Micro and Smart Systems
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Lecture - 21
Micro machined Gyroscopes: Part -2 Modelling of Coupled Electrostatic Microsystems: Part -1

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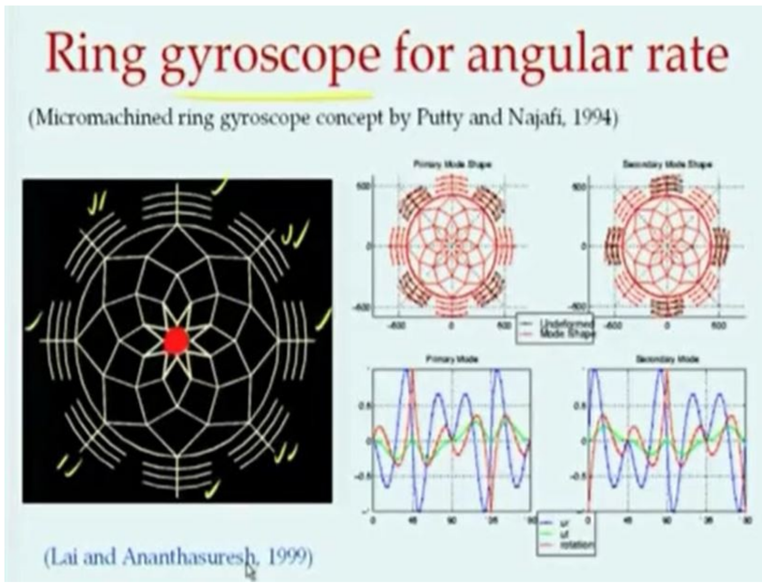

So this is a different kind of a ring gyroscope. Here, we have the same central post as it is shown. Here, it is fixed only there, in the rest of the structure is free to move. Here we see that, to the ring there are lot of these cones attached to it and there are 2 degenerate mode shapes for it. One shape is such that this one, this one, this one and this one. That is these 4 sets of cones oscillate that is as shown over here.

They oscillate about this point, this point, this point and this point, that is one mode shape and the other mode shape is as shown here, where these 2 things that I am checking with 2 arrows, this one, this about that point, about this point, about this point and about this point, it will go like this as it shown.

So when we set, let us say this one into oscillation, that is our drive mode, the sense mode, the other set will start oscillating by measuring the capacitance of these cone fingers that is, that one, that one, that one and that one, we can sense if there is any angular rate. Again it happens because of coriolis acceleration transferring energy from one mode to another mode. So, this is the principle of micro machined gyroscopes.

This is also the principal for macro machined ones, but in micro machined gyroscopes we have 4 different ways of measuring, which we discussed in this lecture and all of these have been made into micro machined devices, some of them are also commercially available especially the dual mass one is commercially available. Ring gyroscopes are not yet commercially available but they also have shown a lot of promise in terms of research. Thank you.

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Hello as part of the micro and smart systems course, today we are going to begin another topic which is very important in the micro systems which is modeling of Electrostatic-elastic coupled problems. Electrostatic actuation is quite popular in micro systems because it scales very well with miniaturization.

That is the force here, at the micro scale electrostatic force is quite large, we do not do that that means that we do not use electrostatic force at macro scale to make our actuators. We use their electromagnetic force quite a lot, but electrostatics has this very nice favorable scaling that the force is enormous at that micro scale and it has an interesting issue with regard to modeling and that is because it couples with the elastic deformation of the structure. We will discuss that in today's lecture.

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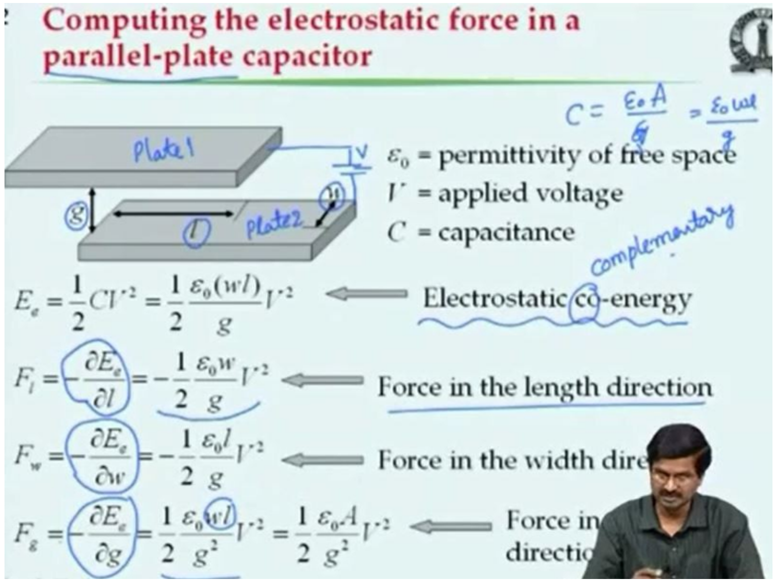

Let us consider a very simple problem of 2 plates that are parallel to each other, which we call parallel plate capacitor. Parallel plate capacitor that means there are 2 plates that are parallel to each other. This is Plate 1 and this is plate 2, based on what we have seen in micro systems until now, you can relate to this type of parallel plate capacitor arrangement in many devices.

One is the electrostatic cone drive, where we have these fingers which are interdigitated the other ones coming in between like this. So here between this and this I have a parallel plate capacitor and if you imagine a (()) (04:41) diaphragm the electrode underneath again, I have a parallel plate capacitors. And in the case of an accelerometer, we have something that moves back and forth this way, there is a (()) (04:54) that moves, there is electrode underneath if I am measuring capacitance there, again I have parallel plate.

So we have many examples, we have such parallel plates, models exist. If I want to know what is the force between these 2 parallel plates, whenever we apply a voltage potential between these 2 plates. Let us say we have connected a battery between these 2 plates, let us say the potential is V for this battery, then what will be the force and the force can be obtained like we had done in the case of mechanical modeling, if we recall we had a theorem called Castigliano’s theorem, where if you take the partial of strain energy, partial derivative strain energy with respect to the displacement, you will get the force in that direction.

Similarly, here, if you want to get the force, we have to take an energy but that energy is called electrostatic co-energy or co here stands for complementary energy, co-energy and that is given by 1/2 CV Square. Normally, you all know that energy stored in a capacitor is 1/2 CV Square, where C is capacitance and V is the potential between the 2 plates of the capacitor, so that is called the electrostatic co-energy.

Now, if you take partial of, that is respect to the displacement here we have 3 variables shown, one is the length, that is the overlapping area between the plates, there is a length, there is a width w, there is a length l, there is a width w and there is also a gap, so gap is g, length is l and w is the width. If I take derivative of this co-energy, if I take partial of the co-energy with respect to l here as it is shown and takes a negative sign that gives me the force in the length direction.

So, if I have 2 plates like this and apply a potential between them, they try to align with each other. Let say I fix the bottom plate, let the top plate move, top plate will move so that the overlapping length will increase and that force is given by negative dou E e, E sub e is the electrostatic co-energy by dou l which = what is given here which is -1/2 Epsilon 0 w/g V Square.

Epsilon 0 here is the permittivity of the free space that is there between the 2 plates, like air for example. Because capacitance of a parallel plate capacitor is given by Epsilon 0 A/gap g, so Epsilon 0, A here is w times l, width times length/g, that is what we have put here, so that is the force in the length direction.

Similarly, if I want to know the force in the width direction that is negative of dou E e/dou w as it is shown here. That =-1/2 Epsilon 0 l/g V Square and there is also a force in the gap direction, which tries to bring the plates together and that is – 1/2 dou E e/dou g, which turns to be this value. Again if we recognize that w l is simply A. We can write it as 1/2 Epsilon 0 A/g Square times V Square that is the force.

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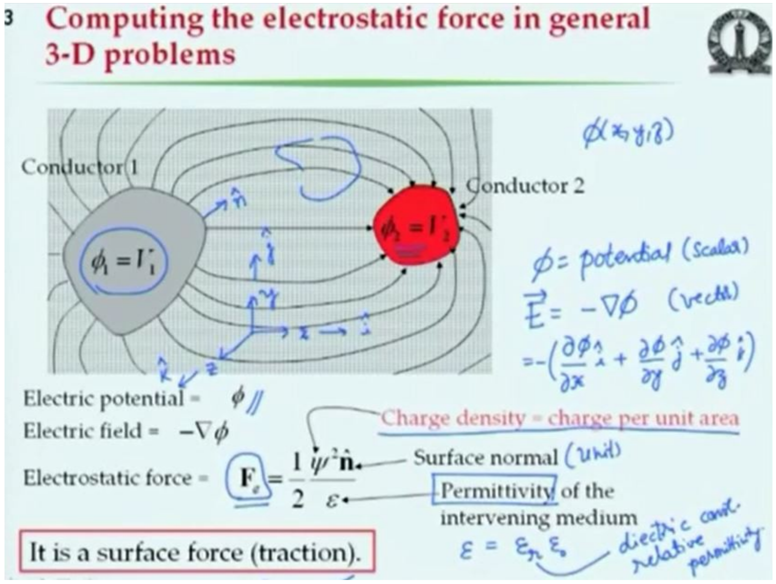

How do we get this formula? What is the basis for it? For that we need to review the basics of electrostatics itself, a very quick review. First, whenever we have conductors as it is shown generally in this, so here we have conductor 1 and there is conductor 2 here. Both of them are at different potentials, the first one is at potential V1 and the second one is at potential V2 and as you know, let us say V1 > V2 then the field lines going from here to there.

And if you have the field lines or field curves shown, we can see that there is a force that acts between these 2 bodies, which is electrostatic force. First of all, the field has a potential which we call Phi, we denote it by Phi, Phi is potential, electric potential or simply what we call the voltage and E the electric field is the negative of the gradient of this Phi that means that this is a vector, Phi is a scalar.

E is a vector, the gradient symbol here stands for, if I take this Phi, that Phi is going to be different because the potential here is V1, is V2 in-between it is going to vary, so that variation dou Phi/dou x, so if I take a co-ordinate system in this, this is x axis, this is y axis, out of plane is let us say z axis. Dou Phi/dou x in the i th direction that is their unit vector in the x direction+dou Phi/dou y j+dou Phi/dou z k all with the negative sign.

So that is i, that is j, this unit vector j and this unit vector k that is the gradient. So once you know potential which is a function of x, y and z. If you take gradient and put a negative sign, you get the electric field, those are the lines that are schematically shown here, is not the exact lines for these problem they are schematically shown, how the electric field is going to be for this problem.

Now, what you want to know is electrostatic force, that electrostatic force is given by this formula, this is also a vector and here what we see is 1/2 Psi Square surface normal, this is a unit normal that is everywhere if I take the surface here, they will be normal to the surface and Hat is what we denoted divided by Epsilon, which is the permittivity intervening medium that is permittivity of this medium.

Permittivity is a property of the material and that permittivity is given by Epsilon and sometimes we write it as Epsilon r times Epsilon 0, epsilon 0 is the permittivity of free space and Epsilon r is called the dielectric constant or we also call it relative permittivity. It is a basic slide but note that whenever you have 2 conductors there can be more than 2 also, we need to first determine the electric potential which will vary from point to point inside the conductor potential will be the same.

Entire conductor will be at the same potential like we have shown V1 and V2 here and in between it is going to vary and if you take the gradient of the potential put the negative sign, we get the electric field. Once we know the electric field, we can compute what is called the Psi, which is a surface charge density, charge per unit area will determine what it is in terms of the electric field now and we have to square it divide by 2 Epsilon.

And that is a magnitude of the force that acts on this conductor or any other conductor and this force is going to act only on the surface, It is a surface force like water pressure if you dip a body into water, this is going to pressure that, the body will feel only on the surface. So this is also a surface force, so it is going to be force per unit area, that F e we have here it is the force per unit area, it is a magnitude, which = Psi Square over 2 Epsilon and the direction along the normal to the surface at that point. Sometimes we call this surface force as traction.

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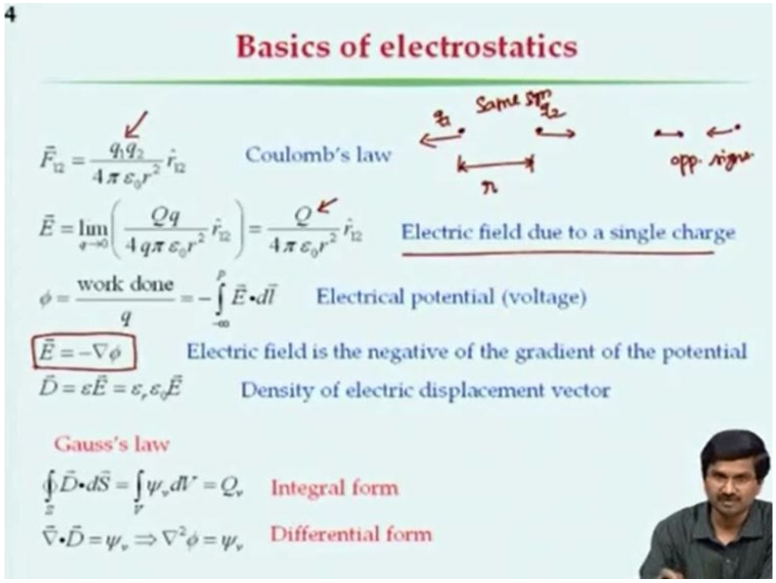

Now, let us see a little bit about electrostatics. We have to start with Coulomb's law that is all of us know, that if there are 2 charges q1 and q2 as shown here, the force between them is given by q1 q2/4 Pi Epsilon 0 times r Square where r is the distance between the 2 charges, so here if I have charge 1 and charge 2 between these if there is distance r, the force is given by let us say this is q1 charge, q2 charge, q1 q2/4 Pi Epsilon 0 r Square times r 12 meaning that between this charge and this charge.

If they have the same sign then they will repel meaning that the force applied by this charge on this will be in the direction and this charge by this on this direction, you have the same sign, they have opposite signs, this is the same sign for the 2 charges. If they have opposite sign, it will be an attractive force, this is opposite signs. You know that like charges repel and unlike charges attract.

Now here if you say what is electric field to a point charge, this is the formula for it, you have charge Q divided by 4 Pi Epsilon 0 r Square and then it will be from the point joining this charge location to the point, where you want electric field that is your r12 and the magnitude of that is r, the electric field due to the single charge and potential due to that we have to say how much work is to be done to bring a unit charge from infinity to that point that is a potential.

And if you use these 2 concepts you can derive that electric field is negative of the potential that we wrote in the previous slide. Now there is another concept, which is D, which is called the electric displacement vector. It is simply Epsilon 0 multiplied by E, Epsilon is the permittivity of the medium multiplied by electric field and that is electric displacement.

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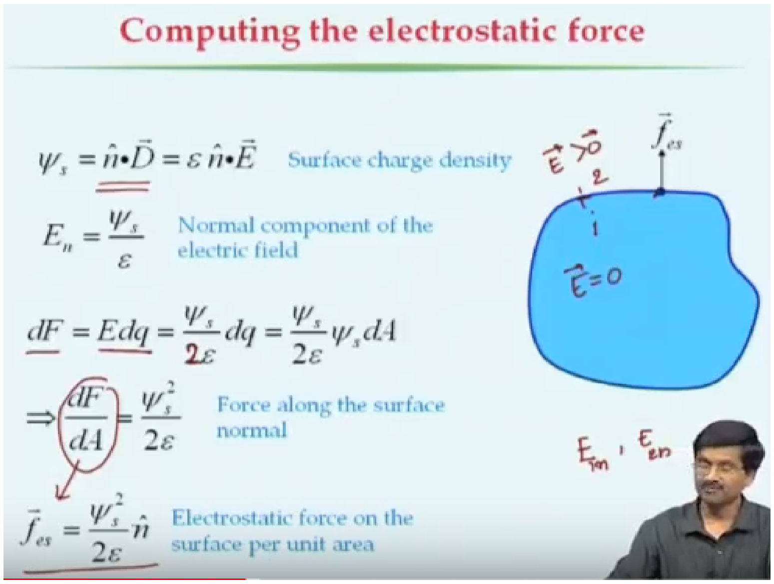

This electric displacement if I take the normal component of that, because electric displacement is going to act everywhere in the domain because electric field everywhere when you come to the boundary of the conductor that electric displacement if I take the normal of this D, dot product with the, if I take normal component of D, which is simply the dot product of D and the normal vector what we get is nothing.

But the surface charge that was there in the formula for force that we just discussed. So let us study force is Psi Square/2 Epsilon in the direction of the normal. In electrostatics, we have a fundamental law besides Coulomb's law, which is Gauss's law. Gauss's law states that the surface integral of this D vector, D. dS then the entire surface = the total charge enclosed in that one.

If I take some S here, let there be lot of conductors, dielectrics or whatever other things that maybe there. If I do this over this surface S, if I take this D everywhere dot with dS and do this integral over the surface, that will be = the total charge contained in this that is the integral form of the Gauss's law. Differential form if you use the Gauss's divergence theorem, where something, which has a surface integral it converts to the volume integral. So this would have been volume integral times dV.

And then this is Psi v dV so we just say that that = that and since d is Epsilon times E and E is minus gradient of Phi, we get that this quantity = simply Del Square Phi and that = Psi V and if there is no charge Psi V here is volumetric charge, that is charge per unit volume. If that = 0 which is most of the time we will not have charge there, what we need to do is we have to solve this equation Del square Phi = 0.

It means that in order to solve for Phi in this problem we have to, we know the boundary here the entire conductor is at potential V1 this is at V2 in between if I want to get Phi is a function of x, y, z. I need to solve this equation that is Del Square Phi = 0, that is the equation you need to solve in computing Phi. Once you know Phi, we can compute E as negative of gradient of Phi.

And then we can take the electric field multiply by Epsilon get D and take the normal component of D and that will be our surface charge. This normal component of the electric field is En and if you multiply by Epsilon that will become Dn that = surface charge. Now if there is a small charge here, because charge will be distributed on the conductor surface there will not be any charge inside the conductor, now due to this charge there will be a force that will induce at other places, so electric field is nothing but force per unit charge.

If you want to get the force, you have to multiply electric field by differential charge dq, Edq, we just said that E normal component is Psi s/Epsilon, and there is a 2 that is coming here, that comes because there is whenever the electric field is 0 inside the body, where as it is not 0 outside the body, when we take that interface one side to Other Side, this half comes because there is a sudden jump in electric field from 0 value to either value.

So there will be half the contribution this side, half the contribution on that side, of course these arguments have to be carefully done in order to understand where this half comes from. For now It is (()) (20:59) to say that the En component normally, the books will say there is E1n and then

E2n that is there is one side, this has side 1 and side 2, where you take the fact that electric field suddenly jumps and that gives you the 2 here in this formula, Psi s/2 Epsilon times dq and we see that dq is surface (()) (21:28) small area dA, Psi s/dA.

If we take dA here this becomes force per unit area that = Psi s Square/2 Epsilon. So this is the force that we get, that is how we compute the force acting on the body, which is in the electrostatic field.

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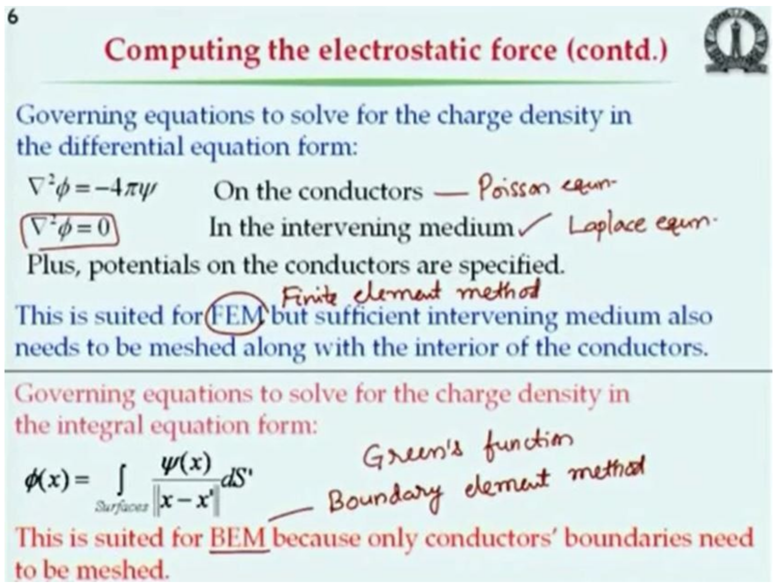

Now we already said that for the intervening medium, we need to solve this equation, which we call Laplace equation and inside the conductors there usually no charge, if they are worked to be charged or on the conductor there want to be some charge we have to use this one, Del Square Phi = some non-0 value, which we call Poisson equation. So that we can compute the potentials and then from the (()) (22:27) electric field from that, the vector D and then the surface charge and then the force.

There is an alternate way to do this, which is to use the integral form of the Gauss's law and find what is called the Green's function solution, green's function for this one that is this is one of the solutions for the integral form of the gauss's law, which we have seen in this slide. This is the integral form of the Gauss's law.

The solution for this, there is a Green's function solution, which is shown here you can use this and solve it using BEM which is Boundary Element Method that can be used to solve this or if you solve the differential equation we can use the Finite element method to solve the differential equation, either one is fine. We can use Finite element method and solve the differential equation or use Boundary element method and solve the integral equation.

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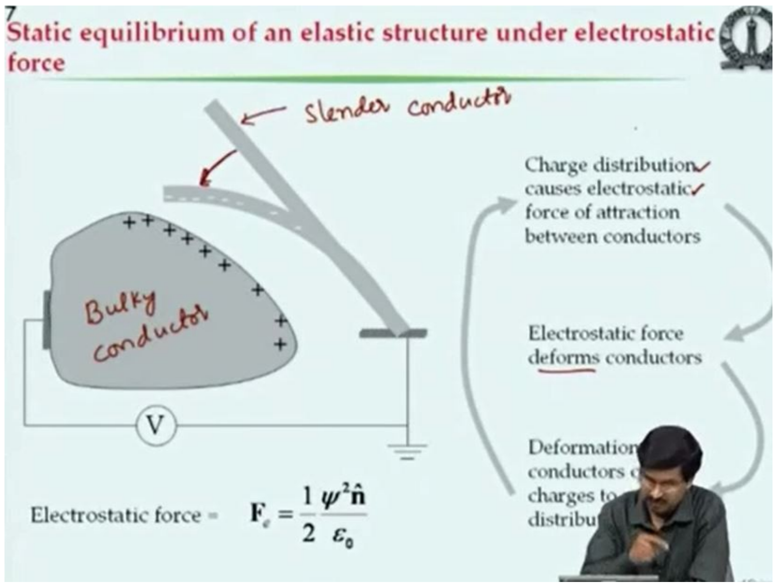

Now let us see how there is coupling between electrostatic field and the elastic field. For that let us take a bulky conductor, let us call this a bulky conductor, intentionally shown to be very stiff and rigid, other one is the slender conductor such as the one shown here. Normally in the micro machined structures the substrate will be like a bulky conductor and the one that moves either it is a beam or a plate or a membrane, whatever that we have there, which is free to move or which is very flexible, there is Slender conductor or there is a bulky conductor.

Now if we apply a potential between these 2 points, so we are showing some V between this conductor and that conductor, then we can solve either the differential equation or the integral equation to compute Phi and then electric field and then D and then surface charge and the surface charge can be shown. Let us say it is something like this positive here and negative over there and when there is charge, there will be force between the 2 electrostatic forces and that force will cause the slender structure conductor to deform as shown.

So it has deformed from there to here. Now you see that because of the deformation the problem has to be solved again in the electrostatic domain because originally it was over here and it has come from there till here. So we have to solve this problem again then check charge distribution and then we have to repeat it.

So we have to first take the problem and do the charge distribution causing electrostatic force of attraction between the 2 and apply that force on the conductors, deform it and then deformation has got changes there, charge distributions you have to go back here and keep on doing this several times until we get a self-consistent solution between electrostatics and the elastic deformation.

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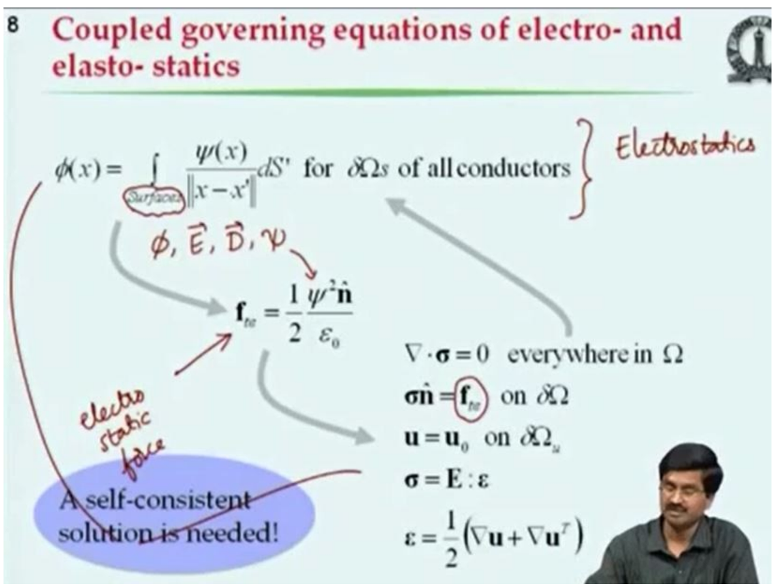

Let us see that in the form of equations. Here we are showing the integral form of the electrostatic equation. This is electrostatics and once we solve it we will get the potential and then we will get the electric field and then will get the electric displacement, we get a surface charge density Psi and that Psi goes here we get the electrostatic force.

This is electrostatic force and that force goes in here as the traction or surface force of the elastic body then the deformation changes and that has to be fed back here because the surfaces would have changed because of you, surface geometry would have changed, you have to redo this and do this again, again and again until this equation and this set of equations are satisfied together that is we want a self-consistent solution between the 2 sets of governing equation between elastic and electrostatic fields.

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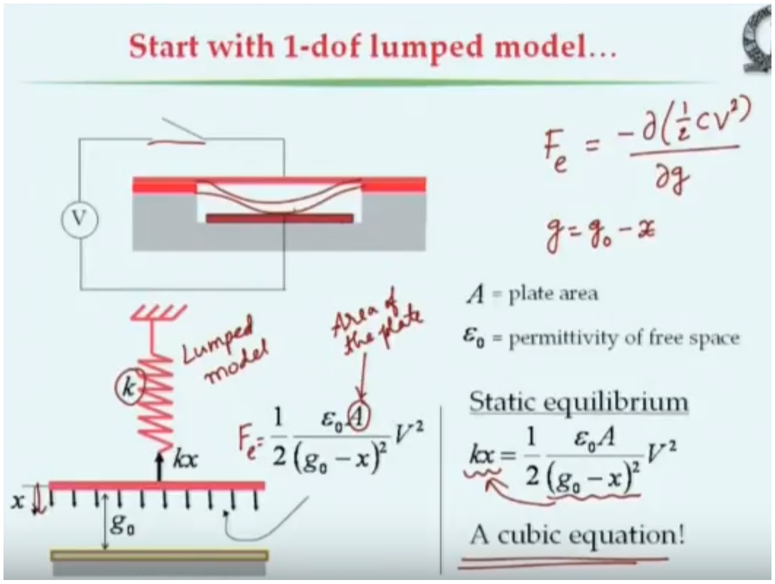

All this may sound quite complicated at the first site indeed it is, in fact that is why the modeling software for electrostatic micro systems are quite sophisticated and at the same time they do take considerable amount of competition time if not analyze a complicated structure. And it turns out that this coupling leads to some catastrophic phenomenon meaning that suddenly something that you do not anticipate happens. So let us take a very simple one-dimensional lumped model to analyze what is going on in this couple problem.

For that let us imagine like a pressure sensor, where there is a diaphragm and there is an electrode. Now if I apply voltage between these 2 that is a closed switch what happens to this because of the force this going to deform and we need to analyze by solving all those equations instead let us model this plate or a diaphragm as a spring of spring constant k and make this diaphragm or a plate or just a flat plate that does not deform because we have captured the deformation as this lumped spring.

This is a lumped model. We just have put a spring and this plate, in the bottom plate that is this one, we are simply giving it is a plate. There is a gap g 0 between them and A is the plate area, A is the area of the plate and here the force, this is the electrostatic force is 1/2 Epsilon 0 A/g 0-x square, where x is the displacement of this plate, this is x. Earlier we had seen that between 2 plates, parallel plates the force is how did we compute? That force was F e, electrostatic force is negative of dou 1/2 C V Square/dou g. g here is whatever g 0 that we have-the displacement that is shown downwards.

Original cap is g 0, where this plate moves by an amount x. The gap is going to be g 0 – x. That is the way we have got this. Now, for equilibrium between the elastic and electrostatic problem, we need to have the mechanical force, which is k times the displacement. k x is the force to the spring that should be = the electrostatic force. If we see, there is k x here and there is g 0 – x square at the denominator, if I take it to this side, I get a cubic equation.

A cubic equation can have all 3 real roots or one real root and 2 complex conjucative roots. In any case, there will be 3 roots, sometimes all real, sometimes a real root and 2 imaginary roots. But, a real model such as this one, which solution does it take? Because, does it take the first root or the second root or the third root? If I take this (()) (30:52) voltage is going to deform to one position. It is not going to have any big duty whether as it should take this or that or that.

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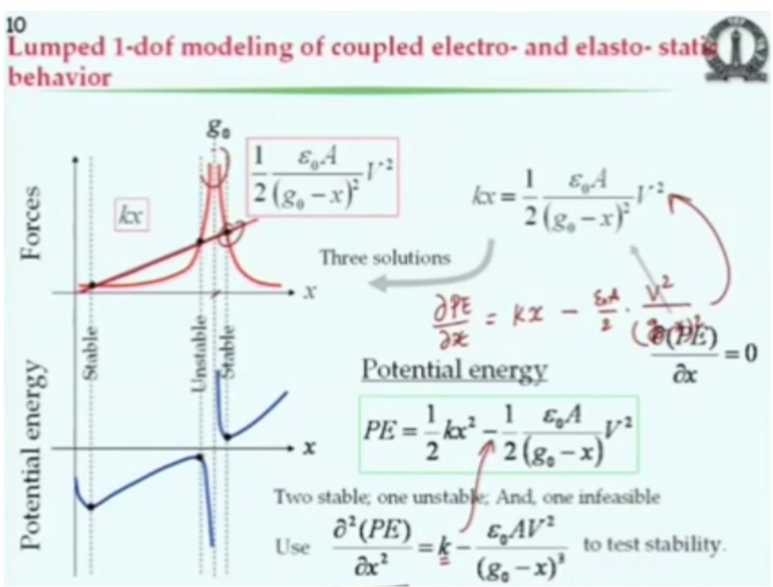

So, you have to analyse that we need to discuss the stability of the equilibrium solutions and to see the stability, let us look at the forces. The mechanical force, which is shown as a straight line here and the other curved one is the electro static force. You can see that when x = g 0, it is going to blow up as it is (()) (31:25) over there. Now, as we saw the nature of the equation cubic, these 2 have 3 solutions.

Here is 1, here is 2, here is 3. The third one is a material, because this point corresponds to g 0, this is more that, that means that this plate has to penetrate this ground electro, which is not feasible, so you do not need to worry about the solution, the third solution. Between these 2, which one would we take, when we talk about stability?

In order to discuss about stability, let us plot the potential energy of the system, which is given by the spring mechanical force half kx square for this lumped model minus this epsilon 0 A/(g 0-x), the half here and then V square, basically half CV square the negative sign because when you take gradient and put a negative sign you get static force, whereas for the mechanical force you do not need to put a negative sign. So, we take care of that in potential energy with a negative sign here.

If I take the derivative of this potential energy, I get back this equation. Let us see that because we have PE, if I do dou PE/dou x, I will get half kx square. When I take the derivative, just becomes kx and then if I take derivative of this, epsilon 0 A/2 remains as it is. 1/(g 0-x), it will become (g 0-x) square and then -x will lead to -1. 1 over would have given 1- (-1), those 2 get cancelled. So this –A remains the same and then we have the V square as it is.

That is the equilibrium equation that is just force balance. To discuss stability, we need to take a second derivation of the potential energy. So we need to take the second derivative, which is dou square (PE)/dou x square. That will from here, it will simply be k and this will be epsilon 0 A V square/ (g 0-x) cube. This have 2 in the denominator, will go away when you take derivative of 1/(g 0-x) and that is what we get for stability.

If this is positive, that cuts most to the minimum of the static energy. If it is negative, it corresponds to the maximum. So between the 2, we know that that one that is the minimum is a solution because potential energy when it is minimized that corresponds to the stable equilibrium. The maximum corresponds to unstable equilibrium.

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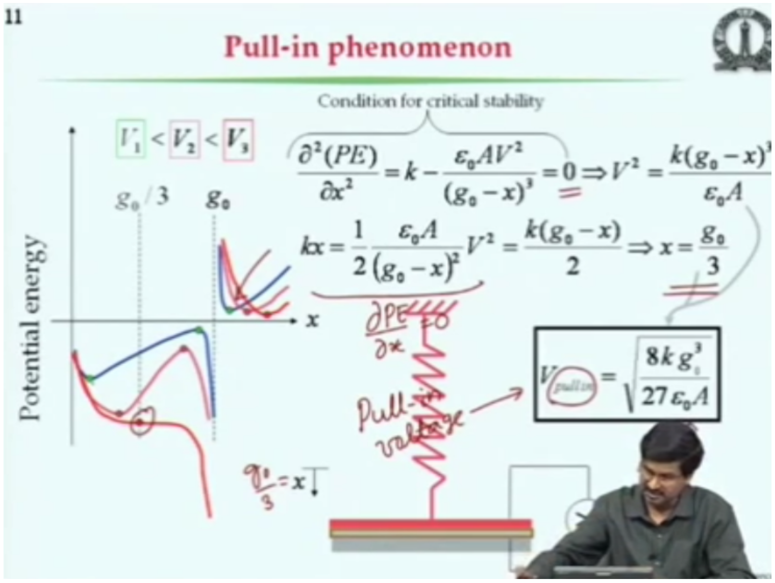

So it is clear that between these 2 problems, that is between that these 2 solutions, which is the first solution, second solution, the plate will choose this one that is closer with x = 0, the plate is moving and it is (()) (34:25) this. And we have already said that third solution is more than g 0, so we do not worry about it. Now let us say I increase this voltage from V 1 to let us say V 2, which is larger than V 1.

So, this placement has to be more, so this stable route will move to the right and it so happens that the unstable route will move to the left. So, when I keep on increasing this voltage to let us say volt V3, then this minimum and the maximum there, quails into one point. And that is the transition point or a catastrophic point, where beyond which there is no real route other than one that is more than g 0.

These 2 become compressed conjugate and we do not have a solution, so what will happen then is the plate, the moving plate, let us say the bottom plate is fixed, it will just move down and touch the bottom plate and it will cause short circuit. So, in order to analyze that critical condition for stability of this, we have to take the second derivative and equate it to 0. And also we have the original equation, which is equilibrium as dou PE/, the first equation is dou square PE/dou x square = 0.

The second equation, which is just one is dou PE/dou x = 0. When you solve these 2 simultaneously, we will notice that x at this critical point is one third of the gap. And then the potential, which is called the pull-in voltage, this is the pull-in voltage, that transferred to be 8 times g 0 cube/ 27 epsilon 0 A within square root. And this is the pull-in voltage. So as we increase, if we go back to this problem, if I have this spring and when I apply voltage, when it is pull-in, it will just move, x = g 0/3. And there will be gap, which is 2 times g 0/3. It will move there.

If it is more than the pull-in voltage, it will simply come and collapse, if it is more than the pull-in voltage. Computing this pull-in voltage is very important because we do not want our micro structure to be unstable. So we have to do it in such a way that whatever voltage is applied on it or this structure is going to experience, should be below the pull-in voltage. Otherwise, there would be catastrophic phenomenon, where the plates come together by themselves.

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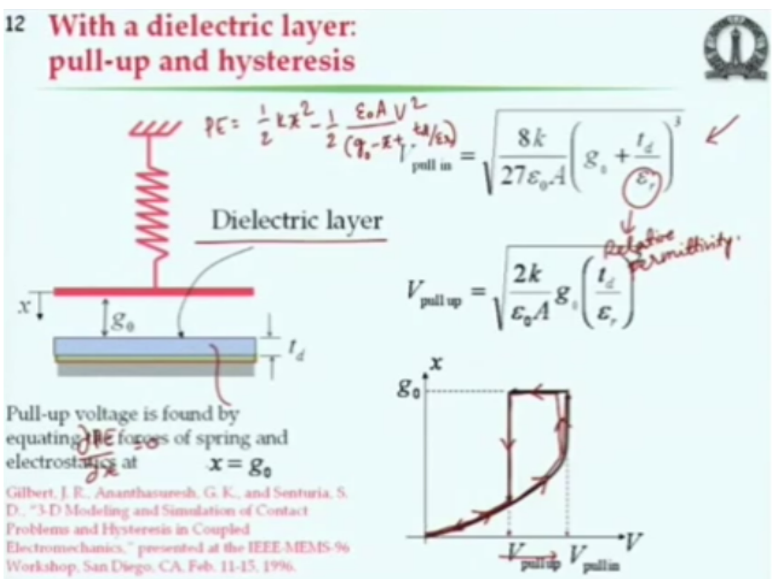

Now what happens if there were to be a dielectric layer, as it is shown here? So we have a dielectric layer. This part is already marked, it is a dielectric layer, the thickness is td. Then if you have to redo this calculation, meaning that we say dou PE/dou x = 0, where our PE is ½ kx square as before – ½ C, C now is epsilon 0 A/the gap (g 0-x)+also this td should be there. But that td will be, epsilon r has to divide this td times V square. Epsilon r here, this is the relative permittivity of the dielectric layer.

If we do that, pull in voltage will have a different formula as it is show here. Now the reason we consider this is to explain another settle point here, which is, first as we increase the voltage, as we are doing here, the displacement that is the displace of the plate keeps on increasing and suddenly it pulls in right. Now, if this direct relay will not be there, it will go and touch the bottom electrode, short circuit happens, that is the end of the story. But now, since there is a dielectric, it has to stop there mechanically and there is no short circuit.

Now, if we decrease the voltage, what would happen? Would it just jump back from there? Actually it does not happen. It will stay there at that point for a while, that is we have to have to reach another voltage called pull up voltage, which is over here, which is smaller than the pull in voltage, comes back and then it will go to 0. So when I am increasing the curve will go like this, increasing the voltage and then the pull in occurs.

At that point, if you decrease the pull in voltage, there will be a point, where there will be pull up voltage, it will move there and then it will follow back from here. So it goes from here like this, like this jump up and then go from here to there. And this is like a hysteresis. It is not really hysteresis, in terms of there being a loss of energy. It is just that the going forward and backward has a different behavior in this case.

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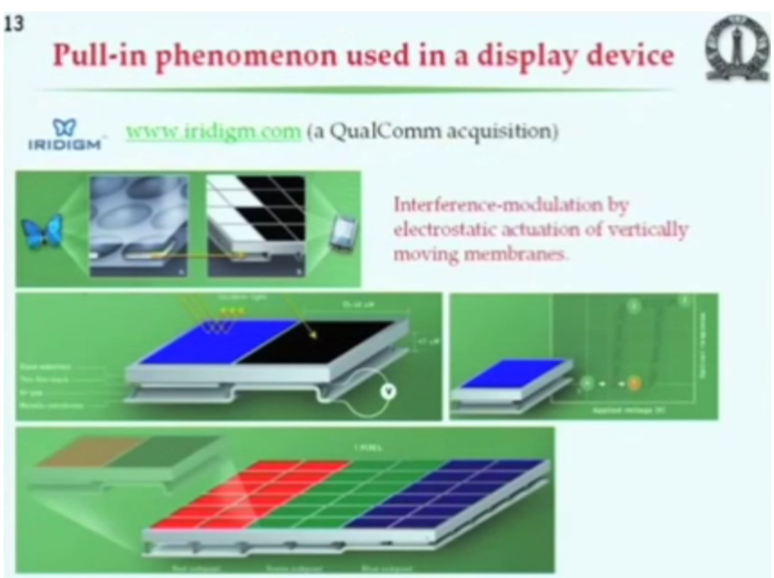

And in fact, it has been exploited in some of the commercial devices and one of them is shown here, where this company says that, if you think of the butterfly, why it has the iridescent colors, that is because on its surface they say there are the small membranes that gap underneath and even when that gap closes, the optics comes in here, where the gap between the top electrode and bottom electrode is lambda/4, thickness of this is lambda/4, so from here to here it is lambda/4, from here to lambda/4, the total is lambda /2.

The light falling off of this and going, other one that reflects out the bottom one and going, they both will be in phase or out of phase, depending on whether the plate has moved in as it is here or not. That is actuated or not actuated, we can have constructive and destructive interference and make the majors, the pixel correspondence to this portion to be dark or grey.

And if you adjust this thickness and gaps according to the wavelength of red, green and blue, the primary colors, we can actually create a starting image. So by addressing each of these mirrors that is shown here, just like they say the butterfly gets its wonderful colors. We can also create starting images, they say commercial product. In fact, the same principle or similar principle is used in the electrostatic micro mirror.

There again in order to tilt the mirror, this way or that way, we have to apply a voltage that is beyond pull in voltage. Of course, there is a k, it is like a torsional spring because there were, we have 2 torsional beams, there is a mirror in between that can tilt one way or the other. There you have to get torsional spring constant and do the same analysis we did, in order to get pull in voltage. And if we apply voltage that is more than pull in voltage it will just tilt that way and this way, we can tilt the beam very fast.

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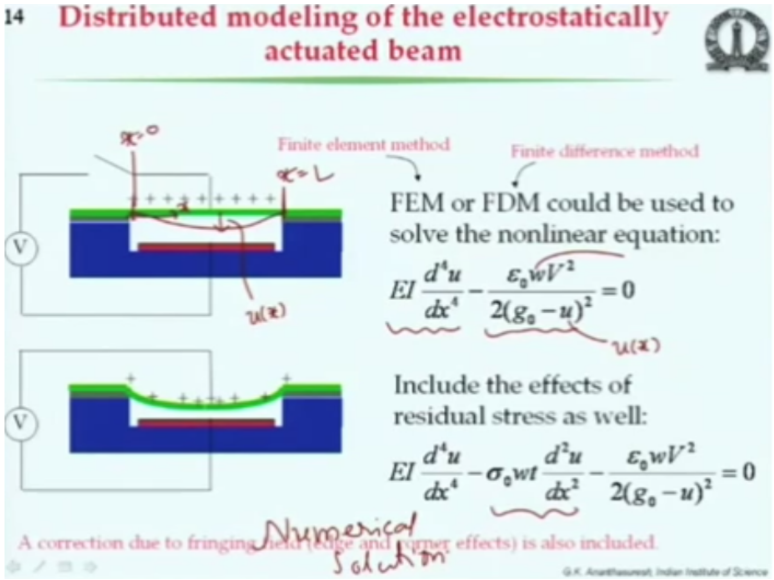

Now if we start moving away from our 1D model, we look that 1D model, meaning one dimensional just lump spring and parallel plate, both in electrostatics and elastic feel, it is a lump model. Now, if you want to get more accurate result, then you have to solve differential equation. There is no other way, which we have discussed. Now let us say that we want to have a domain in between, so it is not full scale tridimensional analysis of electrostatics and elastic field nor it is as simple as just having a spring and a parallel plate approximation.

In between, let us say we take this as a beam, then we have earlier discussed the governing equation for a beam, which is given as EI times 4th derivative of the transverse displaced. If this, we assume that it is a beam, it is going to deform, so that is u affix everywhere, 4th derivative of the respective x, so x is here in this direction, that is x, 0 here and L here, so here x = 0 and here x = L. So EI d to the 4 u/dx to the 4 and this is the electrostatic force.

So here if you notice, instead of A, we have put w that is for unit length. Length is this direction. We have put epsilon 0 wV square/2(g 0-u). Earlier we have x. Note that this u is the function of x. So we have to solve this equation. If you also want to put the residual stress, then we need to include that as found, concept residual stress. Concept residual stress, the concept residual stress we have discussed in one of the earlier lectures, we have to put this, this equation we need to solve. Clearly this equation cannot be solved generically, so you have to get a numerical solution of this equation.

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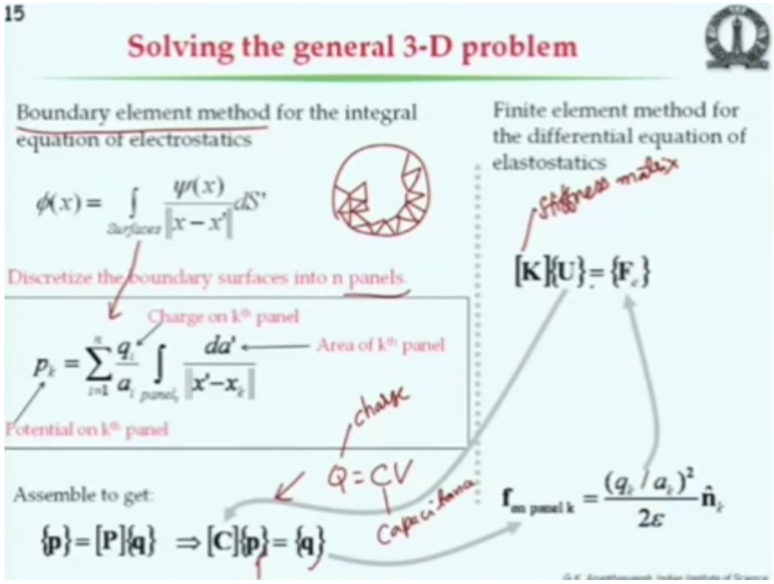

And if you want to solve the general 3D problem, if you want to know how the commercial micro system modeling software programs solve this one, we have to look at the equation that we had earlier, where we said integral form can be used solved using boundary element method, something like this. And if you have to, in the boundary element method, in pantelment method, we would discrete as the entire domain into small elements.

Here also we divide into small elements, but only on the surfaces. That is advent in the boundary element method, we only work with the boundaries. So there the elements, we can call them panels, small triangles. If I have some domain like this, I would divide this whole thing into small triangles like this, some panels, something like this. We can divide this into small triangles.

Each panel is a boundary element. Then we can make this integration, become summation or this, because si s q/area of the panel, q i/a i and then this greens function can be written in this form, thus = potential of that phase, which we know, potentials are known, we need to compute the charges on each panel, we need to solve this equation. If you work this out and assemble in the form of a matrix that is going to look like this.

The p, the potential vector = the P matrix and the charges, which you do not know. So if you want to solve it, we have to get this or C trans p. If we arrange it, p inverse goes that side, there is nothing but the capacitance because we know that for a pallet freight capacitor, Q = CV. So this is the high dimensional version of Q = CV. Q is the charge, V is of course the voltage, this is the capacitance, this is the capacitance in the higher dimensions. So we can, once p's are known, we can compute q, using C p = q relationship.

If you come to the elastic domain, this force, this is Si square by 2 epsilon in the normal direction to the surface there. And if you put this into our loading of the problem for mechanical, just as we had C p = q, the linear modeling, this elastic for linear modeling K U = F, K we call this as stiffness matrix and U is the displacement as you call to the applied loads. We can compute it, but then because of U.

This thing will change again because capacitance depends on the geometry. We need to solve it again, go here, go here, go here, so by going back and forth between these 3, we can solve this problem eventually.

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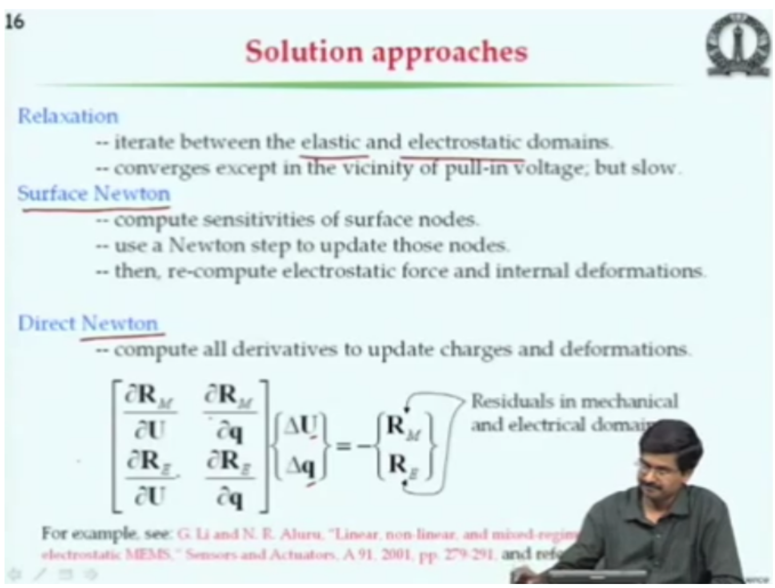

But if you want to do it little bit more efficiently, that is this kind of relaxation, that is we have to solve the refractive problem, solve the mechanical problem, go back and forth, this is called the relaxation approach, that is finally as we literate between elastic and electrostatic, after a while it would converge, but it will take a very long time to converge and especially in the vicinity of the pull in voltage, when the voltage that you applied is very close to pull in voltage of that particular system, then it would take a long time to converge.

So you can, you need to find better approaches for doing it, which is done by taking derivatives, which is one what is called a surface Newton, where you compute the gradients only on the surfaces of the boundaries, surfaces of the conductors. And there is another one, where you can go the Newton method, where you take the sensitivity is not only surface nodes, but also the interior nodes.

So you can have the coupling turns between the mechanical and electrostatic fields as it is shown here and we need to protop the displacements or the charges on the 2 and try to finally make these 2 residuals 0. So you can use these also, which is what is being done in the commercial micro systems modeling software now.

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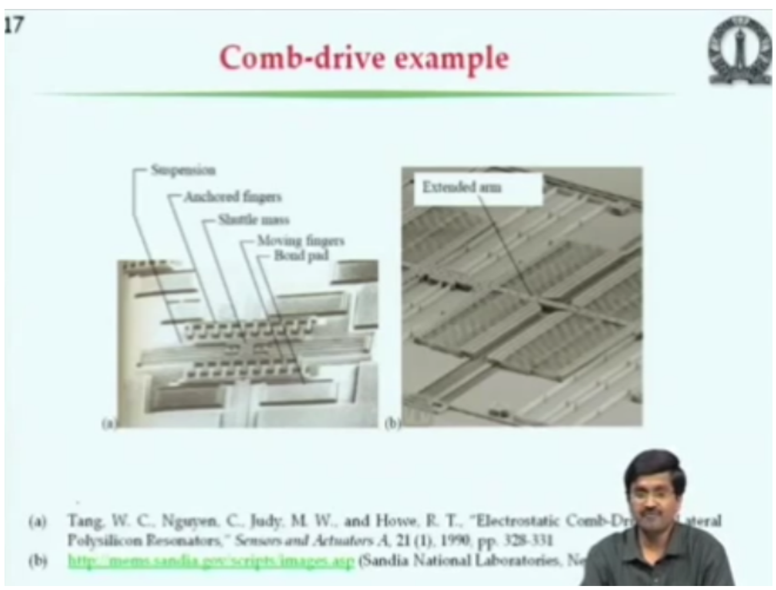

So if we have a comb drive, a structure accelerometer, gyroscope, comb drive, pressure sensor, RF switch, any of these devices, if you take, if they are using electrostatic force or they are using capacity sensing technique, we need to use the modeling that we have just discussed.

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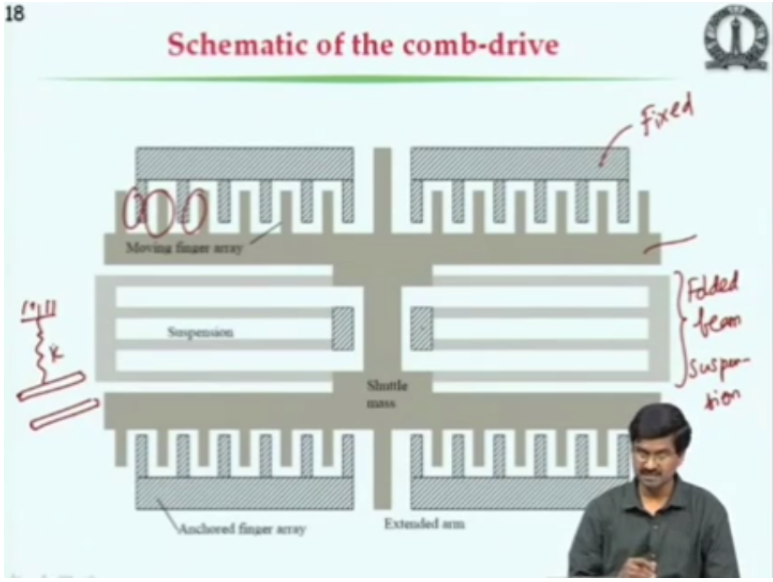

So if we take the comb drive, there is just one view of the comb drive, where whatever is hatched here is anchored, it is fixed. Rest of the portion, which is in this color, is going to deflect. And this portion is what we called folded beam suspension. Again that is anchored here, that is anchored here. Now if there is voltage potential applied between this moving structure and the ground, it experience static force at every pair of this comb fingers, here, here, everywhere. For that, we know much force will be exerted, when things move inside like this.

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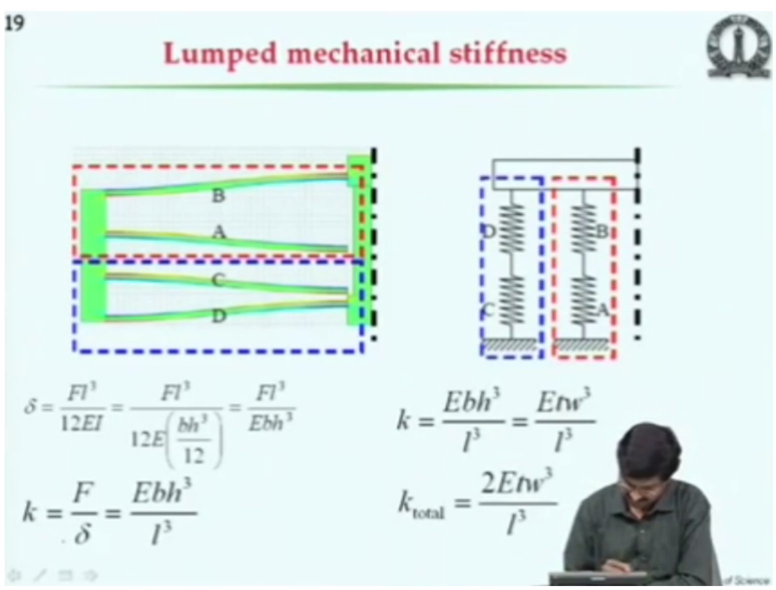

For that, we need to first estimate, let us say before we go to derivational modeling, we have to just estimate the pull in voltage, for that we try, we treat this suspension as 4 beams on this side, which are fixed and guided, fixed and guided, a lump modern technique that we have discussed in one of the earlier chapters. And look at whether the series are parallel, finally reduced this hole suspension into just one spring or spring constant k and then we will have a plate for the electrostatic one, a parallel place capacitor, so just as k is a lumped constant of the mechanical behavior of the structure. Parallel place capacitor is the lumped model for the electrostatic behavior.

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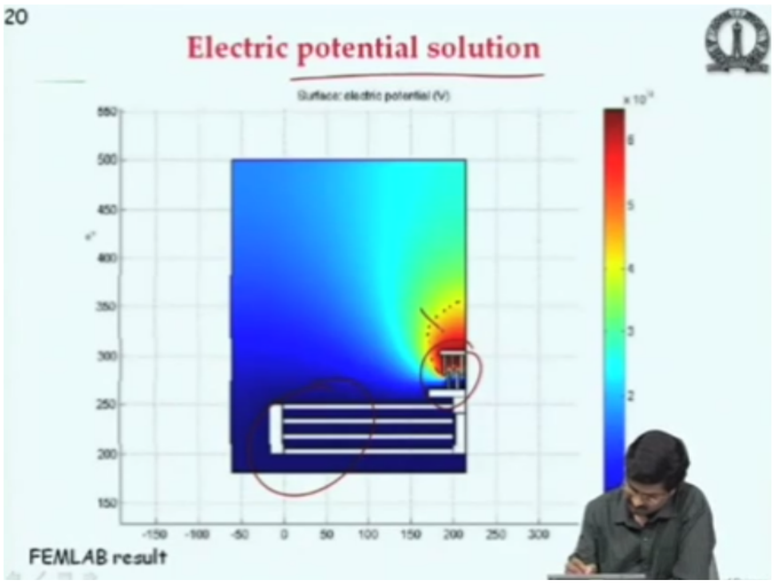

So if you compute that k and try to see what the pull in voltage is, we get a value and if we go to faradenmal software and plot it, here what we are plotting is potential here, so we have electrostatic comb drive, which has a suspension, which is this portion and then there are combs over here. When we apply a potential between these 2, there will be a potential that will exist in the entire domain and also the electric field, then the force and so forth.

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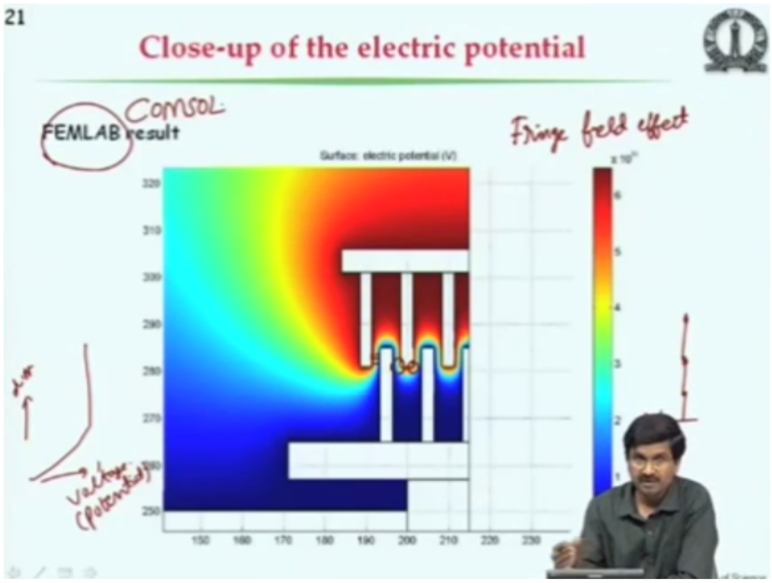

So on to compute, the pull in voltage for the 3 dimensional structure, such as this, we have to start with some initial V value, voltage we have to start some value and then raise it to another value, raise to another value and then see how this placement looks like. If the pull in is going to occur, so what we would find is that this placement initially will increase like that and sudden at some point, it just goes to infinity.

So this is displacement verses voltage potential. Where does it occur? We have derived the lumped model. If I take this spring constant here, I know the thickness, width of the beams, length of the beams, if I know this k, then using the formula for pull in voltage, we can compete the pull in voltage as well as pull up voltage that were to be electric and then we can go back and do detailed fundamental stimulation such as this one and get the result.

What is shown here, the potential or equi potential line, if I take this yellow, it is in the same line, there is an equi potential line. And the electric field always will be normal to this. And between the 2 co fingers, if we really zoom in or here we can see that field lines here are going to be parallel. That is perpendicular to the conductors, but field lines themselves will be parallel to each other.

And around the corners, we have some crowding of the field line, which is called the Fringe field effect. And there are waste to account for this fringing field effects in the lump model by putting a correction factor, but in general, if we have access to a parent element or bond element solver, we can solve the problem numerically, as it is shown here in a commercial software called Femlab, its name now is Comsol, there are lot of others answers and also they are customized modeling software for micro systems called Coventoware, Intelisweet and Softmemes and many others. And we can use them to solve the coupled equation.

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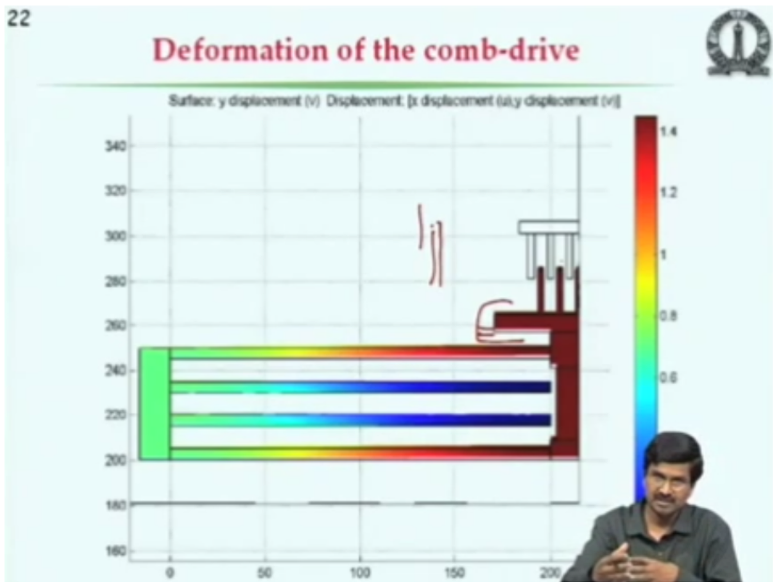

What we have done in this lecture now is to discuss, how we can start with a Columb's equation, Columb's law and go all the way to electrostatic force, we get the couple problem, so that in a comb drive problem such as this, if I apply voltage between these 2, we can see, if you focus on this portion, where it was and where it has moved. That is, this comb fingers, which were at some point like this, after a while this moves up here, this finger would have moved up little to that.

And that is what we see for solving this. This is interrelative process, you have to solve the electrostatic problem, do the elastic problem and again do this and do that. There is a very strong coupling between the electrostatic field and the elastic field because electrostatic field is dependent on the geometry of the conductors. But then, due to the force, electrostatic force, the geometry of the conductors change.

That is the deform and distort of the field, so we have to recompute that and then the first changes you would recompute the deformations and then that could have changed the electrostatic field and then we have to keep on inter relating between the 2 fields, that is why we have a very strong coupling.

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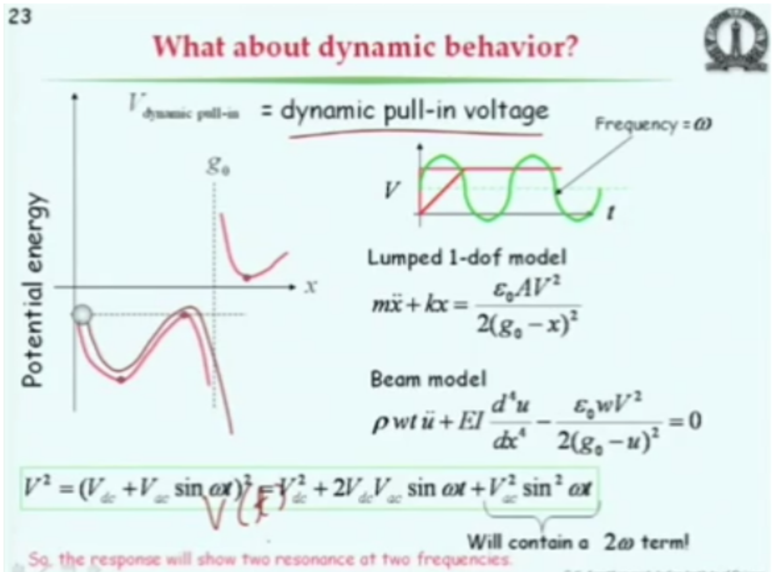

And in fact, there is this pull in phenomenon that we discussed earlier, where you can think of this. This is the potential energy curve that we have seen, that there is a minimum and a maximum. But if you think of what happens in reality, if there is a ball here and that energy = this maximum energy. It will roll down here, go there and if its voltage is slightly more than that, it will just flop it over.

That is there is something called a dynamic pull in voltage. The formula for that also can be derived, just as we have derived the static pull in voltage formula. Here remember that the energy at the beginning = the energy at the maximum. Some way that, we can do and we look at the dynamics of this model, which we will discuss in the next lecture. Now we have discussed the statics and the pull in occurs. But now in the next lecture, we will discuss the dynamics, where we will consider what happens, if the voltage is applied itself, is time varying V(t)

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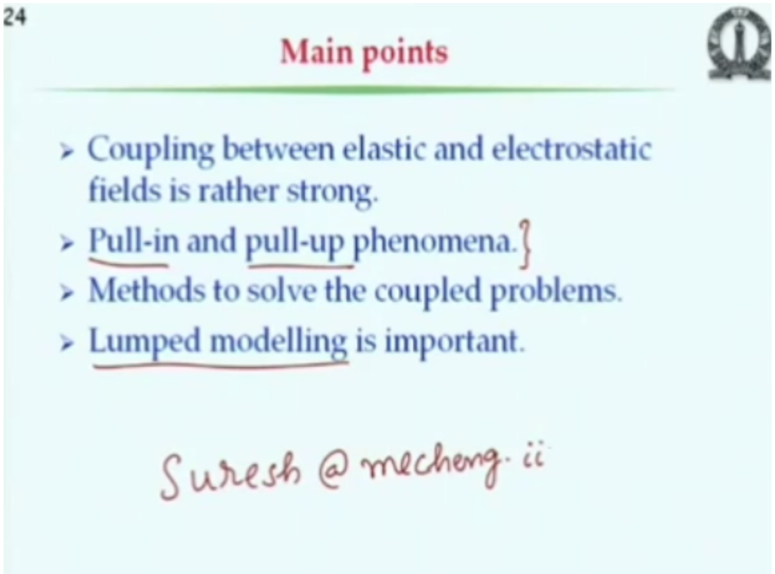

Just to summarize what we have discussed today. Coupling between elastic and electrostatic fields is very strong and it leads to this strange phenomena called pull in and then pull up, hysteresis like behavior, which is exploited in certain devices. And we discussed the methods to solve this problem, both with 1 D model, to understand why this happens and how to solve it. And in the general be equation or the 3D models, what equation it will be solved in order to compute this pull in voltage or pull up voltage.

And we find that lumped modeling is a very useful tool to get in sighted to the problem and also in design. So next lecture, we will discuss the dynamic effects of a coupling between elastic and electrostatic problem. If you have any questions, you can send me in email at suresh@mecheng.iisc.ernet.in. Thank you.