

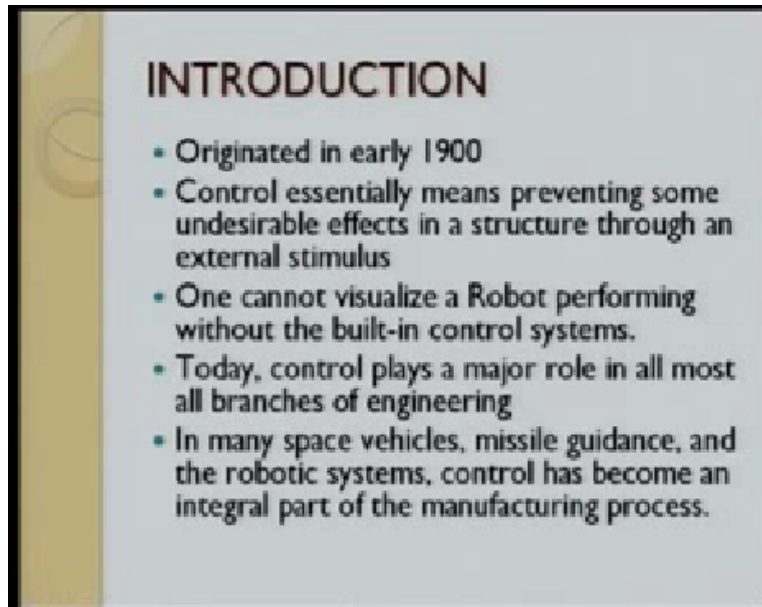
Micro and Smart Systems
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Lecture – 34
Control and Microsystems

So in this lecture, we will talk about control system. So control system is a part and parcel of many smart systems that we design. So control system essentially is designed to perform certain actions which the smart structure has to perform. So in this lecture, we will try to look at what all the ingredients of the control system, what are the mathematical models that are required for control system.

And what are the basics stability concept of a control system and some small basic control system design concepts we will actually study in this lecture.

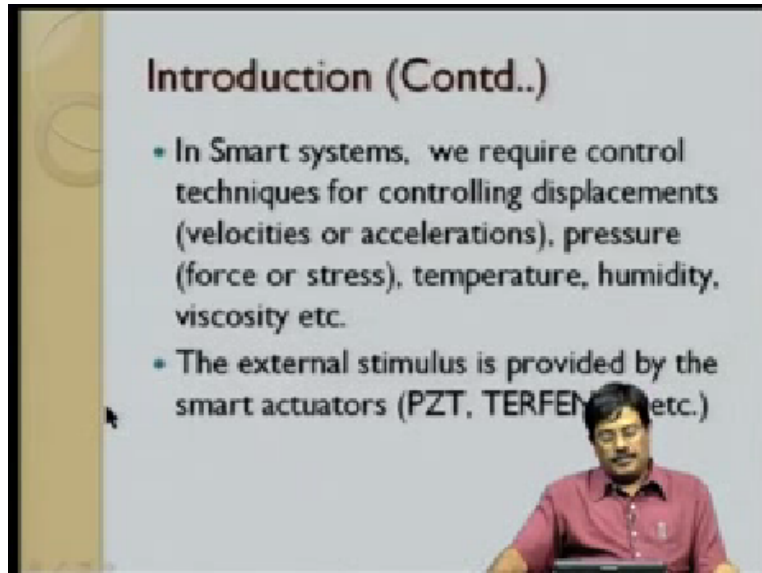
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So to give you some introduction on control system, basically the first control system was started way back in 1900. Control essentially means preventing some undesirable effects in a structure through an external stimulus, okay. One cannot visualise a Robot without a control system. Robot does multiple actions and these actions are possible basically by control systems.

Today, control plays a major part in almost all branches of science and engineering. In many space vehicles, missile guidance and robotic system, control has become an integral part of the manufacturing process.

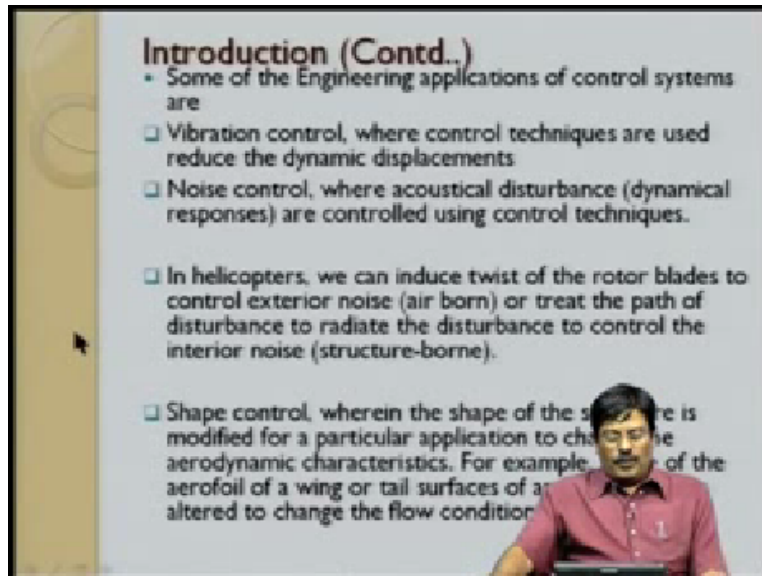
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Where does the control system fit in smart systems. So in smart systems, we require control techniques for controlling what we call displacements and its derivatives like velocities and accelerations, pressure and its derivatives like force and stress, temperature, humidity, viscosity and many more parameters that are basically part and parcel of this smart systems. The control system always requires an external stimulus.

And this external stimulus is provided by smart actuators basically through what we call the constitutive laws and some of the actuators that we studied in previous lectures or the piezoceramic actuators like PZT, magnetoelastic actuators Terfenol-D, etc.

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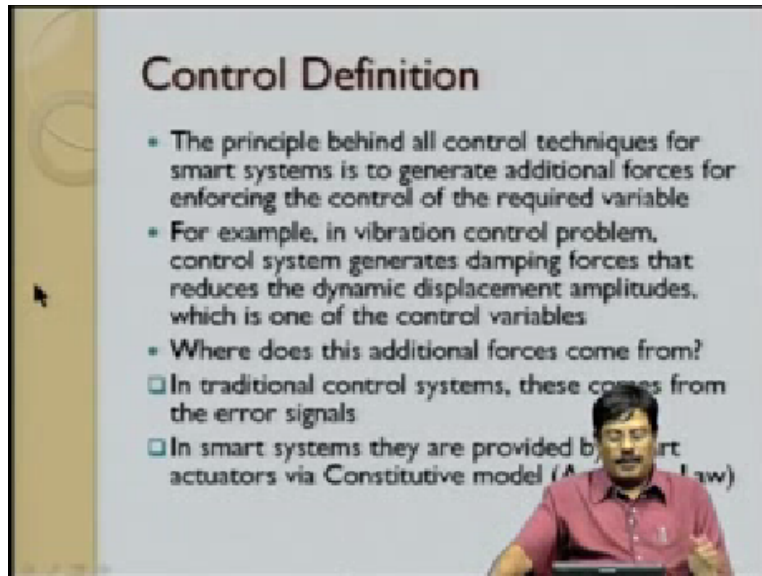


So some of the engineering applications of control systems where we would like the smart systems to perform. One is the vibration control where we control the dynamic displacement using control techniques. We have noise control, excessive noise is not desirable in many of the automobiles and aircrafts. So where the acoustical disturbances are controlled through control techniques.

In helicopters, where helicopters always produce excessive noise because of the rotor displacement. So we try to control it by actuating the flaps. By doing so, we control the exterior noise or sometimes we try to design a control system to treat the noise path so that it does not reach the helicopter cabin so that their cabins are quieter. Many a times we also design a control system to change the shape, a flat plate can be bent and in aircraft.

We can change the aerofoil shape, aerofoil controls the flow. By changing it, we can avoid vortex which basically is one of the root cause for aircrafts to stall.

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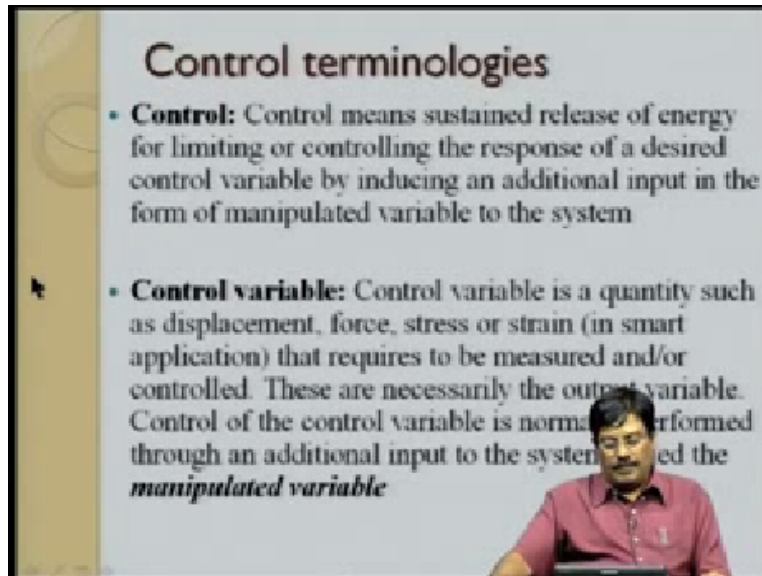
Control Definition

- The principle behind all control techniques for smart systems is to generate additional forces for enforcing the control of the required variable
- For example, in vibration control problem, control system generates damping forces that reduces the dynamic displacement amplitudes, which is one of the control variables
- Where does this additional forces come from?
 - In traditional control systems, these comes from the error signals
 - In smart systems they are provided by smart actuators via Constitutive model (A... Law)

So now let us go to control definition. The principle behind all control techniques for smart system is to generate additional forces for enforcing control of the required variable. So why we need, we need to control something. So what we control is a control variable, for example in vibration control, control system generates damping forces that reduces the dynamic displacements or dynamic amplitudes, which is one of the control variables.

So now the question is, where does this additional force come from especially in smart systems or in the traditional non-smart systems. In traditional control system, in the non-smart systems, these comes from what we call the RF signals. We will talk about RF signals little later in this lecture. In smart systems, they are provided by the smart actuators through constitutive law.

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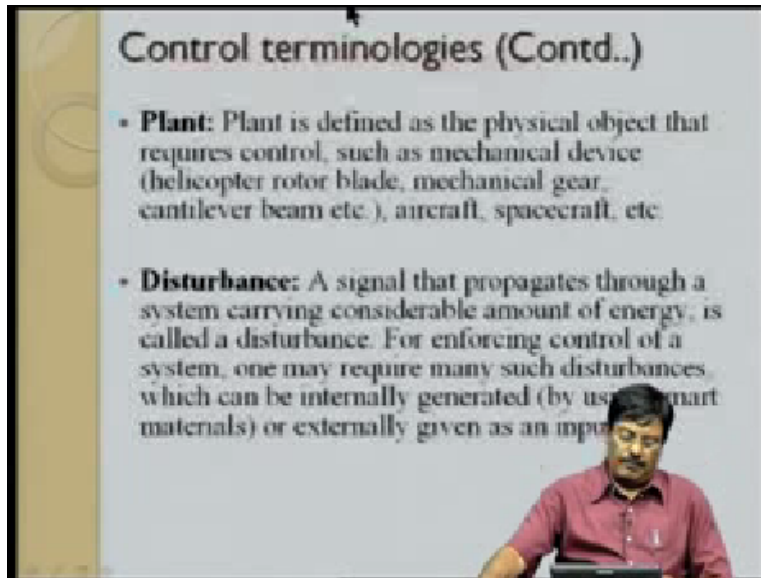
Control terminologies

- **Control:** Control means sustained release of energy for limiting or controlling the response of a desired control variable by inducing an additional input in the form of manipulated variable to the system
- **Control variable:** Control variable is a quantity such as displacement, force, stress or strain (in smart application) that requires to be measured and/or controlled. These are necessarily the output variable. Control of the control variable is normally performed through an additional input to the system called the *manipulated variable*

So let us define some of the control terminologies which we use traditionally in control system design. Let us first begin with what do we mean by control. So control means sustained release of energy for limiting or controlling the response of a desired control variable by inducing an additional input in the form of manipulated variable. So now the manipulated variable is something we input to the system.

Now we next define what is control variable. So we need to control something in the control system. So what is that something that we are going to control, that is the control variable. So the control variable is a quantity such as displacement, force, stress, strain, pressure, temperature, etc. that requires to be measured or controlled. These are necessarily an output variable and control of the control variable is normally performed through an additional input that is provided to the actuator in the smart system and which is called the manipulated variable.

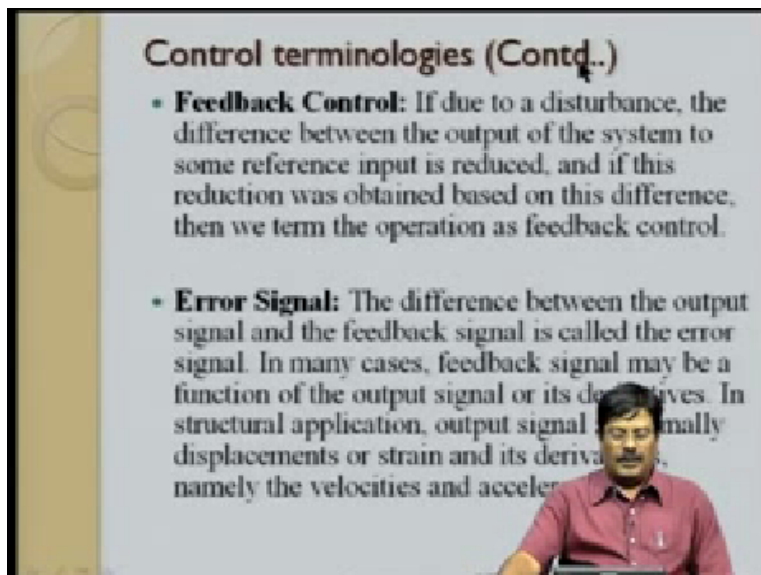
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Next, we define what is called a plant. Plant is defined as a physical object that requires control such as the mechanical device, helicopter blade, mechanical gear, cantilever beam, etc. aircraft, spacecraft. These all represent what we call the plant which requires control. Next, we define what is disturbance. A signal that propagates through a system carrying considerable amount of energy is what we call disturbance.

For enforcing control of a system, one may require many such disturbances which can be internally generated especially in a smart system through smart actuator or externally given as an input as in the case of traditional control systems.

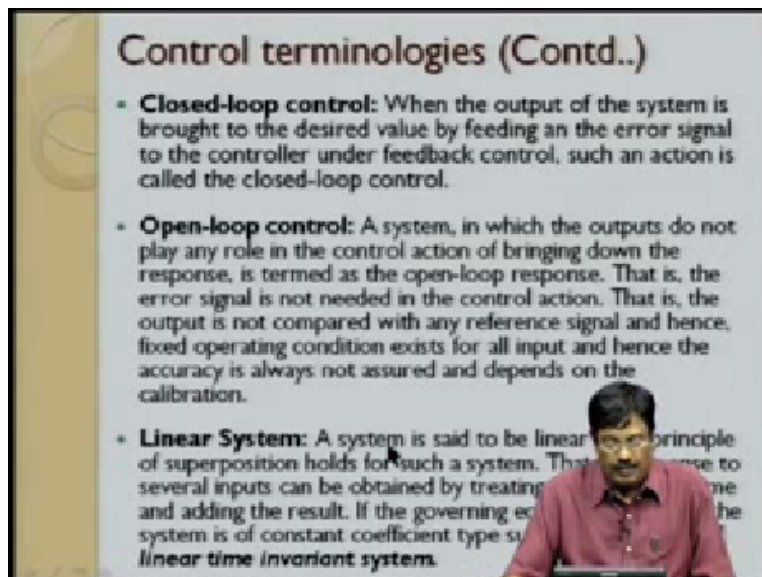
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Next, we define one of the very important parameter in control what we call the feedback control. If due to a disturbance, the difference between the output of the system which we are trying to control to some reference input is reduced and if this reduction was obtained based on this difference, then we term this operation as feedback control. So one of the factors on which the feedback control depends, is the error signal. So now we will redefine what is an error signal.

The difference between the output signal and the feedback signal is what we call the error signal. In many cases, the feedback signal may be a function of the output signal and its derivatives. In structural applications such as vibration control, noise control, the output signal is normally displacements or strain or its derivatives namely velocities and acceleration.

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Control terminologies (Contd..)

- **Closed-loop control:** When the output of the system is brought to the desired value by feeding an the error signal to the controller under feedback control, such an action is called the closed-loop control.
- **Open-loop control:** A system, in which the outputs do not play any role in the control action of bringing down the response, is termed as the open-loop response. That is, the error signal is not needed in the control action. That is, the output is not compared with any reference signal and hence, fixed operating condition exists for all input and hence the accuracy is always not assured and depends on the calibration.
- **Linear System:** A system is said to be linear if the principle of superposition holds for such a system. That is, the response to several inputs can be obtained by treating each input separately and adding the result. If the governing equations of the system is of constant coefficient type such a system is called a **linear time invariant system**.

Next, we will define what is called closed-loop control. So the control system we design is normally closed-loop control system which is the most efficient control system. So it is defined as follows, when the output of the system is brought to the desired value by feeding the error signal to the controller under the feedback control, such an action is termed as the closed-loop control.

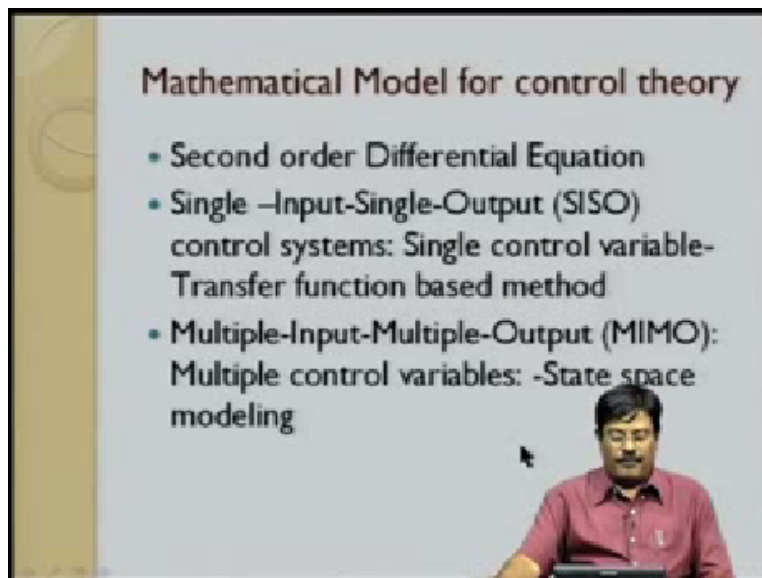
There is an alternative to closed-loop control which is used in some cases but sparingly, which is called the open-loop control. A system in which the outputs do not play a major role in the control action, is termed as the open-loop control. So that is the error signal is not needed at all in

the open-loop control as opposed to the closed-loop control.

That is the output is not compared with any reference signal and hence fixed operating condition exists and hence the accuracy is not always assured in the open-loop control. So we have defined the major control terminologies now. Next we will define what is we call the linear system, before most of our control system design is based on the linear system. A system is said to be linear if the principle of superposition holds.

So what do we mean by principal of superposition, that is the response to several inputs can be obtained by treating one input at a time, getting the response and adding the total response together. So if the governing equation describing a system which is of constant coefficient type, such a system is called linear time invariant system.

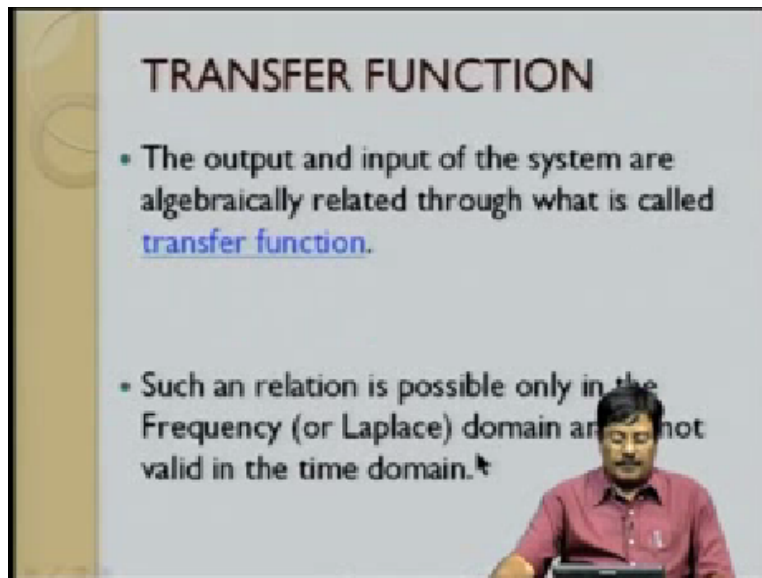
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So now we have defined what all the basic terminologies that are required for control systems, that are involving in design of the control systems. Next we will talk about what are the mathematical models that are required to design the control systems. The fundamental to most of the control system is, we need to have a differential equation which is in most cases is of a second order linear system, second-order linear differential equation and there are 2 kinds of control systems we can design.

One is called Single-Input-Single-Output SISO system, which is essentially based on transfer functions, determination of the transfer function because there is only 1 control variable here and the second method is the Multiple-Input-Multiple-output control system where more than 1 control variable will be there and normally we use what is called the state space approach modeling. We will talk about both of these modelling in this lecture.

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Let us now come to transfer function. What is transfer function, how do we define it. So in a traditional control system, any traditional dynamic system, you have an output, you have an input. The algebraic relationship between the output and input is what we define as transfer function. Such a relation is possible only in the frequency domain or in the Laplace domain which is also a frequency domain and it is not valid in the time domain.

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TRANSFER FUNCTION (Contd..)

- If the output of the system in the transformed frequency domain is $\hat{y}(x, y, z, \omega)$ (where x, y, z are the spatial dimensions and ω is the circular frequency at which the output is sampled) and $\hat{x}(x, y, z, \omega)$ is the input, then the transfer function is obtained by the equation:

$$G(\omega) = \frac{\hat{y}}{\hat{x}}$$
- In the design of controllers, it is necessary to obtain a transfer function, which is normally characterized using Laplace transforms. Use of Laplace transforms is limited to smaller SISO systems.

If the output of the system in the frequency domain is given by y which is a function of the spatial quantities x, y and z and also frequency defined in radian per second and if the input given to the system is x , okay, then we will define the transfer function G in frequency domain, $G(\omega)$ which will be equal to \hat{y}/\hat{x} , okay. It is a simple thing. For every frequency, there is a relation and this is what we call the transfer function.

So in the design of controllers, it is necessary to obtain a transfer function which is normally characterized using Laplace transform and there is a straightforward relationship between Laplace transform and Fourier transform. The use of Laplace transform however is limited to only single control variable that is it is valid for only SISO systems.

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Laplace Transform

$$L\{f(x, y, t)\} = \hat{f}(x, y, s) = \int_0^{\infty} f(x, y, t) e^{-st} dt \quad (9.2.1)$$

$$L\left[\frac{df(x, y, t)}{dt}\right] = s\hat{f}(x, y, s) - f(x, y, 0)$$

$$L\left[\frac{d^2 f(x, y, t)}{dt^2}\right] = s^2 \hat{f}(x, y, s) - sf(x, y, 0) - \dot{f}(x, y, 0) \quad (9.2.2)$$

- Use of Laplace transformation on a differential equation, reduces the same to an algebraic equation.



So let us now talk about Laplace transform. So the Laplace transform of a function f , a time domain function f , which is also spatially dependent, is given by this expression 9.2.1, okay. So we have the Laplace transform defined by this equation. The Laplace transform of the derivative of this function is also defined by this expression here which also depends upon the function evaluated at time $T=0$, so that should be known before.

The second derivative of this function, the Laplace transform of that is given by this equation here which depends upon not only the value of the function at time $T=0$ and also the derivative of the function at time $=0$, first derivative of the function. Like that we can write the Laplace transforms which can be extended to the n th derivative. So what does this Laplace transform do, why it is so powerful, why it is so useful. So basically, Laplace transform transforms the differential equation into a set of algebraic equation which are more easier to handle.

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- Let y be the output variable and x be the input variable. The linear differential equation that has n th order temporal derivative and m th order temporal input derivative, where $n > m$. This can be written as

$$A_n \frac{d^n y}{dt^n} + A_{n-1} \frac{d^{n-1} y}{dt^{n-1}} + \dots + A_1 \frac{dy}{dt} + A_0 y = B_m \frac{d^m x}{dt^m} + B_{m-1} \frac{d^{m-1} x}{dt^{m-1}} + \dots + B_1 \frac{dx}{dt} + B_0 x \quad (9.2.3)$$

So now let us come back to our control system. Let y be the output variable and x be the input variable. The linear differential equation of the n th order temporal derivative and n th order temporal input derivative where n is larger than m , can be written by this n th order differential equation. The left-hand side is basically the differential equation. Right-hand side is essentially the input because y is the output, x is the input.

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Applying Laplace transformation reduces the above equation to the algebraic equation. Transfer function that relates to the output to the input under the zero initial condition, is then given by

$$\frac{\tilde{y}(s)}{\tilde{x}(s)} = \frac{B_m s^m + B_{m-1} s^{m-1} + \dots + B_1 s + B_0}{A_n s^n + A_{n-1} s^{n-1} + \dots + A_1 s + A_0} \quad (9.2.4)$$

In the above equation, both numerator and denominator can be factorized as

$$\frac{\tilde{y}(s)}{\tilde{x}(s)} = \frac{(s + \alpha_1)(s + \alpha_2) \dots (s + \alpha_m)}{(s + \beta_1)(s + \beta_2) \dots (s + \beta_n)} \quad (9.2.5)$$

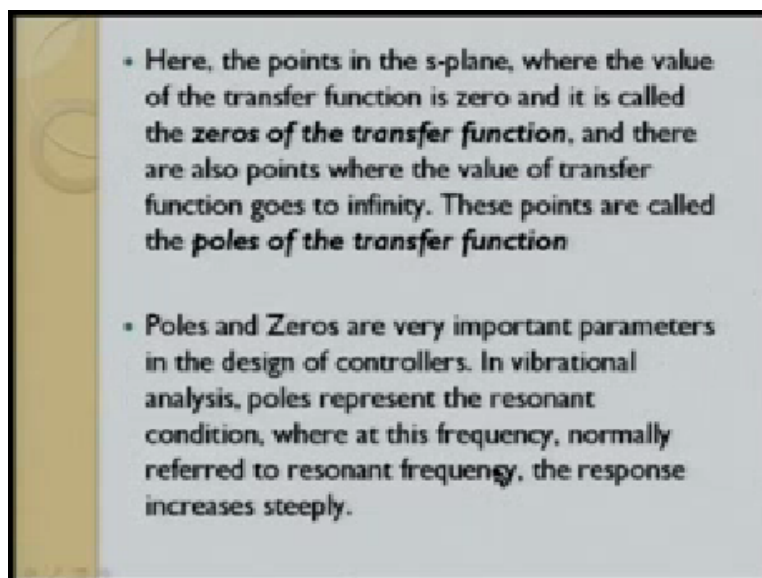
So when we apply the Laplace transform, okay and assume that zero initial condition, that is the value of the function at time $T=0$ and its derivative is basically and all the higher derivatives at time $T=0$ that is the 0 initial condition, we can write the governing equation into a set of algebraic equation where y is the output in the Laplace domain, x is the output in the Laplace

domain and we can get this, okay. So this 9.2.4 is basically an algebraic relation which has a numerator as well as the denominator.

So basically what we do here is, we can factorize the numerator, we can factorize the denominator as given here. So basically the numerator can be factorized as $s + \alpha_1$ multiplied to $s + \alpha_2$ multiplied to $s + \alpha_m$ and similarly the denominator $s + \beta_1$ multiplied to $s + \beta_2$ to $s + \beta_m$. So the transfer function will be 0 at values $-\alpha_1, -\alpha_2, -\alpha_3$, etc. The denominator will be 0 at $-\beta_1, -\beta_2, -\beta_3$.

So when the numerator is 0, we say there is a 0 in the transfer function, when the denominator is 0 which makes the transfer function infinity, we say these are the poles.

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So basically poles and zero are the important parameter in control system. So basically poles are very important for the design of controllers. In vibration analysis, poles represents the resonant condition where the driving frequency will equal to the natural frequency which basically increases the displacement or the vibrational amplitude to enormously large extent which has to be avoided at any cost.

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Single degree of freedom system

$$m\ddot{x} + c\dot{x} + kx = f(t) \quad (9.2.6)$$

- In the above equation, m denotes the mass of the system, c is the viscous damping coefficient, and k represents the effective stiffness of the system.
- Here, x , \dot{x} , \ddot{x} represents the displacement, velocity and acceleration of the system. $f(t)$ is the force and is considered as



So let us come back to a single degree of freedom system. What is a single degree of freedom system where there is only one predominant motion that means there is only one control variable. So this is basically given by a second-order differential equation where \ddot{x} , \dot{x} , x are the derivative of the x which represents acceleration, velocity and x basically represents the vibrational amplitude and f of t is basically the forcing function.


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Single degree of freedom system

$$(ms^2 + cs + k)\hat{x}(s) = \hat{f}(s)$$

$$\frac{\hat{x}(s)}{\hat{f}(s)} = \frac{1}{ms^2 + cs + k} \quad (9.2.7)$$

$$\frac{\hat{x}(s)}{\hat{f}(s)} = \frac{1}{(s + \alpha_1)(s + \alpha_2)}$$

$$\alpha_1 = -\frac{c}{2m} + \sqrt{\left(\frac{c}{2m}\right)^2 - \frac{k}{m}}, \alpha_2 = -\frac{c}{2m} - \sqrt{\left(\frac{c}{2m}\right)^2 - \frac{k}{m}} \quad (9.2.8)$$


So when we take a Laplace transform of this, we get $ms^2 + cs + k$ multiplied with \hat{x} of s which is nothing but the Laplace transformation of x of t which is equal to \hat{f} of s . So \hat{x} of s is the output, \hat{f} of s is the input, the ratio of these will give us the transfer function, that is $\frac{1}{ms^2 + cs + k}$.

So basically, the transfer functions, the numerator is just 1 value, so it cannot be factorized, that is there is no zeros in this transfer function where as the denominator can be factorized since it is a quadratic, so it has 2 poles with the value alpha 1 and alpha 2 and alpha 1 and alpha 2 are given by this equations here.

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- From the above equation, we have no zeros and two poles at α_1 and α_2 , which can be real or complex.
- The real or complex value of α_1 and α_2 depends on the value of the radical under square root
- For the design of controller, it is necessary that the value of real part of α_1 and α_2 should be negative.
- From Equation (9.2.7), when we substitute $s=i\omega$, we can transform the problem from Laplace domain to the Fourier domain. That is, the transfer function becomes

$$\frac{j(\omega)}{f(\omega)} = \frac{1}{-m\omega^2 + ic\omega + k}$$

FRF (Frequency response function)
(9.2.9)

So from the above equation, we have seen that there are no zeros but 2 poles at -alpha 1 and -alpha 2, the real or complex value of alpha 1 and alpha 2 depends upon the value of the radical, the value of this radical, if $c/2m$ is greater than k/m , then alpha 1 will be real and if $c/2m$ is less than k/m , it is going to be imaginary. So for the design of the controller, one important property is for the stability of the control system, it is necessary that the real part of alpha 1 and alpha 2 should always be negative.

Otherwise, the control system will be unstable. So from the equation above, if we substitute instead of the Laplace parameter s by i times omega, we can transform the problem from the Laplace domain to Fourier domain, then the transfer function is given in terms of omega which is given here and the quantity that is on the right-hand side, $1/-m \omega^2 + ic \omega + k$ represents what we call the frequency response function.

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Transfer Function from Finite Element Method

$$[M]\ddot{x} + [C]\dot{x} + [K]x = \{F\} \quad (9.2.10)$$

- Here, $[M]$, $[C]$, and $[K]$ are the mass, damping and stiffness matrices of size $n \times n$ and x , \dot{x} and \ddot{x} are the displacement, velocity and acceleration vector of size $n \times 1$ and $\{F\}$ is the applied force vector.
- Applying Fourier (or Laplace) transformation to the above equation and assuming the initial conditions to be zero, we get

$$[\hat{K}]\hat{x} = \{\hat{F}\}, \quad [\hat{K}] = -\omega^2[M] + i\omega[C] + [K] \quad (9.2.11)$$

where, $[\hat{K}]$ is the frequency dependent dynamic stiffness matrix. To obtain the transfer function, we apply unit impulse at the desired degree of freedom and the Equation (9.2.11) is solved for this unit impulse. Such a degree of freedom will have n poles (resonant frequency). If n is very large, control design becomes extremely difficult and one has to reduce the FE system. This can be accomplished by Modal Order Reduction.

So now how do we determine transfer function from finite element method because which is normally used for any control system design. The finite element equation is given by 9.2.10 which is $M \cdot \ddot{x} + C \cdot \dot{x} + K \cdot x = F$ where M , C and K are matrixes of size n by n , that is it has n control variables, okay. So and x , \dot{x} and \ddot{x} are basically the dynamic amplitudes, x is the displacement, \dot{x} is the velocity and \ddot{x} is the acceleration and F is the applied force vector.

So we apply either Laplace transform or Fourier transform to this, we reduce this equation into $\hat{K} \cdot \hat{x} = \hat{F}$, all in the transform domain where \hat{K} is given by the equation 9.2.11 and it is a frequency dependent matrix which is called the dynamic stiffness matrix. So basically we solve 9.2.11 by giving a unit impulse in the place where you require the transfer function and solve for the unit impulse, this matrix equation and whatever the output you get, \hat{x} is essentially the transfer function. So it has to be solved numerical.

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State Space Modeling

- A system is said to be in **state** if for a given input, the response can be completely determined for all future times with minimum amount of information. Mathematically, a dynamic system is defined by a differential equation of the form given by

$$\frac{d^n y}{dt^n} + a_1 \frac{d^{n-1} y}{dt^{n-1}} + \dots + a_{n-1} \frac{dy}{dt} + a_n y = R(t) \quad (9.2.12)$$

- The above differential equation of n th order and hence all the n derivatives are defined and it requires n initial conditions for its solution

Now let us come to the State Space Modeling. So a system is said to be in state space if for a given input, the response can be completely determined for all future times with minimum amount of information. Mathematically, a dynamic system is defined by a differential equation which is given n th order differentially. Here y is basically the output variable that we are looking at, R is going to be which is basically the input, the R input may contain the input time function or its derivatives

Let us first assume that R of t contains no derivatives of the input function.

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State Space Modeling (Contd..)

- The above differential equation of n th order and hence all the n derivatives are defined and it requires n initial conditions for its solution
- We may choose to call each of the variables y and each of first $(n-1)$ derivatives as **state variables**.
- The number of state variables required to model a differential equation is equal to the order of the differential equation.
- The fundamental to the state space modeling is to provide a systematic mathematical approach to analysis of the characteristics of the system by reducing a single differential equation into a coupled set of first order differential equations with each equation defining one state. This set of equations is called the **state equations**.

So in the above equations the n th order and all n derivatives should be defined and it requires n

initial conditions. We may choose to call all the variables y and each of the $n-1$ derivatives as state variables, okay. The number of state variables required to model a differential equation is equal to the order of the equation. So why do we need state space approach.

Basically fundamental to state space modelling is to provide a systematic mathematical approach to the analysis of the characteristics of the system by reducing a single n th order differential equation coupled set of differential equation into a set of first-order differential equation with each equation defining one state. The set of equations, such set of 3 equation is called the state equation.

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State Space Modeling (Cont..)

- In Equation (9.2.12), $R(t)$ represents the forcing term. Let us first assume that $R(t)$ does not involve any derivative terms and is equal to $f(t)$. We can now assume a set of n state variables. Let us now define

$$\begin{aligned}
 x_1 &= y, & x_2 &= \frac{dy}{dt} = \frac{dx_1}{dt} \\
 x_3 &= \frac{d^2 y}{dt^2} = \frac{dx_2}{dt}, & x_4 &= \frac{d^3 y}{dt^3} = \frac{dx_3}{dt} \\
 &\vdots & & \vdots \\
 x_{n-1} &= \frac{d^{n-2} y}{dt^{n-2}} = \frac{dx_{n-2}}{dt}, & x_n &= \frac{d^{n-1} y}{dt^{n-1}} = \frac{dx_{n-1}}{dt}
 \end{aligned}
 \tag{9.2.13}$$

So let us now assume as I said $R(t)$ does not contain any derivatives, then we can define the state variables as given by 9.2.13 where x_1 will be the first state variable which is equal to y , my output; x_2 will be the derivative dy/dt which is equal to dx_1/dt ; x_3 will be the second derivative which will be the first derivative of x_2 ; x_4 will be third derivative of y which will be first derivative of x_3 and so on. We can write $n-1$ and n th derivative.

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State Space Modeling (Cont..)

The n th state equation is obtained by using above definition in Equation (9.2.12). That is

$$\frac{dx_n}{dt} = f(t) - A_n x_1 - A_{n-1} x_2 - \dots - A_{n-2} x_{n-1} - A_n x_n \quad (9.2.14)$$

Equations (9.2.13) and (9.2.14) can be put in the matrix form as

$$\dot{x} = -[A]x + [B]f \quad (9.2.15)$$

So the n th state equation is obtained by using the above definition, that is we can write as given in 9.2.14. The equations 9.13 and 14 can be put in matrix form as given here, okay.

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State Space Modeling (Cont..)

$$\dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_n \end{bmatrix} = [A] \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} + [B] \begin{bmatrix} f \\ \vdots \\ f \end{bmatrix} \quad (9.2.14)$$

$$y = \begin{bmatrix} 1 & 0 & \dots & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = y = [C]x \quad (9.2.14)$$

[A]=State Matrix [B]=Input Matrix [C]=Output Matrix


y=Output

So which then expanded will be shown here, x is a vector of all state variables, a is called the state matrix, b is called the input matrix. Again y can be related to the state variable as $y=C*x$. So these 2 together form what is called the state space equation which is basically used for our controls system design.

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State Space Modeling (Cont..)

- Now let us consider a case where the right hand side of the Equation (9.2.12) has in addition to the forcing function $f(t)$, also contains its time derivatives. That is

$$R(t) = b_0 f(t) + b_1 \frac{df}{dt} + \dots + b_{n-1} \frac{d^{n-1} f}{dt^{n-1}} + b_n \frac{d^n f}{dt^n} \quad (9.2.17)$$


Now if we consider that my input on the right-hand side is also a function of the derivative of the input function f , then $R(t)$ is defined by equation 9.2.17 which contains the derivatives, all higher derivatives. If we use this, our previous definition of the state variable is not valid because it does not eliminate these derivatives.


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State Space Modeling (Cont..)

- Earlier definition of state variables given in Equation (9.2.13), will not yield unique solution. In this case, the state variables must be so chosen such that they will eliminate the derivatives of input $f(t)$. This can be accomplished if we define the n state variables as

$$\begin{aligned} x_1 &= y - c_0 f, & x_2 &= \frac{dy}{dt} - c_0 \frac{df}{dt} - c_1 f - \frac{dx_1}{dt} - c_1 f \\ x_3 &= \frac{d^2 y}{dt^2} - c_0 \frac{d^2 f}{dt^2} - c_1 \frac{df}{dt} - c_2 f - \frac{dx_2}{dt} - c_2 f \\ &\vdots \\ x_n &= \frac{d^{n-1} y}{dt^{n-1}} - c_0 \frac{d^{n-1} f}{dt^{n-1}} - c_1 \frac{d^{n-2} f}{dt^{n-2}} - \dots - c_{n-1} f - \frac{dx_{n-1}}{dt} \end{aligned} \quad (9.2.18)$$

where

$$\begin{aligned} c_0 &= b_0, & c_1 &= b_1 - a_1 c_0, & c_2 &= b_2 - a_2 c_0 - a_1 c_1 \\ &\vdots \\ c_n &= b_n - a_n c_0 - \dots - a_{n-1} c_{n-1} - a_n c_0 \end{aligned}$$


So we need to define a new set of the state variables. So these are given here in the equation 9.2.18 where x_1 will be instead of y , we also introduce $c_0 \cdot f$; my state of second variable, x_2 will be the first derivative of $y - c_0$ into first derivative of the input and the actual function; then x_3 will be second derivative of y minus the second derivative of the function, the first derivative of the function and the function itself, input function, like that we can write the n th derivative.

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State Space Modeling (Cont.)

Choice of these state variables lead to

$$\begin{aligned} \frac{dx_1}{dt} &= x_2 + c_1 f \\ \frac{dx_2}{dt} &= x_3 + c_2 f \\ &\vdots \\ \frac{dx_{n-1}}{dt} &= x_n + c_{n-1} f \\ \frac{dx_n}{dt} &= -a_n x_1 - a_{n-1} x_2 - \dots - a_1 x_n + c_n f \end{aligned} \tag{9.2.20}$$

So once we write this, we can chose the new state variable questions as shown below. So the first equation where $dx_1/dt=x_2+c_1f$ and the nth one will be given by here, this equation.

(Refer Slide Time: 24:11)

State Space Modeling (Cont.)

• This equation in matrix form is given by

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_{n-1} \\ \dot{x}_n \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 1 \\ -a_n & -a_{n-1} & -a_{n-2} & \dots & -a_1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{n-1} \\ x_n \end{bmatrix} + \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_{n-1} \\ c_n \end{bmatrix} f$$

OR

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 1 \\ -a_n & -a_{n-1} & -a_{n-2} & \dots & -a_1 \end{bmatrix} x + c_n f$$

State space representation of the original nth order Governing Equation

$$\begin{aligned} \dot{x} &= [A]x + [B]f \\ y &= [C]x + Df \end{aligned} \tag{9.2.21}$$

So in doing so, we can write these equations in matrix form which is a relating to the state vector on the left-hand side, the state matrix and the input matrix and the output and input can be derived here. So in doing so, we also introduce an additional parameter D which is given by c_0*f that enters into picture. So this form is convenient for us to design the state variable when the input that is given to the system is also dependent on its derivatives.

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State Space Modeling (Cont..)

- One can now obtain the transfer function of the system from the state equation (9.2.21). This can be done if one takes the Laplace transform of Equation (9.2.21). That is

$$\begin{aligned} s[\hat{x}(s)] - [x(0)] &= [A][\hat{x}(s)] + [B]\hat{f}(s) \\ \hat{y}(s) &= [C][\hat{x}(s)] + D\hat{f}(s) \end{aligned} \quad (9.2.22)$$

- Here, $\hat{x}(s)$ and $\hat{f}(s)$ are the Laplace transform of the state vector $\{x(t)\}$ and the forcing function $f(t)$. Transfer functions are normally derived assuming zero initial condition. From first of Equation (9.2.22), we have

$$[\hat{x}(s)] = [sI - A]^{-1} [B]\hat{f}(s) \quad (9.2.23)$$

So once we do that, now we take the, next step is to see how we transform this into frequency domain. So we take a Laplace transform of this where we write the equation here in the x domain which also contain x of 0 that is the value of the state vector at time t=0. So when we do that, from the first equation, we eliminate this x of s and substituting the second of the equation 9.2.22, then we will get a relationship between the output and the input which will become a transfer function matrix.

So this is given here, output/input. So which requires inversion of s-A matrix.

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State Space Modeling (Cont..)

Using the above in the second of Equation (9.2.22), we can relate the output to the input. That is, the transfer function is given by

$$\frac{\hat{y}(s)}{\hat{f}(s)} = G(s) = [C][sI - A]^{-1} [B] + D \quad (9.2.24)$$

That is, the transfer function computation involves computation of $[sI - A]^{-1}$. Hence, determinant of matrix $[sI - A]$ will give the characteristic polynomial of the transfer function and the eigenvalue of matrix $[A]$ will give the poles of the system.

So the transfer function is given by this equation here, okay. So basically the determinant of s-a

will give me the characteristic polynomial of the control system and the matrix A will give us the poles that is going to be there in this control system for which we have to design our control system.

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SDOF System

$$m\ddot{x} + c\dot{x} + kx = f(t) \quad (9.2.25)$$

- For state space representation of the system we define $x_1(t) = x(t)$, $x_2(t) = \dot{x}(t)$ as state variables.
- Using these state variables, Equation (9.2.25) reduces to following two first order equation (state equations) written in matrix form

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{c}{m} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} f \quad y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad (9.2.26)$$

So let us see how we can apply this to a simple single degree of freedom system. We again come back here, this we studied few slides before. This is again a single degree of system where x is the control variable basically. So $m\ddot{x} + c\dot{x} + kx = f$ of t. Now to represent this in state space, we need to choose the state variables. We take the first state variable $x_1 = x$ and $x_2 = \dot{x}$ double dot

So we can rewrite this, reduce this into 2 sets of equations, that is x_1 . x_2 . is related to this matrix a and x_1 and x_2 is the state vector plus B and again output is given by this matrix c times the state vector. So we can easily represent this single degree of freedom system into a matrix contain 2x2 state vector and vector containing the input matrix, output matrix and the output vector.

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SDOF System (Contd..)

The above equation is in the conventional form of state equations given by Equation (9.2.21). Substituting the matrices **[A]**, **[B]**, **[C]** and **D** derived from the above equation in Equation (9.2.24), we can write the transfer function as

$$G(s) = \frac{1}{ms^2 + cs + k} \quad (9.2.27)$$

This is same as what was derived in Equation (9.2.7), obtained by taking Laplace transformation on the governing equation.

So from the above equation, as in the conventional form of the state equation, substituting the matrix A B C and D in the derived above equation, we can write the transfer functions when we do that in this form which is basically same what we derived before by using our regular transfer function methods instead of using the state spacer approach.

So now what we have done is we have taken the single degree of freedom system to show how we can derive the transfer function both from your conventional method of transforming directly these equation into frequency domain or using the states space approach.

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State Space Model from FEM

- In designing controllers for multi-input-multi-output system, especially for structural applications, one will have to depend extensively on discretized mathematical model as that derived from FE techniques. The discretized Finite Element governing equation of any structure is of the form

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} = \{f\} \quad (9.2.28)$$

Hence, the state vectors for the FE equation are $\{x_1\} = \{x\}$ and $\{x_2\} = \{\dot{x}\}$. The reduced state space form of Equation (9.2.28) and its corresponding output vector is given by

$$\begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{Bmatrix} = \begin{bmatrix} [0] & [I] \\ -[M]^{-1}[K] & -[M]^{-1}[C] \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} + \begin{bmatrix} [0] \\ [M]^{-1} \end{bmatrix} \{f\}$$

$$y = \begin{bmatrix} [I] & [0] \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} \quad (9.2.29)$$

How do we do the state space model from FEM. Again, in designing the controller for multi-

input-multi-output system, especially for structural application, one will have to depend extensively on the discretized model because the number of degrees of freedom and number of control variables will be so large. The discretized finite element equation is given by 9.2.28 which is given by $M\ddot{x} + C\dot{x} + Kx = f$ where M , C and K are matrices, x is a vector, f is the input, all are of size n by n and x the vectors of size n by 1 .

So here we choose the state vector as x_1 vector is equal to the actual control variable x , x vector and x_2 will be the derivative of x_1 . So when we do that and apply our state space model, we get the control equation in terms of this state matrix by equation 9.2.29 which is given here. So once we know that, we can clearly identify what is the A matrix B matrix and C matrix which will be very useful in the control design.

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State Space Model from FEM (Contd..)

The above is in the standard form given in Equation (9.2.21), wherein we can clearly identify matrices **[A]**, **[B]** and **[C]**, respectively. Equation (9.2.29) represents $2n \times 2n$ system. That is, an $n \times n$ second order system (Equation (9.2.28)), when reduced to state space form, becomes $2n \times 2n$ of first order system. Also, the input and the output are related especially for the feedback by

$$\{f\} = [G]\{y\} \quad (9.2.30)$$

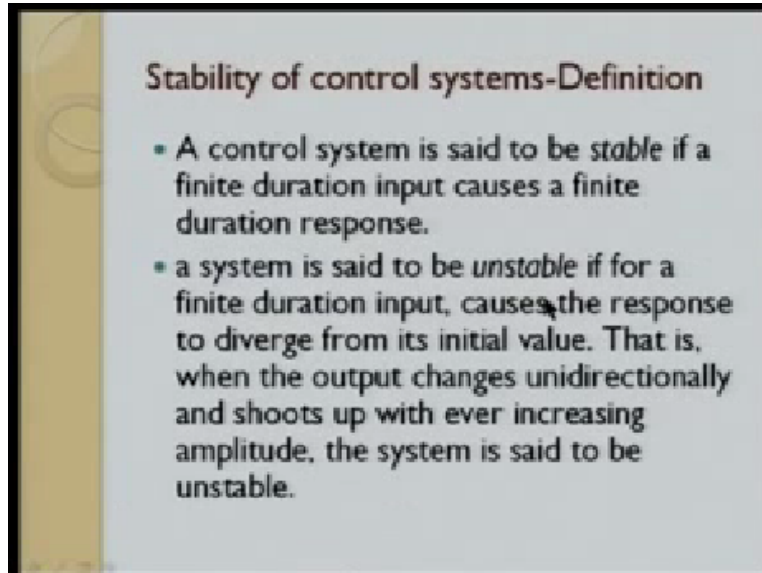
In the above equation, **[G]** is the gain matrix of size $n \times r$, when r states are chosen for input feedback to reduce the response, especially for vibration control applications.

So what we now learnt is if there is a system which is of order n by n , the state matrix will be of the size $2n$ by $2n$. So we increase the size, okay. In addition, if you want to do a control system, in addition to getting the A B C matrix, we also have to get the gain matrix. What is a gain matrix. Gain matrix is essentially relates the input to the output, y is the output, f is the input. $F=Gy$ where G is the gain matrix which is of the size $n \times r$.

What is this r , r is the number of the states that you need to control. So in terms of vibrational frequency if you want to control 10 frequencies, r will be 10. Then n will be maybe 100, 1000,

10,000. So basically it is the number of control variable that you want to control, it is a control state, r represents that.

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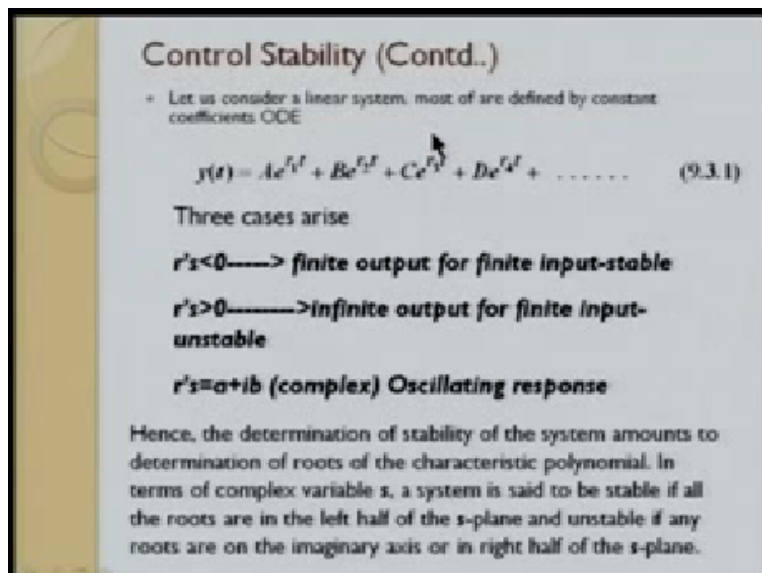


Stability of control systems-Definition

- A control system is said to be *stable* if a finite duration input causes a finite duration response.
- a system is said to be *unstable* if for a finite duration input, causes the response to diverge from its initial value. That is, when the output changes unidirectionally and shoots up with ever increasing amplitude, the system is said to be unstable.

Now let us define another important parameter for the control system that is the stability. Every control system has to be stable and when is the control system stable. A control system is said to be stable if a finite duration input causes a finite duration response. A system is said to be unstable if for a finite duration of input, it causes response that diverges phenomenally from the initial value that is when the output changes unidirectionally and shoots up with ever increasing amplitude, the system is said to be unstable.

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Control Stability (Contd.)

• Let us consider a linear system, most of are defined by constant coefficients, ODE

$$y(x) = Ae^{r_1x} + Be^{r_2x} + Ce^{r_3x} + De^{r_4x} + \dots \quad (9.3.1)$$

Three cases arise

- $r's < 0 \longrightarrow$ finite output for finite input-stable
- $r's > 0 \longrightarrow$ infinite output for finite input-unstable
- $r's = a+ib$ (complex) Oscillating response

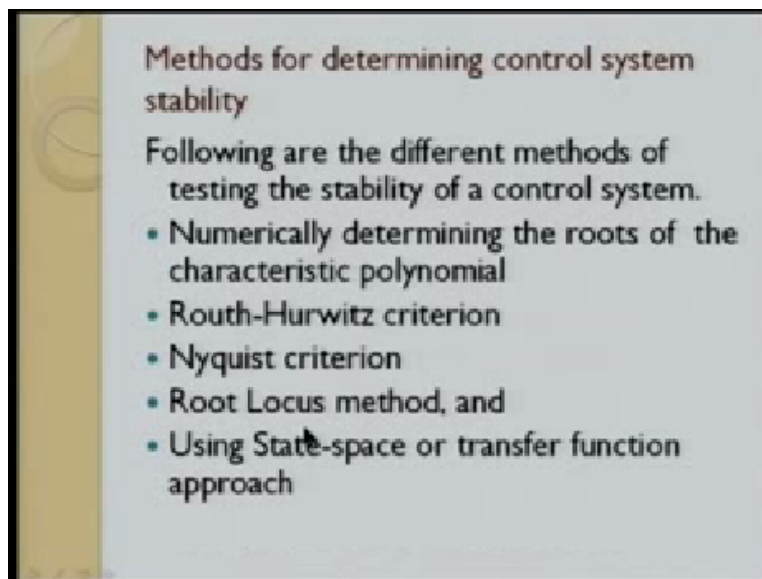
Hence, the determination of stability of the system amounts to determination of roots of the characteristic polynomial. In terms of complex variable s , a system is said to be stable if all the roots are in the left half of the s -plane and unstable if any roots are on the imaginary axis or in right half of the s -plane.

So when does it happen. So we studied until now that most second-order systems are of constant coefficients which has exponentials as solutions. So the solution of such will be of the form y of t is the solution for which you are looking will be $Ae^{r_1 t} + e^{r_2 t}$, etc. okay and it is necessary that for the r 's can be less than 0, greater than 0, or r 's can be complex.

So r in 3 cases, these are the three cases that arises. If r 's are less than 0, the finite output to a finite input, then we say it is stable. When r , this is greater than 0, there is an infinite output your finite input that means it is unstable. When r 's are complex, then we will have an oscillating response because basically they will be of the form sines and cosines. Hence the determination of stability of the system amounts to determination of the roots of the polynomial.

In terms of complex variable s , the system is said to be stable if all the roots are on the left half of the s -plane. So if you plot the s versus frequency, then it has to be in the left, so basically that is why in the starting of this lecturer, I said all the roots should be on the negative so that this is on the left side of the s -plane or the imaginary axis.

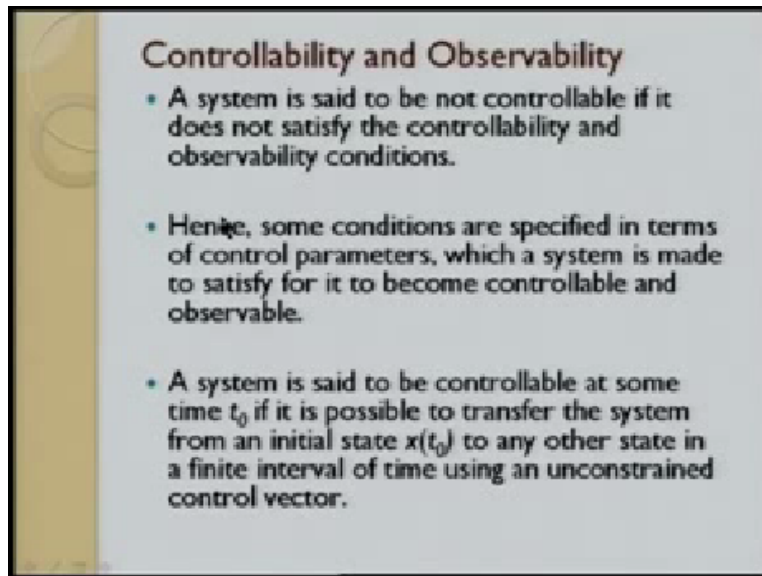
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How would we determine the methods of the control. Following are the different methods that are used. One of the methods is numerically determining the roots of the characteristic polynomial, that is find when the degrees of freedom or number of control variable is small, so that you can factorize.

But if the system is large even if we have more than 3 or 4 variables, it becomes very difficult to factorize, then we have to use certain numerical criterion such as Ruth-Hurwitz criterion, Nyquist criterion, Root Locus method or using the state space and transfer function approach.

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There are 2 other important parameter that determines the stability. One what we call the controllability and observability. A system is said to be not controllable if it does not satisfy these 2 conditions, namely the controllability and the observability condition. Hence some conditions are specified in terms of control parameter which a system is made to satisfy or to become controllable or observable.

A system is said to be controllable at some time t_0 if it is possible to transfer the system from an initial state x of t_0 to any other state in finite interval of time using the unconstrained control vector.

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Controllability and Observability (Contd.)

- A system, which is in state, is said to be observable at some time t_0 , if it is possible to determine this state from the observation of output over a finite interval of time.
- Using the above definition, we can derive the condition for both input and output controllability
- The condition of controllability of the input is that the vectors $[B], [A][B], \dots, [A]^{n-1}[B]$ are linearly independent and the matrix given by

$$\begin{bmatrix} [B] & [A][B] & \dots & [A]^{n-1}[B] \end{bmatrix} \quad (9.3.2)$$

which is of rank n or is not singular

A system is said to be in a state which is in state, is said to be observable at sometime t_0 if it is possible to determine the state from the observation of the output over the finite interval of time. So using the above definition, we can derive the condition for input and output controllability. So the controllability has to be both because the input has to be controllable and the output has to be controllable.

So the condition for the controllability of input is that the vectors that is the output vector B , A times B , that is a multiplication of the state matrix into B and such products up to $A^{n-1}B$, if you have an n by n system are linearly independent or in linear algebra terms, in matrix terms, the matrix containing these matrix that is B AB , the product $A^{n-1}B$, should be of their rank n . It should be for n by n system. That is this matrix should not be singular. If it is singular, then input is not controllable.

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Controllability and Observability (Contd...)

= we can state the condition of output controllability of the state equation through a matrix representation as

$$\begin{bmatrix} [C|B] & [C|A|B] & [C|A^2|B] & \dots & [C|A^{n-1}|B] & [D] \end{bmatrix} \quad (9.3.3)$$

The above matrix is of the order $m \times (n+1)r$, where matrix **[A]** is $n \times n$, vector **[B]** is $n \times r$, **[C]** is $m \times n$ and **[D]** is $m \times r$. The condition for output controllability is that the matrix given in Equation (9.3.3) is of the rank m .

Observability Condition

That is the state equation given by Equation (9.2.21) is observable, if and only if the matrix given by

$$\begin{bmatrix} [C^T] & [A^T]C^T & \dots & [A^{n-1}]^T C^T \end{bmatrix} \quad (9.3.4)$$

which is of rank n or is not singular

Now we have to check the output controllability. So what is the output controllability. So we need to take the output matrix C that require the output with the input, so that is the C, the product of CB CAB CA square B and CA n-1 B and D and this matrix should not be singular. So here, the output is basically depends upon how many control states that we are controlling, that is why the term the size of the matrix enters a parameter r comes into the picture.

So in the above matrix, the r is of the order $m \times n + 1 \times r$ where the matrix A is of n by n, B is $n \times r$, C is $m \times n$ because there are m inputs and D is $M \times r$. So the condition for output controllability is this matrix given in equation 9.3.3 should be of the rank m, okay. So this is as far as the controllability conditions are there.

Now the observability condition, that is the state equation given by equation 9.2.1 that is the state space equation which we derived initially, is observable is only when the output matrix, that is C transpose A times C transpose product and A transpose n-1 of the n-1 state vector into C transpose, this matrix should be of order n and should not be singular.

So these are the 2 important conditions that a control system has to satisfy to make sure that the design control system is stable.

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Controllability and Observability (Contd...)

- We can also state the conditions for complete controllability and observability in s-plane. That is, the system is not completely controllable or observable if there exists common factors in the transfer functions in the numerator and denominator. For example, a transfer function given by

$$G(s) = \frac{(s+1)(s+4)}{(s+1)(s+2)(s+3)}$$

and it is not completely controllable or observable due to common factor $(s+1)$ in both numerator or denominator of the above equation.

We can also state the conditions for complete controllability and observability even by looking at the transfer function. This is only of course true for only some simple systems where the control parameters or control variables are small, that is the system is not completely controllable or observable if there exists common factors in the transfer functions in the numerator and denominator.

For example, if the transfer function given by this equation here, G of $s=s+1*s+4$ divided by $s+1*s+2*s+3$ is not completely controllable or observable because there is a common factor $s+1$ on both the numerator and denominator. So even such a system should be avoided.

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Control Design Concepts.

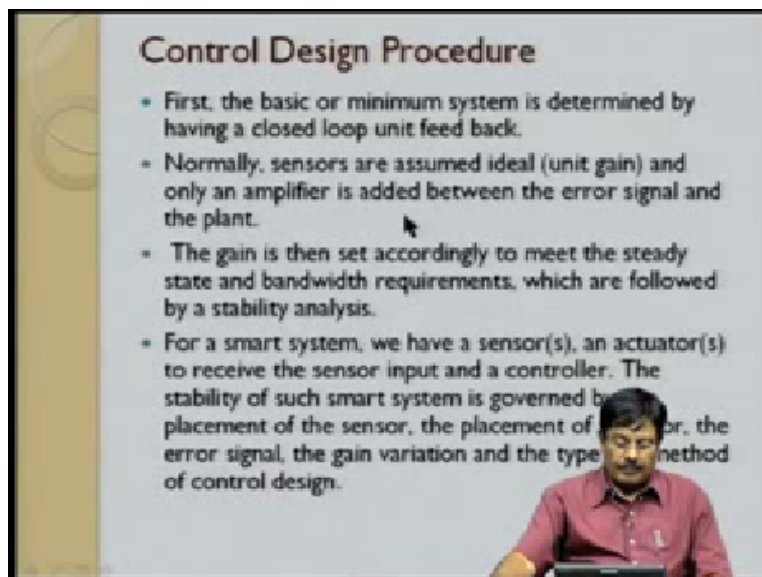
- The fundamental to the design of the control system is to place the poles at the appropriate position so that the stability of the system is ensured.
- Plant is a part of the control system that has unchangeable parts and it is described by the transfer functions or state variables.
- The poles can be shifted to the appropriate position by closing a loop around a plant with a feed back signal with appropriate gain.
- The gain matrix is the one that relates the output vector to the input vector. The gains can be constant or variable depending upon the control system design.

Next, this is the last part of the lecture. Let us take what we know some of the important concepts in the control design. Fundamental to the design of the control system is to place the poles that is from the transfer function, we find the poles to place these poles at their appropriate position so that the stability is ensured. Plant is a part of the control system that has unchangeable parts, we cannot change.

Because when we want to control the aircraft, we cannot change the parts and the plant is described by the transfer functions or the state variables. The poles can be shifted at the appropriate position by using a close loop control around a plant with the feedback gain signal. So basically, we give a feedback gain signal from the smart actuator in the case of smart system to appropriately shift the poles to a location where the stability can be ensured.

Another important parameter again for a closed-loop control system, we need to have a gain and this gain matrix is the one that relates the output vector to the input vector and the gains can be constant or variable depending upon the control design.

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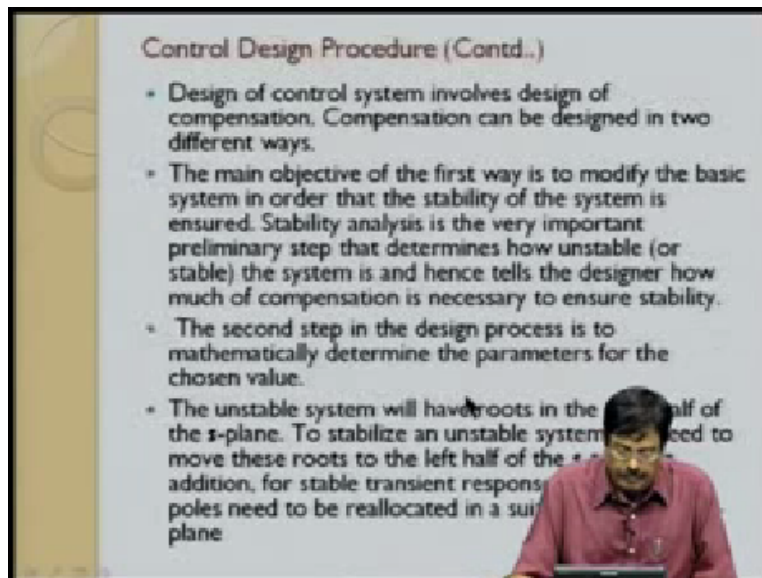
Control Design Procedure

- First, the basic or minimum system is determined by having a closed loop unit feed back.
- Normally, sensors are assumed ideal (unit gain) and only an amplifier is added between the error signal and the plant.
- The gain is then set accordingly to meet the steady state and bandwidth requirements, which are followed by a stability analysis.
- For a smart system, we have a sensor(s), an actuator(s) to receive the sensor input and a controller. The stability of such smart system is governed by the placement of the sensor, the placement of the actuator, the error signal, the gain variation and the type of control design.

So the control design procedure is as follows. How do we go about doing it. Firsts the basic or minimum system is determined by having a closed-loop unit feedback, okay. Normally sensors are assumed ideal, that is they have unit gain and only an amplifier is added to the error signal and the plant. The gain is then set accordingly to meet the steady state.

So once we know a unit gain, we look at the responses, then we increase or decrease the gain based on to see that when the system acquires the steady state response and also the bandwidth requirements which are followed by stability analysis. For smart systems, we have sensors and actuators to receive the sensor input and a controller. The stability of such system is governed by the placement of sensor, the placement of actuator, the error signal, the gain variation and the method of the control design. What are the different methods.

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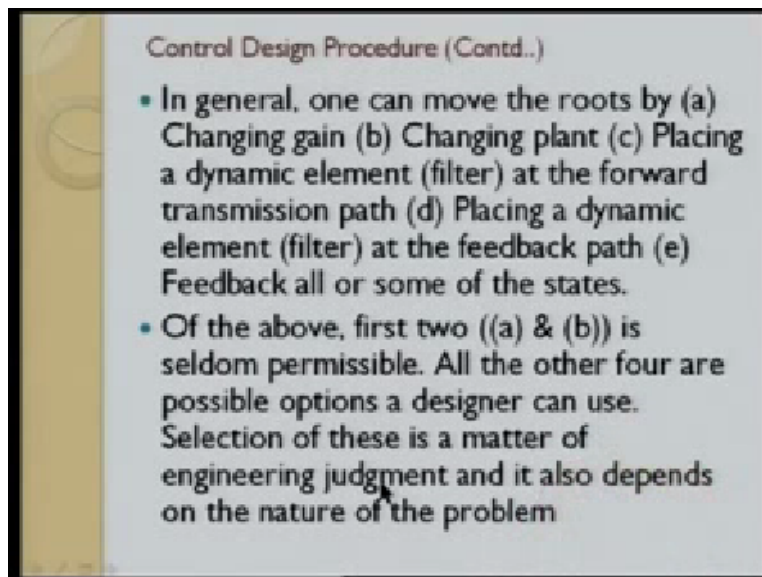


The design of the control system involves design of what we call compensation. What is compensation. Compensation can be defined in many ways. There are 2 major ways that we can design the compensation which is always prescribed in the control design. The first way to design a compensation is to modify the basic system in order that the stability of the system is ensured.

Stability analysis is very important preliminary step that determines how stable or unstable the system is and hence tells the designer how much of compensation is necessary to ensure the stability. The second step in the design process is to mathematically determine the parameter for the chosen value that is what we are looking at where the poles are, is there any 0's, how do we avoid or what is a kind of gains we need and these are some of the things which we need to determine.

The unstable system will have roots on the left-hand side of the s-plane. I told you before itself that if you have the roots on the right half of the plane then it is unstable. For the stability, we need to ensure that their roots were always on the left half of the s-plane basically. So we need to ensure that. So for stable response, these poles have to be moved from the right half to the left half. So for which we need to design our error signal such a way that this can happen.

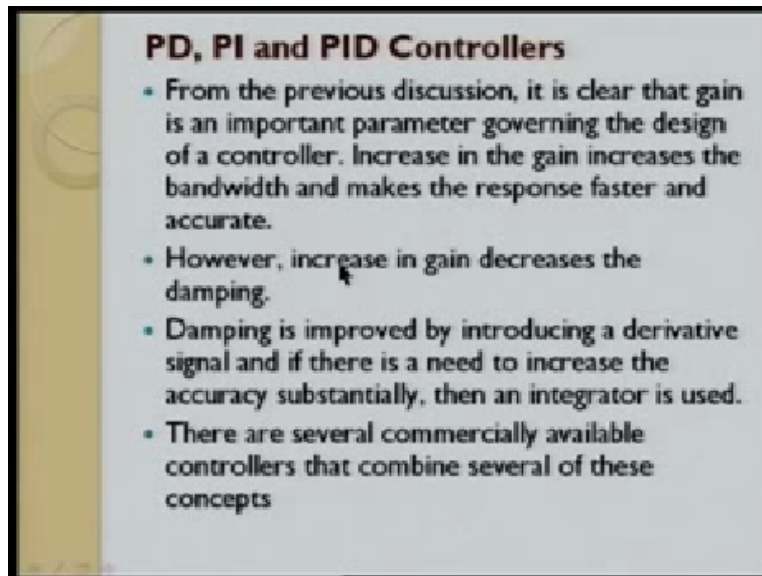
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In general, one can move rules by many methods. Following are some of the methods. One is by changing the gain, changing the plant, placing the dynamic element at the forward transmission path such as filters, placing a dynamic filter element at the feedback path or feedback all are some of the states. From the above, it is not possible to change the plant, it is impossible.

So some of the other options given here are a possibility and which is more suitable, it is only the designer, an experienced designer will be able to tell depending upon problem to problem. So selection of these is a matter of engineering judgement and also depends upon the nature of the problem.

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PD, PI and PID Controllers

- From the previous discussion, it is clear that gain is an important parameter governing the design of a controller. Increase in the gain increases the bandwidth and makes the response faster and accurate.
- However, increase in gain decreases the damping.
- Damping is improved by introducing a derivative signal and if there is a need to increase the accuracy substantially, then an integrator is used.
- There are several commercially available controllers that combine several of these concepts

So now let us talk about some of the very simple controllers for the linear systems that are there that are commonly used, successfully used in many of the systems, in many of the disciplines in engineering. These are called PD controller, PI controller, or PID controller. PD controller is proportional derivative controller. PI is proportional integral controller. PID is proportional integral and derivative controller.

From the previous discussion, it is clear that gain is an important parameter governing the design. Increasing gain, increases the bandwidth and makes the response very fast; however, we have to be very careful. Increase in gain decreases the damping which is our major motivation of control, we have to damp out the response. So it has to be optimal so that we get faster response at the same time, we get an optimal damping to control the control variable to our desired level.

Damping is somehow improved by introducing a derivative of a signal. So that is why PD is more useful and if there is a need to increase the accuracy substantially then integrator is used. So that is why the combination of PID will ensure faster response, better damping and better accuracy. There are several commercially available controllers that combine several of these concepts very effectively.

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PD, PI and PID Controllers (Contd..)

- PD Controller = Proportional + Derivative

$$\Rightarrow G(s) = K_p + K_d s$$
- PI Controller = Proportional + Integral

$$\Rightarrow G(s) = K_p + \frac{K_i}{s}$$
- PID Controller = Proportional + Integral + Derivative

$$\Rightarrow G(s) = K_p + \frac{K_i}{s} + K_d s$$
- K_p, K_d, K_i Gain for proportional, derivative and integral controllers, which are adjustable
- Among the above PID controllers are extremely popular and successful and have been used in many applications such as autopilot in ships and aircraft.

So now let us describe each one of them much more detail. We have what is called PD controller which is proportional from plus the derivative. So normally, the transfer function of this controller is given by what is given here G of s= K_p+K_d times s in the Laplace domain. So it has no numerator no denominator. It is one number, algebraic relation.

If you look at the PI controller, that is proportional plus integral, the transfer function G of s is given by K_p+K_i/s , where K_p is the gain of the proportional controller, K_d is the gain of the derivative controller, K_i is the gain of the integral controller. So the PID controller, the transfer function is given by G of s = $K_p+K_i/s+K_d$ times s. So as I said K_p K_d K_i are the gain of the proportional derivative and integral controllers which are adjustable. We can adjust, we can vary, we can make it constant, we can vary.

Among the above as I said because the differential increases the damping, integral increases the accuracy, PID controllers are extremely popular and successful and have been used in many applications, especially in aircraft like autopilot and also in ships and in space vehicles and even in the helicopters.

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Controller Design (Contd...)

- Let us now consider the transfer function of PD controller. This is given by

$$G(s) = K_p + K_d s = K_d \left(1 + \frac{K_p}{K_d} s \right) \quad (9.4.1)$$

This controller simply introduces a free zero and the design requires a zero be placed at the appropriate location and adjust the gain accordingly.

- Let us now consider the transfer function of PI controller. This is given by

$$G(s) = K_p + \frac{K_i}{s} = \frac{K_p (s + K_i / K_p)}{s} \quad (9.4.2)$$

The proportional gain K_p is an adjustable amplifier gain parameter. In many systems, it is responsible for process stability. In many systems K_i is responsible for driving error signal to zero. However, if it is set too high, there will be oscillation or instability or integrator windup or actuator saturation.

PI controller introduces zero at $s = -K_i / K_p = 0$

So now let us consider the transfer function of a PD controller. This is given by $G(s) = K_p + K_d s$, we write this transfer function K_d , this is equal to $K_d s + K_p / K_d$. So K_p / K_d is the ratio of the gains of the proportional and derivative controllers. This controller simply introduces a free zero and the design requires a zero be placed at appropriate location to adjust the gain accordingly so that you get the desired control of the control variable.

Let us now consider the transfer function of a PI controller. The PI controller is basically given by $K_p + K_i / s$, so that means which can be written as $K_p s + K_i / K_p$ divided by s . So what we do here is, the proportional gain K_p is an adjustable amplifier gain parameter. In many systems, it is responsible for the process stability. In many cases, K_i is responsible for the error signal going to 0; however, if it is set too high, there will be oscillation and instability or integrator windup or actuator situation, okay.

However, one thing we can clear is, it introduces a zero that is the transfer function becomes zero when $s = -K_i / K_p$ and it has a pole at $s = 0$. So it has both poles and zero, so we can adjust, the pole location is fixed, the zero location can be adjusted depending upon the values of K_p and K_i , so we can vary it to see that.


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Controller Design (Contd...)

- Let us now consider the **PID** controller. The transfer function in this case is given by

$$G(s) = K_p + \frac{K_i}{s} + K_d s = \frac{K_d s^2 + K_p s + K_i}{s} \quad (9.4.3)$$

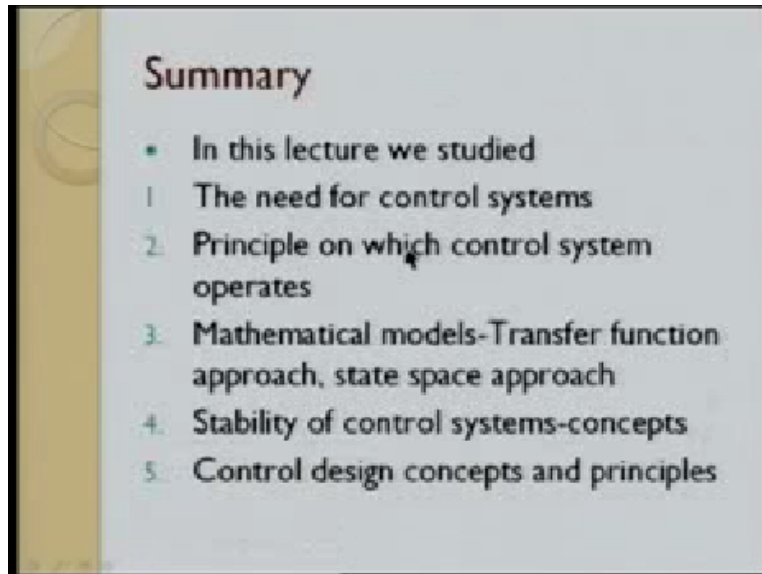
This requires a pole be placed at the origin and two zeros at the desired location for adjustment of the dynamic response. The two zeros may be real or complex depending on the gains used and it will always be on the left half plane. PID controllers can be digitally implemented with microprocessors.



Now let us consider the last of these controllers that is the PID controller. The transfer function in this case is given by $K_p + K_i/s + K_d \times s$. That is you have both proportional, integral and derivative and this can be written as $K_d s^2 + K_p s + K_i$ times s . So we clearly see that it has a pole at $s=0$ that is the origin, so the pole is fixed and the numerator can be factorized as a quadratic equation, it can be factorized, so it has 2 locations where we can introduce 0.

So this requires a pole to be placed at the origin and 2 zeros at the desired location for adjustment of the dynamic response. The 2 zeros may be real or complex depending upon the gains used and it will always be on the left half of the s -plane so that stability is insured. PID controllers can be digitally implemented with microprocessors, that is one of the reasons why it is very, very popular among the control engineers, okay.

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So this is the final slide. So let us summarise what we have studied in this lecture. So far we introduce the need for control, especially in smart systems. We cannot find a smart system without control, the control algorithm. So because we want smart system to perform certain smart functions, like vibration control, noise control, shape control, temperature control, so we need to design a control system to give sufficient inputs which alters the control variable and that is what the control system is.

We also discussed the principal on which the control system works, that is it requires the state matrix, it requires the output matrix, it requires the input matrix, the relationship between the state vectors and the output, output and the input, the gain matrix, the feedback control, the open-loop control. Some of these concepts were introduced in this lecture. We also discussed about the mathematical models that are required for the design of the control system.

So we described 2 models, one is based on transfer function and one is based on the state space which is an extended transfer function, I would call it as an extended transfer. In the transfer function model, we basically defined how do we get the relationship between the input and output for a single input and single output system where there is only one control variable to be controlled.

And in the second approach where there are multiple state variables that means you may arise a

situation where multiple state variables need to be controlled, then we used to develop this state space model. So in the state space model, basically we have generated the mathematical model to construct the state matrix, the input matrix, the output matrix and how we can actually generate these matrices when we have a finite element type model which is a large model, where it is a multi-input-multi-output models.

Next we introduced what are called the stability of the control systems. So what are the parameters that you need to look, what are the things that you need to ensure for the stability of the control system, how the poles affect the stability of the control system, what are the methods to actually move the poles to the position where the system is stable and what is the controllable and observable condition and what are the conditions to make the controllability of the input, controllability of the output and the observability conditions.

And finally we outline what are the design concepts, the design procedures that we need to adopt, basic simple design procedures and we also reviewed some of the very commonly used controllers like the PI controllers, PD controllers and PID controllers, what would be the transfer function of such controllers, what are the advantages of each of these and how do we implement all these things with this we end this lecture.