3 different ways.

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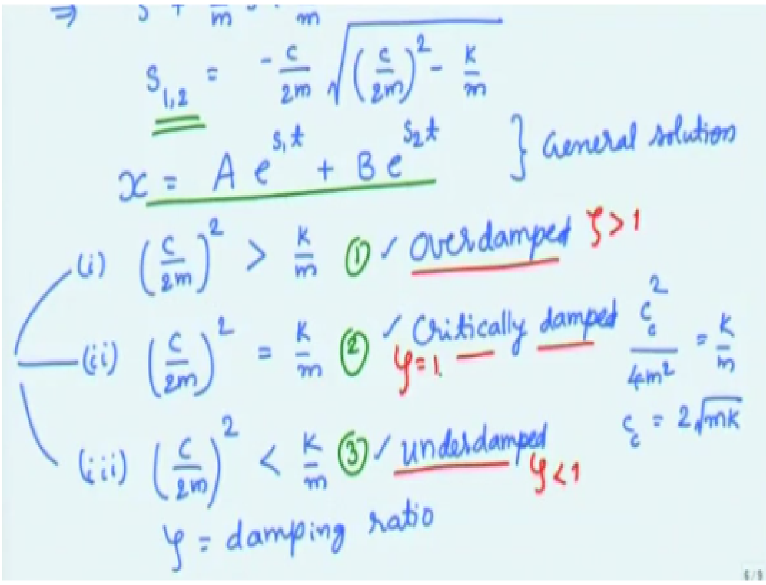

One over damped, two critically damped and three under damped. Defined the damping quotient, coefficient which is damping ratio which we said is given by the quantity that we have indicated here where c square cc square critically damped ratio that square divided by 4 m square and it is = km that will be damping quotient 1. Anything else it will be more than 1 if there is more damping less than 1 if there is less damping.

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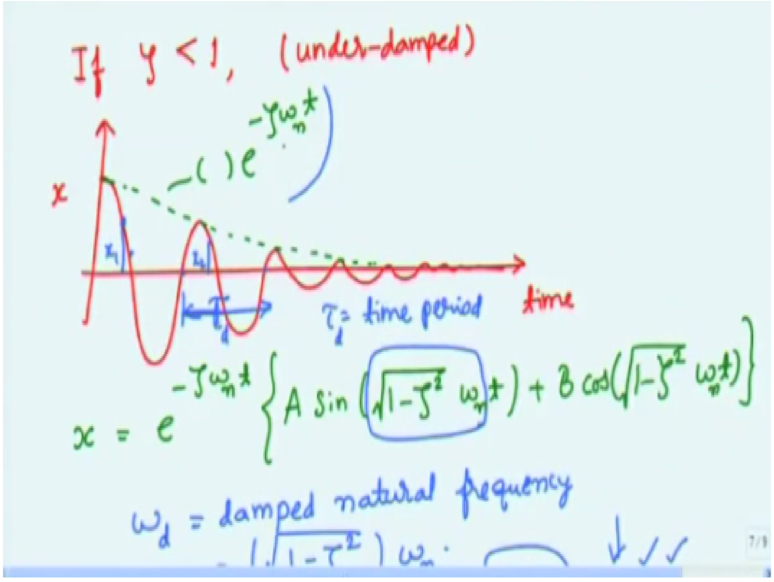

So, we had defined this damping ratio and we actually saw the importance of this damping ratio. This is the second order system if I pert up that a little bit like in a free vibration then it will start at some value and amplitude as it oscillates is going to reduce and that is where this damping ratio applies. There is omega n which is the natural frequency which is dependent on only K effective on inertia effective, m effective.

But as damping ratio decides how this amplitude actually decreases. And we had seen that analytically as well as graphically here and then defined a concept of damped natural frequency and how we can determine this in experiments for that we defined the concept of logarithmic decrement which is simply the ratio of successive displacements in one period time.

That is if I take a point here exactly after some time this point may correspond to somewhere here. So, this time period if I take, okay if I see the displacement x1 and x2 here, if I take the natural logarithm of that, that is equal to this damping ratio times omega n and then the time period itself, okay. So, based on this we can compute once we know this damping ratio we can compute the damping coefficient which is the c.

As we saw in the previous slide damping ratio = 1 when this c the damping coefficient is critically damped. So, we say 1 this divided by whatever damping coefficients, so if once you know cc if it is more than 1 we will get the appropriate damping coefficient c for the system.

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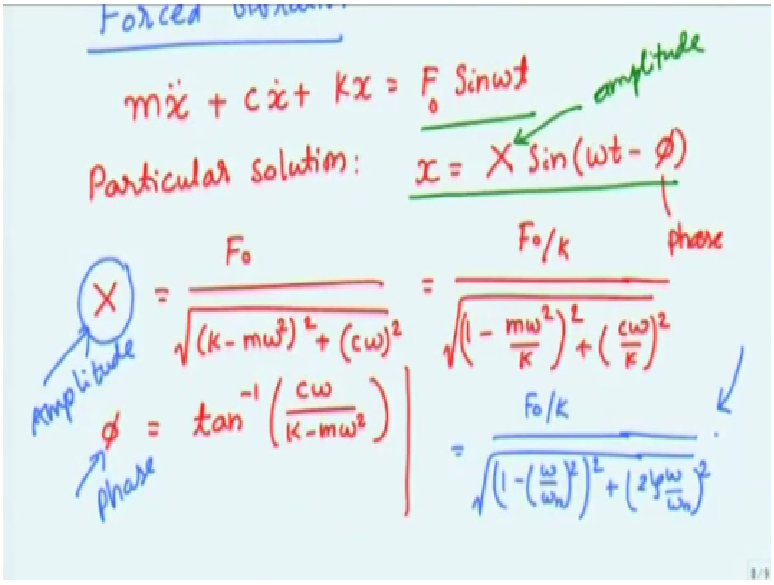

And then we started talking about forced vibration. When there is a force on the right hand side as supposed to being 0 in the phi vibrations now we have a non-0 value for the force. Then we noted the analytical solution for that where there is amplitude and phase because analytical solution will have x = this is the amplitude x and then we have a phase. If omega is the applied natural frequency will be offset from that because of force applied someway and then there will be offset in the resulting displacements.

So, the expressions are given over here and then we try to normalize it by dividing by natural frequency. And we try to see how that curve is going to look like. That is where we ended the last lecture.

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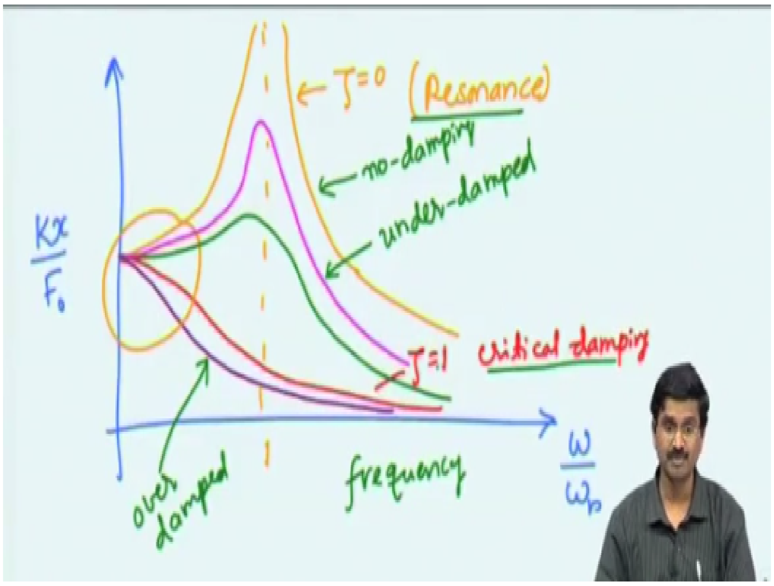

So, it is a quick review of what we did earlier because that is important for what we are going to discuss today. So, if I do that amplitude ration kx divide by force that is like normalized quantity if I plot it with frequency on the x axis, okay, it is going to look like this. The first one here is over damped and then there is this is over damped and this is critically damped the red one that we have said is critical damping.

And then there is all this is zeta = 0 damping ratio 0 is actually no damping. When there is no damping you see that we go to infinity. The amplitude is going to really high we used to call we call resonance. The term that we have discussed last time that most of you must be familiar with and then all these other curves are under damped. They are under damped meaning that they are damped less than their critical damping that is damping ratio being = 1.

So, this has lot of significance when it comes to sensor design for that let us just go to the place where we had left it, okay.

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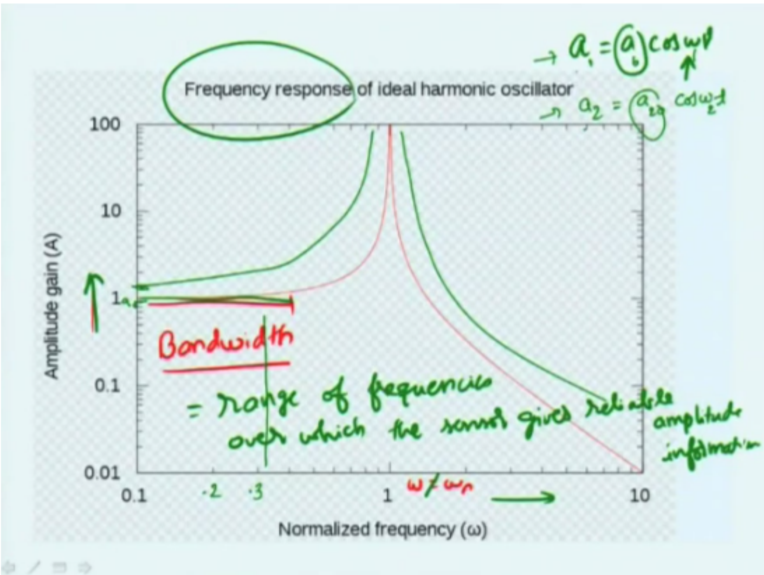

So, the same ratio that kx/F0 if I plot it on the log scale as shown over here. We see that, okay so, let us okay that is log scale and so is this. This is the frequency omega divided by omega n natural frequency it is one that is where the resonance occurs and you see when you log-log plot. If you do that there is a portion which is constant and then it goes like this.

Real experimental results also look like this when you were to take a frequency responds of a dynamic system you would normally see this. There could be more than more natural frequency then you will see more peaks like this. But important here what we call bandwidth is the portion of the frequency response that is independent of the frequency. So, if you have an accelerometer, if accelerometer will sense acceleration.

But the axle that is sensed, if it is going to vary with time let us say the acceleration that I am sensing has a a0 and then cos omega t. So, it has its own frequency dependence, right. But I am interested in measuring this acceleration because I wanted to measure vibration. Vibration always will have this harmonic motion. So, I want to measure this, a0, right. But since that is varying with time harmonically because I am vibrating and I am putting my accelerometer over it.

Then I am not interested in the frequency part I am interested in the amplitude part. But if I have a sensor that gives me this a magnitude as it varies from – a0 to + a0 because of cos omega t here. I am not interested in that I am interested in this amplitude. That is why we need to pick a sensor that is independent. Now you see whatever is the frequency omega this amplitude ratio is constant and that is called the bandwidth.

So, this is the range of frequencies bandwidth refers to range of frequencies over which the sensor gives correct or reliable amplitude information, okay. So, imagine that if I had 2 signals this is a1 let us say this is, let us call it a1 0 and then let us say have another signal a2, a2 0 cosine omega 2t let us say this is 1t. Both of them at some point can have the same a1, a2 value whereas a1 0 and a2 0 may be different.

If my interest is to measure these that difference will show up in this case because if this is a1 0, okay a2 0 if it is more it is going to be something like this, right. So, I have precise difference between a1 0 and a2 0 even though a1, a2 at different points may have the same amplitude, right. So, amplitude meaning that this is the signal amplitude. So, the bandwidth is an important concept in sensors.

It is not just accelerometers but for any other sensor, pressure sensors, humidity sensors and gas sensors anything that you take if the signal itself is time varying then bandwidth is an important concept. As a thumb rule we normally take about one third because the log-log scale here is where the 2 is there and 3 up to that point usually it is linear. So, if you take since this is 0.2 this is 0.1, 0.2, 0.3 this goes to 1 log scale.

So, about one third of the natural frequency you can take it as a bandwidth in practice as a thumb rule. And that is how when you design it if you pay attention to the natural frequency of your system the first natural frequency one third of that is a bandwidth. Over that frequency range starting from 0 to that value you will have amplitude information reliably given by your sensor. So, this is the concept that we had discussed in the last lecture.

Today, we will continue with that vibration concept but we will look at a related concept called Gyroscopes. We have Micro Machined Gyroscopes, micro machine is an important let us go back, let us get the pen, yeah, Micro Machined Gyroscopes. Gyroscopes are available already. They are used to do in initial navigation and many other applications. But now we are talking about micro machine gyroscopes where things are really small.

To give you an idea of the size that we are talking about I have a chip, gyroscope chip with me today. I am going to show that you now. So, here is that gyroscope chip. You can see it on the tip of my index finger, very small chip hard to see but it is right here, okay. It is very very small, my index finger. So, let me, yeah it is just a dot on my index finger just like a mole. It is very small.

Its size would be you can let us say see it about 5 millimeter/5 millimeter and then thickness is about 2 millimeters very small. Inside that the sensor will be much smaller. We need a microscope to see it properly but the package device is what I am showing. Now if I mount it on something let us I put it on my hand as I tilt my hand like this, it is going to tell me at what rate my hand is being tilted.

Similarly, if I mount it on a car and if the car is going to roll over then we will detect that before it happens. So, corrective action can be taken. Similarly, if I put it in airplane as airplane is going along it will tell me how much it is rolling pitching and yawing if you have multi axis gyroscopes. Like accelerometers you can have angular rate senses to sense 1 axis, 2 axis and all 3 axis. So, we talk about these micro machine gyroscopes today.

One thing that we note is that when you measure linear motion we always use acceleration. We always use acceleration sensor. We measure acceleration because that effect will be seen as acceleration times for mass will give you the force and the effect of force comes as a deformation or displacement which you measure and relate to acceleration. But when it comes to angular motion we do not measure angular acceleration.

Whether it is at micro scale or macro scale we do not measure it at,

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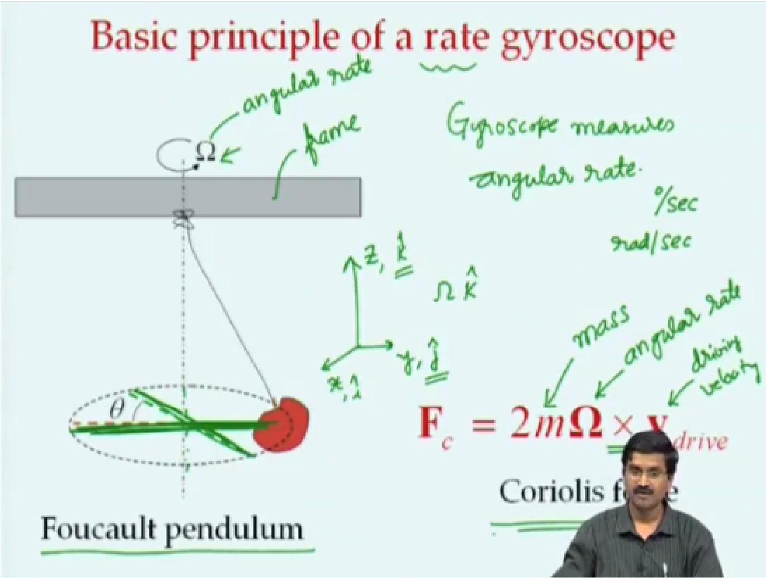

We do not measure angular acceleration, instead we measure angular rate. We measure as it is shown here we measure angular rate. A gyroscope is a sensor which measures the angular rate. Gyroscope, gyration is tilting like if you are dancing that is kind of gyrating. So, gyroscope measures angular rate that is some degree per second r radiance per second. That is the unit of angular rate.

It is a pertinent question to ask why do we measure angular rate instead of angular acceleration, okay. There are a few reasons for it. One reason we will see it mathematically today. For that we will first consider the simplest angular rate sensors and that is called a Foucault pendulum. Pendulum designed and used by a person name Foucault a French engineer who noted that if you want to measure the angular rate of the earth's spinning.

So, earth is rotating as we know if you want to measure the angular rate of the earth what do you do being an earth you want to measure it. So, for that he built this very long pendulum the very long wire he hung a very large weight. But here I am showing it is just like an apple or a mango tight to a string which is fixed here to a ceiling and then it is hanging and it is said to oscillate let us say along the red line.

The red line I have shown here, is just a pendulum it is just oscillating, okay. Now if this frame, this is the frame to which this one is attached if that frame starts rotating, okay with an angular rate omega. So, this is angular rate. If this frame starts rotating then what happens to this mango that is hung from this celling. We have set it to vibrate along a line like this in a plane, okay.

If you come back after sometime that says it is going from here to redline and back and forth it is going like this. What I have done is the red line I made it green now. But what happens after sometime when this starts rotating is that it will change its plane of motion and start going there that is plane of oscillation rotates when angle theta dot If you measure this theta we can correlate this to this omega, okay. Why does it happen?

Normally in science museums there will be this huge pendulum attached to a high ceiling and it set it to motion like this. Let us say in the morning at 8 o' clock, if you come back after one hour it would have changed its plane of motion to something like this and then after sometime something like this. It depends on the earth's rotation and based on the earth rotation that is the time measurement because earth finishes one spin in 24 hours.

So, we can do a clock like this with Foucault pendulum. That tells us at what rate the earth is rotating. What is the principle behind this and that is what is called Coriolis force or Coriolis acceleration. The Coriolis with acceleration for the simple case is given by the mass of this mango or apple, okay m is the mass and this is as we already said is angular rate and this is the driving velocity.

What is driving velocity? We said that we took this pendulum and started to put set it to motion. So, there is certain velocity for this mango as it goes back and forth here, right. That is the V drive, driving velocity and this is angular rate. And this Coriolis acceleration as you can see we have use the cross product. When you have put this omega and V in boldface letters because omega is a vector, it has an amplitude omega and also a direction.

If I call this direction upwards as my z axis this omega will be written as omega amplitude times the unit vector along this z axis where x and y axis can be here the right handed system. And this velocity let us say originally we said this is how it is going if I call that y axis or its unit vector j hat okay and this is i hat. What we get is that omega which has the K along the z axis unit vector. Velocity v drive has j if I take the cross product of K cross j I will get a component in the ith direction or depending on the amplitudes the negative ith components.

That is what is oscillating like this suddenly starts having component in the perpendicular direction. So equivalent of that it will start tilting. That is if I have motion set like this because of Coriolis acceleration it is along the z axis here I start having a component like this. I said the motion like this and now I start having a component of this drive velocity that we give in this direction starts going in this direction.

So, as a result it will start going across like this, okay. So, I was set it like this and there is a component here it goes at some angle as it is shown in the diagram. That is the principle of the gyroscope and that is the principle of Coriolis, acceleration Coriolis force.

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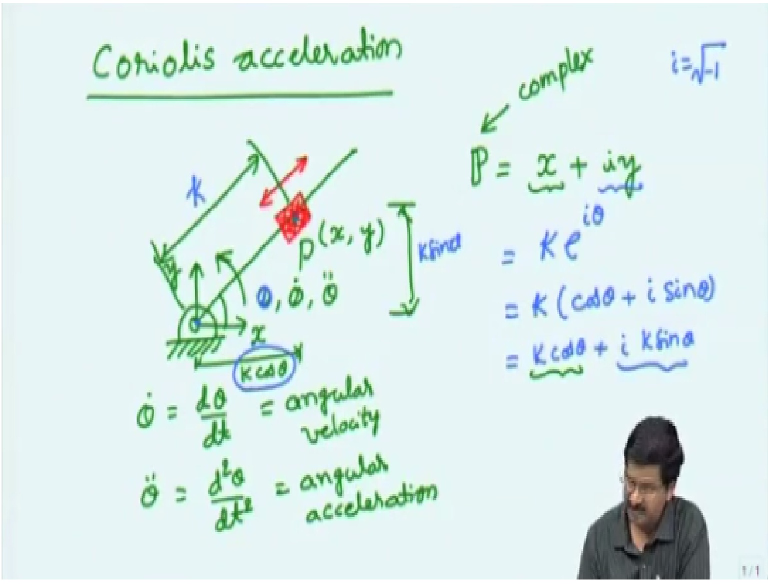

In order to see this Coriolis force let us switch to our note pad and see what we can do about this Coriolis acceleration. So, now let us start from the very beginning to discuss what this Coriolis acceleration really is, okay. Let us say, we have a rod and on that we put a bar on that we put a little slider. So, let us say this red one is a slider which can go back and forth along this rod. Let us say this rod is pivoted to a reference frame, okay.

So, that means that this thing can rotate. This rod can rotate rigid rod and this slider can slide along this, okay. If I take a point over here and say the coordinates of that points are x and y where this is let us say our y axis this is our x axis, okay. Let us say this point is p and that has coordinates x and y. Now if this rod has angular motion theta and angular rate theta dot and angular acceleration theta double dot. So, theta dot is d theta/dt and theta double dot is d square theta/dt square.

Okay, this is the angular rate which you want to measure. We still have not answered why we want to measure angular velocity I suppose to theta double dot which is angular acceleration but when we finish the calculation that we are going to do on this simple system will have an answer to that question. Okay, now I am going to also tell you a simple tool for analyzing kinematics of planar system.

This is a planar system in the sense that this rod is moving in the plane and this slider is moving along the rod which remains to be the same plane. If you have that the position P we can represent that as a complex number. It is a very simple concept where the position x and y components of this because this is the coordinate system. This thing is x here and this thing is y here because this is a coordinates, seen that as a complex number.

The advantage of representing like this is that if I were to indicate the distance along the rod where the slider is located if I call that K and this angle of cos is already theta for us. We can write this P also as K*e raised to i theta, okay. So, where K is the distance from the origin to this point and times e raised to i theta because we know that e raised to i theta is cos sin theta + i*sin theta, where i of course is square root of negative one, okay.

Now this means that K cos sin theta + i K sin theta and this K cos sin theta is nothing but our x because if this were to be K if this angle is theta then this horizontal this displacement here is K cos sin theta dot Right, similarly this one is going to be K sin theta, so according to what we had done earlier that is what we got here, okay. So, it is legitimate to say that position P of this point we can write it as K*e power i theta dot.

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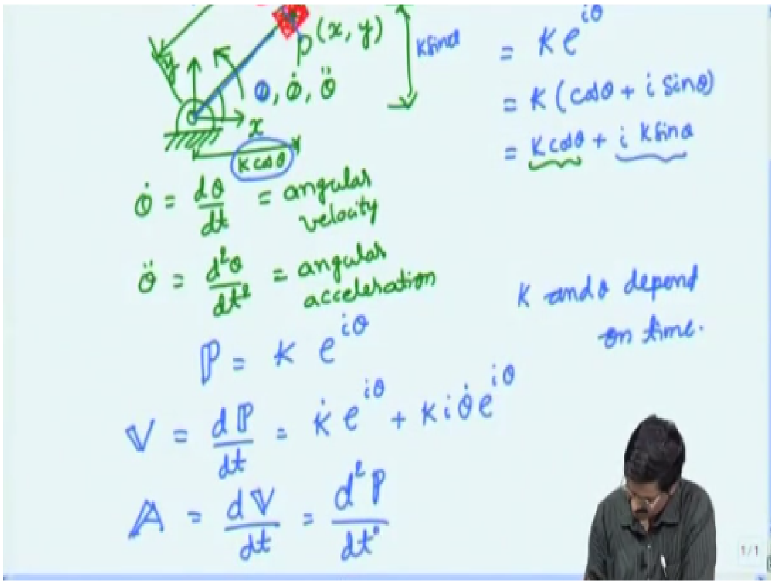

You will see why we are doing this, okay. Now if I want to know the velocity of that point P, I have to take differentiation of it. So, if I want to say the velocity if it is a complex number I am just adding this double line at the left side. If I want to get this V then I have to do derivative of this P with respect to time. So we have derivative, so we have K because it is sliding along the rod, K depends on time theta depends on time.

So, K and theta depend on time. They vary with time, okay. They depend on time means that they are variable with time. So, I have to take differentiation using product rule. I will first say K. e power i theta and then K*i theta dot E power i theta dot That means that first I have taken product rule. First dK/dt that is K. then e power i theta as it is then kept K as it is and i take derivative of e power i theta dot

Which is e power i theta and then we have to take derivative of this which is i theta dot d theta/dt that is what we get that is the velocity. So, you have a component along the radial direction that is radial direction is this, okay because slider is going like this. And then this rod is also rotating there is also tangential component because e power i theta. will be perpendicular to the one that has not have i, okay.

Because that is what we saw x and y they are perpendicular to each other components what has i and does not have i will be perpendicular to each other. So, now if I want to get the acceleration, okay that is also a vector. So, I will have represented as a complex number. I have to do dV/dt are d square P/dt square, okay.

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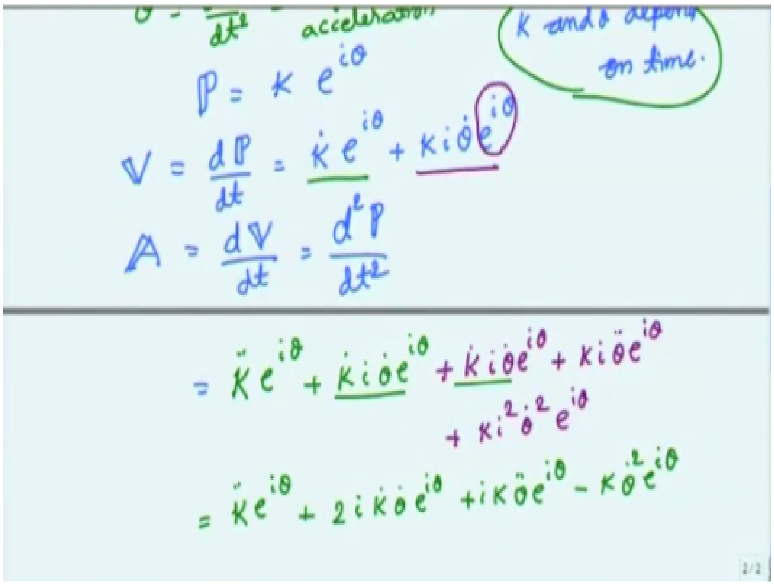

So, let us write that that is = we already have 2 terms here, okay. Now note again that both K and theta depend on time or vary with time. So I had to take derivative of when I take derivative of this you have to keep in mind that K. also may vary with time theta will vary with time. So, I will first have K double dot e power i theta that is product rule one and then we will have K. i theta dot e power i theta dot.

So, that takes care of two terms that come because of this one and that is used a different color for terms due to this one now, okay. So, that this thing was the green one now we will write the purple for this term, okay. So, there are K theta dot and e power there are 3 terms. So, we will have 3 terms coming here. First we will take k., okay and then i theta dot e power i theta remain the same.

That is the first term taking derivative now will take the second term that will have K i theta double dot. e power i theta and then there will be another terms which will be for this portion that will be K i square theta dot square e power i theta dot Why do we have square we already have i theta dot and take derivative of this quantity I will get e power i theta times i times theta dot So we will get this.

So, if I write this back again, so I will have k double dot e power i theta plus these 2 terms are the same. So, I can write it as 2 i K. theta dot e power i theta plus I have K, i k theta double dot e power i theta - because this i square because square root of - 1 is i, i square is - 1 K theta dot square e power i theta dot So, now we got 4 components for the acceleration. The first term that you see K double dot times e power i theta is what we can call linear acceleration, okay.

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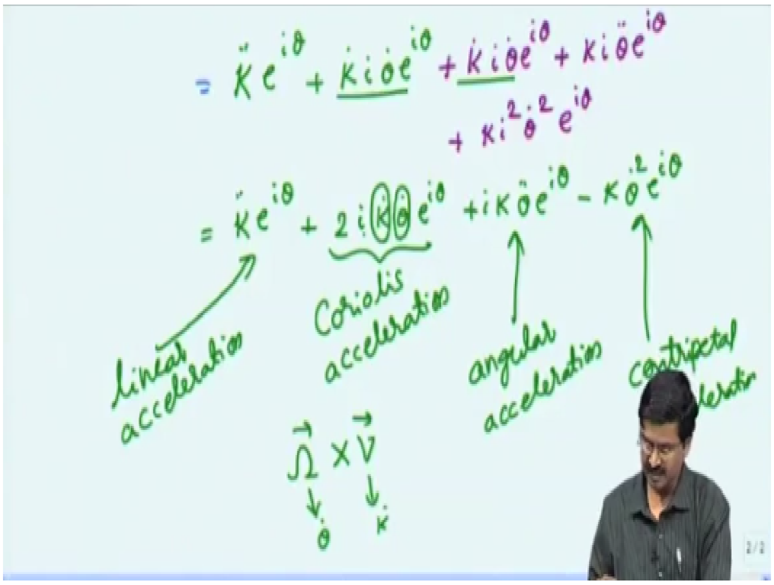

This is linear acceleration. Remember that rod is sliding, the block is siding along the rod and that is linear acceleration and this portion which you can recognize as our centripetal acceleration. Notice that these 2, the first and the fourth term do not have i in them. It is just e power i theta is the direction they just going along the rod but in these other two terms have i in them. They will be perpendicular.

So, this one theta double dot is there this is angular acceleration. And then we have this term which is called the Coriolis acceleration. So, Coriolis acceleration we saw earlier we had something like omega. just omega crossed with V, okay. So, we have that theta dot is our omega and then K. is our V. So, this is related to K. this is related to theta dot We have the cross because direction is already has come here.

We had taken the angular rate perpendicular to the plane in what is above, okay if I have. The angular rate is perpendicular to the plane of this screen here when I take dot product of that with this velocity which is here is right handed rule. So, we have this thumb going up along the angular rate and then we have this going this way, okay. That is this is our velocity direction this thumb is the upward angular velocity direction.

Then this one right handed rule cross product is going to be our oscillation. So, this is the angular rate and this is the V and this is the perpendicular one to both of them is the Coriolis force direction. And now we can see we come back to this equation, right. So, if you look at this, this Coriolis acceleration depends on the angular rate as well as K dot. So, instead of measuring the effect of the angular acceleration which is stated double dot.

Which is only K changing the position of something is more difficult than changing the velocity especially if something is moving we can control its velocity better than the position. So, here we can think of this K. which we said in the Foucault pendulum we took the pendulum and set it to motion and then when there is angular rate of the frame it started changing the plane of oscillation.

So, that was a function of K. how much what rate it goes depends on this K. as much as it depends on theta dot . So, K. can be seen as a gain in the system to measure. So, if you were to take this system and the rotating frame even it has some angular acceleration the contribution of that will be much smaller compared to the contribution of the Coriolis acceleration. That is usually the case.

But we were actually designing a gyroscope whether is macro machine or micro machine we have to take care of that the contribution of the angular acceleration term let us go down to what we wrote. This contribution should be less than this term. That is our interest Coriolis acceleration is our interest. This should be negligible. So, we have to make sure that that is not going to dominate.

In other words, position displacement K is not much that how far the block is from the pivot of the rod whereas this K. is the velocity how fast is it moving, okay. So, if we go back to the Foucault pendulum example now. Let us say what we have this example. The velocity at which this moves, okay matters for us, okay. So, the velocity at which this moves is going to come in this V drive as we have already discussed.

There is angular rate that force comes on this changes the plane of oscillation to that. But we are not going to use the pendulum in a micro machine system such as the chip that I had shown you a few minutes ago instead we use like a mass as shown here.

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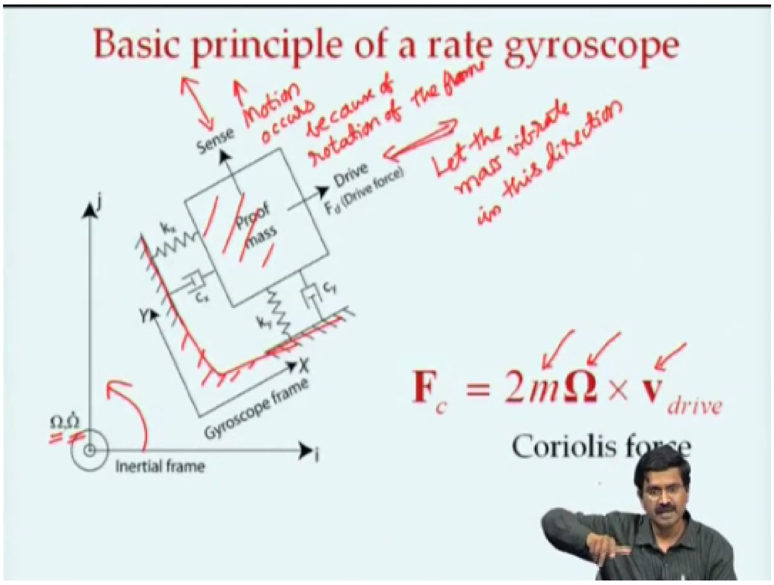

The basic principle of angular rate gyroscope simply lies in the fact that there is a proof mass such as this and that is set to be free to move in 2 directions. We can call this x axis that is in this direction and another is the y axis local and we have intentionally shown that the thing is rotated. If I think of this frame, okay this frame if we think of a box or there a thing that is actually rotating. That is what we shown here.

Rotating with an angular rate omega an angular acceleration omega. . Both the terms we saw in the little derivation that we did. But we thought that if we use Coriolis force there is mass that is important and angular rate that you want to measure is coming along and angular and the linear velocity that we are setting into motion. When you say this a drive one you first take this proof mass just like the pendulum and let it vibrate in this direction, okay.

We let the mass vibrate in this direction, okay and if the frames start rotating, okay. Then we start seeing a motion here, okay. This motion in this direction occurs because of rotation of the frame. Rotation of the frame whose rotation rate is what we wanted to measure. So, we set into motion if we do not see any motion in a perpendicular direction then you can conclude that this frame is not rotating.

When you start seeing the motion when you know that it is rotating and how much motion occurs will tell you at what rate this frame is rotating. That is the principle of the angular rate sensor. That happens because of the Coriolis force that we discussed. So, when there is motion in one direction and there is angular rate in another direction let us say angular rate happens to be in this direction that is this direction and motion in this direction.

Right hand rule if you follow the perpendicular one is going to be the motion that comes. In other words, the energy put into one axis gets transferred to the perpendicular axis that is why the Foucault pendulum changes its plane of oscillation, okay.

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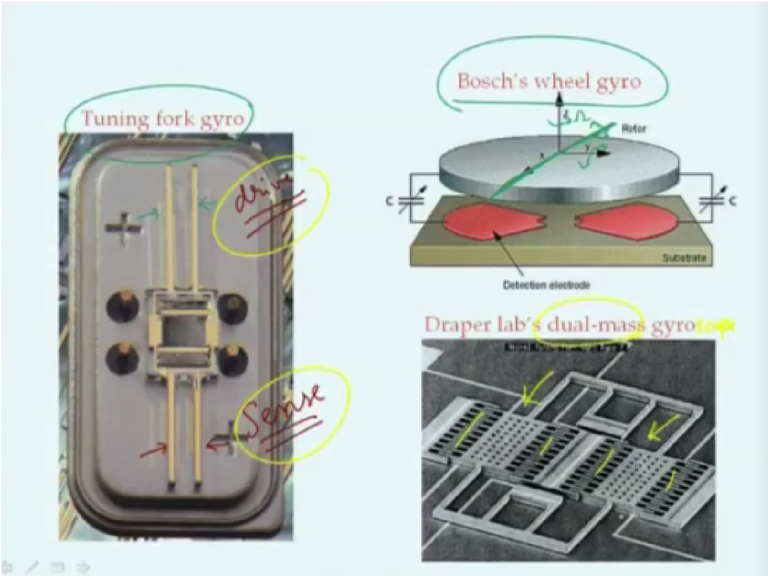

So, how does it look like in reality for a micro machine gyroscopes such as a small chip that I showed a few minutes ago. That might have an element such as what is shown here in 3 different ways, okay. Let us first look at this wheel gyro as it is called, okay this is the wheel gyro. This is Bosch company makes a micro machine wheel gyro and in fact that is what we may be familiar with as a gimbal wheel.

Gimbal wheel is we have a disc that can rotate about let us say x axis, it can rotate about the y axis and it can also rotate about z axis. So, you have a disc that can rotate about all 3 axis that is the gimbal wheel which is used to demonstrate experiments in physics and also in some of the macro scale gyroscopes. So, now if there were to be the rotation set along x, okay. So, you take this disc and let it oscillate about the x axis, okay.

So, it starts oscillating like this whenever this omega that we want to measure happens than that rotation will get transferred to rotation from the y axis. So, x and y are equivalent axis. You are driving the x axis and when there is angular rate y axis motion starts seeing. Once you know that there is motion on the y axis rotation about the y axis then you would say that oh, my frame is rotating.

The frame that is this substrate which is attached to a car or a aircraft or something is actually tilting and that is what we would try to measure. Similarly, we have another one here which is called Tuning fork gyroscope. Gyro is a short for gyroscope. This has a tuning fork as you can see. There are two here and here, okay on this side also. Now what happens is if you start let us say this is the drive side that is you take these things and set into motion this start moving and they have couple one so they also start moving.

So, you do not see they both of them will be just moving tuning forks. If have 2 fingers, these fingers are set into motion either in face like this, okay are out of face like this, okay. If I have such a thing now when there is angular rate about let us say the axis perpendicular to this chip, okay. I am moving like this suddenly there is angular rate about this axis.

So, if I use right hand rule my, so first let us say that this 2 times can move in face like this or they can go out of face like this, tuning fork. Now let us say I have put them in face like this. Now, if there were to be an angular rate about this axis. So, this is the angular rate and I have motion this way let us say one twine is moving like this then that twine will experience a force due to Coriolis force upwards.

Other one moving like this that is by flip it that will be downwards. So, these two twines are well demonstrated with my hands. If I am setting motion like this, okay. So, when both of they were going like this both will experience a force to go out of plane like this. But if I take them move like this, so they are moving like this.

Now because of Coriolis effect if there is angular rate in this direction that is pointing like this then one of them will start experiencing force like this other one will go like this, they will go. So, they are moving like this. The once there is angular rate while they are moving like this they will also moving like this. They will go like this, okay. And that you can measure on the other side this is the sense side this is the drive side, okay.

Let us use a different color. This is the drive side this is a sense side, okay. We can measure that with a tuning fork or you can have a dual mass a draper lab in the US has this dual mass gyro or gyroscope to finish it that is the short form gyro. Here there are 2 masses there is mass 1 and mass2 and hey are set into motion and oscillation the same way. So, both masses will be because of the **com** drive that we had discussed earlier.

So, we have the com drive here, here, here, here, okay. That will make them 2 masses move like this, okay. Now when there is angular rate about the axis perpendicular to the plane when one moves like this other is moving like this. Let us consider the one that moves like this. If I apply the right hand rule then this is the motion this is there it starts having the motion like this, okay. Then it starts moving this way other will start move that way.

But the way this works is different, this is the axis gyroscope that is they are moving like this, if there is an angular rate about this axis in this is where we have said the motion and this is the angular rate then one starts going up the other one I should not use left hand but I just need to turn it like this, right this motion goes like this then it starts going up. So the 2 masses going like this.

When there is angular rate about this axis then one starts going up other starts going down. So, in addition to going like this they will start doing this. While going it is hard to do, they will start doing this kind of electrical motion but out of face. When this is up this is down when that is up this down, okay. That is how the gyroscope works. You need 2 components that all set into motion and one of them goes one way other goes the other way.

So, we have 3 concepts the wheel gyro, dual mass and a Tuning fork.

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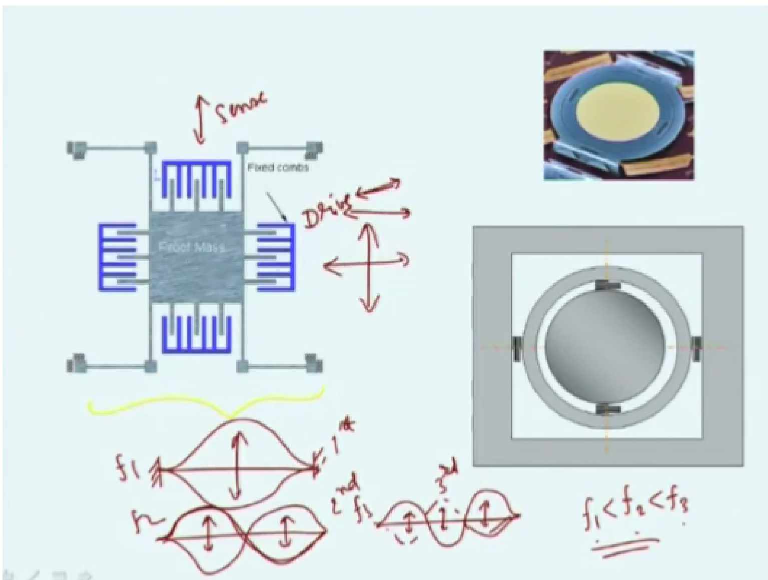

There also more kinds that quick in have one element which is what we kept showing in many lectures that it is a gyroscope. This gyroscope has matched frequencies and it is important to have the match because otherwise at resonance we put it a resonance to have maximum amplitude with minimum effort and the other one also should have the same frequencies.

So, the energy from this mode natural mode which we talked about natural frequency when the context of vibrations there is also a mode meaning that if I take a beam let is a fixed it at both ends, okay. In the first mode fundamental mode this beam is going to vibrate like this. It will go up and down, okay. It will go up and down like this that is the first mode. The second mode for this is going to be -- and it is going to be like this.

With 0 slope here goes up and comes down and that way, okay. That is going to vibrate like this and so forth. Third one will have one more time that is its 0 slopes goes down goes up and comes like this. In other words, it will oscillate back and forth like that, okay. This is the first mode, second mode and third mode. These mode shapes if I have a mass that has the frequency corresponded such as f1, f2, f3, so f1 will be less than f2 is less than f3.

It is the first mode, second mode third mode but if I have 2 of these equal which is what is case here because it has a symmetric suspension in x and y directions. So, it is the frequency of oscillation in this direction will be same as in this direction. If I set it to motion in one direction because of angular rate, there will be there perpendicular to the plane of this one then what we see is that it will start moving in the other directions.

So, by measuring capacitances in this direction if this is the drive let us say drive direction is horizontal, okay horizontally this way and this is the sense direction, okay. If there is angular rate above again if you apply the right hand rule this angular rate perpended to the proof mass plane and this is the drive one they will start having motion in the sense direction.

So, these again the angular rate direction perpendicular to the mass this is the drive and there will motion in this direction that is sense direction. We can measure that and get the angular rate. And we already discuss the reason because then we will have how much velocity we can give this direction and also the mass of it and the angular rate as gains and this is a jumbled up arrangement.

This is a 2 axis mirror and if I put this third axis if I put a similar torsion springs for this so that it can tilt about this other axis. Then I can use this also a wheel gyroscope, okay.

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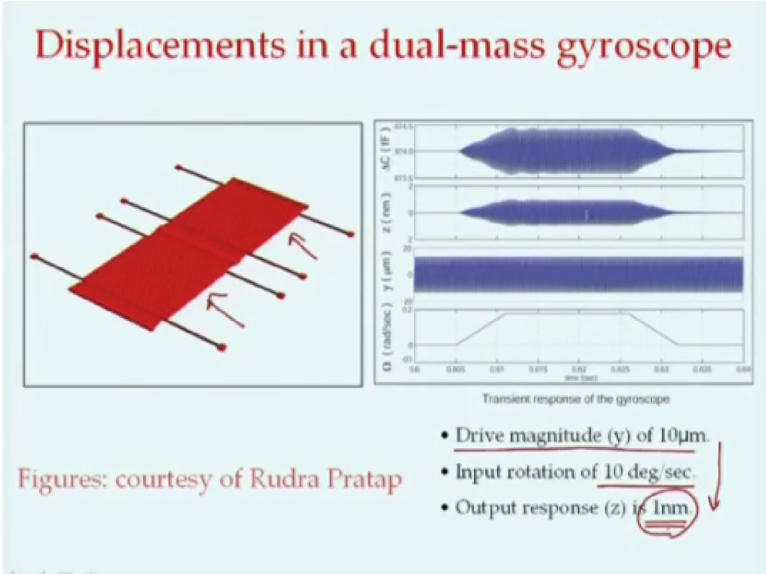

There also other ways that you can sense and one of them will see the dual mass here first. There is a mass one here and mass two both of them can oscillate and they set into resonance even more in the presence of angular rate they start going other way one goes this way other goes that way you can sense that and try to measure the angular rate. There also other ways that you can see

And just while you were adjust slide let us see that drive magnitude of this is the order of 10 microns very small displacement but high frequency. It can sense input rotation of let us say 10 degree per second and output response that case will be 1 nanometer. So, 10 microns if you put you get much less displacement due to Coriolis because the masses here are very small angular rate 10 degree per second is also not very high.

But the mass the fact that the mass is very small and the drive magnitude 10 microns and frequency may be high that may be higher but for typical response may 1 nanometer very small displacement and due to 1 nanometer displacement you have to see what capacity change occurs and we able to measure it. So, people have always been on the lookout for different concepts for the gyroscopes.

Let us look at these ring gyroscopes which is popular. People have made these ring gyroscopes. Let us discuss that ring gyroscope concept by going back to our note pad. \

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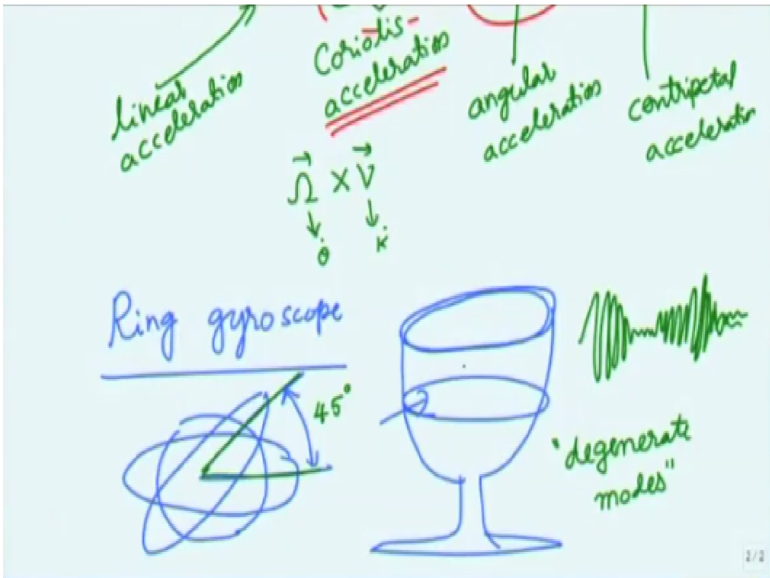

Ring gyroscope are people for short they call it ring gyro the way this works is if I have let us say a wine class, okay a circular one if I just hit it we hear a sound, okay. The sound is because this is actually like a ring, okay. You can take several slices everywhere these starts vibrating. That is if I take a perfectly circular ring, okay set it into motion free vibration which is what we discussed in the last lecture and reviewed today for 10 minutes.

If I take this in vibrations if I look at one mode of this is going to make the circle into ellipse and there will be another mode which is also an ellipse but that will be at 45 degree angle.

So, the angle between these 2, okay is going to be, so angle between major ellipse and this ellipse put 40 degrees. That is if we take a wine glass if we hit it there are 2 modes that have the same frequency both are ellipses. That one case this wine glass becomes ellipse this way other case it becomes ellipse at a 45 degree angle. That is why we see a ringing sound the beat sound ting ting like this.

That is the beats you are familiar. There are 2 frequencies combine into one frequency will see a phenomenon called beats, so it will increase decrease and then increase and then decrease the ringing phenomena, okay. That is what happens here because there are 2 degenerate modes. We call if 2 frequencies are the same there called degenerate modes or degenerate frequencies.

Over all of these because of x and y we had this frequency equal the same thing will be there for the ring gyroscope also.

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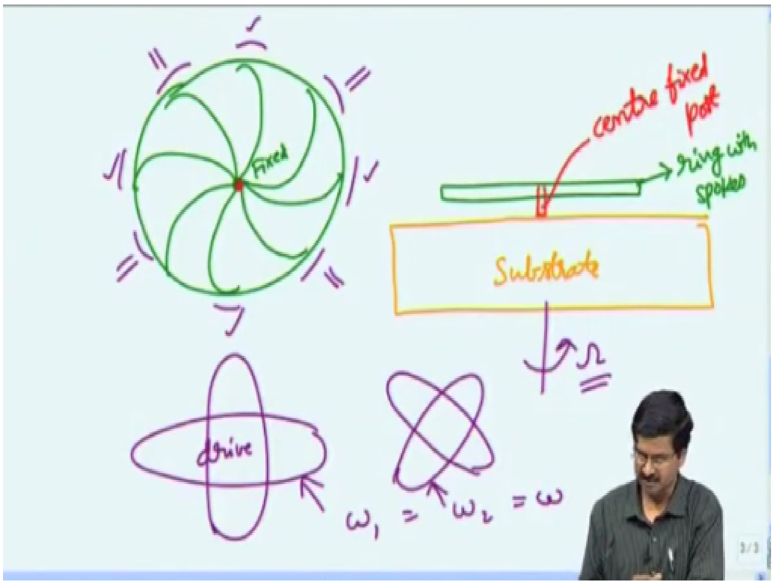

So, if I take a ring, okay and let us say ring cannot be free when use it in a device so we will take a central post and attach it with some suspension. Normally people use it with semicircular spokes. You can use straight spokes also, the advantages in semicircular one, okay. Let us have this 8 spokes like this and it is fixed at the center. So, the center post is the one that is fixed other than this the red portion will mark it.

Everything else is free to move. Imagine that we have a sub straight where you made a ring with these spokes at only the center. That is if I look at the side view of this I am going to have this ring that I will see from the side that the central post here will be slightly larger and here I can have my sub straight. This is the sub straight and this is the center fixed or anchored post like a pillar and then we have this as a ring with spokes.

Spokes here are semicircular that is what people have found it to be convenient in terms of design performance. Now if I put some electrode over here all around, okay let us say I set this into motion. This is my drive mode that is I use this, this, this and this, my electrodes activate them. They starts going like this. Going like meaning it goes like this and like this because the ellipse along at this way that way it starts vibrating.

If the substrates have this angular rate omega then what will happen is that this ring will start having a mode at 45 degrees as we just discussed, okay. Both omega here omega 1 and mega

2 both are equal to omega. Sorry, this is the natural frequency so we will use different symbol rather than this, okay. So, we will have omega 1 for this, omega 2 for this both are = omega, so this is this omega that you want to measure.

So, when that angular rate happens they start rotating like this. Then if you measure capacitance along these stator electrodes and this moving electrode here then we know that angular rate has occurred. That is the principle of the angular rate gyroscope and there are many ways to do that one of the ways is shown here.

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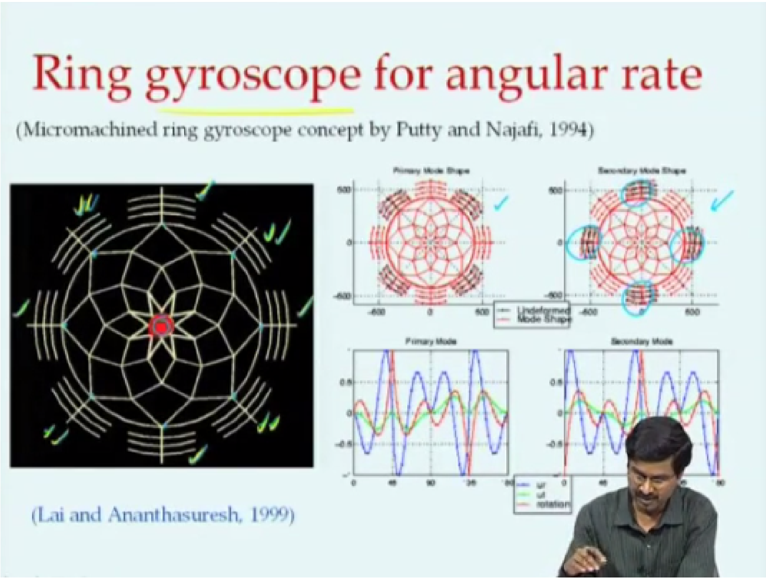

See here what we see the different one. We have the ring and there is a central post just like what we had earlier. So, let us use the red color. This is fixed here other than this everything else if free to move. Now this has a special property that first if we excite this, this, this and this meaning that you make them oscillate like this. These are curved chromes just like the comb drive with the linear comb. You can also have a curved combs here.

It can go and then if you set them into motion the other ones that is these will start oscillating as is shown here. It is fixed only there and the rest of the structure is free to move. Here we see that to the ring there are lots of these combs attached to it and there are 2 degenerate mode shapes for it. One shape is such that this one, this one, this one and this one that is these 4 sets of combs oscillate, okay. That is as shown over here. They oscillate about this point, this point, this point and this point.

That is one mode shape and the other mode shape is I shown here were these 2 things that I am checking with 2 arrows these for this about that point about this point this point and about this point it will go like this as it shown. So, when we set let us say this one into oscillation that is our drive mode. The sense mode the other set will start oscillating by measuring the capacitance of these com fingers that is that one, that one, that one and that one.

We can sense if there is any angular rate. Again it happens because of Coriolis acceleration transferring energy from one mode to another mode. So, this is the principle of micro machine gyroscopes. These also the principle of our macro machine ones but in micro machined gyroscopes we have 4 different ways of measuring which we discussed in this lecture.

And all of these have been made into micro machined devices. Some of them are also commercially available especially the dual mass one is commercially available. The ring gyroscopes are not at commercial available but they have also shown a lot of promise in terms of research. Thank you.