Micro and Smart Systems
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Lecture – 34
Control and Microsystems

So in this lecture, we will talk about control system. So control system is a part and parcel of many smart systems that we design. So control system essentially is designed to perform certain actions which the smart structure has to perform. So in this lecture, we will try to look at what all the ingredients of the control system, what are the mathematical models that are required for control system.

And what are the basics stability concept of a control system and some small basic control system design concepts we will actually study in this lecture.

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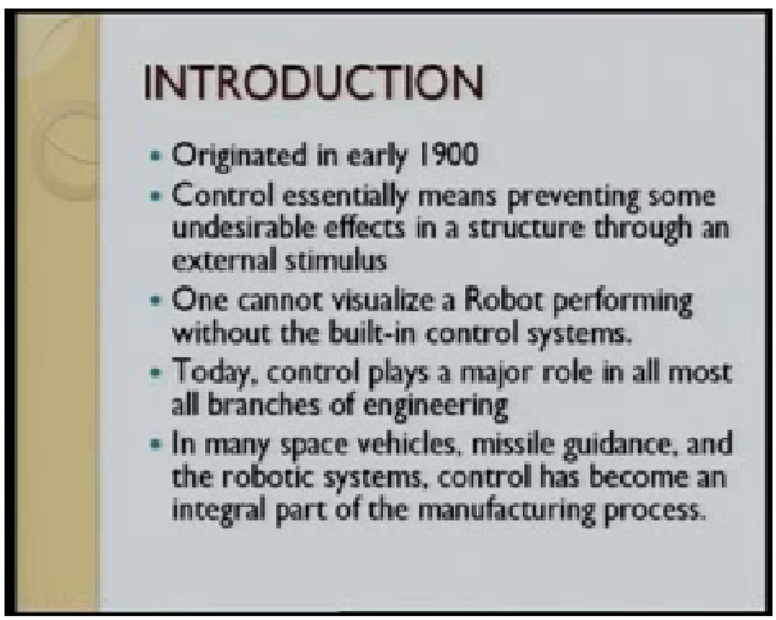

So to give you some introduction on control system, basically the first control system was started way back in 1900. Control essentially means preventing some undesirable effects in a structure through an external stimulus, okay. One cannot visualise a Robot without a control system. Robot does multiple actions and these actions are possible basically by control systems.

Today, control plays a major part in almost all branches of science and engineering. In many space vehicles, missile guidance and robotic system, control has become an integral part of the manufacturing process.

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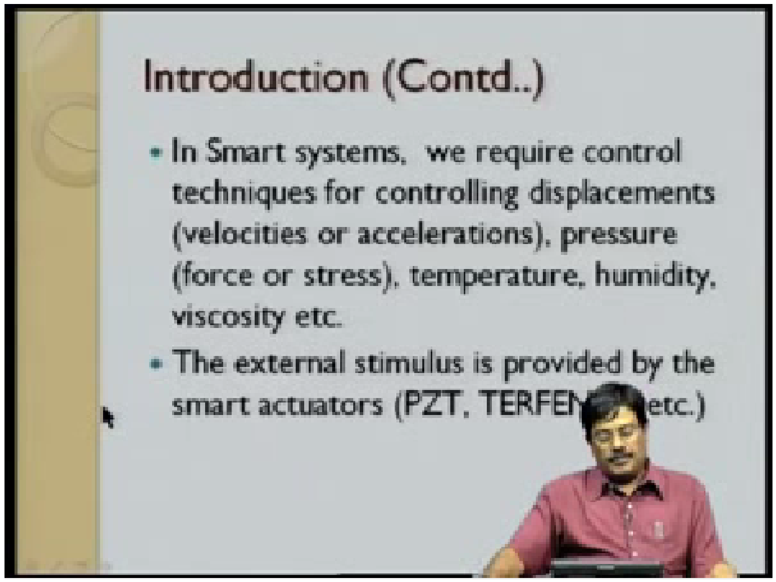

Where does the control system fit in smart systems. So in smart systems, we require control techniques for controlling what we call displacements and its derivatives like velocities and accelerations, pressure and its derivatives like force and stress, temperature, humidity, viscosity and many more parameters that are basically part and parcel of this smart systems. The control system always requires an external stimulus.

And this external stimulus is provided by smart actuators basically through what we call the constitutive laws and some of the actuators that we studied in previous lectures or the piezoceramic actuators like PZT, magnetoelastic actuators Terfenol-D, etc.

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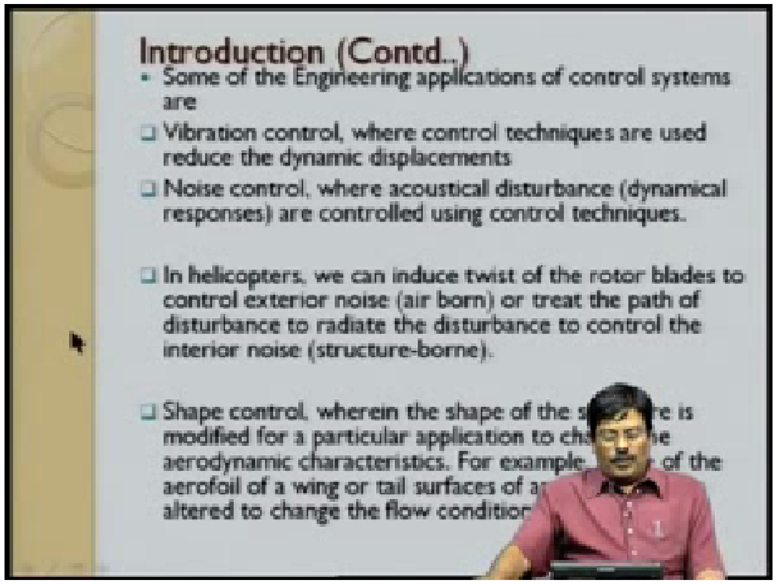

So some of the engineering applications of control systems where we would like the smart systems to perform. One is the vibration control where we control the dynamic displacement using control techniques. We have noise control, excessive noise is not desirable in many of the automobiles and aircrafts. So where the acoustical disturbances are controlled through control techniques.

In helicopters, where helicopters always produce excessive noise because of the rotor displacement. So we try to control it by actuating the flaps. By doing so, we control the exterior noise or sometimes we try to design a control system to treat the noise path so that it does not reach the helicopter cabin so that their cabins are quieter. Many a times we also design a control system to change the shape, a flat plate can be bent and in aircraft.

We can change the aerofoil shape, aerofoil controls the flow. By changing it, we can avoid vortex which basically is one of the root cause for aircrafts to stall.

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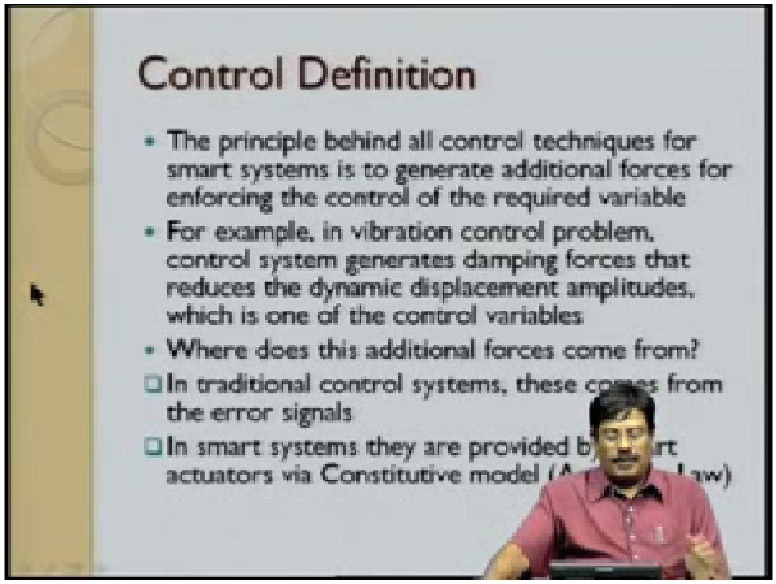

So now let us go to control definition. The principle behind all control techniques for smart system is to generate additional forces for enforcing control of the required variable. So why we need, we need to control something. So what we control is a control variable, for example in vibration control, control system generates damping forces that reduces the dynamic displacements or dynamic amplitudes, which is one of the control variables.

So now the question is, where does this additional force come from especially in smart systems or in the traditional non-smart systems. In traditional control system, in the non-smart systems, these comes from what we call the RF signals. We will talk about RF signals little later in this lecture. In smart systems, they are provided by the smart actuators through constitutive law.

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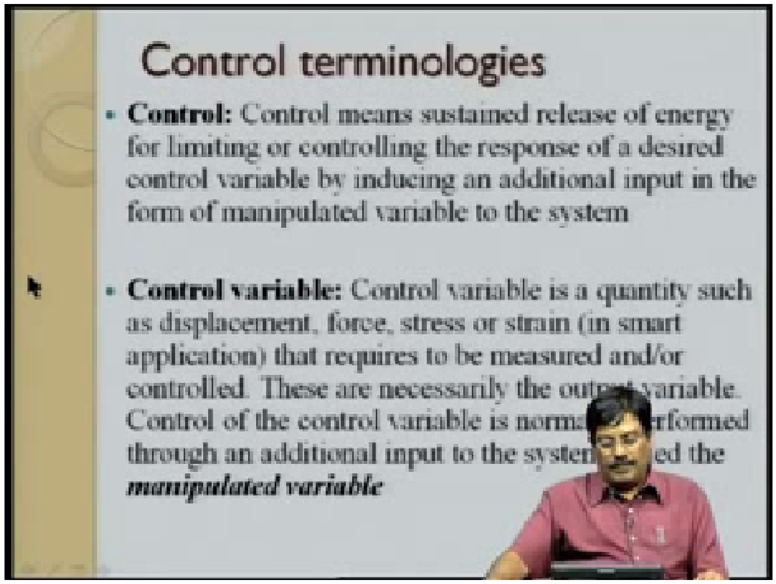

So let us define some of the control terminologies which we use traditionally in control system design. Let us first begin with what do we mean by control. So control means sustained release of energy for limiting or controlling the response of a desired control variable by inducing an additional input in the form of manipulated variable. So now the manipulated variable is something we input to the system.

Now we next define what is control variable. So we need to control something in the control system. So what is that something that we are going to control, that is the control variable. So the control variable is a quantity such as displacement, force, stress, strain, pressure, temperature, etc. that requires to be measured or controlled. These are necessarily an output variable and control of the control variable is normally performed through an additional input that is provided to the actuator in the smart system and which is called the manipulated variable.

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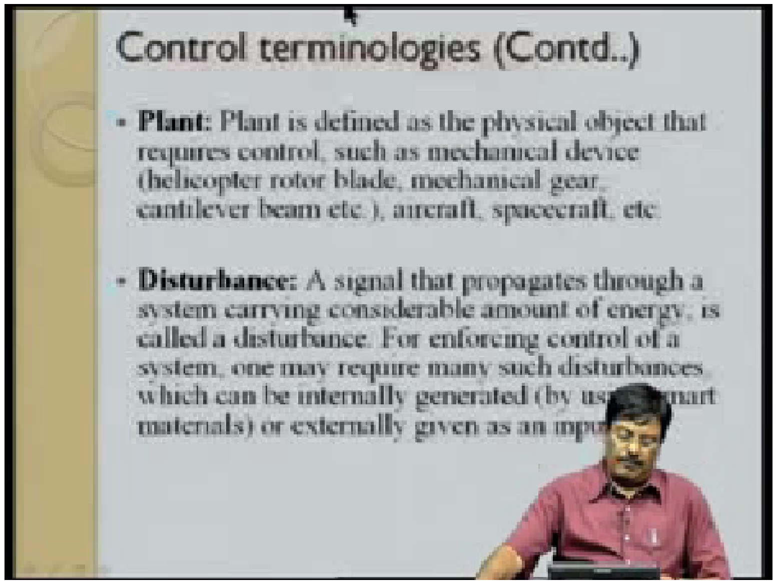

Next, we define what is called a plant. Plant is defined as a physical object that requires control such as the mechanical device, helicopter blade, mechanical gear, cantilever beam, etc. aircraft, spacecraft. These all represent what we call the plant which requires control. Next, we define what is disturbance. A signal that propagates through a system carrying considerable amount of energy is what we call disturbance.

For enforcing control of a system, one may require many such disturbances which can be internally generated especially in a smart system through smart actuator or externally given as an input as in the case of traditional control systems.

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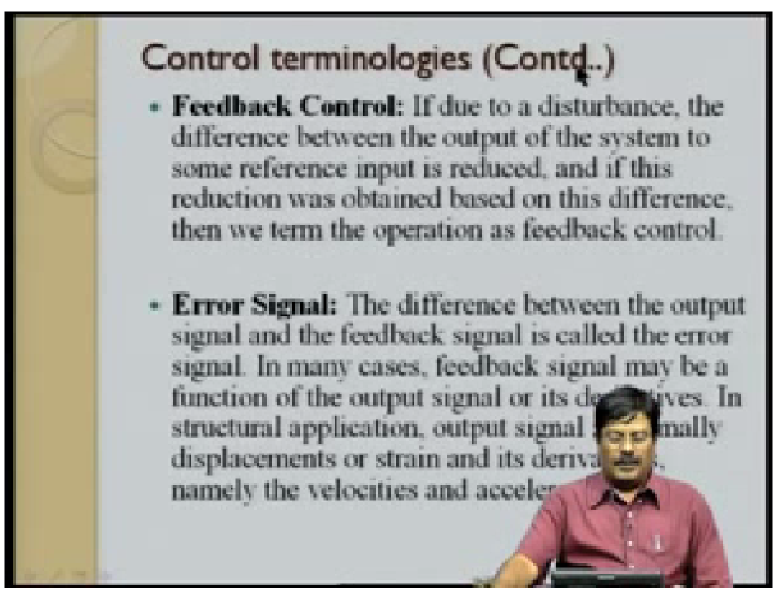

Next, we define one of the very important parameter in control what we call the feedback control. If due to a disturbance, the difference between the output of the system which we are trying to control to some reference input is reduced and if this reduction was obtained based on this difference, then we term this operation as feedback control. So one of the factors on which the feedback control depends, is the error signal. So now we will redefine what is an error signal.

The difference between the output signal and the feedback signal is what we call the error signal. In many cases, the feedback signal may be a function of the output signal and its derivatives. In structural applications such as vibration control, noise control, the output signal is normally displacements or strain or its derivatives namely velocities and acceleration.

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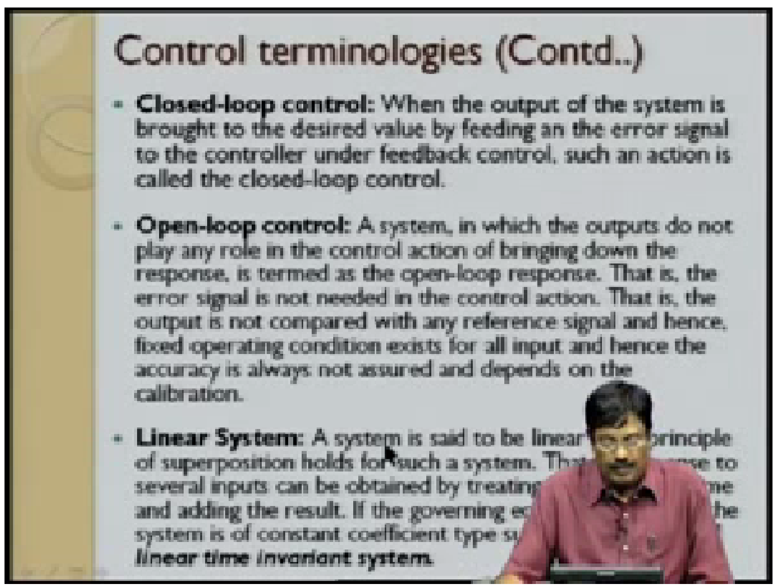

Next, we will define what is called closed-loop control. So the control system we design is normally closed-loop control system which is the most efficient control system. So it is defined as follows, when the output of the system is brought to the desired value by feeding the error signal to the controller under the feedback control, such an action is termed as the closed-loop control.

There is an alternative to closed-loop control which is used in some cases but sparingly, which is called the open-loop control. A system in which the outputs do not play a major role in the control action, is termed as the open-loop control. So that is the error signal is not needed at all in the open-loop control as opposed to the closed-loop control.

That is the output is not compared with any reference signal and hence fixed operating condition exists and hence the accuracy is not always assured in the open-loop control. So we have defined the major control terminologies now. Next we will define what is we call the linear system, before most of our control system design is based on the linear system. A system is said to be linear if the principle of superposition holds.

So what do we mean by principal of superposition, that is the response to several inputs can be obtained by treating one input at a time, getting the response and adding the total response together. So if the governing equation describing a system which is of constant coefficient type, such a system is called linear time invariant system.

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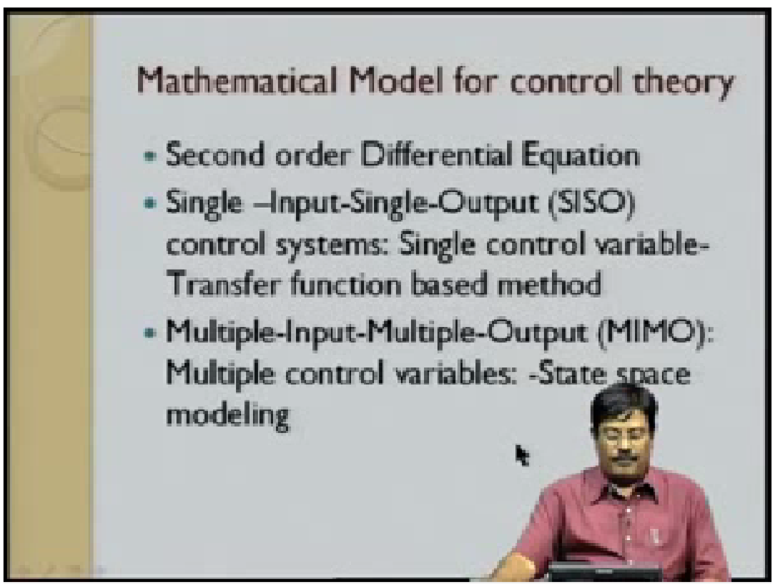

So now we have defined what all the basic terminologies that are required for control systems, that are involving in design of the control systems. Next we will talk about what are the mathematical models that are required to design the control systems. The fundamental to most of the control system is, we need to have a differential equation which is in most cases is of a second order linear system, second-order linear differential equation and there are 2 kinds of control systems we can design.

One is called Single-Input-Single-Output SISO system, which is essentially based on transfer functions, determination of the transfer function because there is only 1 control variable here and the second method is the Multiple-Input-Multiple-output control system where more than 1 control variable will be there and normally we use what is called the state space approach modeling. We will talk about both of these modelling in this lecture.

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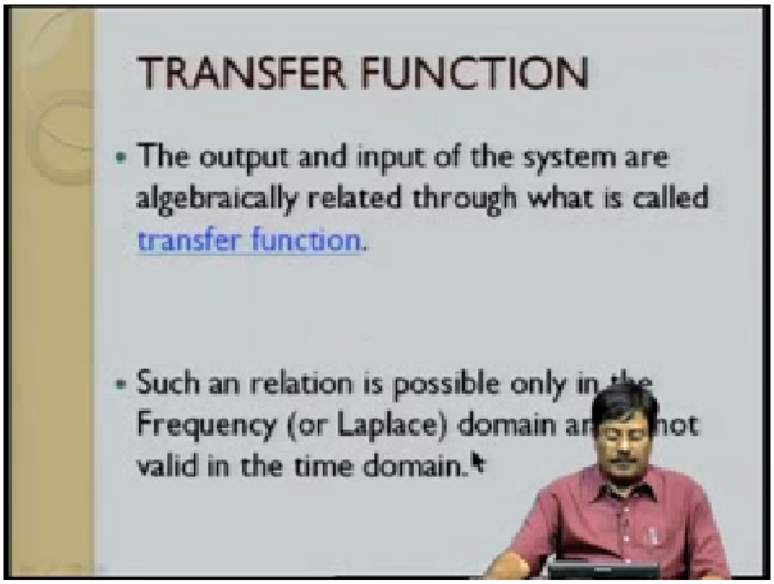

Let us now come to transfer function. What is transfer function, how do we define it. So in a traditional control system, any traditional dynamic system, you have an output, you have an input. The algebraic relationship between the output and input is what we define as transfer function. Such a relation is possible only in the frequency domain or in the Laplace domain which is also a frequency domain and it is not valid in the time domain.

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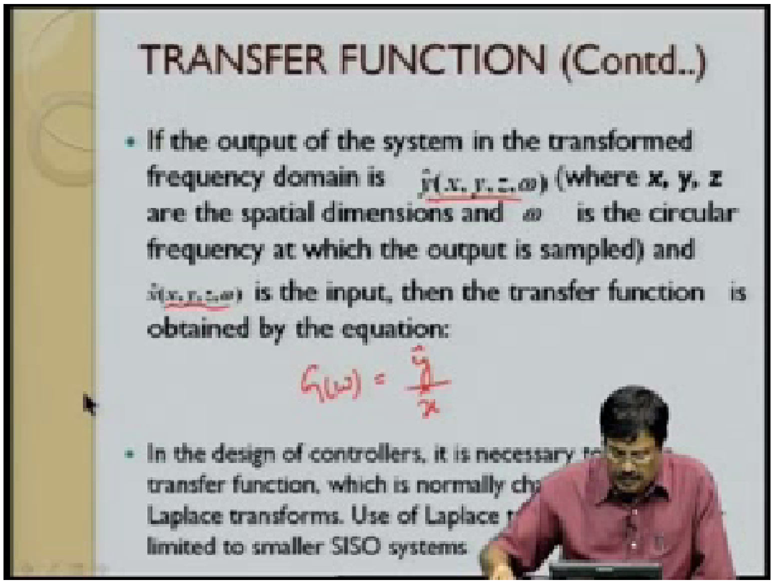

If the output of the system in the frequency domain is given by y which is a function of the spatial quantities x, y and z and also frequency defined in radiance per second and if the input given to the system is x, okay, then we will define the transfer function G in frequency domain, G omega which will be equal to output y hat/x hat, okay. It is a simple thing. For every frequency, there is a relation and this is what we call the transfer function.

So in the design of controllers, it is necessary to obtain a transfer function which is normally characterised using Laplace transform and there is a straightforward relationship between Laplace transform and Fourier transform. The use Laplace transform however is limited to only single control variable that is it is valid for only SISO systems.

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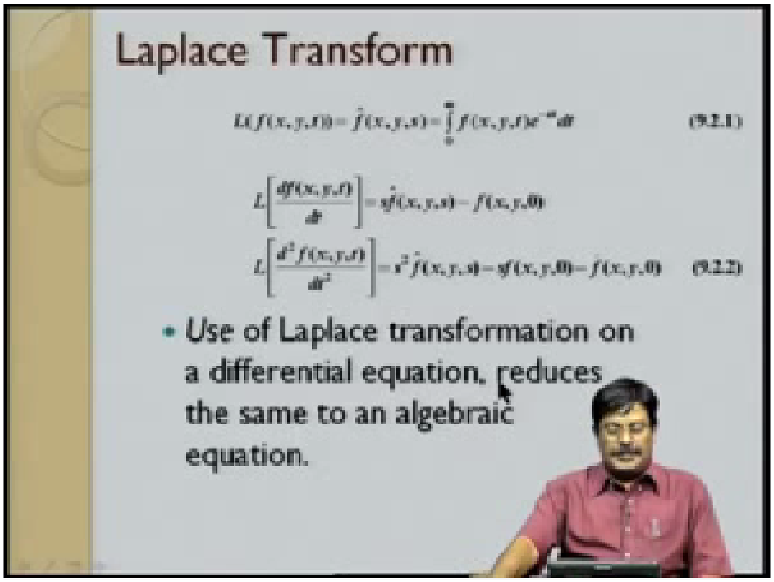

So let us now talk about Laplace transform. So the Laplace transform of a function f, a time domain function f, which is also spatially dependent, is given by this expression 9.2.1, okay. So we have the Laplace transform defined by this equation. The Laplace transform of the derivative of this function is also defined by this expression here which also depends upon the function evaluated at time T=0, so that should be known before.

The second derivative of this function, the Laplace transform of that is given by this equation here which depends upon not only the value of the function at time T=0 and also the derivative of the function at time=0, first derivative of the function. Like that we can write the Laplace transforms which can be extended to the nth derivative. So what does this Laplace transform do, why it is so powerful, why it is so useful. So basically, Laplace transform transforms the differential equation into a set of algebraic equation which are more easier to handle.

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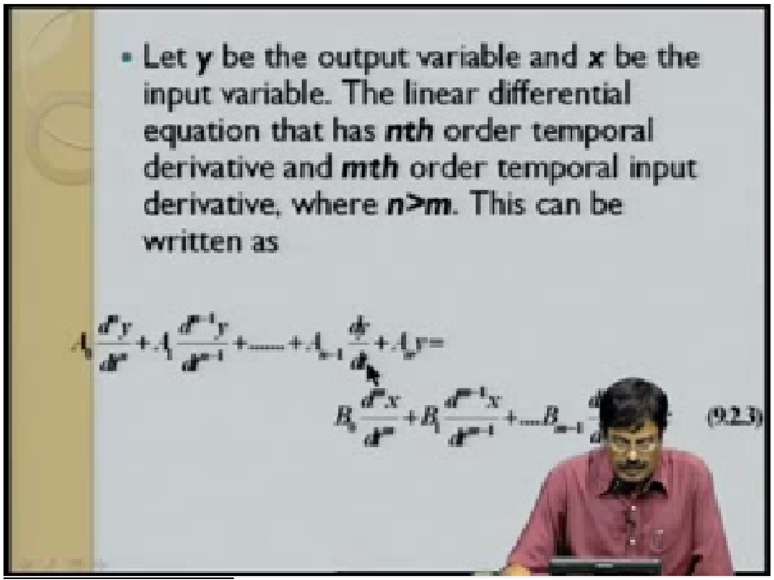

So now let us come back to our control system. Let y be the output variable and x be the input variable. The linear differential equation of the nth order temporal derivative and nth order temporal input derivative where n is larger than m, can be written by this nth order differential equation. The left-hand side is basically the differential equation. Right-hand side is essentially the input because y is the output, x is the input.

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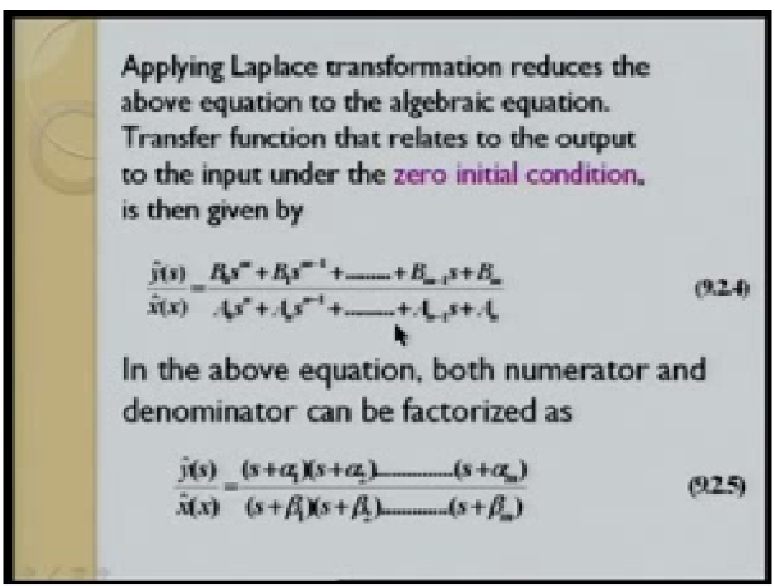

So when we apply the Laplace transform, okay and assume that zero initial condition, that is the value of the function at time T=0 and its derivative is basically and all the higher derivatives at time T=0 that is the 0 initial condition, we can write the governing equation into a set of algebraic equation where y is the output in the Laplace domain, x is the output in the Laplace domain and we can get this, okay. So this 9.2.4 is basically an algebraic relation which has a numerator as well as the denominator.

So basically what we do here is, we can factorize the numerator, we can factorize the denominator as given here. So basically the numerator can be factorized as s+alpha 1*s+alpha 2 multiplied to s+alpha m and similarly the denominator s+beta 1*s+beta 2 to s+beta m. So the transfer function will be 0 at values -alpha 1, -alpha 2, -alpha 3, etc. The denominator will be 0 at -beta 1, -beta 2, -beta 3.

So when the numerator is 0, we say there is a 0 in the transfer function, when the denominator is 0 which makes the transfer function infinity, we say these are the poles.

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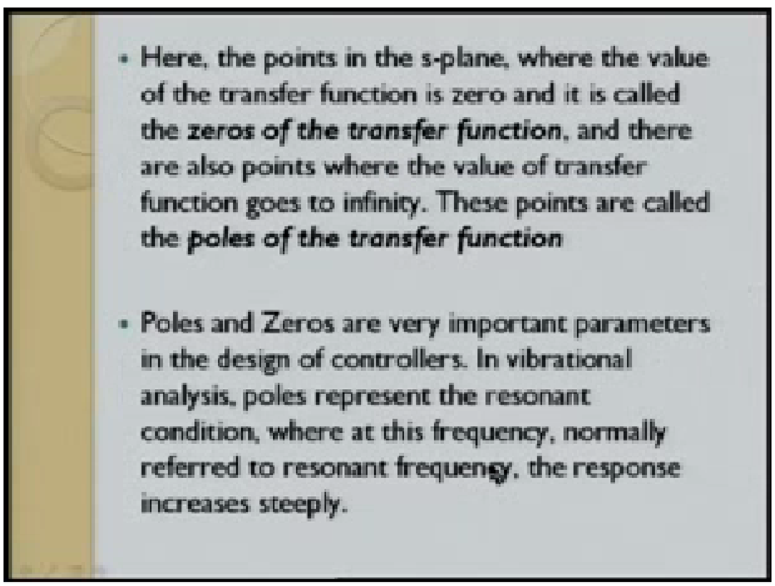

So basically poles and zero are the important parameter in control system. So basically poles are very important for the design of controllers. In vibration analysis, poles represents the resonant condition where the driving frequency will equal to the natural frequency which basically increases the displacement or the vibrational amplitude to enormously large extent which has to be avoided at any cost.

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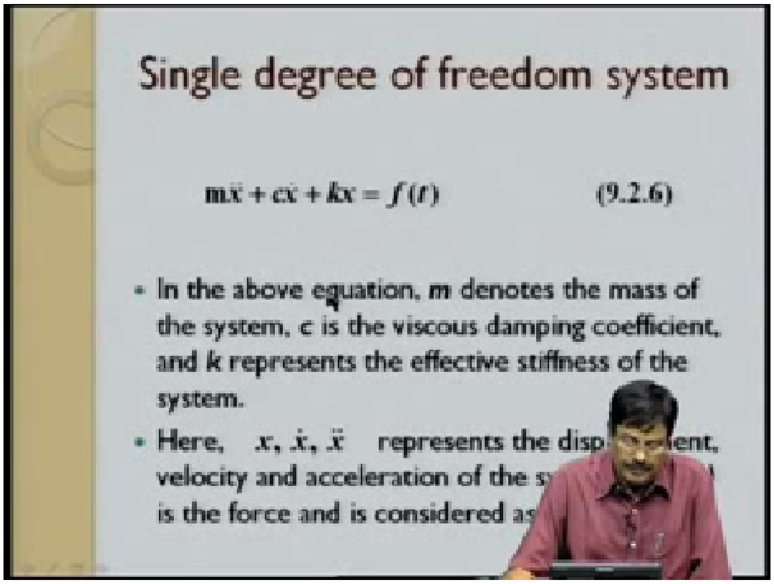

So let us come back to a single degree of freedom system. What is a single degree of freedom system where there is only one predominant motion that means there is only one control variable. So this is basically given by a second-order differential equation where x double dot, x. are the derivative of the x which represents acceleration, velocity and x basically represents the vibrational amplitude and f of t is basically the forcing function.

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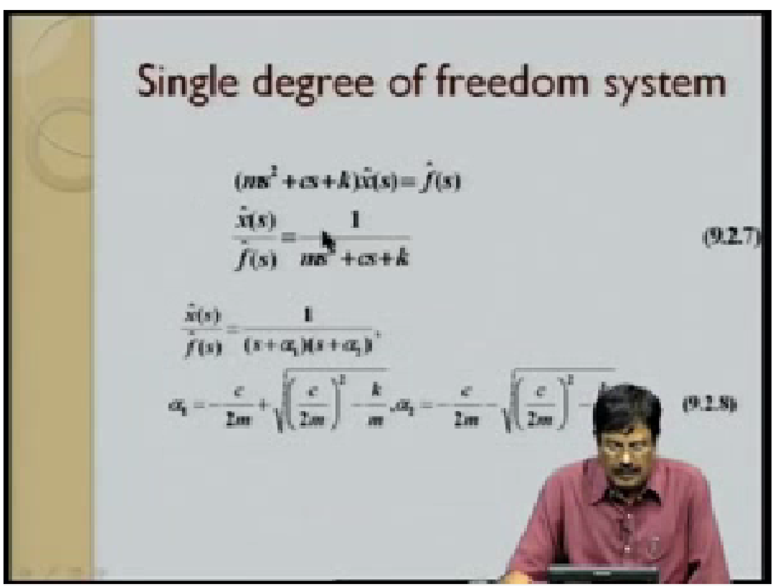

So when we take a Laplace transform of this, we get ms square+cs+k multiplied with x hat of s which is nothing but the Laplace transformation of x of t which is equal to f of s. So x of s is the output, f of s is the input, the ratio of these will give us the transfer function, that is ms square+cs+k.

So basically, the transfer functions, the numerator is just 1 value, so it cannot be factorized, that is there is no zeros in this transfer function where as the denominator can be factorized since it is a quadratic, so it has 2 poles with the value alpha 1 and alpha 2 and alpha 1 and alpha 2 are given by this equations here.

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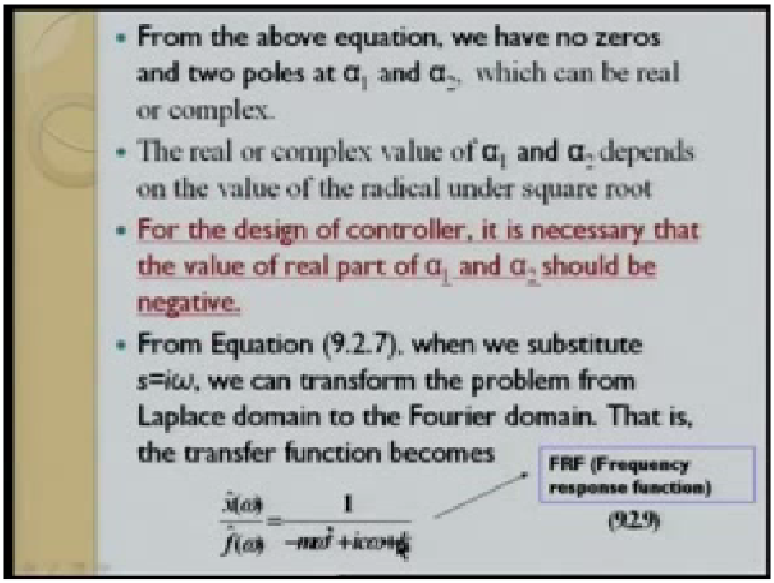

So from the above equation, we have seen that there are no zeros but 2 poles at -alpha 1 and -alpha 2, the real or complex value of alpha 1 and alpha 2 depends upon the value of the radical, the value of this radical, if c/2m is greater than k/m, then alpha 1 will be real and if c/2m is less than k/m, it is going to be imaginary. So for the design of the controller, one important property is for the stability of the control system, it is necessary that the real part of alpha 1 and alpha 2 should always be negative.

Otherwise, the control system will be unstable. So from the equation above, if we substitute instead of the Laplace parameter s by i times omega, we can transform the problem from the Laplace domain to Fourier domain, then the transfer function is given in terms of omega which is given here and the quantity that is on the right-hand side, 1/-m omega square+ic omega+k represents what we call the frequency response function.

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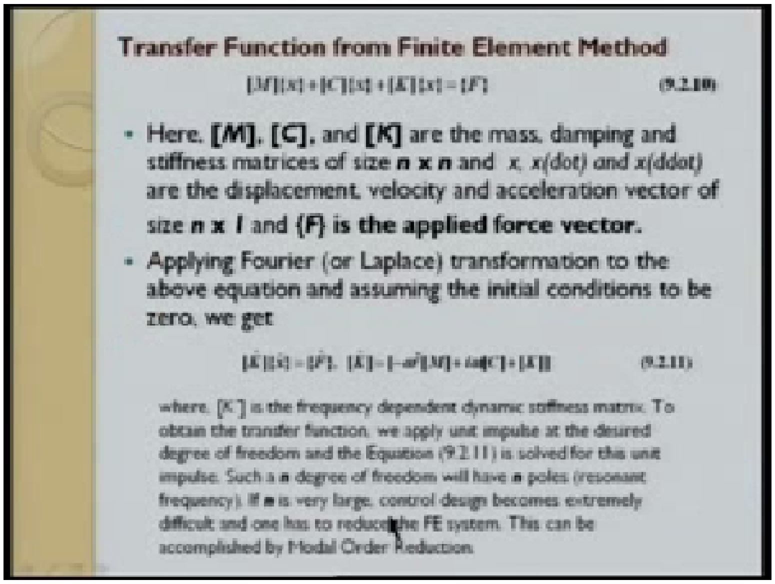

So now how do we determine transfer function from finite element method because which is normally used for any control system design. The finite element equation is given by 9.2.10 which is M*x double dotC*x.K*x=F where M, C and K are matrixes of size n by n, that is it has n control variables, okay. So and x, x. and x double dot are basically the dynamic amplitudes, x. is the velocity and x double dot is the acceleration and F is the applied force vector.

So we apply either Laplace transform or Fourier transform to this, we reduce this equation into K hat*x equal F hat, all in the transform domain where K hat is given by the equation 9.2.11 and it is a frequency dependent matrix which is called the dynamic stiffness matrix. So basically we solve 9.2.11 by giving a unit impulse in the place where you require the transfer function and solve for the unit impulse, this matrix equation and whatever the output you get, x hat is essentially the transfer function. So it has to be solved numerical.

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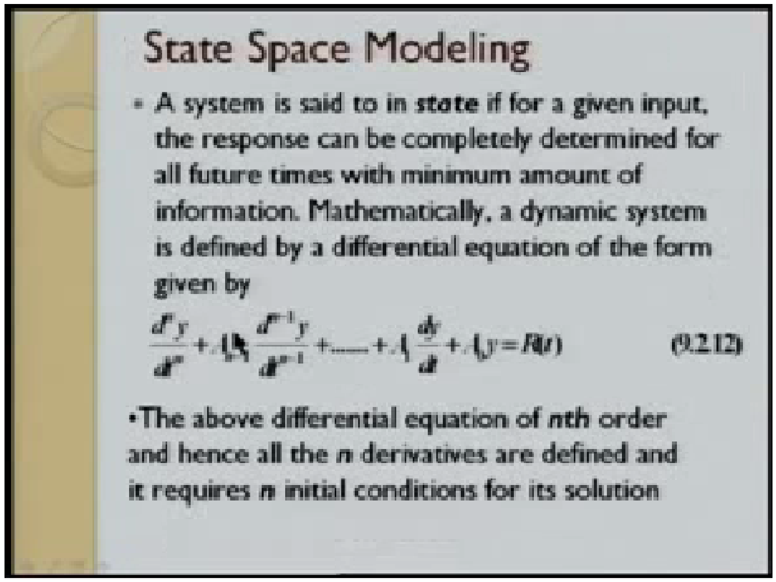

Now let us come to the State Space Modeling. So a system is said to be in state space if for a given input, the response can be completely determined for all future times with minimum amount of information. Mathematically, a dynamic system is defined by a differential equation which is given nth order differentially. Here y is basically the output variable that we are looking at, R is going to be which is basically the input, the R input may contain the input time function or its derivatives

Let us first assume that R of t contains no derivatives of the input function.

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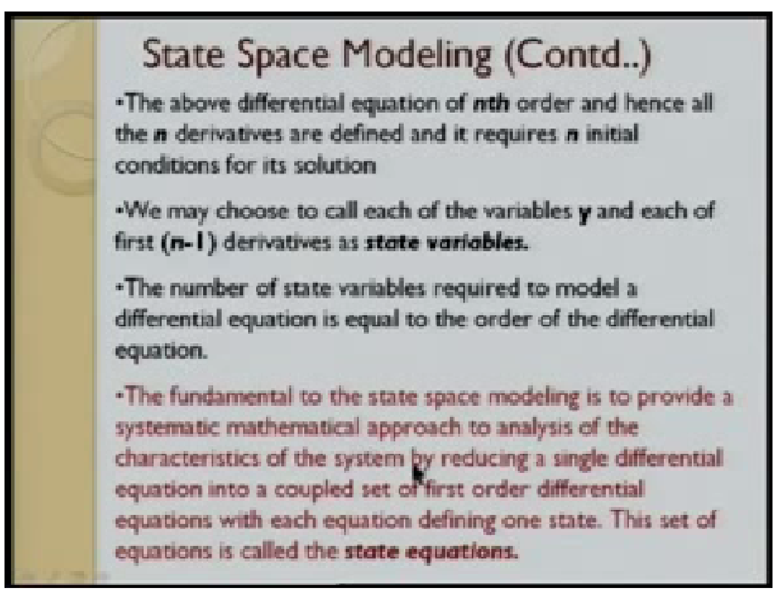

So in the above equations the nth order and all n derivatives should be defined and it requires n initial conditions. We may choose to call all the variables y and each of the n-1 derivatives as state variables, okay. The number of state variables required to model a differential equation is equal to the order of the equation. So why do we need state spacer approach.

Basically fundamental to state space modelling is to provide a systematic mathematical approach to the analysis of the characteristics of the system by reducing a single nth order differential equation coupled set of differential equation into a set of first-order differential equation with each equation defining one state. The set of equations, such set of 3 equation is called the state equation.

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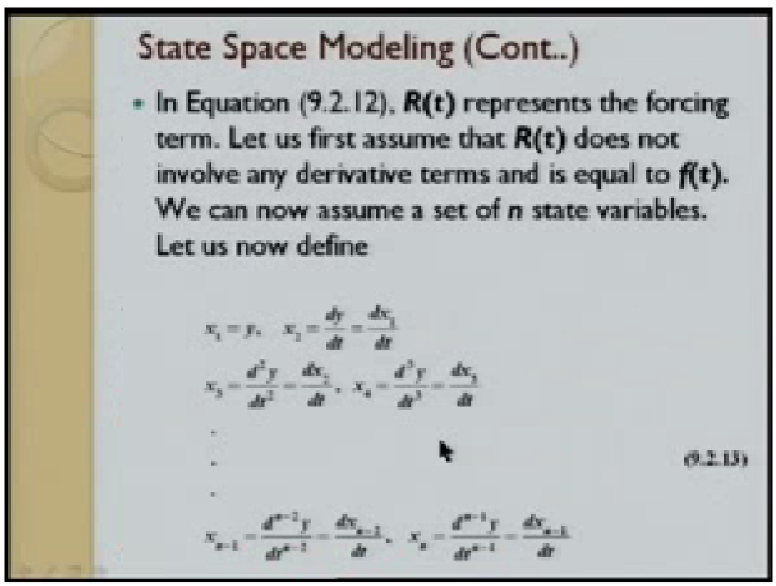

So let us now assume as I said R t does not contain any derivatives, then we can define the state variables as given by 9.2.13 where x1 will be the first state variable which is equal to y, my output; x2 will be the derivative dy/dt which is equal to dx1/dt; x3 will be the second derivative which will be the first derivative of x2; x4 will be third derivative of y which will be first derivative of x3 and so on. We can write n-1 and nth derivative.

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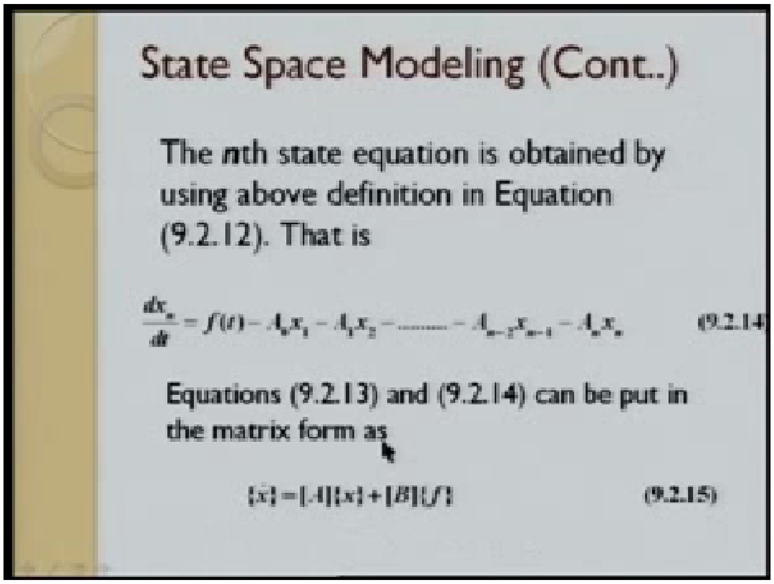

So the nth state question is obtained by using the above definition, that is we can write as given in 9.2.14. The equations 9.13 and 14 can be put in matrix form as given here, okay.

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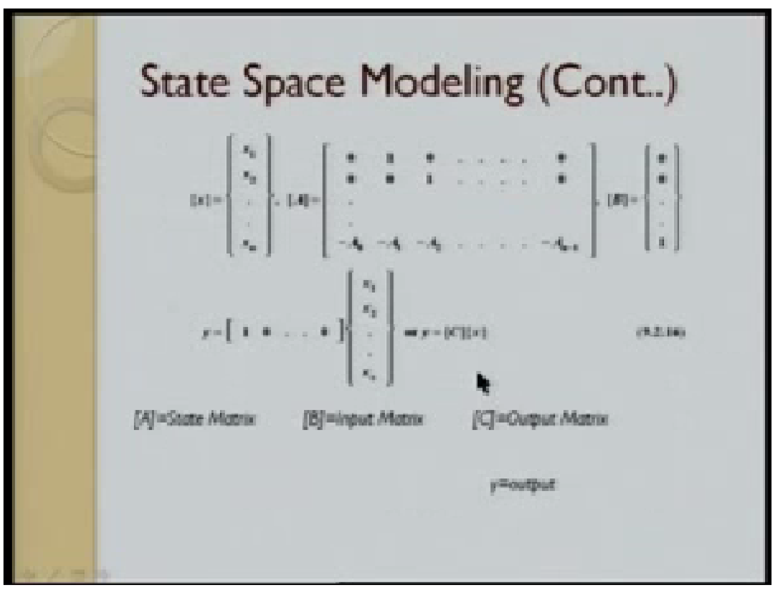

So which then expanded will be shown here, x is a vector of all state variables, a is called the state matrix, b is called the input matrix. Again y can be related to the state variable as y=C*x. So these 2 together form what is called the state space equation which is basically used for our controls system design.

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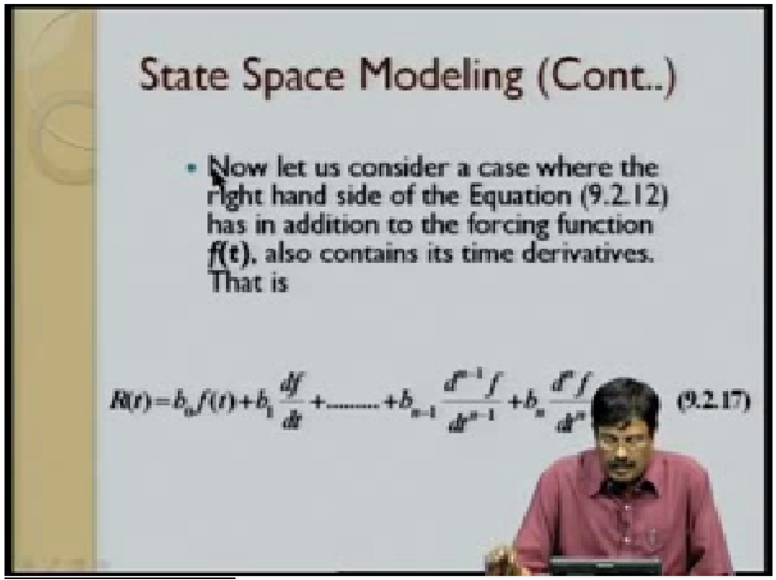

Now if we consider that my input on the right-hand side is also a function of the derivative of the input function f, then R t is defined by equation 9.2.17 which contains the derivatives, all higher derivatives. If we use this, our previous definition of the state variable is not valid because it does not eliminate these derivatives.

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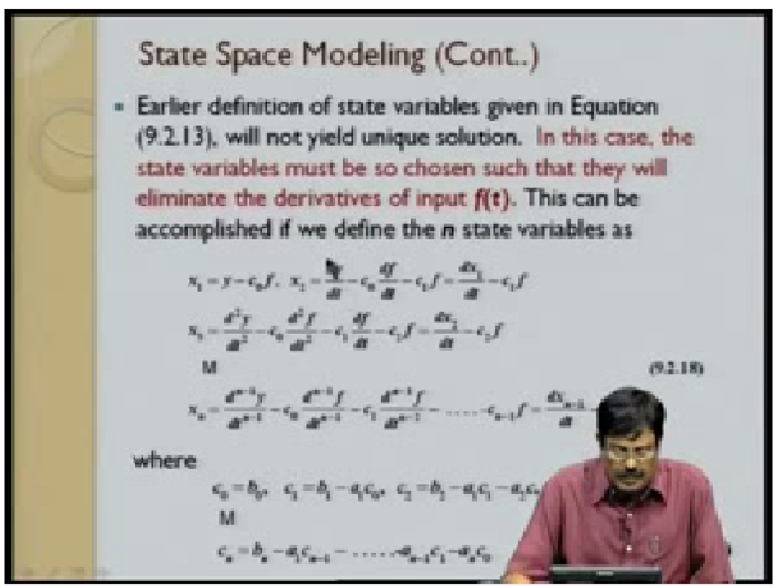

So we need to define a new set of the state variables. So these are given here in the equation 9.2.18 where x1 will be instead of y, we also introduce c0-f; my state of second variable, x2 will be the first derivative of y-c0 into first derivative of the input and the actual function; then x3 will be second derivative of y minus the second derivative of the function, the first derivative of the function and the function itself, input function, like that we can write the nth derivative.

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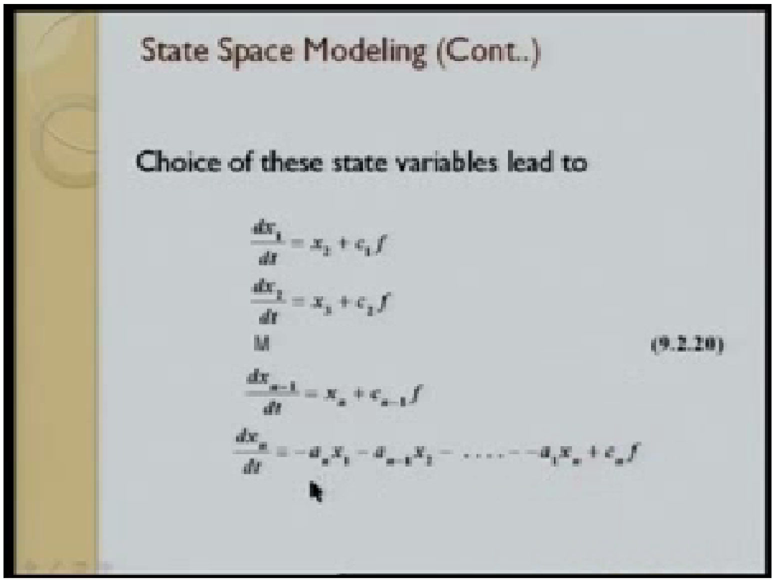

So once we write this, we can chose the new state variable questions as shown below. So the first equation where dx1/dt=x2+c1f and the nth one will be given by here, this equation.

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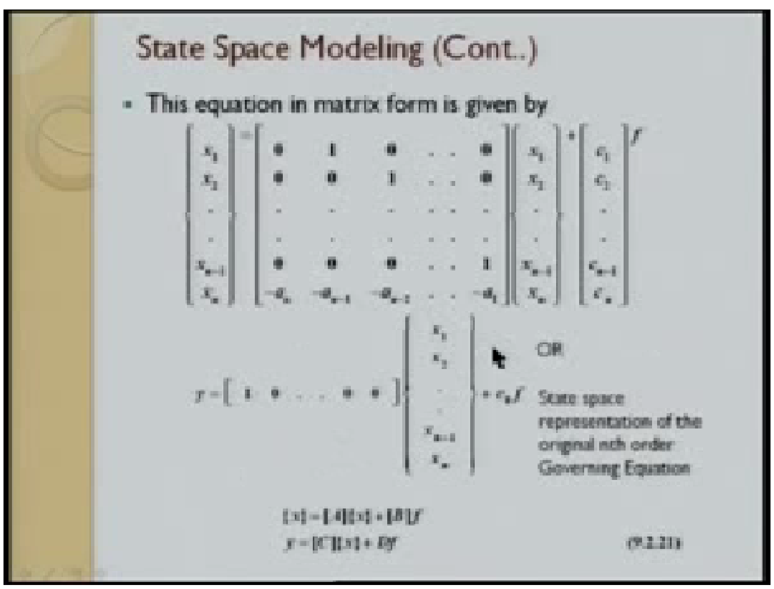

So in doing so, we can write these equations in matrix form which is a relating to the state vector on the left-hand side, the state matrix and the input matrix and the output and input can be derived here. So in doing so, we also introduce an additional parameter D which is given by c0*f that enters into picture. So this form is convenient for us to design the state variable when the input that is given to the system is also dependent on its derivatives.

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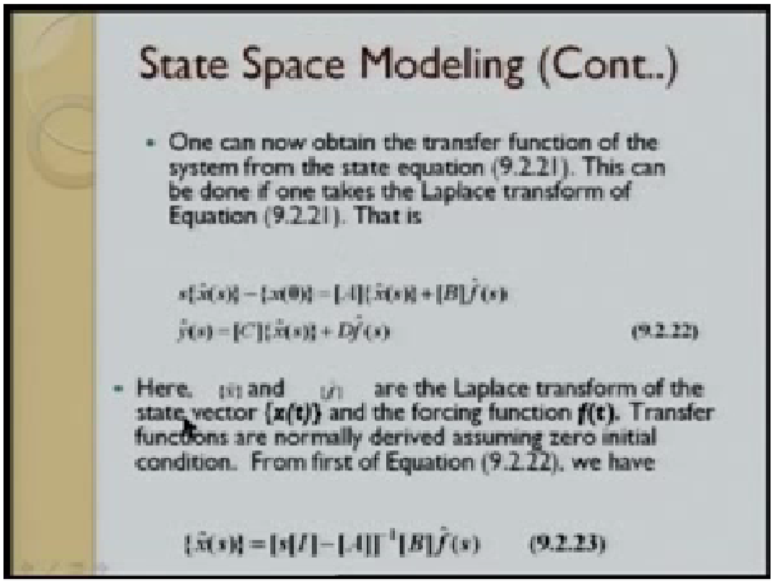

So once we do that, now we take the, next step is to see how we transform this into frequency domain. So we take a Laplace transform of this where we write the equation here in the x domain which also contain x of 0 that is the value of the state vector at time t=0. So when we do that, from the first equation, we eliminate this x of s and substituting the second of the equation 9.2.22, then we will get a relationship between the output and the input which will become a transfer function matrix.

So this is given here, output/input. So which requires inversion of s-A matrix.

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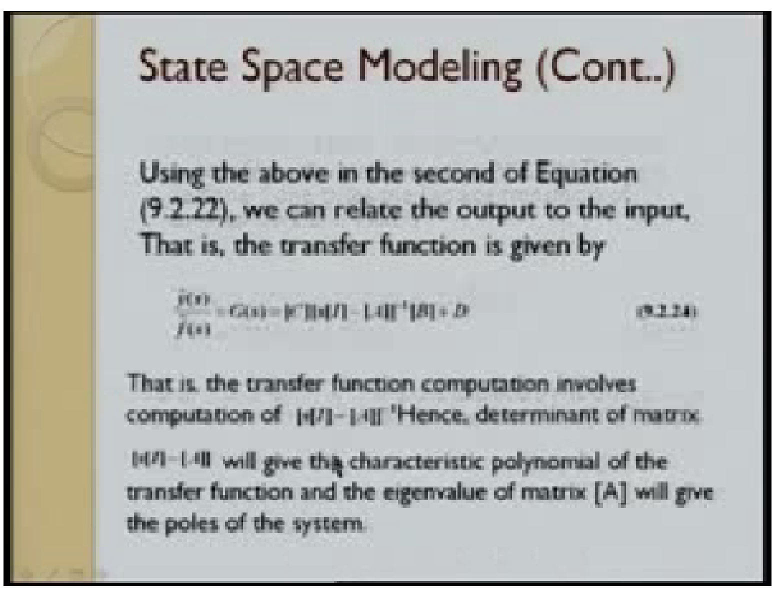

So the transfer function is given by this equation here, okay. So basically the determinant of s-a will give me the characteristic polynomial of the control system and the matrix A will give us the poles that is going to be there in this control system for which we have to design our control system.

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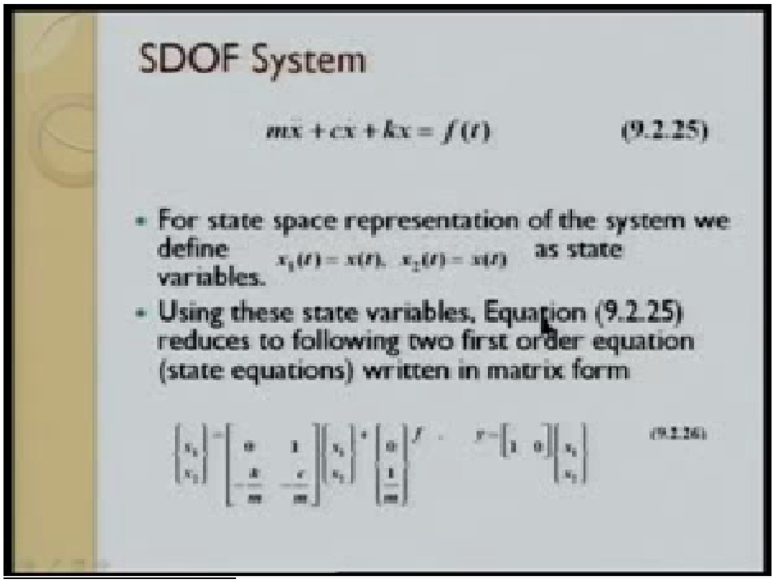

So let us see how we can apply this to a simple single degree of freedom system. We again come back here, this we studied few slides before. This is again a single degree of system where x is the control variable basically. So mx double dot+cx.+kx=f of t. Now to represent this in state space, we need to choose the state variables. We take the first state variable x1=x and x2=x double dot

So we can rewrite this, reduce this into 2 sets of equations, that is x1. x2. is related to this matrix a and x1 and x2 is the state vector plus B and again output is given by this matrix c times the state vector. So we can easily represent this single degree of freedom system into a matrix contain 2x2 state vector and vector containing the input matrix, output matrix and the output vector.

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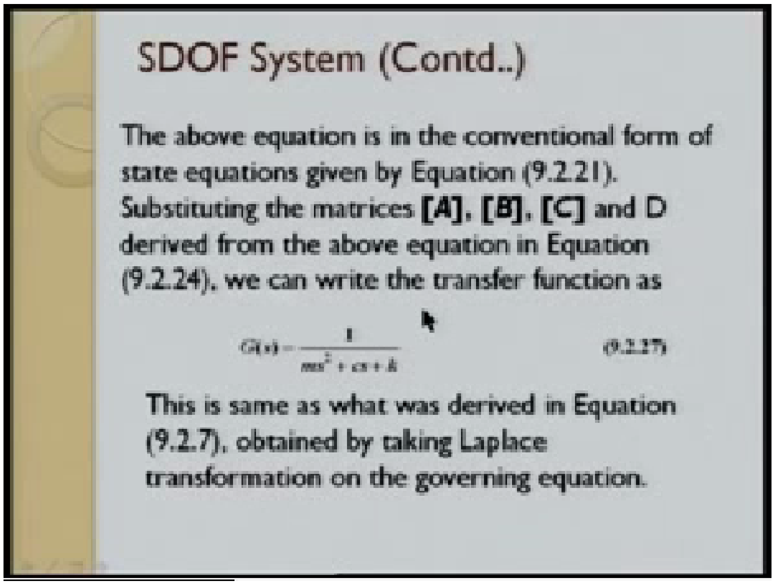

So from the above equation, as in the conventional form of the state equation, substituting the matrix A B C and D in the derived above equation, we can write the transfer functions when we do that in this form which is basically same what we derived before by using our regular transfer function methods instead of using the state spacer approach.

So now what we have done is we have taken the single degree of freedom system to show how we can derive the transfer function both from your conventional method of transforming directly these equation into frequency domain or using the states space approach.

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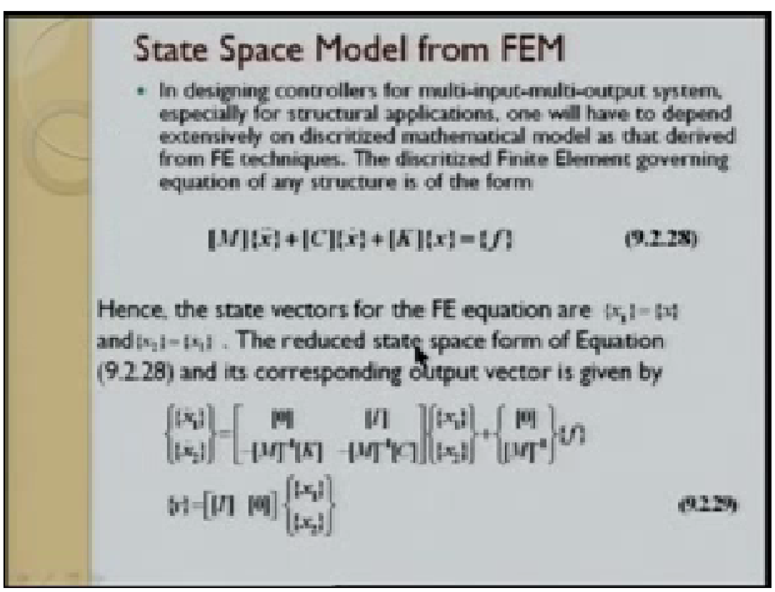

How do we do the state space model from FEM. Again, in designing the controller for multi-input-multi-output system, especially for structural application, one will have to depend extensively on the discritized model because the number of degrees of freedom and number of control variables will be so large. The discritized finite element equation is given by 9.2.28 which is given by Mx double dot+Cx.+kx where M C and K are matrixes, x is a vector, f is the input, all are of size n by n and x the vectors of size n by 1.

So here we choose the state vector as x1 vector is equal to the actual control variable x, x vector and x2 will be the derivative of x1. So when we do that and apply our state space model, we get the control equation in terms of this state matrix by equation 9.2.29 which is given here. So once we know that, we can clearly identify what is the A matrix B matrix and C matrix which will be very useful in the control design.

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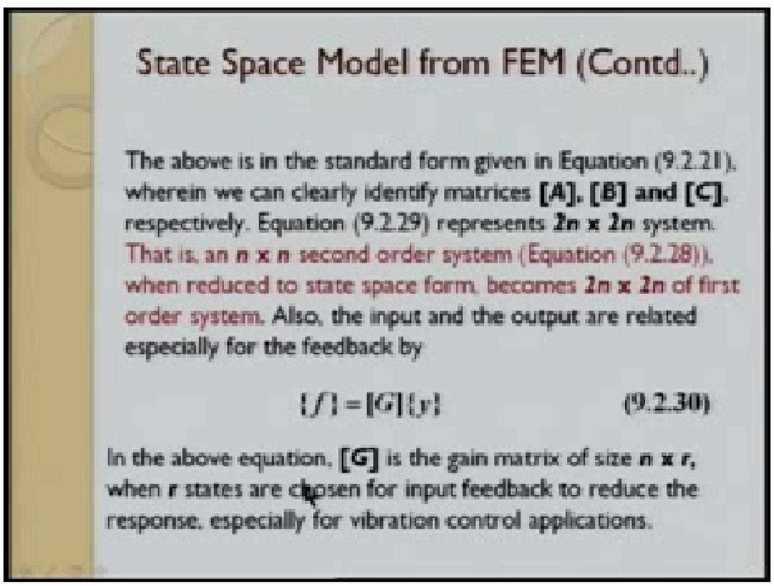

So what we now learnt is if there is a system which is of order n by n, the state matrix will be of the size 2n by 2n. So we increase the size, okay. In addition, if you want to do a control system, in addition to getting the A B C matrix, we also have to get the gain matrix. What is a gain matrix. Gain matrix is essentially relates the input to the output, y is the output, f is the input. F=Gy where G is the gain matrix which is of the size n*r.

What is this r, r is the number of the states that you need to control. So in terms of vibrational frequency if you want to control 10 frequencies, r will be 10. Then n will be maybe 100, 1000, 10,000. So basically it is the number of control variable that you want to control, it is a control state, r represents that.

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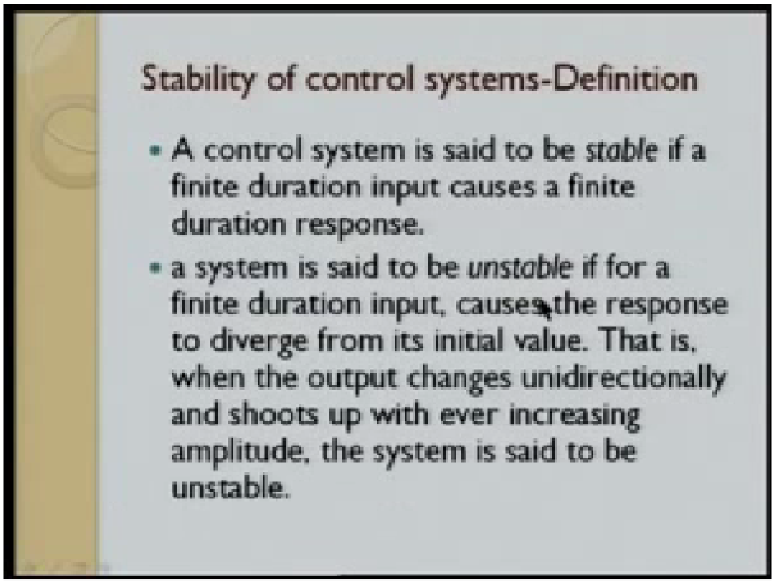

Now let us define another important parameter for the control system that is the stability. Every control system has to be stable and when is the control system stable. A control system is said to be stable if a finite duration input causes a finite duration response. A system is said to be unstable if for a finite duration of input, it causes response that diverges phenomenally from the initial value that is when the output changes unidirectionally and shoots up with ever increasing amplitude, the system is said to be unstable.

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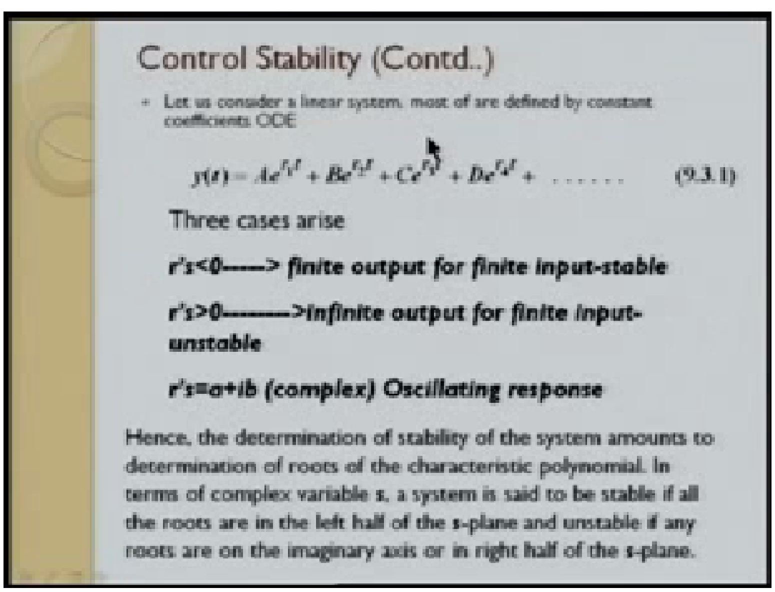

So when does it happen. So we studied until now that most second-order systems are of constant coefficients which has exponentials as solutions. So the solution of such will be of the form y of t is the solution for which you are looking will be Ae to the power of r1t + e r2t, etc. okay and it is necessary that for the r's can be less than 0, greater than 0, or r's can be complex.

So r in 3 cases, these are the three cases that arises. If r's are less than 0, the finite output to a finite input, then we say it is stable. When r, this is greater than 0, there is an infinite output your finite input that means it is unstable. When r's are complex, then we will have an oscillating response because basically they will be of the form sines and cosines. Hence the determination of stability of the system amounts to determination of the roots of the polynomial.

In terms of complex variable s, the system is said to be stable if all the roots are on the left half of the s-plane. So if you plot the s versus frequency, then it has to be in the left, so basically that is why in the starting of this lecturer, I said all the roots should be on the negative so that this is on the left side of the s-plane or the imaginary axis.

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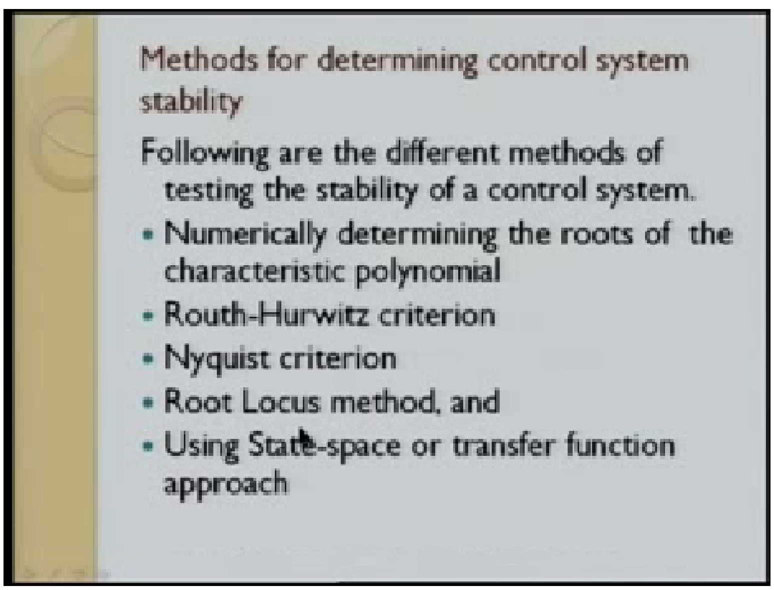

How would we determine the methods of the control. Following are the different methods that are used. One of the methods is numerically determining the roots of the characteristic polynomial, that is find when the degrees of freedom or number of control variable is small, so that you can factorize.

But if the system is large even if we have more than 3 or 4 variables, it becomes very difficult to factorize, then we have to use certain numerical criterion such as Ruth-Hurwitz criterion, Nyquist criterion, Root Locus method or using the state space and transfer function approach.

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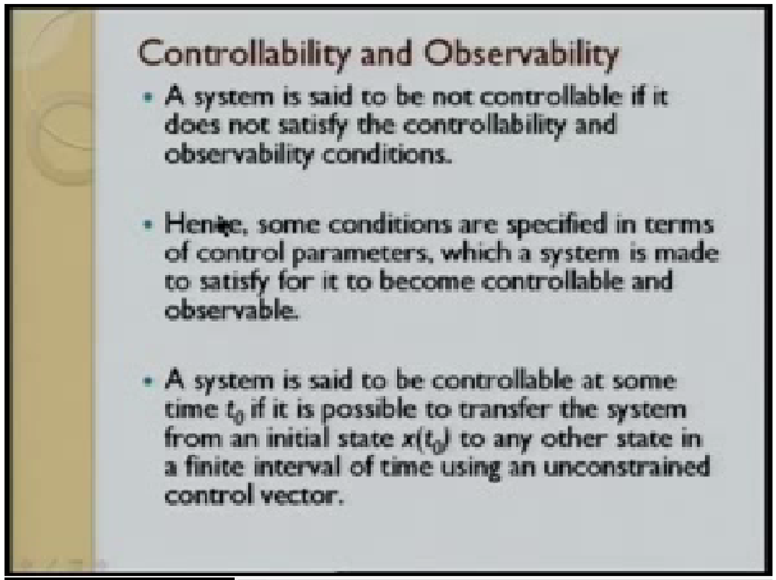

There are 2 other important parameter that determines the stability. One what we call the controllability and observability. A system is said to be not controllable if it does not satisfy these 2 conditions, namely the controllability and the observability condition. Hence some conditions are specified in terms of control parameter which a system is made to satisfy or to become controllable or observable.

A system is said to be controllable at some time t0 if it is possible to transfer the system from an initial state x of t0 to any other state in finite interval of time using the unconstrained control vector.

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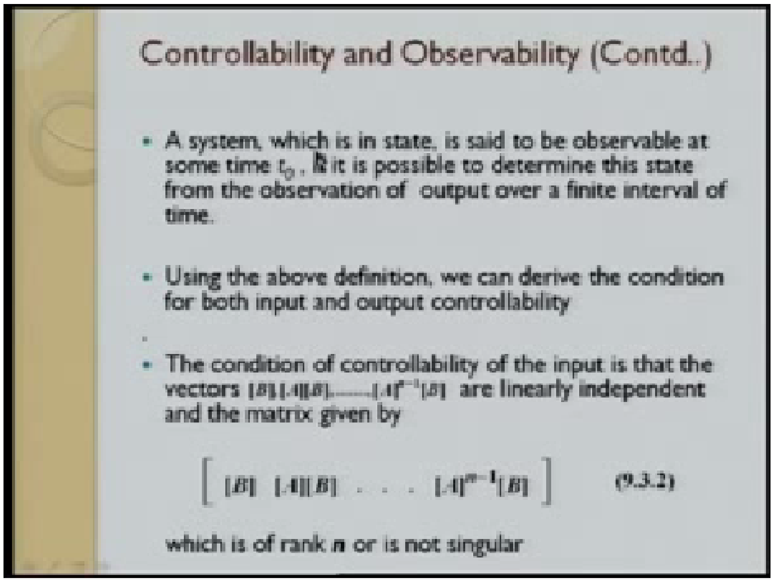

A system is said to be in a state which is in state, is said to be observable at sometime t0 if it is possible to determine the state from the observation of the output over the finite interval of time. So using the above definition, we can derive the condition for input and output controllability. So the controllability has to be both because the input has to be controllable and the output has to be controllable.

So the condition for the controllability of input is that the vectors that is the output vector B, A times B, that is a multiplication of the state matrix into B and such products up to A n-1*B, if you have an n by n system are linearly independent or in linear algebra terms, in matrix terms, the matrix containing these matrix that is B AB, the product A n -1*B, should be of their rank n. It should be for n by n system. That is this matrix should not be singular. If it is singular, then input is not controllable.

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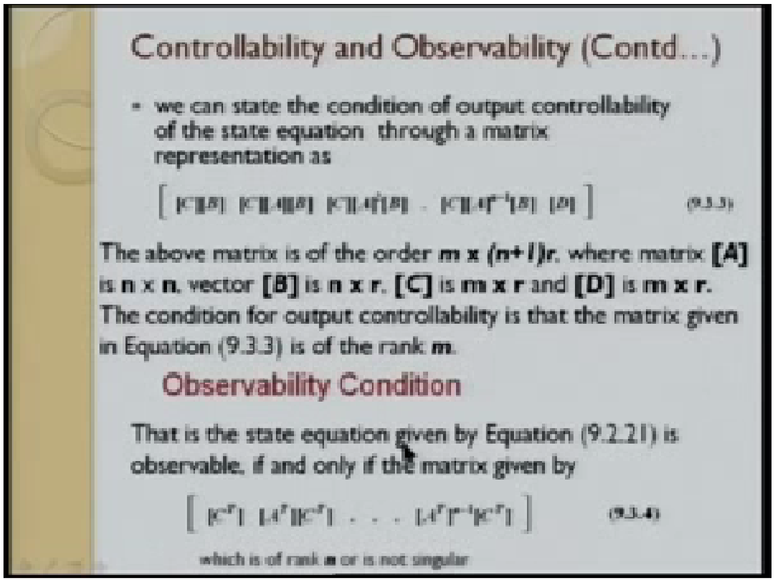

Now we have to check the output controllability. So what is the output controllability. So we need to take the output matrix C that require the output with the input, so that is the C, the product of CB CAB CA square B and CA n-1 B and D and this matrix should not be singular. So here, the output is basically depends upon how many control states that we are controlling, that is why the term the size of the matrix enters a parameter r comes into the picture.

So in the above matrix, the r is of the order m*n+1*r where the matrix A is of n by n, B is n*r, C is m*r because there are m inputs and D is M*r. So the condition for output controllability is this matrix given in equation 9.3.3 should be of the rank m, okay. So this is as far as the controllability conditions are there.

Now the observability condition, that is the state equation given by equation 9.2.1 that is the state space equation which we derived initially, is observable is only when the output matrix, that is C transpose A times C transpose product and A transpose n-1 of the n-1 state vector into C transpose, this matrix should be of order n and should not be singular.

So these are the 2 important conditions that a control system has to satisfy to make sure that the design control system is stable.

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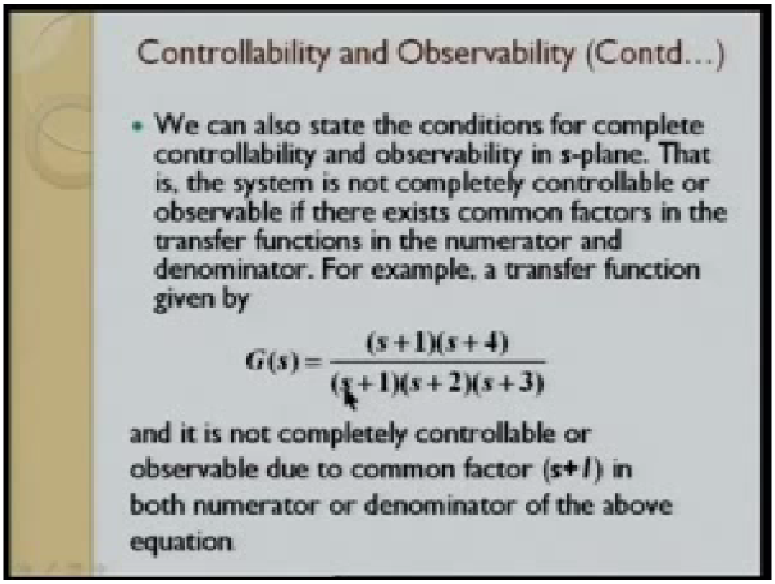

We can also state the conditions for complete controllability and observability even by looking at the transfer function. This is only of course true for only some simple systems where the control parameters or control variables are small, that is the system is not completely controllable or observable if there exists common factors in the transfer functions in the numerator and denominator.

For example, if the transfer function given by this equation here, G of s=s+1*s+4 divided by s+1*s+2*s+3 is not completely controllable or observable because there is a common factor s+1 on both the numerator and denominator. So even such a system should be avoided.

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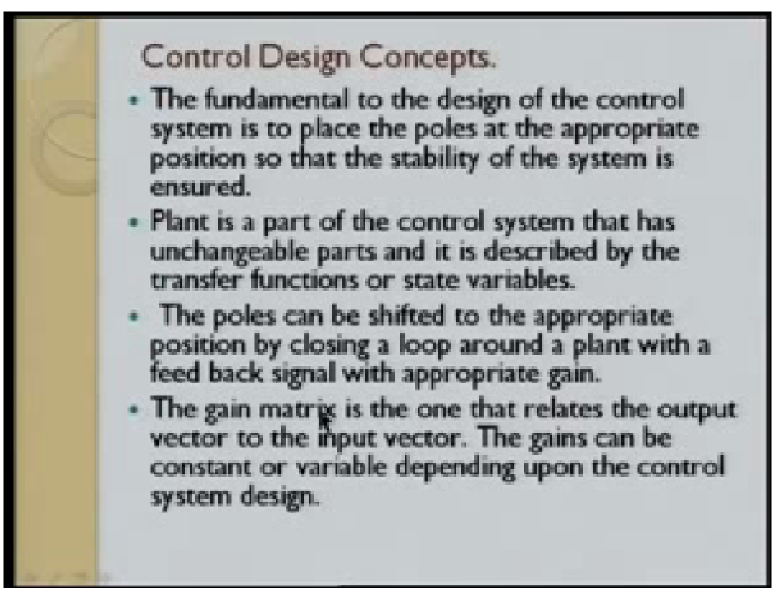

Next, this is the last part of the lecture. Let us take what we know some of the important concepts in the control design. Fundamental to the design of the control system is to place the poles that is from the transfer function, we find the poles to place these poles at their appropriate position so that the stability is ensured. Plant is a part of the control system that has unchangeable parts, we cannot change.

Because when we want to control the aircraft, we cannot change the parts and the plant is described by the transfer functions or the state variables. The poles can be shifted at the appropriate position by using a close loop control around a plant with the feedback gain signal. So basically, we give a feedback gain signal from the smart actuator in the case of smart system to appropriately shift the poles to a location where the stability can be ensured.

Another important parameter again for a closed-loop control system, we need to have a gain and this gain matrix is the one that relates the output vector to the input vector and the gains can be constant or variable depending upon the control design.

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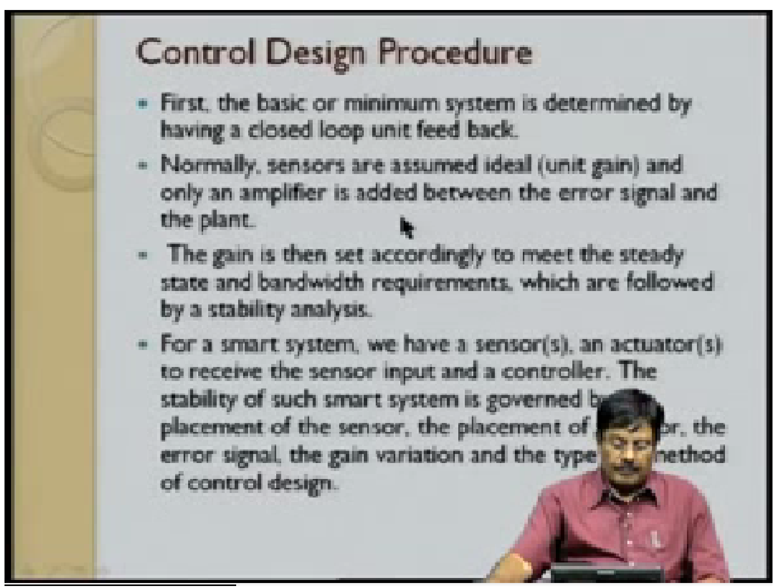

So the control design procedure is as follows. How do we go about doing it. Firsts the basic or minimum system is determined by having a closed-loop unit feedback, okay. Normally sensors are assumed ideal, that is they have unit gain and only an amplifier is added to the error signal and the plant. The gain is then set accordingly to meet the steady state.

So once we know a unit gain, we look at the responses, then we increase or decrease the gain based on to see that when the system acquires the steady state response and also the bandwidth requirements which are followed by stability analysis. For smart systems, we have sensors and actuators to receive the sensor input and a controller. The stability of such system is governed by the placement of sensor, the placement of actuator, the error signal, the gain variation and the method of the control design. What are the different methods.

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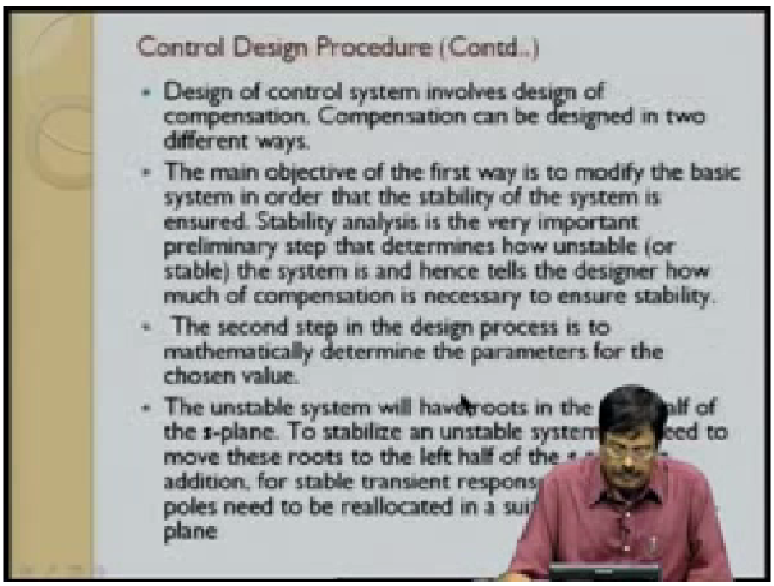

The design of the control system involves design of what we call compensation. What is compensation. Compensation can be defined in many ways. There are 2 major ways that we can design the compensation which is always prescribed in the control design. The first way to design a compensation is to modify the basic system in order that the stability of the system is ensured.

Stability analysis is very important preliminary step that determines how stable or unstable the system is and hence tells the designer how much of compensation is necessary to ensure the stability. The second step in the design process is to mathematically determine the parameter for the chosen value that is what we are looking at where the poles are, is there any 0's, how do we avoid or what is a kind of gains we need and these are some of the things which we need to determine.

The unstable system will have roots on the left-hand side of the s-plane. I told you before itself that if you have the roots on the right half of the plane then it is unstable. For the stability, we need to ensure that their roots were always on the left half of the s-plane basically. So we need to ensure that. So for stable response, these poles have to be moved from the right half to the left half. So for which we need to design our error signal such a way that this can happen.

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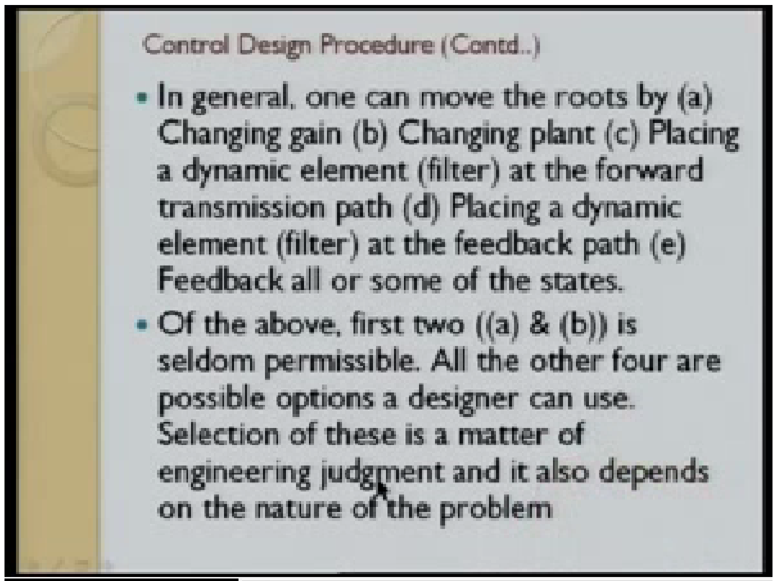

In general, one can move rules by many methods. Following are some of the methods. One is by changing the gain, changing the plant, placing the dynamic element at the forward transmission path such as filters, placing a dynamic filter element at the feedback path or feedback all are some of the states. From the above, it is not possible to change the plant, it is impossible.

So some of the other options given here are a possibility and which is more suitable, it is only the designer, an experienced designer will be able to tell depending upon problem to problem. So selection of these is a matter of engineering judgement and also depends upon the nature of the problem.

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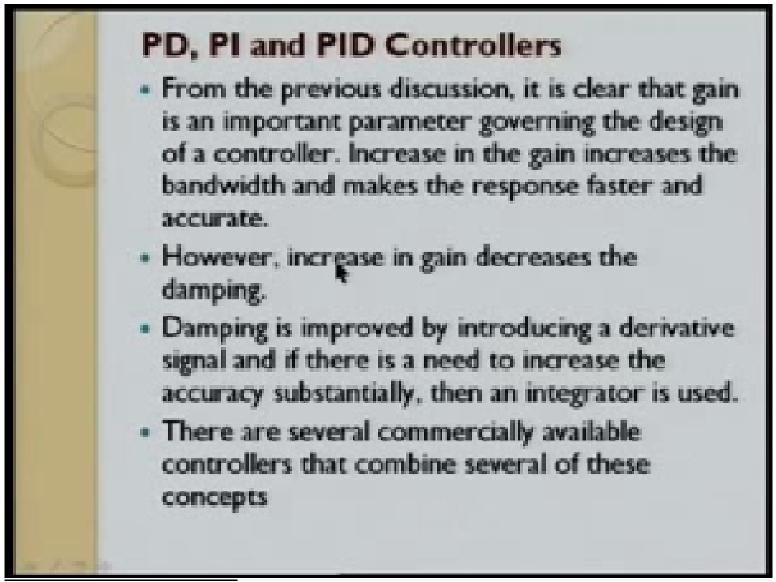

So now let us talk about some of the very simple controllers for the linear systems that are there that are commonly used, successfully used in many of the systems, in many of the disciplines in engineering. These are called PD controller, PI controller, or PID controller. PD controller is proportional derivative controller. PI is proportional integral controller. PID is proportional integral and derivative controller.

From the previous discussion, it is clear that gain is an important parameter governing the design. Increasing gain, increases the bandwidth and makes the response very fast; however, we have to be very careful. Increase in gain decreases the damping which is our major motivation of control, we have to damp out the response. So it has to be optimal so that we get faster response at the same time, we get an optimal damping to control the control variable to our desired level.

Damping is somehow improved by introducing a derivative of a signal. So that is why PD is more useful and if there is a need to increase the accuracy substantially then integrator is used. So that is why the combination of PID will ensure faster response, better damping and better accuracy. There are several commercially available controllers that combine several of these concepts very effectively.

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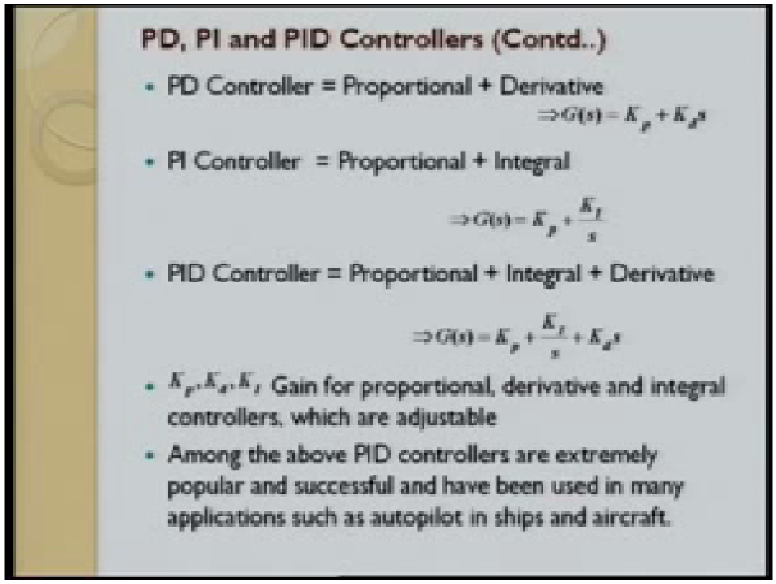

So now let us describe each one of them much more detail. We have what is called PD controller which is proportional from plus the derivative. So normally, the transfer function of this controller is given by what is given here G of s=Kp+Kd times s in the Laplace domain. So it has no numerator no denominator. It is one number, algebraic relation.

If you look at the PI controller, that is proportional plus integral, the transfer function G of s is given by Kp+Ki/s, where Kp is the gain of the proportional controller, Kd is the gain of the derivative controller, Ki is the gain of the integral controller. So the PID controller, the transfer function is given by G of s = Kp+Ki/s+Kd times s. So as I said Kp Kd Ki are the gain of the proportional derivative and integral controllers which are adjustable. We can adjust, we can vary, we can make it constant, we can vary.

Among the above as I said because the differential increases the damping, integral increases the accuracy, PID controllers are extremely popular and successful and have been used in many applications, especially in aircraft like autopilot and also in ships and in space vehicles and even in the helicopters.

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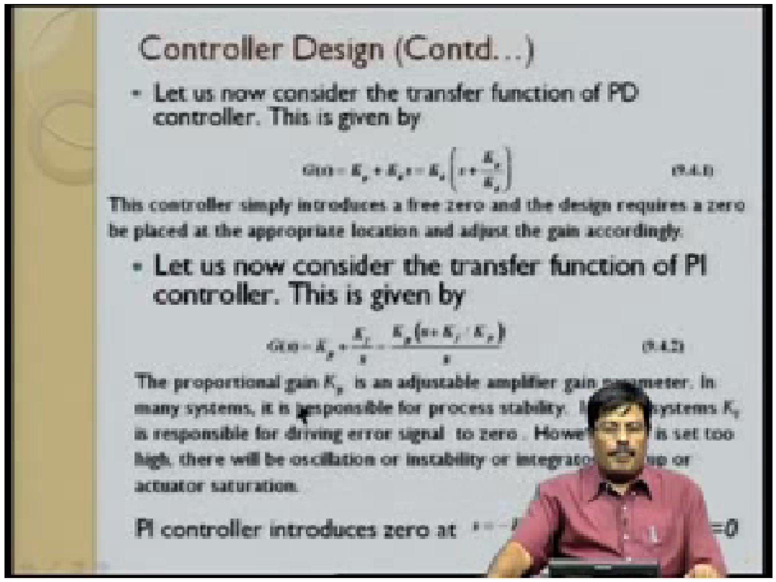

So now let us consider the transfer function of a PD controller. This is given by G of s=Kp/Ks, we write this transfer function Kd, this is equal to Kd*s+Kp/Kd. So Kp/Kd is the ratio of the gains of the proportional and derivative controllers. This controller simply introduce a free zero and the design requires a zero be placed at appropriate location to adjust the gain accordingly so that you get the desired control of the control variable.

Let us now consider the transfer function of a PI controller. The PI controller is basically given by Kp+Ki/s, so that means which can be written as Kp*S+Ki/Kp divided by s. So what we do here is, the proportional gain Kp is an adjustable amplifier gain parameter. In many system, it is responsible for the process stability. In many cases, Ki is responsible for the error signal going to 0; however, if it is set too high, there will be oscillation and instability or integrator windup or actuator situation, okay.

However, one thing we can clear is, it introduces a zero that is the transfer function becomes zero when s=-Ki/Kp and it has a pole at s=0. So it has both poles and zero, so we can adjust, the pole location is fixed, the zero location can be adjusted depending upon the values of Kp and Ki, so we can vary it to see that.

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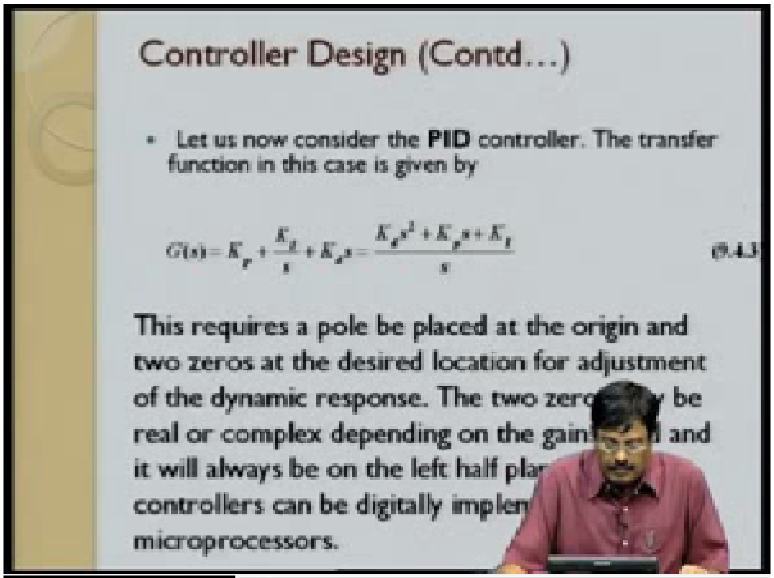

Now let us consider the last of this controllers that is the PID controller. The transfer function in this case is given by Kp+Ki/s+Kd times s. That is you have both proportional, integral and derivative and this can be written as Kd square+Kps+Ki times s. So we clearly see that it has a pole at s=0 that the origin, so the pole is fixed and the numerator can be factorized as a quadratic equation, it can be factorized, so it has 2 locations where we can introduce 0.

So this requires a pole to be placed at the origin and 2 zeros at the desired location for adjustment of the dynamic response. The 2 zeros may be real or complex depending upon the gains used and it will always be on the left half of the s-plane so that stability is insured. PID controllers can be digitally implemented with microprocessors, that is one of the reasons why it is very, very popular among the control engineers, okay.

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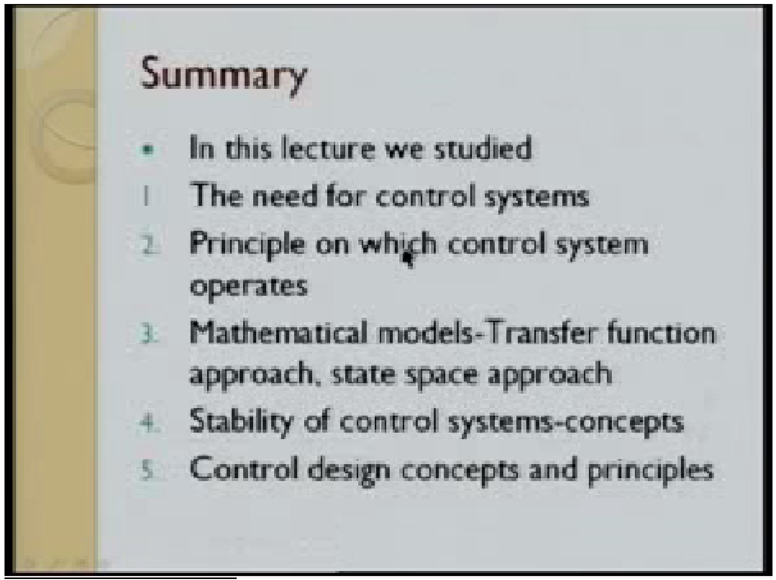

So this is the final slide. So let us summarise what we have studied in this lecture. So far we introduce the need for control, especially in smart systems. We cannot find a smart system without control, the control algorithm. So because we want smart system to perform certain smart functions, like vibration control, noise control, shape control, temperature control, so we need to design a control system to give sufficient inputs which alters the control variable and that is what the control system is.

We also discussed the principal on which the control system works, that is it requires the state matrix, it requires the output matrix, it requires the input matrix, the relationship between the state vectors and the output, output and the input, the gain matrix, the feedback control, the open-loop control. Some of these concepts were introduced in this lecture. We also discussed about the mathematical models that are required for the design of the control system.

So we described 2 models, one is based on transfer function and one is based on the state space which is an extended transfer function, I would call it as an extended transfer. In the transfer function model, we basically defined how do we get the relationship between the input and output for a single input and single output system where there is only one control variable to be controlled.

And in the second approach where there are multiple state variables that means you may arise a situation where multiple state variables needs to be controlled, then we used to develop this state space model. So in the state space model, basically we have generated the mathematical model to construct the state matrix, the input matrix, the output matrix and how we can actually generate these matrixes when we have a finite element type model which is a large model, where it is a multi-input-multi-output models.

Next we introduced what are called the stability of the control systems. So what are the paramilitaries that you need to look, what are the things that you need to ensure for the stability of the control system, how the poles affect the stability of the control system, what are the methods to actually move the poles to the position where the system is stable and what is the controllable and observable condition and what are the condition to make the controllability of the input, controllability of the output and the observatory conditions.

And finally we outline what are the design concepts, the design procedures that we need to adopt, basic simple design procedures and we also reviewed some of the very commonly used controllers like the PI controllers, PD controllers and PID controllers, what would be the transfer function of such controllers, what are the advantages of each of these and how do we implement all these things with this we end this lecture.