Micro and Smart Systems
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Lecture – 35
Vibration Control of a Beam

So in this lecture, we will see the application of control system to a practical application which is a vibration control of a beam, so before actually explaining the control process, let me actually introduce the concept of vibration, vibration of a system.

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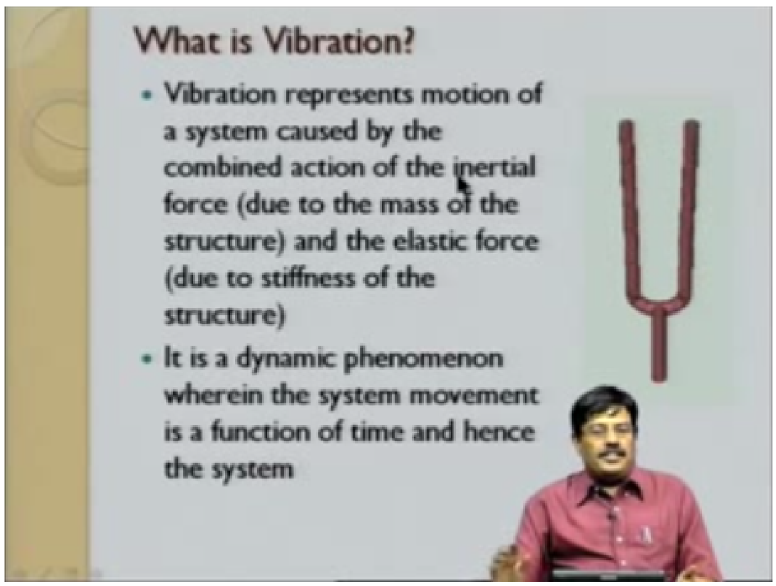

So now the question is what is vibration? So vibration represents a motion of a system caused by the combined action of the inertial force due to the mass of the structure, elastic force due to stiffness of the structure. In dynamic phenomenon, wherein the system movement is a function of both time and the system is also a function of time. So basically, if you look at this tuning fork, when you give an initial disturbance it vibrates and this vibration is causing because of the mass of the structure and also the stiffness of the structure.

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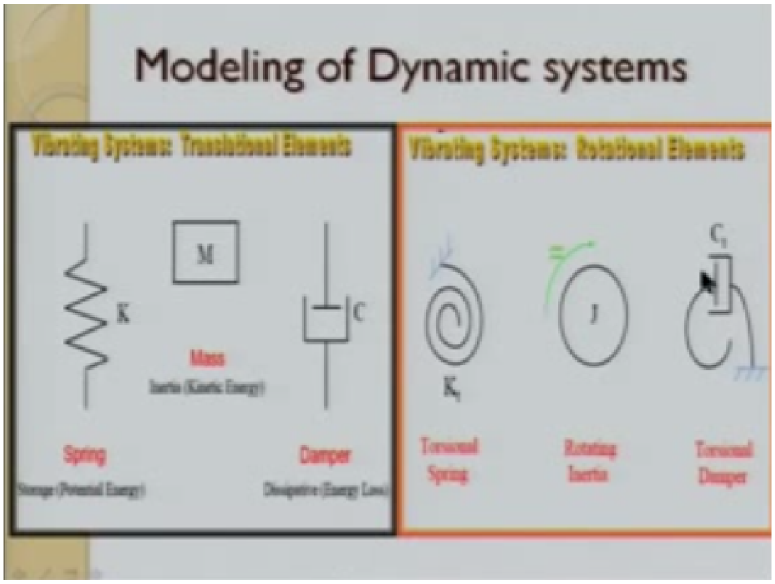

So how we model the dynamic systems, so the dynamic systems are model there are typical regular system, there are translational elements, there are rotational elements. The translation elements are modeled using what we call the spring, the mass element is represented by a block M and there is also going to be damping because when you give an initial disturbance, after some time, it dies out, what is the force that causes this dying out of the response, that is the damping force.

So normally, there are various types of dampers, what we use is the viscous damper, is represented as shows here. There are also some rotational elements, like shafts, where the representation is slightly different. The stiffness of the shaft is represented by a rotational spring with a stiffness KT. The rotational mass is represented by a circular element which is called the rotational inertia and the damping is given by a torsional damper which is represented here.

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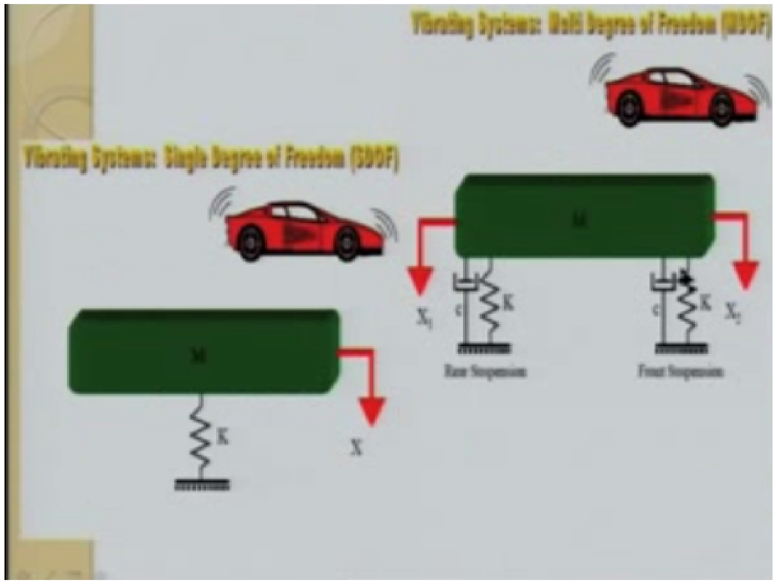

Now let us look at the body how we can (()) (02:32), let us take an automobile. The automobile has stiffness and mass, we can identify this as one single block of mass which is represents the mass of the automobile and the entire stiffness by a spring and this is what we call the single degree of freedom system. We have a little more sophistication and say the rate suspension as one stiffness element, the front suspension is the second stiffness movement which correspond mass or damping element.

And this has two motions one on the rear suspension, one on the front suspension. So this is a 2-degree of freedom module.

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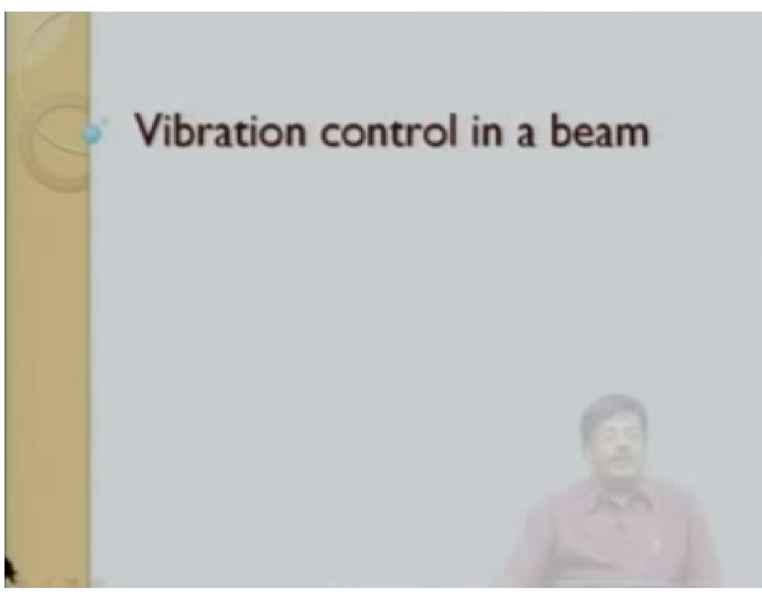

So, in this lecture we will talk about the use of control system on a typical engineering application that is controlling a vibration in a beam.

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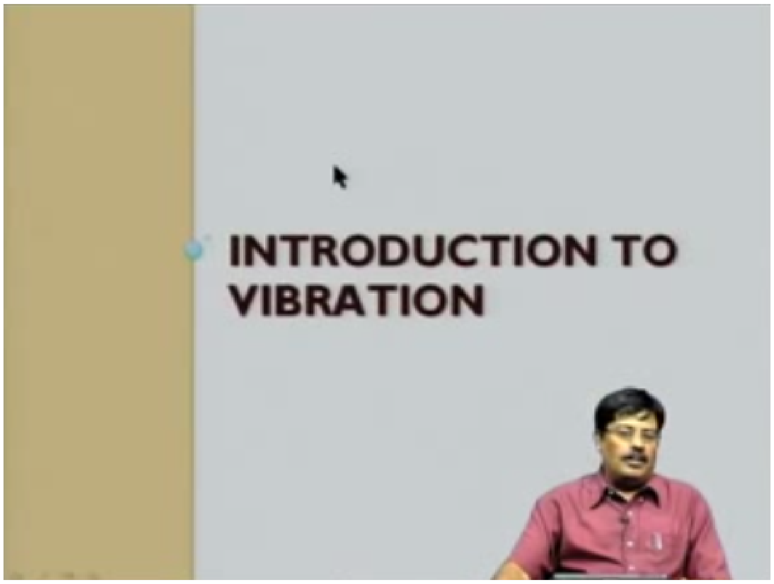

So, before we do that we need to introduce vibrations. So, in this lecture I will first introduce vibration and then go on to talk about this simple control system that we have developed for the vibration of a beam.

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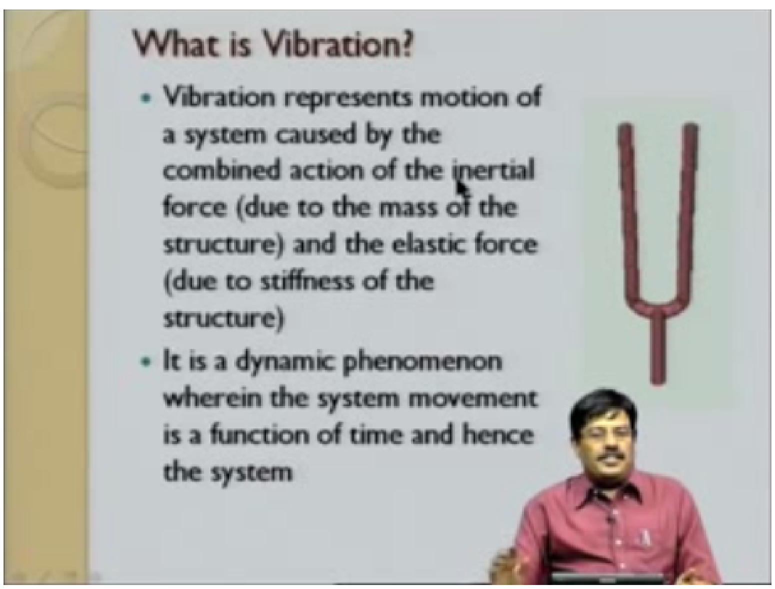

So, what is vibration. So, vibration represents a motion of the system caused by the combined action of the initial force due to the mass of the structure and the stiffness force caused due to elasticity of the structure. So, the combined action gives what is called the vibration. So, it requires an initial movement, as we see in this tuning fork. So, when I give initial movement it moves. Two forks moves up and down vibrates and this is caused by the mass and the stiffness of the structure.

So, this is essentially a dynamic phenomenon where the input as well as the output is time depended.

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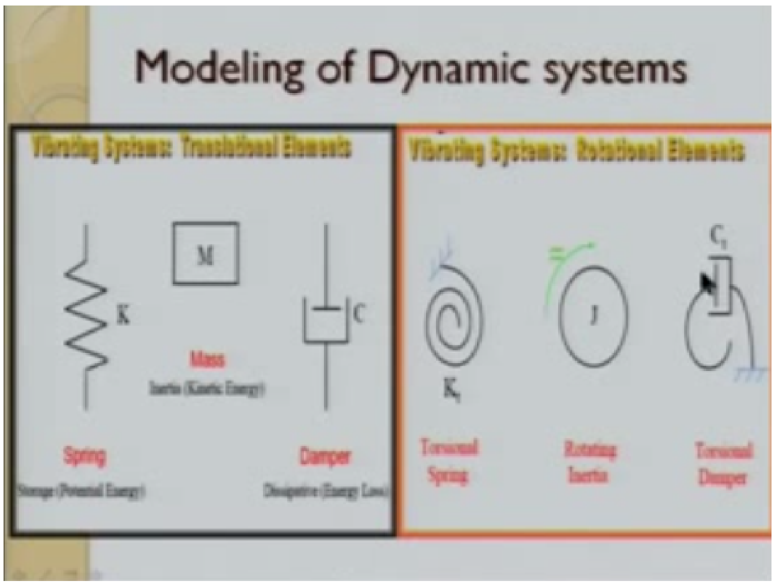

So, how do we model the dynamic systems. So, the modeling requires certain elements. So, if it is a translation elements like a beam or a plate or any such devices we represent the stiffness by a spring of value K, the spring coefficient, the mass is represented by a block of mass represented by M and whenever we give a disturbance, this disturbance does not continue for long and dies after sometime and that is caused by damping force.

There are different kinds of damping and here what we have represented is one particular type of damping called the vicious damping which is represented by a dash part as shown here. If it is a rotational element as in the case of a shaft or any rotational member for which we need to study the vibration, the rotational stiffness is represented by a helical spring of constant KT, the rotational inertia is represented by a circular element with coefficient J which represents the mass movement of inertia and damper again a vicious damper which is called the torsional damper.

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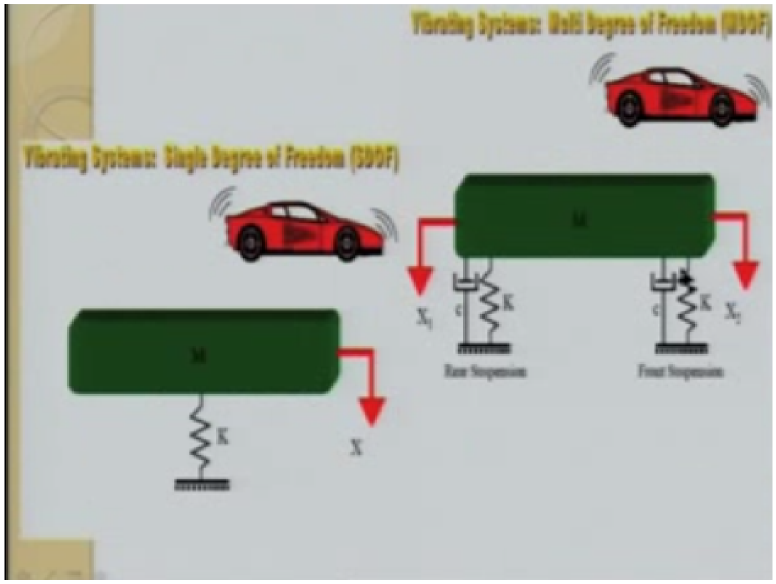

How do we model the system? Let us take an example of an automobile. In this automobile, basically this whole thing can be represented by 1 degree of freedom or 2 degrees of freedom or even many more degrees of freedom. If it is modeled as 1 degree of freedom, then the entire mass of the automobile is blocked as mass M here in green color. Then we have a stiffness K which represents a stiffness of the automobile and only movement is only parallel to the ground basically.

So, that represents the dynamical systems in terms of springs and masses. We can do much more sophistication. We can actually model separately the rare suspension and front suspension by respective spring constant K and the damper element C. So, this will have two motions one on the rare and one on the front perpendicular to the plane of the paper and this represents 2 degrees of freedom system.

We can also model the headrest. We can model the passengers. We can model everything. All in terms of spring and mass and more and more sophistication we can build into vibrating system.

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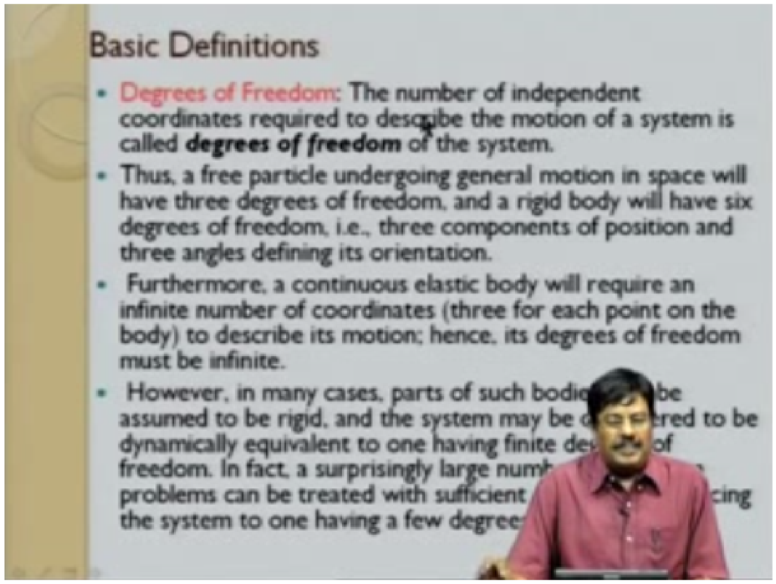

So, let us get into some basic definitions here. So, first definition I would like to define is degree of freedom. How do you define a degree of freedom. It is the number of independent coordinate required to describe the motion of a system is called the degree of freedom. So, thus a free particle undergoing general motion, especially if it is a free body, then it will have 6 degrees of freedom. That is three translation motion and three rotational motions.

So, in aircraft terms it is called yaw, pitch and roll motions that are the rotation and the movement in the three coordinate directions is the three translation motion. Furthermore, if it is an elastic body, we can idealize into N number of masses. So, basically if there are N number of masses, it becomes N degree of freedom. So, typically an elastic body can have infinite degrees of freedom. However, in most cases, especially in control system design, we do not require such sophisticated model.

We can do away with only few degrees of freedom. That is we can model the system with very few degrees of freedom and then still do a very good job in designing the control system.

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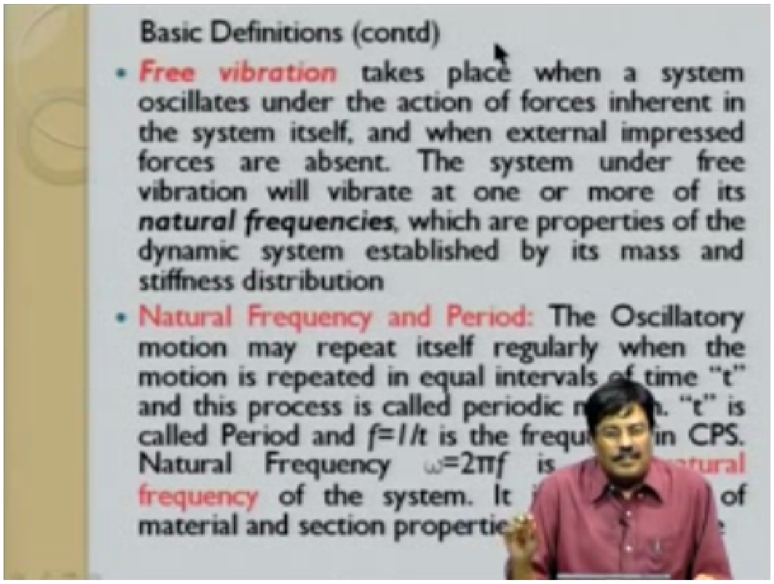

Next, we will define what is free vibration. The free vibration is a vibration that takes place in the absence of external force, i.e., we just give a disturbance. Say for a cantilever beam, then the beam goes up and down repeated motion at certain interval of time and this interval of time by which it vibrates is called the natural frequency at the system and that is what I have defined next and this is related to the time period, the period by which it causes the mean position up and down and this frequency can be expressed in various forms.

It can be expressed in cycles per second or radians per second or it can be expressed by Hertz. Omega is equal to 2 pi F gives the relationship between the frequency in radians per second to Hertz. If F is in Hertz to convert it into radians per second you multiple by 2 pi. Next we will talk about forced vibration. So, forced vibration is vibration taking place due to an applied dynamic force, that is external force is applied.

Then, that force will also be applied at certain frequency, okay which is called the excitation frequency. If the frequency of the excitation or the driving frequency coincides with the natural frequency then a system called resonance takes place, which will basically dangerously large oscillation may result because of this. The failure of the major structures such as bridges, buildings, aeroplanes, even the main structures are due to the existence of resonance.

So, thus the design of the control system basically the polls is basically signifying the resonance which causes instability. At any cost, the resonance should be avoided.

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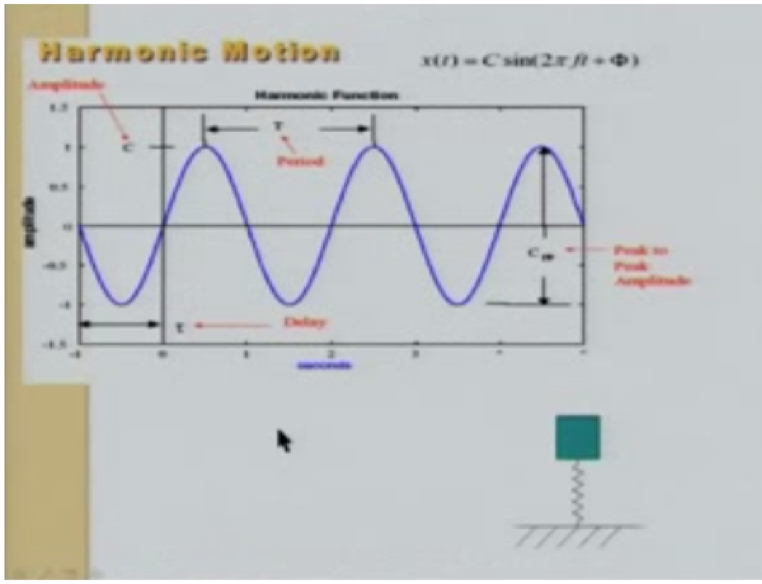

So, now let us look at some of the simple motions. Here is a motion that is shown here that is of the spring mass. This motion when we plot in terms of time, if you measure the amplitude at the tip of the mass, it will look like here. It is a simple sinusoidal motion, okay and such a motion is called the simple harmonic motion. So, it has a period T. It has a peak amplitude C and it continuously keeps going. So, this is called a simple harmonic motion. Most vibratory system is characterized by the synthesis of many such simple harmonic motions.

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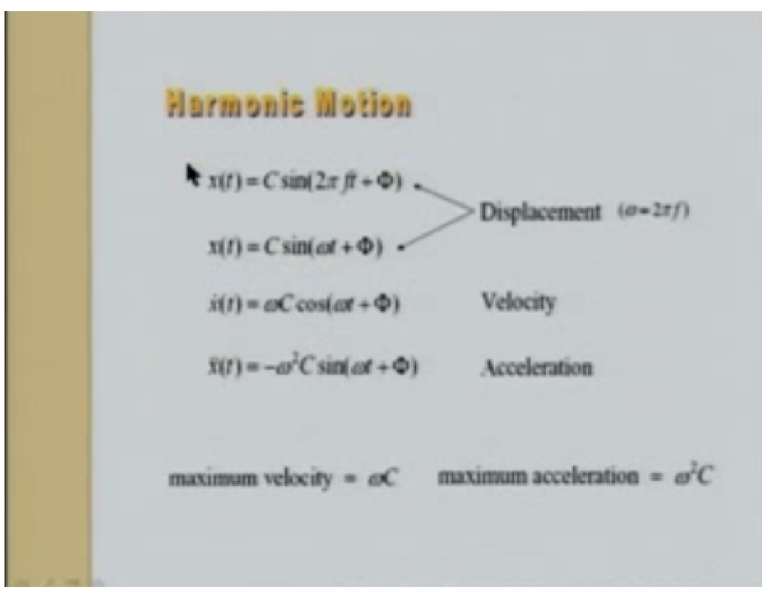

So, let us look at it. So, if X is a simple harmonic motion which is displacement where we can express either in terms F, the frequency in Hertz or in terms of radians per second, omega is 2 pi F. We can get the velocity of that motion. By differentiating this displacement X/DX/DT which is given by here, okay and X double dot is given by here. We also have a parameter phi which is a face between the displacement and velocity.

You could see that the velocity lacks displacement by 90 degrees and acceleration by 180 degrees. So, the peak velocity is omega C and acceleration is omega square C. So, basically displacement in terms of magnitude is the lowest and acceleration is highest.

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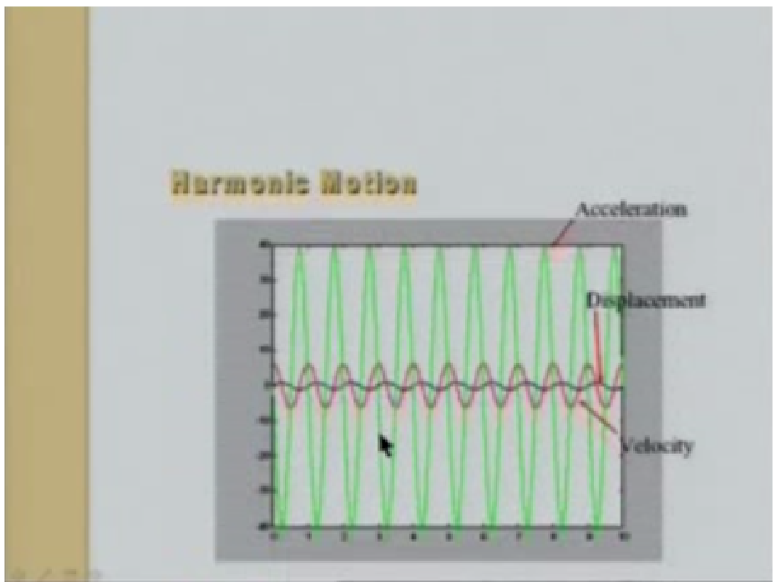

We can pictorially look at it here. So, we see that displacement is many order smaller compared to the acceleration if you plot for a typical motion.

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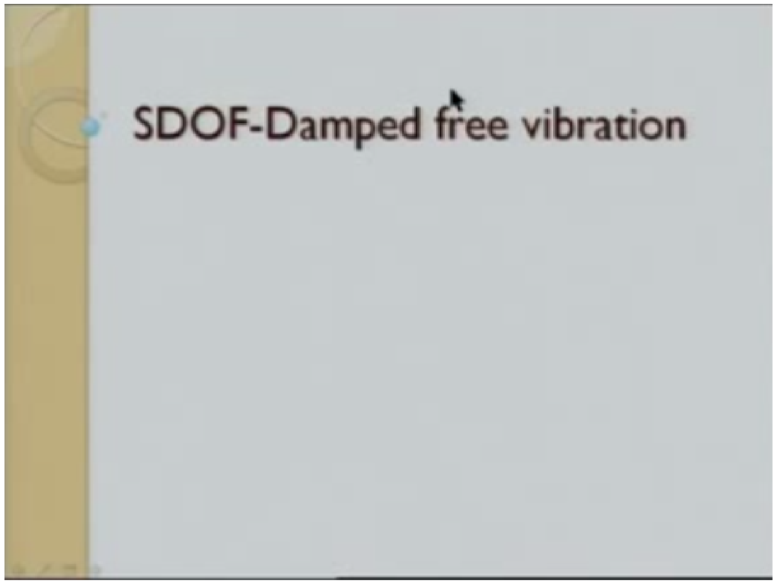

Let us come back to a single degree of freedom system where we do not have any dumping here.

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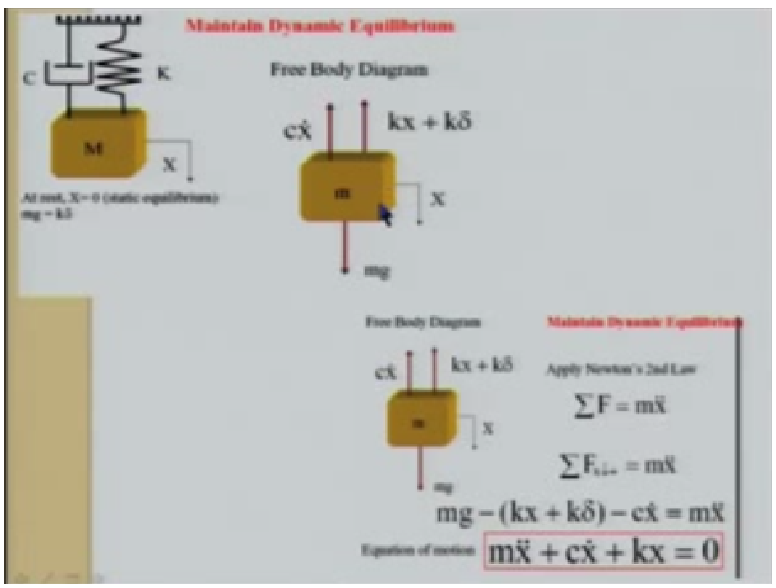

So, this system basically can be represented by mass, spring and a damper. So, we begin the mathematical analysis by isolating this mass and drawing up all the forces and applying the second Newton's law. So, when we isolate this mass and draw the forces, the damping force is always proportional to the velocity of the system in which vibrating, i.e, CX dot and MX double dot is basically the acceleration.

So, when we draw this free body and assume that whenever before vibrating when we isolate the free body, MG cancels with K delta because of the static equilibrium. We get the governing differential equation by using the free body as MX double dot+CX dot+KX=0. So, it is a second-order differential equation.

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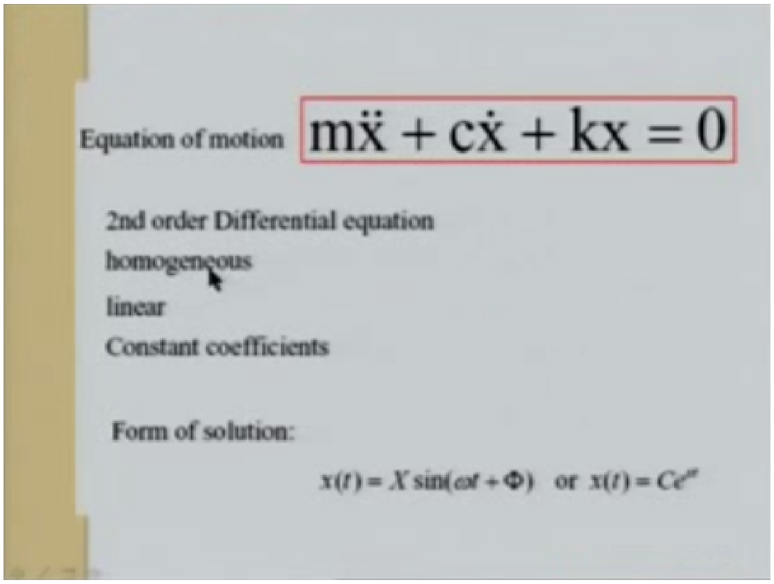

If you look at it, it is a second-order differential equation, it is linear equation, it is homogeneous equation and is of constant coefficients. So, basically it will have exponential solutions, so C=XT. Our objective is to find the two roots of the equation basically and find out the responses. So, the characteristic equation can be got by substituting x (T)=Ce for st here into this equation and getting the velocity and acceleration and substituting it, we get the characteristic equation.

This solution will give us two roots and that forms the solution here.

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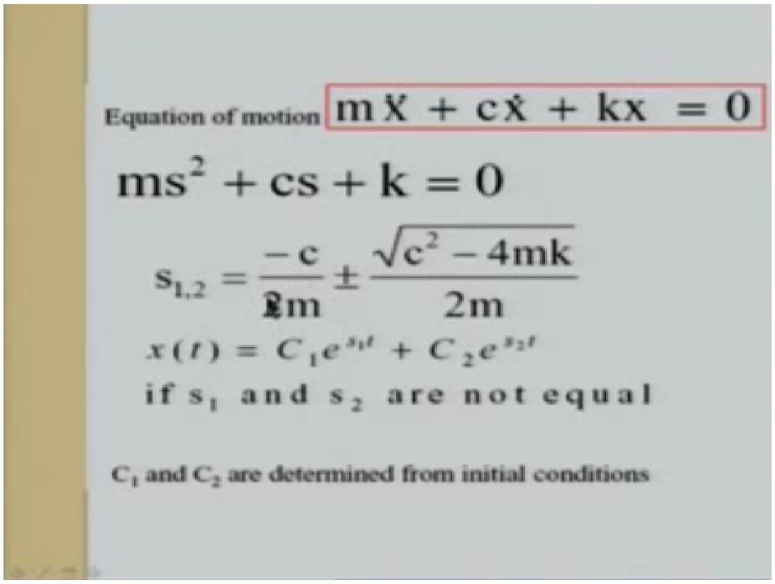

So, the roots of the equation is given by this equation. Because it is a quadratic which can be easily solved. The characteristics of the roots depends upon the term inside the radical. It can real, it can be imaginary, it can be complex. The nature of the roost depends upon the value under this route. So, basically if C square>4MK, then we have all the roots are real. If C square<4MK it is imaginary and if C square=4MK, then what we call it is a critical situation.

Once we get this S1 and S2, the total solution can be got here, and C1 and C2 are basic constant which are determined from the initial condition whether it is at rest when XT=0 and X dot T=0, that is velocity.

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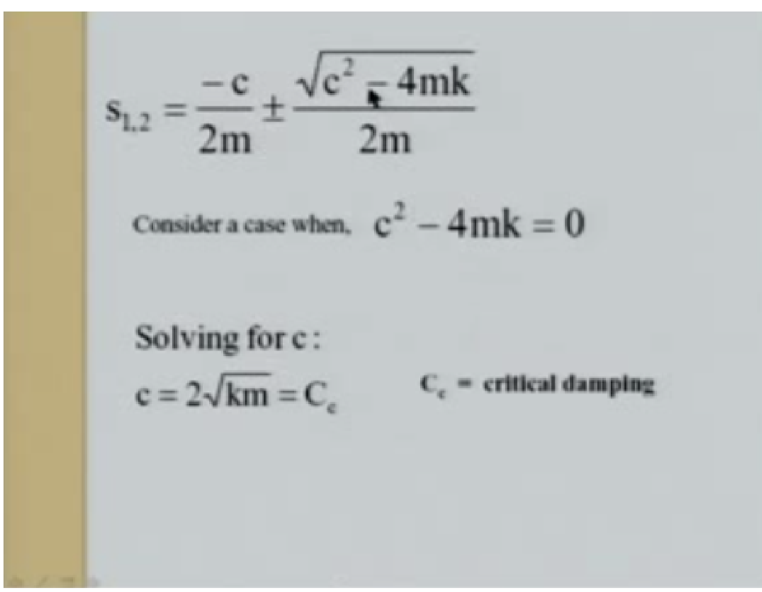

So, now let us spent some time into this equation which determines the root. Let us consider a critical situation when C square=4MK which will give me on solving the value for C. This is the maximum damping that the system can have and that depends upon the characteristics of the system, namely its stiffness and mass and this is called a critical damping. So, we would always like to see how much of the damping is a factor critical damping, i.e., C/CC which is what we call the damping ratio.

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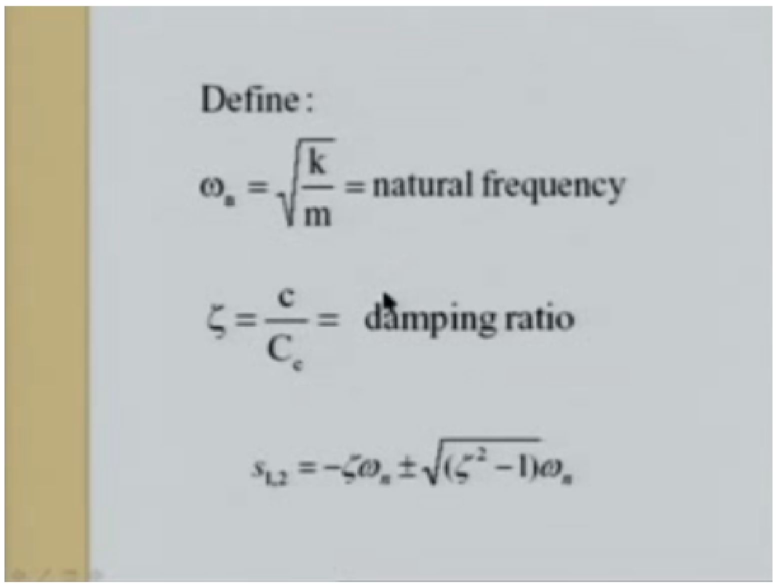

So, that is what we introduced here. So, once we do this, we can say the natural frequency of the system. As I said before, it depends upon both the stiffness and mass which is equal to root of K/M and we can the roots in terms of the natural frequency and damping ratio as given here. So, here we can clearly see psi square>1, then this will be real and if psi square<1 it is imaginary.

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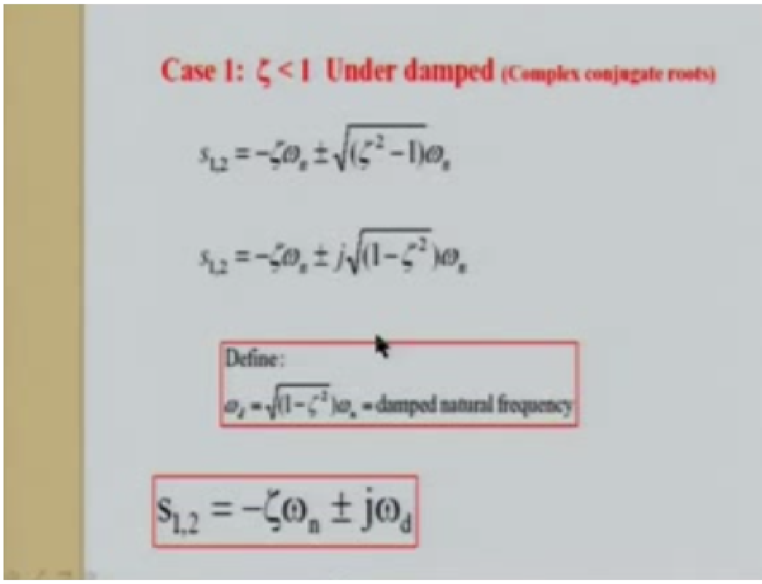

So, let us consider the first case where psi square<1. So, then the roots of this equation can be written in terms of the complex number. We can write this root under the radical here. So, that means the roots will be of the form A+IB, i.e., it has both the real and imaginary part and we define a new variable called the damp natural frequency which is a function of the damping ratio.

This damping ratio is a fraction of the total critical damping ratio, C/CC. Once we do this, we can write the term under this radical is omega D. So, we can write roots as a psi n so omega $\pm$ J times omega D where J is the complex number.

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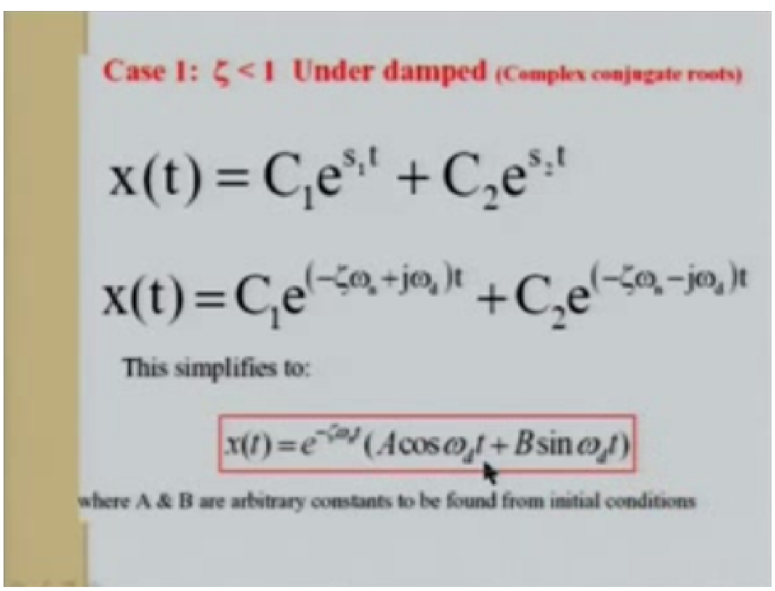

So, once we do this, we can write this equation of this form and we came actually express this complex exponential in terms of sins and cosines which gives me this equation E to the power -psi omega NT x A cos omega DT + B sin omega DT where A and B are arbitrary constants, can be found from the initial condition. So, this is a completely oscillatory. If you look at the term inside, this is an oscillatory function but it is multiplied by an exponential, so it decays.

So, basically the damping affect is clearly seen here.

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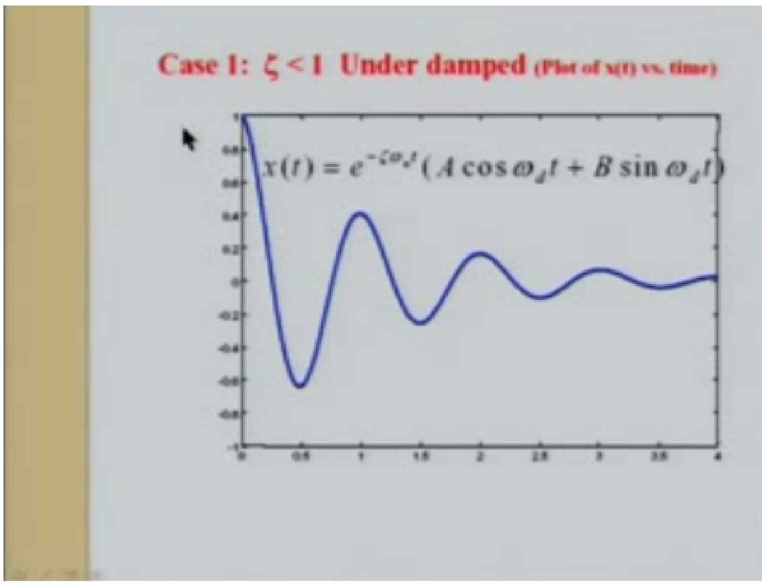

So, this is how it is. So, it starts very high and as the time progresses it that damps out to very zero valve. So, basically the vibration (()) (17:41) and that is mainly because we have included the property of damping here.

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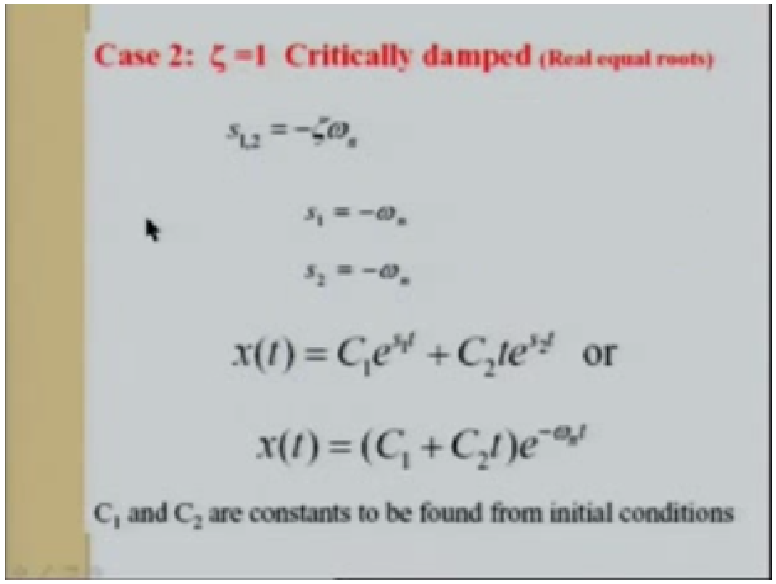

Now, let us take a case where it is critically damped, i.e., the radical is zero. Then, we have two repeated roots, S1 and S2. For repeated roots, the solution becomes here. So, you clearly see that as T increases the response increases linearly with time multiplied with this constant here. So, again here C1 and C2 are determined from the initial condition.

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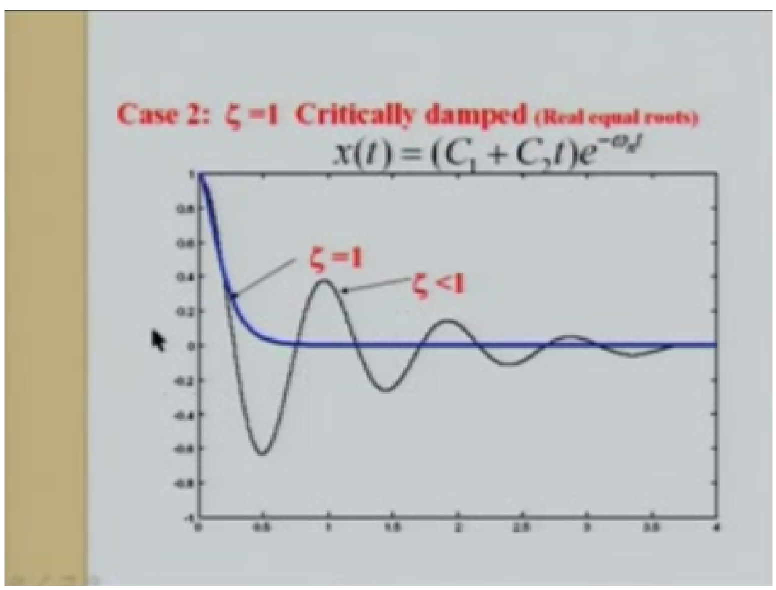

So, if you plot this response as oppose to the oscillatory response, you clearly see after the initial surge of response, it comes to zero practically very fast, okay. So, this is a totally different behaviour when the radical is changed for this kind of solution. Next, let us take a third case when psi, the damping ratio is more 1.

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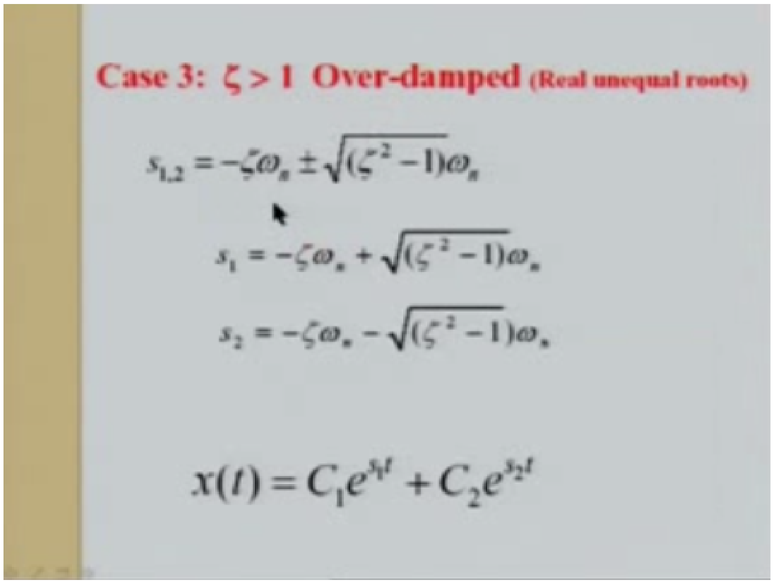

When it is 1, the roots can be written as psi square-1. So, it has two roots, then it has S1 and S2 are real.

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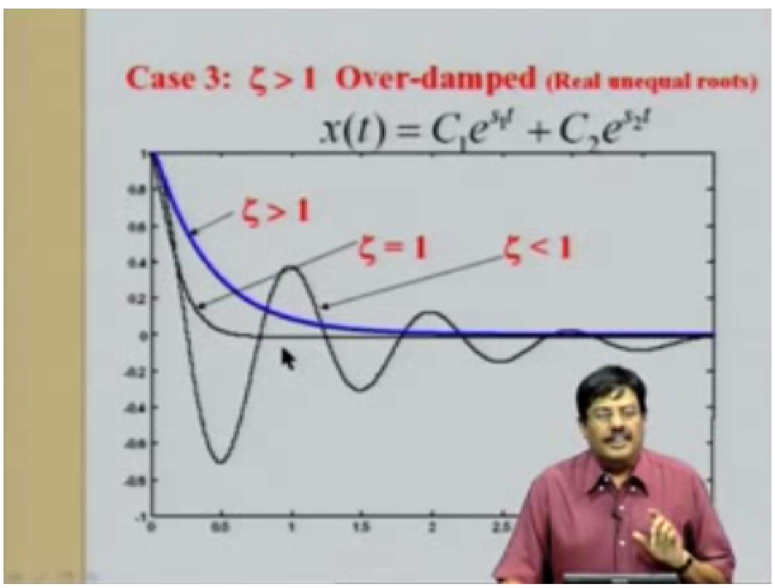

That means basically it has no oscillatory component and the system comes to rest very fast. It is completely over-damp system. So, we have under-damp, critically damp, and over-damp system and this plot basically gives you the overlapping response of these three cases and you see that the critically damped and over-damped are not of much interest to us because it does not vibrate. It comes to rest very fast.

What is of importance to us in the control system is the under-damp case where we still get an oscillatory motion and we should make sure that this oscillatory motion does not increase as the time progress.

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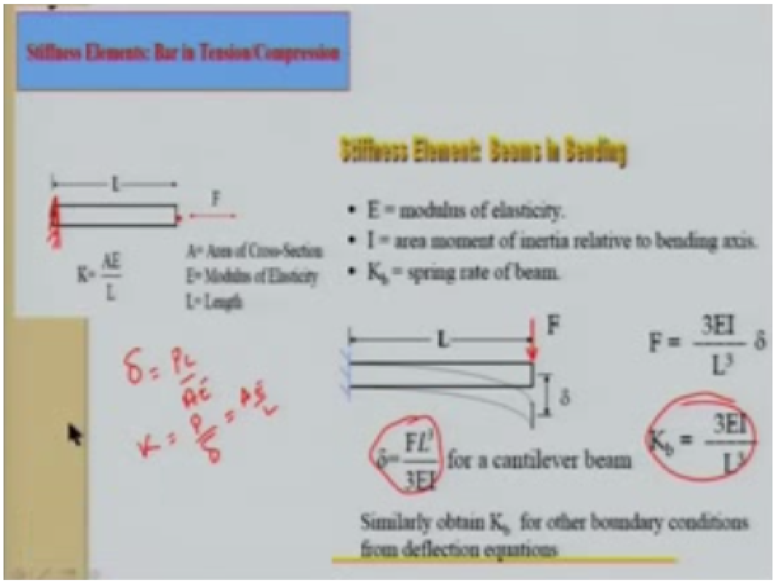

So, let us look at now how do we construct the stiffness elements for the case of the simple structures. For example, if we have a rod here. If the rod is basically say if it is fixed here, let us put a fixity here, then my delta displacement will be PL/AE, okay. Now, basically stiffness is defined as force by deflection which is nothing but AE/L and that is how we get here because this is a predominantly large displacement. All displacements are at the tip here at this position.

Similarly in the case of beam, say cantilever which is fixed at one end and load is applied in the transverse direction, the maximum deflection delta is in the transverse direction delta and using the strength of materials approach, we can easily find that the deflection delta will be equal to FL cube/3EI. So that means basically F/delta is force by display 3EI/L, so that is what the stiffness is 3EI/L.

So, wherever we are trying to find out for the element, the boundary conditions plays a big part. We will see in the next case how the boundary condition plays a big part.

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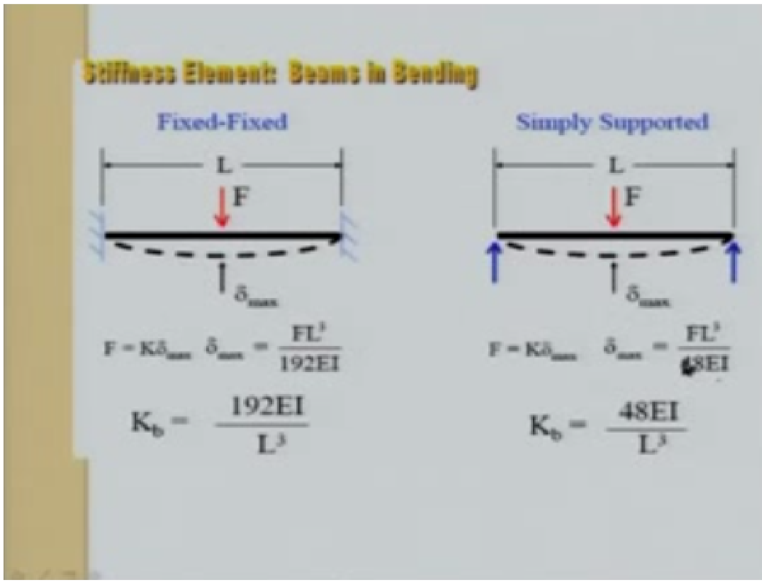

For example, if it is a simply supported beam or a fixed beam. In the case of a fixed beam, we see that the maximum deflection is given by FL cube here, FL cube/92EI. So, F/delta is 92EI/L cube which is totally different from what we had for a cantilever beam. If it is a simply supported beam, maximum deflection from the strength of material approach is FL cube/48EI, so the stiffness for the simply supported case will be 48EI/L cube.

So, basically the boundary condition of the structure determines the equivalent stiffness that you are trying to calculate which is very important, especially if you want to construct a vibratory system from the actual system in terms of springs and masses.

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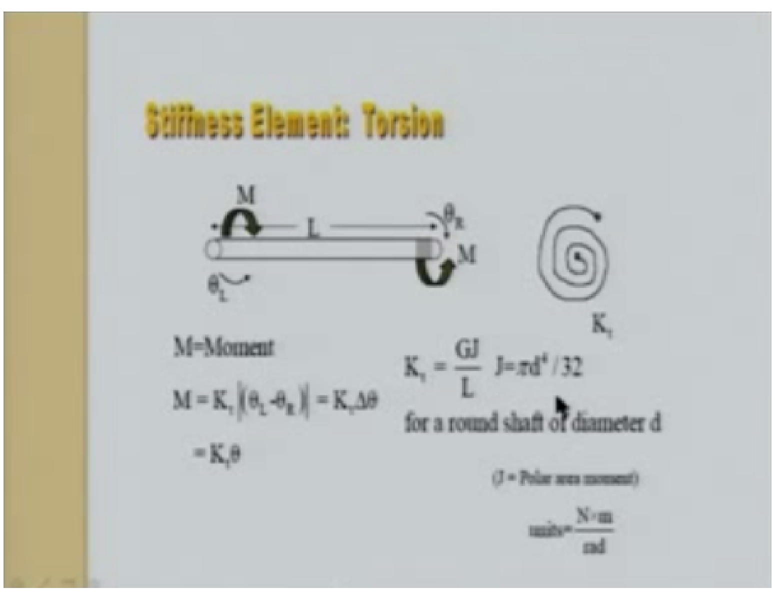

In the case of a torsion, very similar to that of a rod. If it is fixed here, the total of movement will be KT x theta. So, basically this KT will be GJ/L instead of AE/L in the case of a rod where J will be a function of the diameter of the shaft. So, similarly you can find out the equivalent stiffness for many systems in order to construct the spring mass damper which is essential for us to do the controlled design.

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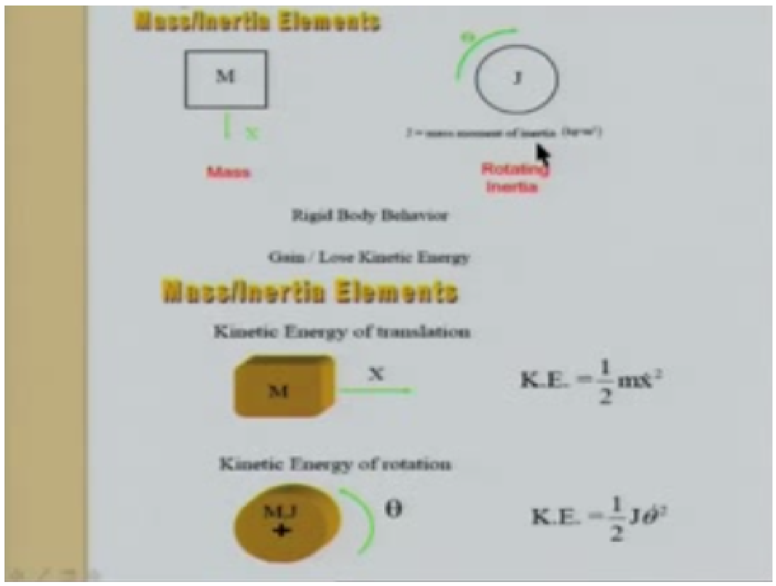

Mass element as I said is represented as mass M for translational motion and J a rotational element for the rotational inertia component and the energy associated. So, whenever you have a mass, there will be a kinetic energy associated which is half mass times velocity. If X is translation motion, X dot is the velocity half MX square. In the case of rotational motion, J is mass movement of inertia.

If theta is the motion and theta dot is the rotational velocity, then the rotational energy will be half J x theta dot square.

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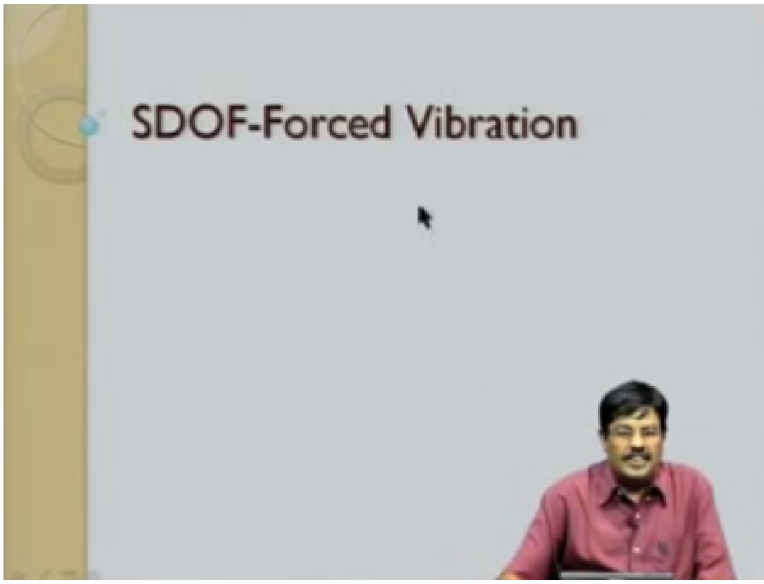

Let us come back to the single degree of freedom force vibration now, because that is where it is very critical, because there will be forcing in many of our system what we are trying to design.

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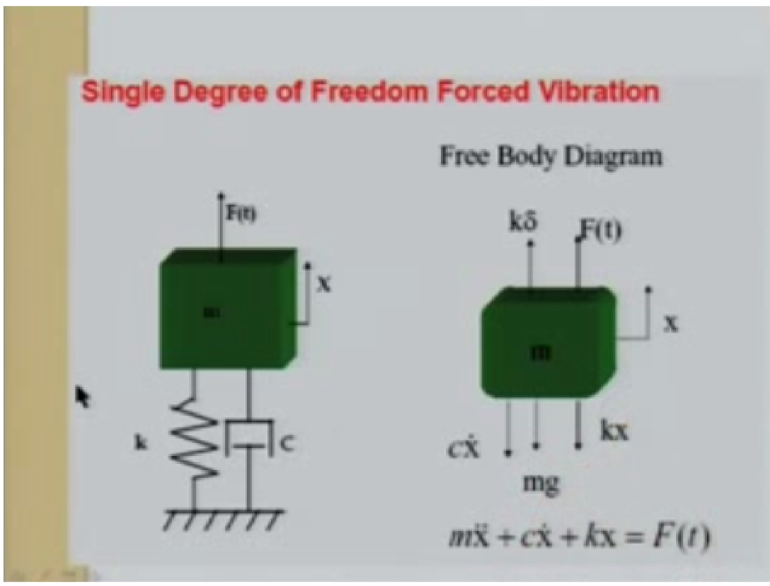

So, this basically is a same spring-mass system where you have spring, damper, single degree of freedom systems. There is only one motion X but in addition to the these, there is an additional force FFT which is a dynamic force that is act on the structure. Again, we isolate the free body. So, when we write the free body and use the Newton's second law, then we get the differential equation of motion and additional term FT come into the right hand side, i.e., MX double dot+CX dot+KX=FFT.

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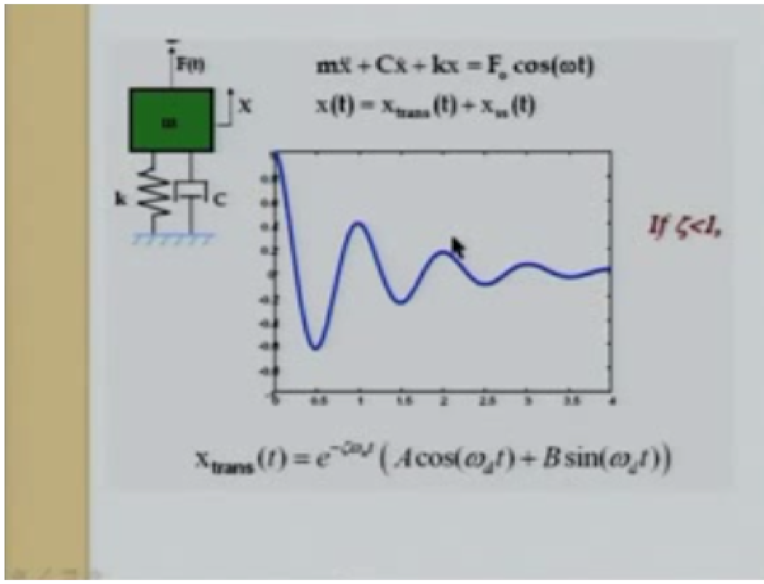

So, here the solution of this is again second-order. So, in the previous case we had zero, here we have this force which I have assumed it as cosine variation. FFT can be F knot x cosine omega T. It can be anything but here we have taken it as cosine omega T where this omega is the driving frequency. So, solution of this has two components; one is the complementary solution which is essentially the free vibration solution.

We call it as a transient solution and the particular solution which we call the steady-state solution and we assume that it is under-damp system because that is of interest to use when psi <1. So, the steady-state solution we have already found out from the free vibration which is given by here E power – psi omega NT x A cosine omega DT + B sin omega DT which we already know.

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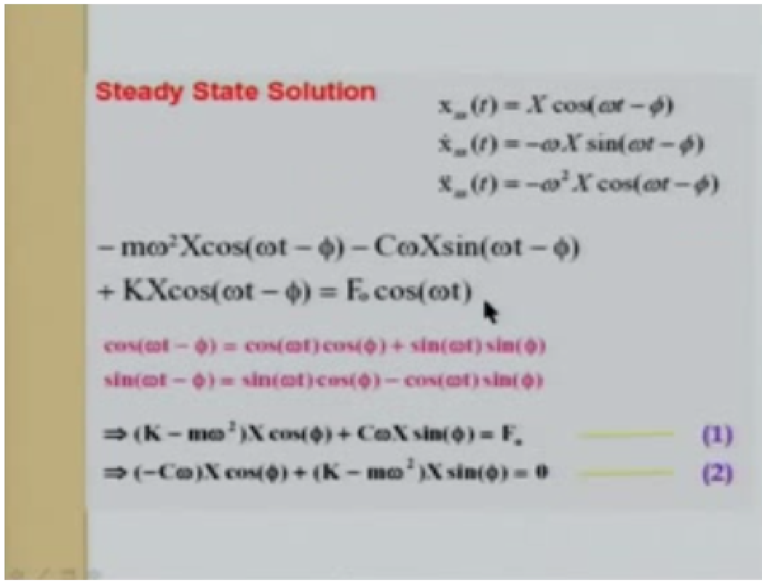

Now let us how we can determine the steady-state solution. So, if you look at this relation, we have F=F knot cosine omega T. We assume that the displacement also follows a cosine part but with a phase angle phi from which we assume the solution of this form where X is the steady-state solution amplitude. X dot is the steady-state velocity and X double dot is a steady-state acceleration.

We substitute it into the equation and then we get two equations, we group the cosine omega T terms and sin omega T terms from here and expand this using the cosine trigonometric identity. We get this following two equations which is given by equation 1 and 2.

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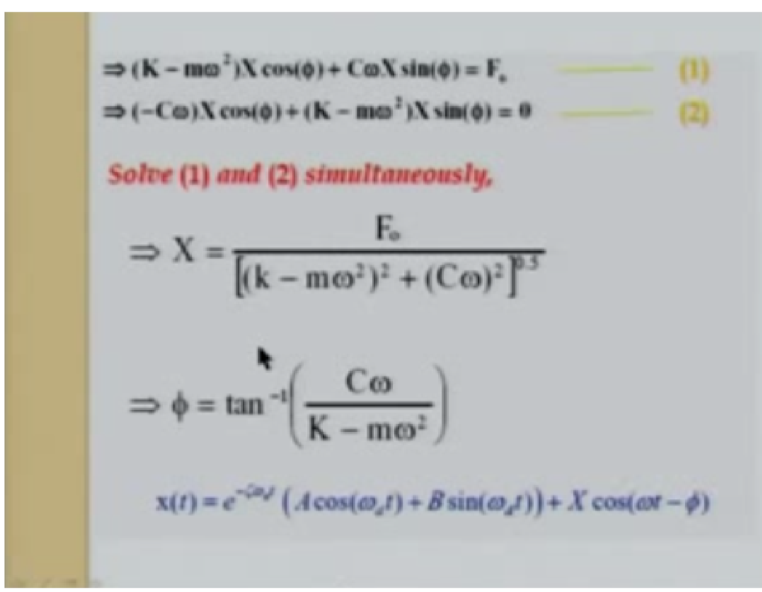

Now, we solve these two equations to get the value of X which is given by this equation. So, F is equal to F knot into this K-M omega square whole square+C omega whole square and this is the phase equation. So, the unknowns are X and phi, so we get this two equations to solve this. So, the total solution will be the transient solution and the steady-state solution. One thing that you can see here if C is equal to 0, okay and we see that when omega N natural frequency is root of K/M.

So, omega M square is K/M. When I take this here, when C is zero, when omega becomes root of K/M, then the X goes to infinite response, that is resonant condition gives excessively high amplitude which has to be avoided at any cost.

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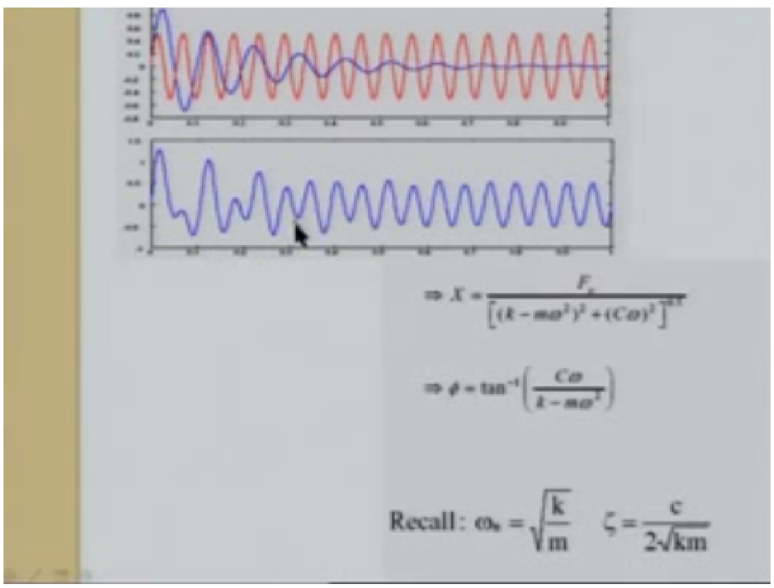

So, here is a response. The red line says the steady-state response, the blue line says the transient response which dies down after some time but the steady state does not die down. It keeps on going. The total response is the sum of these two response which is given here, okay. So, here we take this equation here and we form what is called XK/F knot. We actually take this F knot down and multiply by K and simplify the right-hand side and we get what is called a magnification factor.

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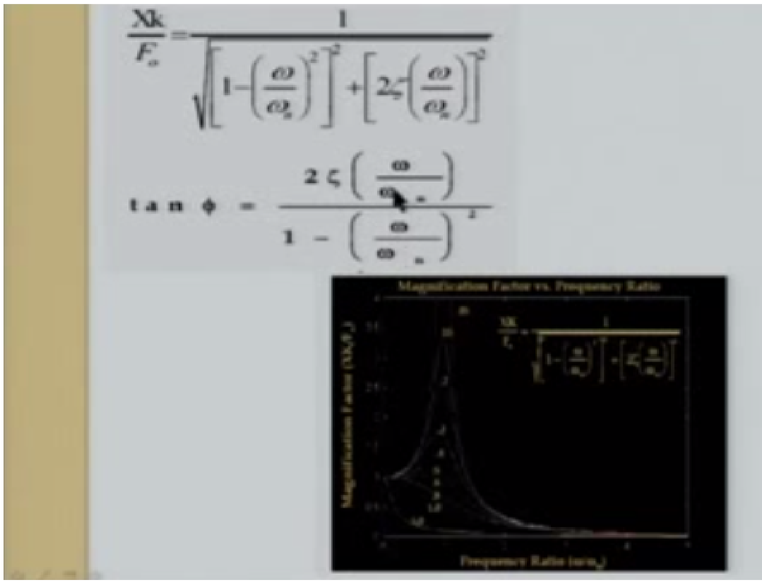

This magnifications factor tells how much of the response is amplified because of the presence of the dynamic amplitude. So, here we can see the plot here which essentially shows the magnifications factor as a function of the frequency ratio omega/omega N. We see that when omega=omega N for small damping there is a steep increase in extreme response which has to be avoided at any cost.

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Now let us come to the basic control concept. How do we use this vibration knowledge to design the control system.

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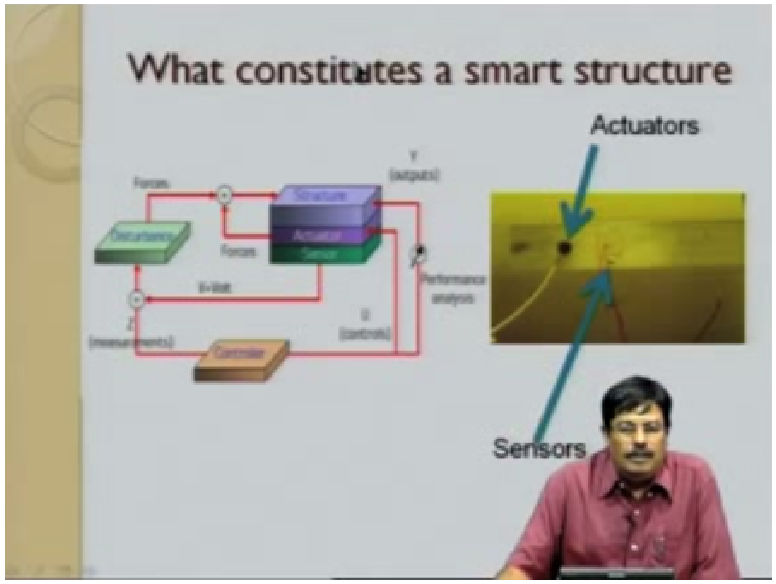

So, here we see that a typical smart structure contains a structure, a smart actuator, a smart sensor, a controller and the disturbance. So, basically what happens the sensor senses it and together with the actuated input forms error signal which is sent to the controller, okay. There is already an applied force. So, when this is applied counteractively to reduce the vibration of the structure.

So, to typically show here is a beam that is shown here and this is the patch which is the sensor patch and this is the actuator. So, the actuator is smart actuator. It can be based on PZT or magnetostrictive or teflon. In this demonstration, we will actually show you how we can construct a smart structure using PZT actuator.

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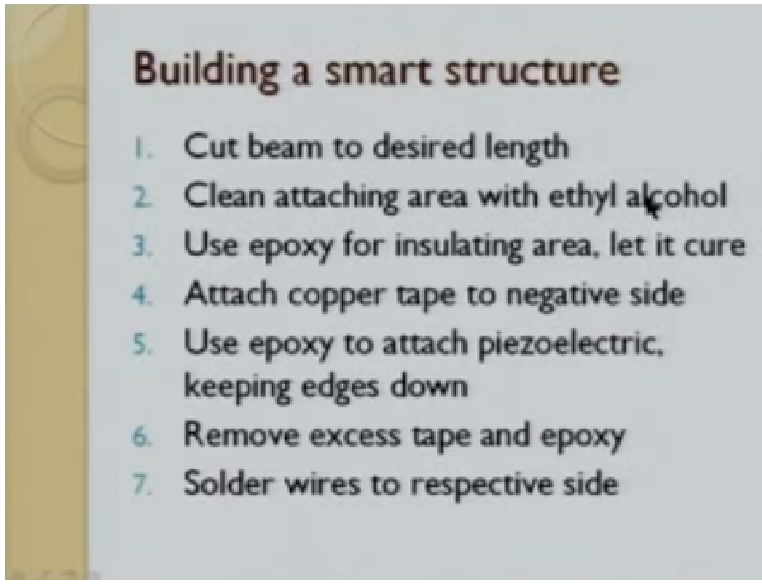

So, first of all the first thing what we have to do for this demonstration of this vibration control is how do we build a smart structure. So, what we do is, we are going to demonstrate it on a beam. So, we are going to take a beam, cut to the desired length. It can be a metallic beam, it can be a composite beam, it can be any other beam. Then, we clean the area where we need to attach this smart patch by using ethyl alcohol, so that any impurities that are there is a removed.

Then, we use epoxy for the insulating area. So, we do a coating of the epoxy and make it let it cure here. Then what we do is in order to build the lead wires at the later part, we attach a copper tape on the backside of the beam. If you are attaching the patch on the front side, we are putting this copper tape at the back and we use epoxy to attach piezoelectric actuators into this region by keeping the edges down.

We remove the excess tape and we soldered the wires in order to connect the control system. So, in order to demonstrate this pictorially, we show this here.

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So this is a beam that we have used it, cut to the approximate length or desired length.

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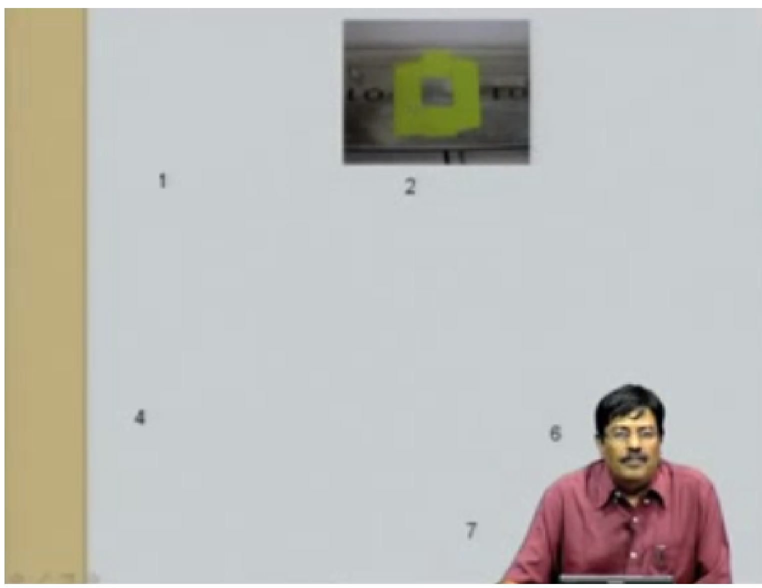

Then, we have isolated the region where we need to attach this piezoelectric patch. So, we have coated with epoxy in this region so that the impurities are removed.

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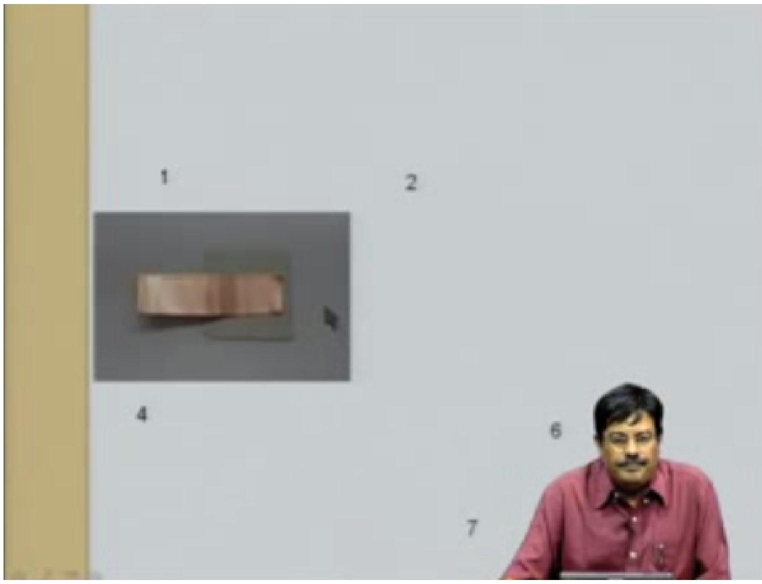

Then, we attach the copper tape on the backside where we need to attach the patches so that it is making more conductive.

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Then, we actually fix the patch using epoxy, form it down, remove the excessive tape around this.

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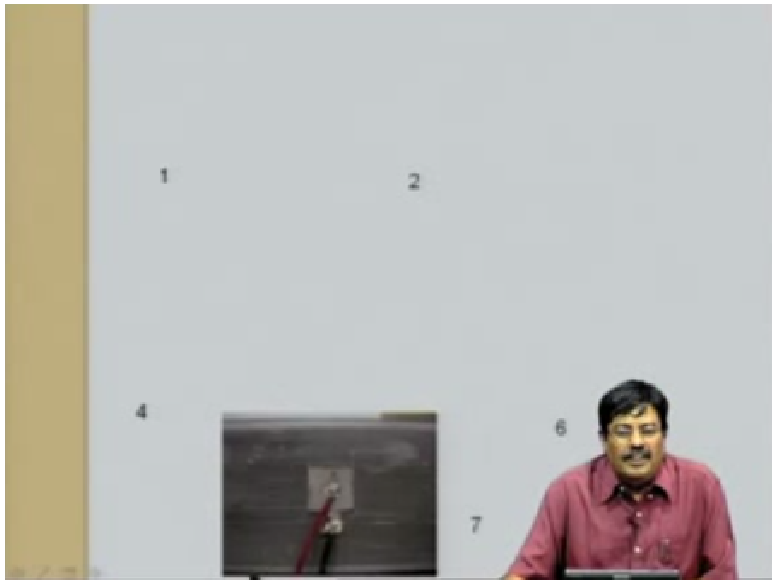

And finally the patch is fixed here and we solder the tape. So, whatever I have explained here we are putting it in the actual structure here.

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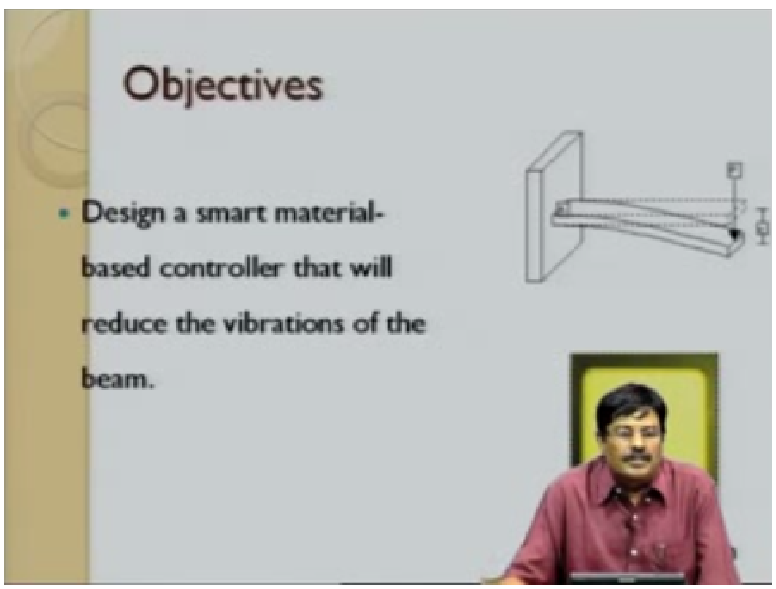

So, now what is the objective here. So, objective here is we have a cantilevered beam which is applied to some dynamic force and our objective is to reduce the vibration due to the presence of the smart patch. So, how does the smart system control is designed basically in order to make sure that this smart patch and parts necessary counterforce to balance this vibration.

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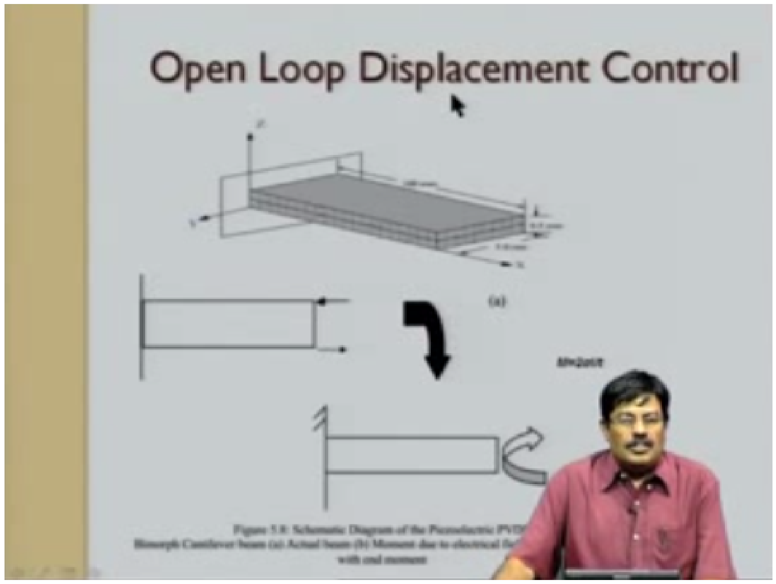

So, before doing that, in our control design we also talked about open loop control where no error signals is there. So, how basically the smart system can be used open loop control without any error signal. So, here we have a bimorph beam. Bimorph beam is a beam which contains the piezo patches or the smart patch both at the top and the bottom surface. So, here is a bimorph PVDF film, so what does this do.

Basically, when we energize this beam basically it creates two forces on the two opposite direction at the tip as shown here which eventually causes a movement. This movement will cause the beam to bend, okay. So, basically if there is a vertical force acting in addition to the effects of this PVDF film we have. So, basically this will counteract the beam bending, okay. So, the effective structure what we are trying to solve is a cantilevered beam with a movement as given here.

The movement is given by two sigma V/T, that is basically due to the voltage that is passed on to the piezoelectric patch here.

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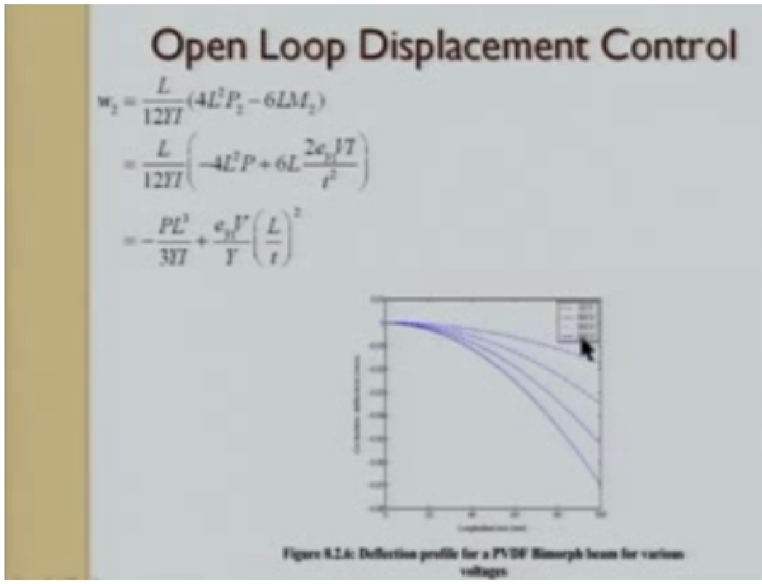

So, we actually derived this in our earlier classes that the tip displacement due to the mechanical load P and the voltage given to the smart patch is given by this expression here. So, basically you see that for a given P, the P is fixed here. If the voltage is increased, this net value keeps on decreasing and it will come to a value such a way that when we can basically make the displacement due to apply load zero by increasing V enormously large.

So, basically no error signal is there. It is just open loop. So, basically we are trying to apply a voltage to counteract the deformation that is caused by the mechanical load. So, here there is a plot which basically says the value of the displacement as a function of voltage. So, when the voltage is 50 volts, this is very large and this becomes smaller and smaller as we increase the voltage, and eventually we came make this displacement zero.

So, that is the smart concepts. So, this voltage V is random. We do not know how much to apply in order to get it the required level. This is avoided by the closed-loop control which we described in the control lecture. So, the closed-loop control along with the error signal can help to reduce this vibration levels to the desired levels. So, let us see how we can design the closed-loop control system.

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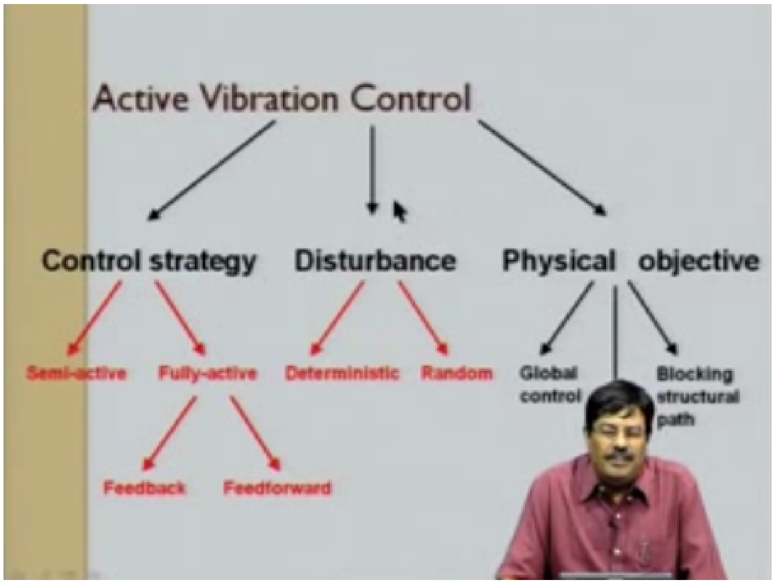

So, now the active control strategy, what are the different. There are different control strategy, there are different kinds of disturbances and there are different physical objectives of the control. So ,if you look at it, the control can be semi-active or fully active. Semi-active is we basically change the stiffness and the damping of the system by some means which are not active. In the fully active system where we actually design a feedback or a feed-forward control using the error signals.

So, these are the different strategies that one can apply. If you look at the disturbance, there are two different disturbance; one is a completely random which is normally in practical cases and one is i deterministic. We give a predetermined input. So, that also depends on the type of control system that we need to design and last but not the least, the physical objectives. There can be a global objective that whole plant we need to reduce the vibration or we have to do only a local control.

So, depending upon the objective, we need to define the control system.

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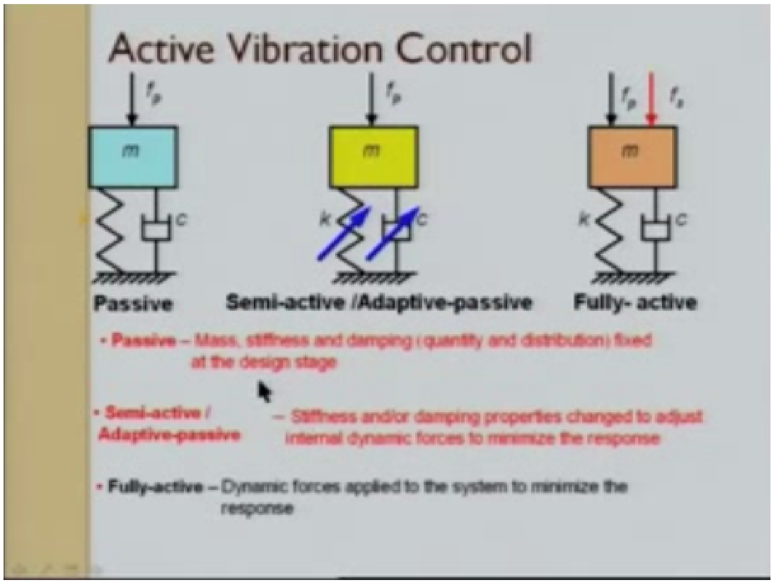

Let us talk about each one of this a little bit more detail. The active vibration control what we are talking about can be passive. In the passive, we basically increase the damping material. So, basically we put a new material which can an extensive higher damping in order to control it. So, there is no change in the stiffness or anything is done.

So, in the semi-active or adaptive passive, we change dynamically the stiffness and the damping of the system by some means, by actually using constrained layer damping or some other means where we can actually do and are fully active is like using a smart rear where we basically energize the structure with different voltage levels which dynamically changes the voltage levels in order to make sure that the vibration levels comes to the minimum level what is desired by the designers.

So, the passive is basically mass, stiffness, damping, quantity and distribution fixed at the design stage. So, we have enormous damping. In the semi-active or adaptive passive, stiffness and damping properties are changed to adjust the internal dynamic force to minimise the response and in the fully active phase, dynamic forces are applied to the system to minimise the response, that is automatically.

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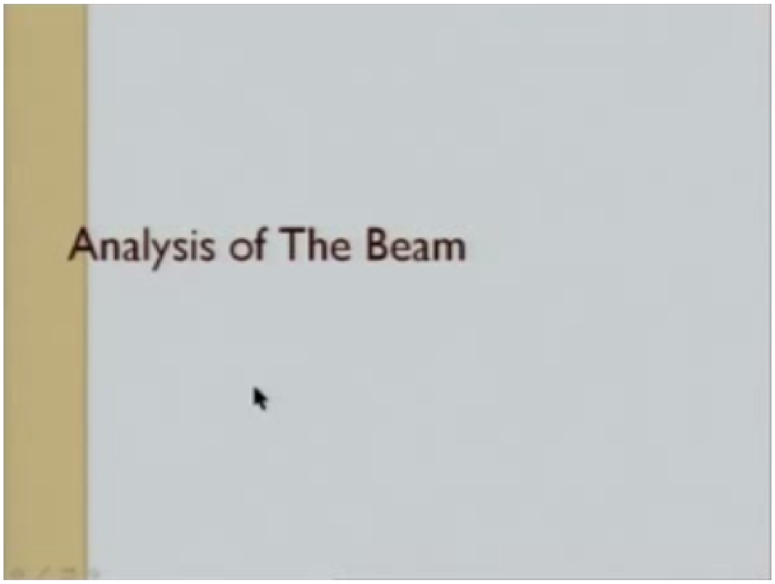

So, let us do the analysis of a beam in this case.

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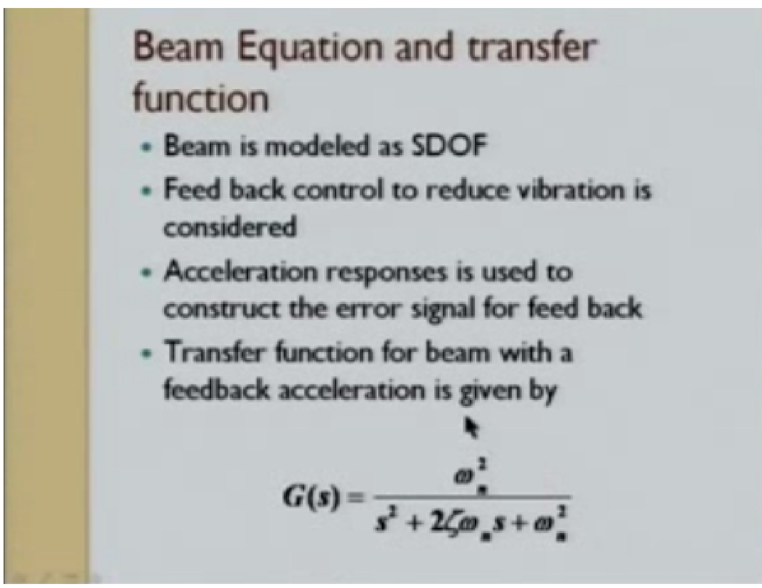

So, the beam equation and transfer function. Here beam is modeled again as a single degree of freedom. It is a very simple control strategy to demonstrate how control system can be designed. Here we are designing a feedback control to reduce the vibration, okay. Now, the sensors, we could have used displacement, we could have used velocity, we could have used acceleration. But what we have used here is basically the acceleration.

Acceleration is basically used to construct the error signal. The error signal is a key component for the feedback control. So, we use the measured acceleration at some point typically the tip as the basic input that goes into construction of the error signal which is fed to the controller to the actuators. So, the transfer function, if we did not have an error signal, it would be 1/this denominator.

Since the error signal is given here, the transfer function is modified because acceleration is proportional to M omega N square. So, the omega N square term is coming here. So, this is basically is the transfer function that requires. So, we can determine the poles, there is no zeros here and we have to see how we can design the control system here.

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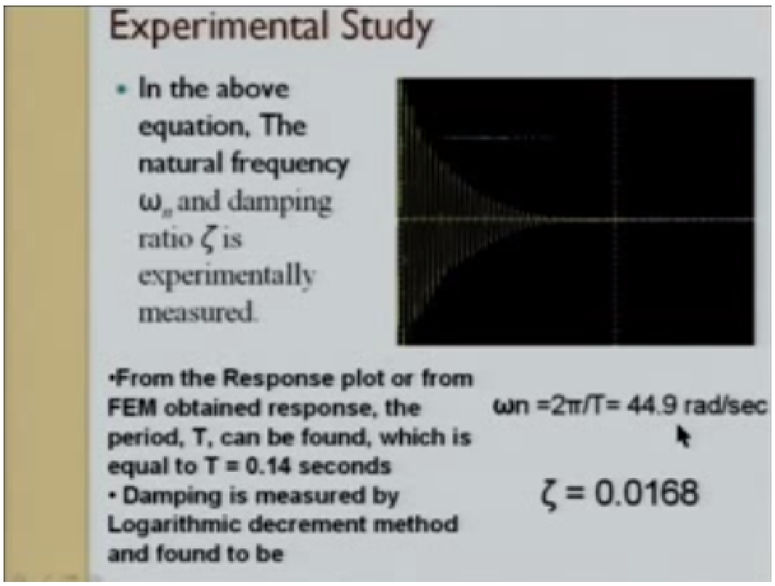

So, first we do an experimental study. So, in the above equation the natural frequency omega and damping is to be known. So, we measured it experimentally. How do we measure it. We can take the time responds. This is a typical time response that is given here, okay. So, this goes up and down here. So, we take the period, find the period and from the period one by period multiplied by two pi will give me the time response which is in our experiment was found to be 44.9 radians per second or it can be expressed also as cycles per second.

We found that T to be the period between two successive peaks to be 0.14 seconds. Then how do we measure the damping. The damping is measured by what is called on the logarithmic decrement which is normally represented as delta. So, delta will be equal to 2 pi psi /root of 1-psi square, okay. So, now what we do is the ratio of two successive amplitudes, so there is a decrease, okay. This is taken, okay.

So, delta is nothing but the log of N/N+1, that is the ratio of two successive amplitudes. So, we measure the two successive amplitudes. So, then we use this equation here to actually find out logarithmic decrement and once we find that, we find the value of psi and when we did this we got this value to be 0.0168.

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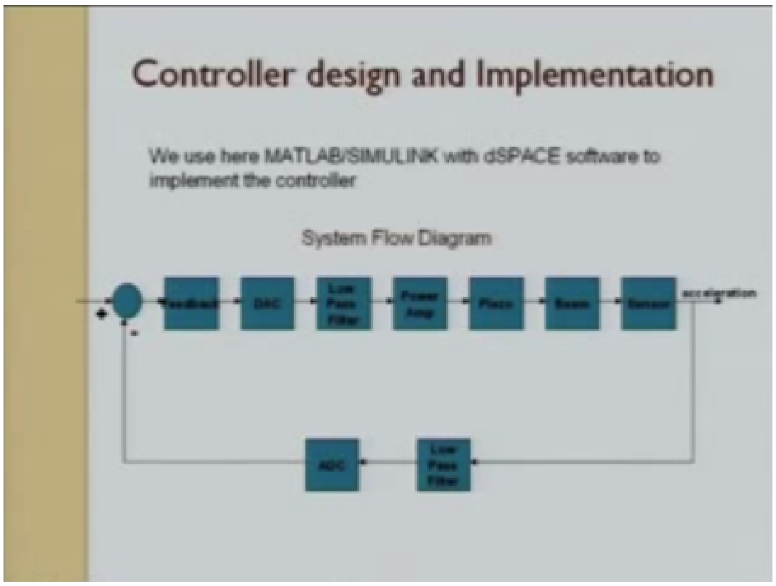

So, we designed this controller a simple PID controller using the Matlab/Simulink. So, Matlab has a very good toolbox called Simulink which is exclusively dedicated to design the control algorithm and there is a dSPACE software which is a data acquisition software which can be linked to Matlab Simulink to enforce the control. So, basically we have a piezo patch is put on a beam which act as a sensor, acceleration is a sensor.

It is passed to a low pass filter, then analog to digital controller to enforce the feedback control. So, this constructs the error signal which is given here and which is amplified. The power has to be amplified due to restrictions in the voltage what we have and then the control is enforced. So, this is a simple flow diagram that shows the control strategy that is employed to just control the vibration of a cantilever beam using piezo patches.

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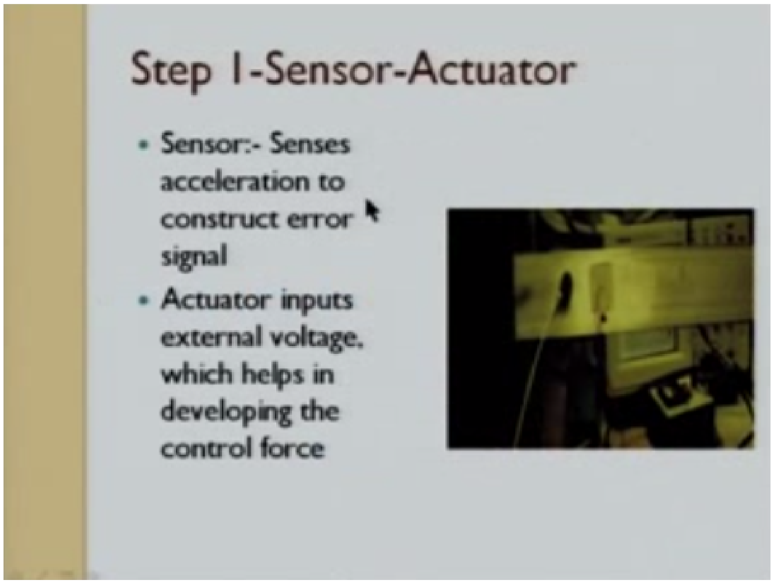

So, let us go step-by-step how we did this. So, first what we did here is first we have to place the sensors and actuators. We have shown your how to pace sensors in the starting of this lecture. How to put the sensors here and once the sensor is placed here and the actuator is needed to input the external voltage that is required for control, the sensor near the tip measures the acceleration and this is used along with the actuator input to construct the error signal which is fundamental to feedback, okay.

The actuator inputs the external voltage which helps in developing the control force because that is what we need and also the error signal.

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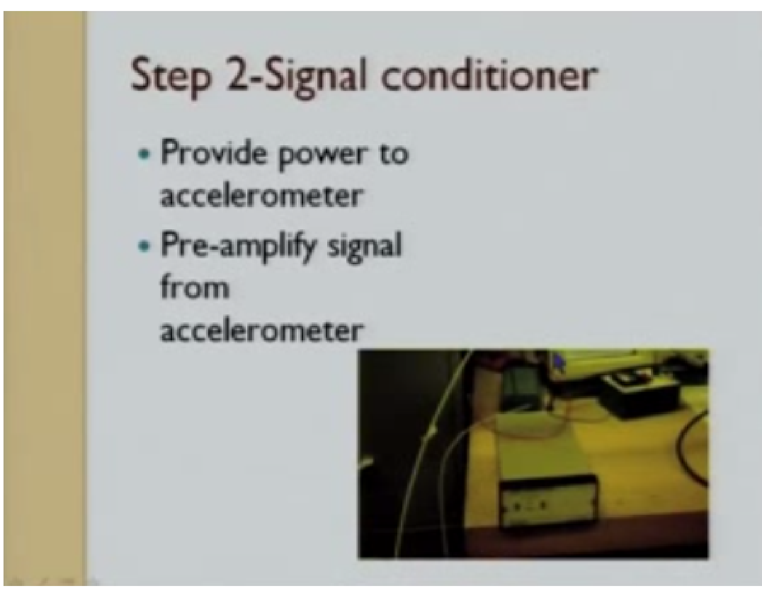

Then, the accelerometer measured has to be pre-amplified, so it is sent as signal conditioner. A signal conditioner is shown here. We provide the power to the accelerometer which helps to pre-amplify the signal because that is required in order to get reasonable responses.

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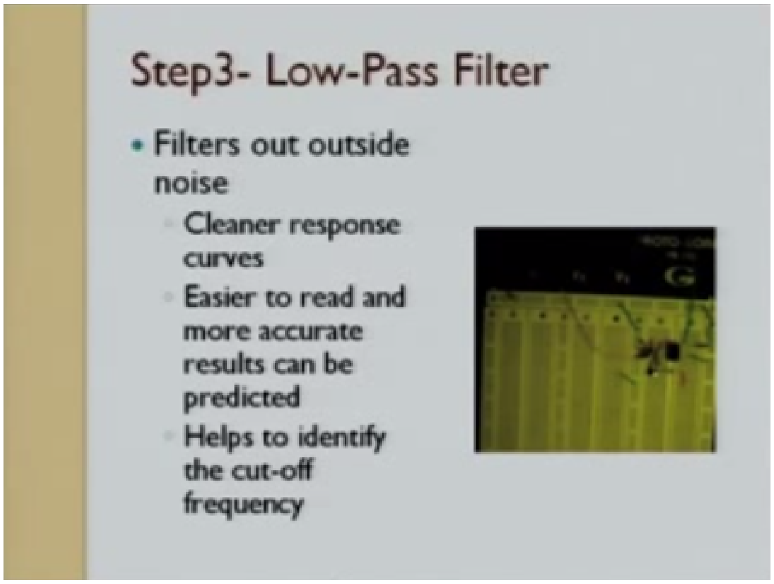

Then, the output from this conditioner is sent to a low pass filter, why do we need this because basically when we measure the experimental measured responses, it is polluted with voice. So, in order to get a cleaner response which are easier to read and more accurate results can be predicted by passing it through a low pass filter. It also helps to identify the cut-off frequency, because it gives us the frequency sweeps.

So, it helps us to identify the cut-off frequency whether we are working in the correct frequency regime for which the control is to be enforced.

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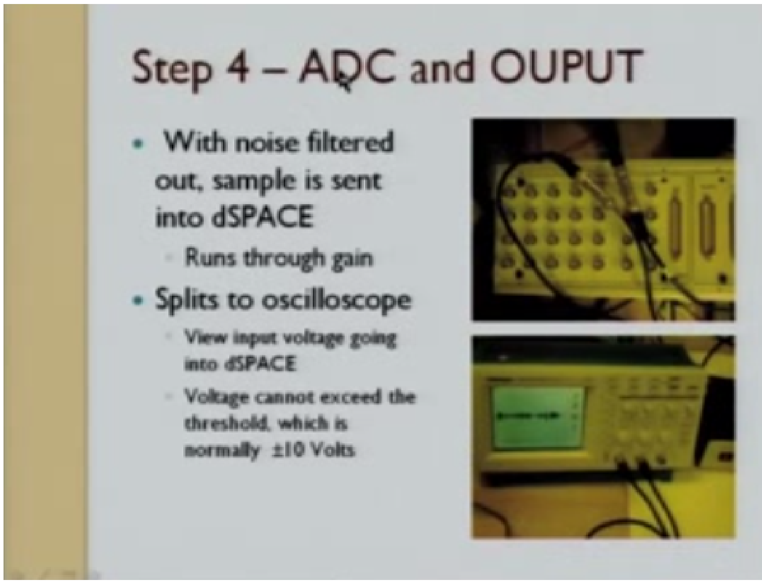

Next, we send analog to digital converter and the output is measured, that is noise filtered out the sample is sent to the dSPACE controller which runs through the gain. So, the gain is another. So, how do we choose a gain. So, there is a parameter in dSPACE where we can change the gain to inputted to that. Because the gain is a relationship between output and input which is an important parameter, so it runs through a gain.

So, first it runs through unit gain and then the gain is constantly increased or decreased in order to see at what point we get the controller. Whatever that is going into this ADC is now viewed in the oscilloscope for the output. The input voltage is first seen. In traditional systems, we cannot exceed more than 10 volts for operating condition. More than 10 volts is more dangerous in many cases like in real structures like aircraft, large voltages are not available.

We have to work with small voltage and then use some kind of amplifier to amplify the voltage. So, we put the threshold of the voltage to be 10 volts in many cases. So, here we have put the threshold to 10 volts and we make sure that the input voltage of the response that is coming out is not exceeded.

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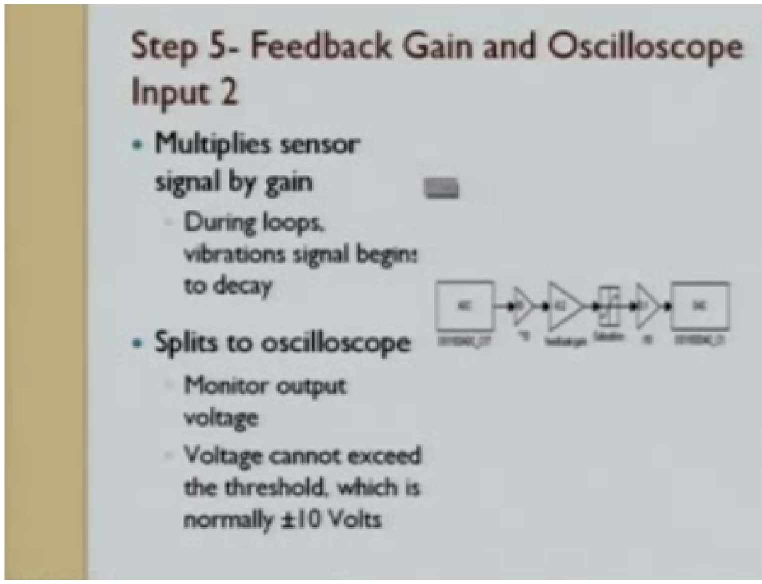

Next, we run through the feedback gain. So, the sensor output is multiplied by the gain, okay and it loops over and over again and see whether the vibration that is initially there due to the applied force is beginning to decay, okay and once this comes here, we look at the output also at the oscilloscope. When we do that, we also make sure that the voltage of the output voltage is also not exceeding to 10 volts. Because that is a threshold that we have put here,

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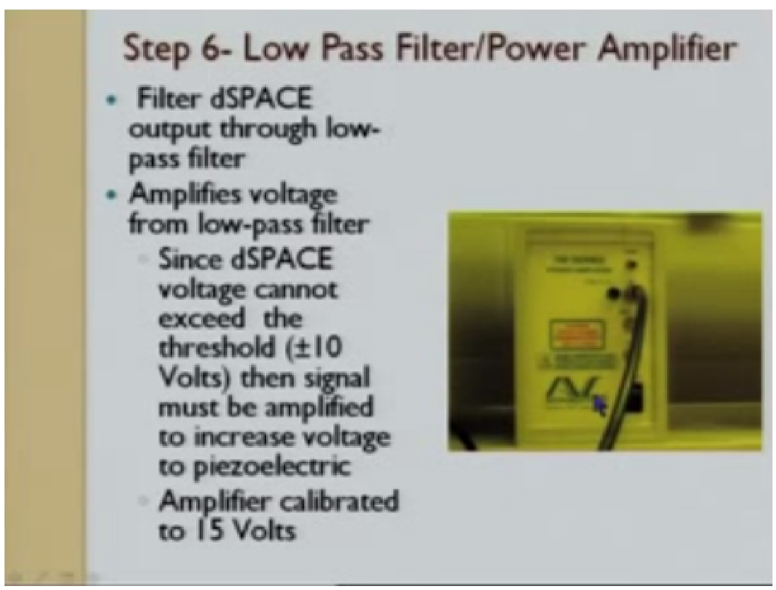

So, the low pass filter now after the output is there is again passed to a low pass filter or a power amplifier because the output may be very small because the voltage levels are very small. So, we require a larger voltage because this voltage has to be amplified in order to give the necessary control force for the decay of vibrations. So, that has to be done basically to make sure that the output is sent through a power amplifier which is shown here.

So, this power amplifier amplifies voltage for low pass. Since dSPACE voltage cannot exceed 10 volts threshold, the signal must be amplified to increase the voltage to piezoelectric volts. So, in this case we need to calibrate the power amplifier when we have done this for about 15 volts. So, finally after passing through after preliminarily cleaning up all the signal, the voltage is applied to the piezoelectric actuator which is shown here.

The piezoelectric actuator will bend or contract to apply the control force on the beam. Then, we can clearly see there is a reduction in the vibration levels that the beam is vibrating due to the applied force.

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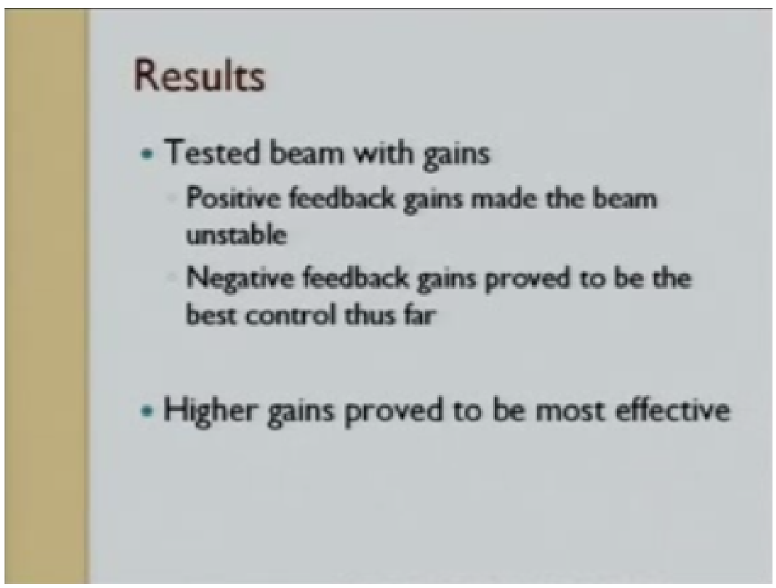

So, let us look at some of the results here. So, what we have found here is when the beam was tested with various gains, the positive feedback gains made the beam unstable. So, the positive feedback in fact, because the omega N square is there, is trying to put the poles onto the right-hand side of S plane which we said that is very bad for the stability of the control system. So, basically the positive gains is not helping in control.

However, when we decrease control and we give the negative, so when we make that negative feedback basically that is helping to put the poles onto the left-hand side of the S plane basically ensuring the control; and on the left-hand side, higher the gains more was control that we could see. Of course, we have put it in a particular positions which was optimised to see that at that position when we feed that the error signal from there onto the actuator, we know that we can get maximum control.

But it also depends upon where you pick the error signal and where you actually feed it to the actuator. So, here in this case the location was such a way that higher the gains more effective was in the reduction of the vibrations.

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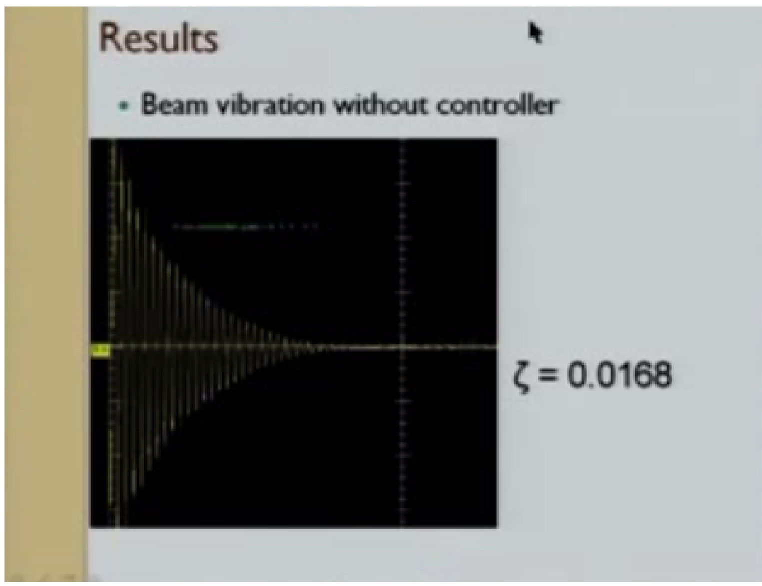

So, this basically gives the response without vibrations. So, you see that there is a peak vibration here starting and starts to decay after some time where the damping was basically 0.0168, that is a ratio of the actual damping to the critical damping.

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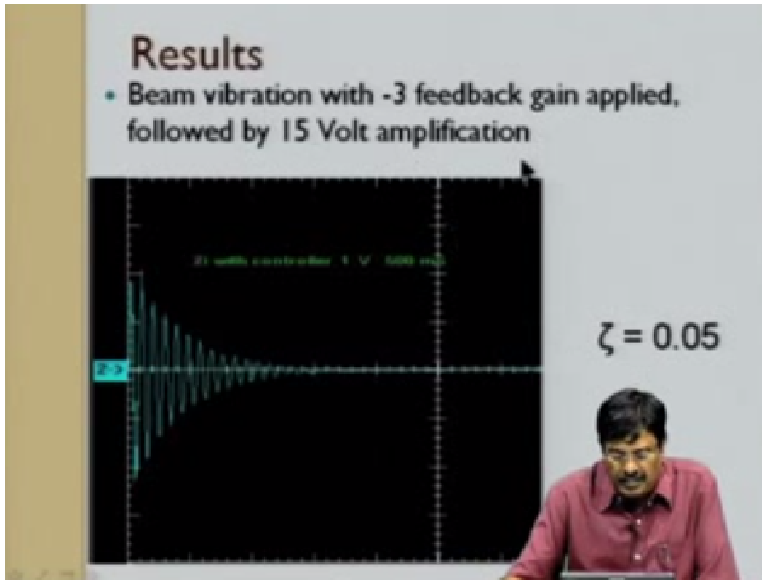

Then after three feedback loops with 15-volt amplification, we have to amplify the voltage whatever that has come near to 15 volts, we see that there is a significant reduction in the vibration levels which is shown here and the damping was increased from 0.01 to 0.05. So, there is significant increasing in the damping which is making the response to come down. So, basically the closed-loop feedback was effective in increasing the damping levels and that is the objective of the control anyway.

We did that have closed-loop feedback system here.

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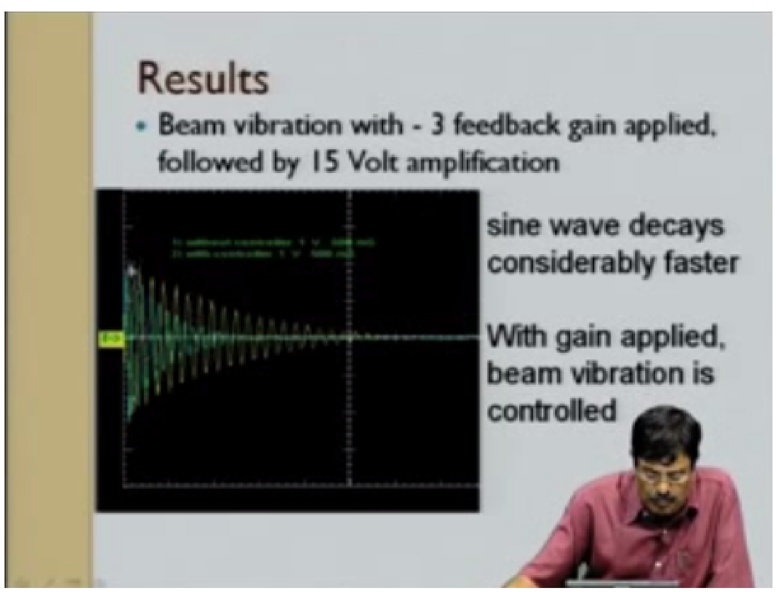

So, here is a plot which is superimposed with the results with no controller and with a feedback

PID controller. So, we see that there is a substantial reduction in the responses as the time progresses. Even though, initially the controls were small at the initial time at very small times. But as the time increases, there is a substantial reduction. So, sine wave decays considerably faster. With gain applied to the beam, we see that the vibration is more or less controlled.

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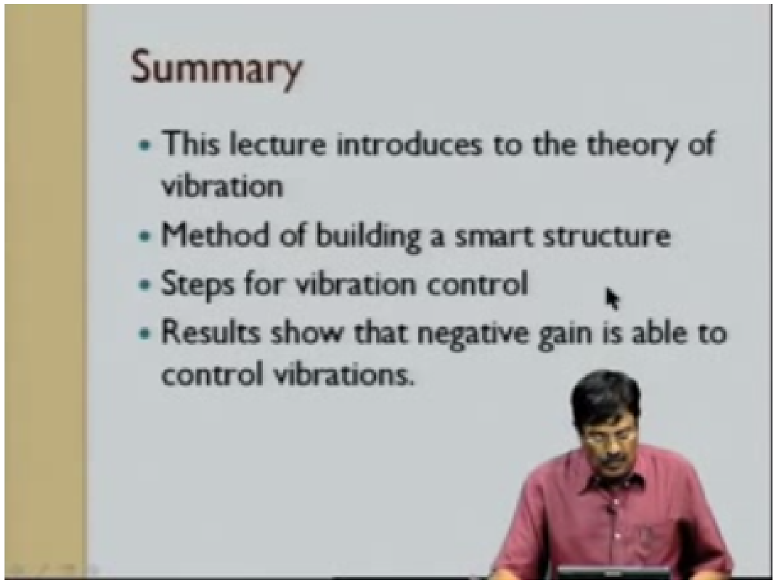

So, in summary, what have we learnt in this lecture. So, basically in this lecture, we understood the concept of vibrations. How do we model the vibrations in terms of springs and masses. Why damping is very important. How the damping reduces the vibration. What is free vibration. What is forced vibration. How do we control the forced vibration. What are the equations there. What is magnification factors. Why is resonance very important.

How is resonance related to poles basically if it is on the right-hand side of the S plane it causes extreme buildup of the responses which we need to avoid and causes unstable control system. We also talked about what is a smart structure. What are the basic ingredients of making a smart structure, i.e., how do we construct a smart beam basically from first principles. How do we do that.

How do we use this vibration experience to actually build the smart system, especially how do we bond the piezoelectric actuator to the structure. What are the things that you need and what are the essential steps that you need to take to build the control systems that is the low pass filter, the analog to digital converter. How do we construct the feedback system. How does the actuator signal goes to the sensor, etc.

So, this is a very rudimentary signal and this can be extended to many bigger system and we found that we also demonstrated the use of this on a simple cantilever beam and we showed the results. In the results, we have said that how we experimentally characterised the damping using logarithmic decrement. How do you measure the natural frequencies of the system also was discussed here.

Finally, the results we found that the feedback or the other signal considerably changes the pole locations. So, basically when we have a positive feedback gain, we said that it was putting the poles onto the right-hand side of the S plane which is completely undesirable for us from the control point of view. So, basically we just gave a negative feedback that is the gains were negative and this basically shifted the poles from the right hand side to the left hand side and started getting control,

So, we showed that there is a significant reduction in the vibration levels by having a negative control and we also said that one of important prospect is the output voltage has to be available in most controls system is small. We have to put a threshold and because of that we have to preamplify voltage, so that the power amplifier is calibrated further and using a 15-volt calibration, we found that there is a substantial reduction in the vibration levels.

So, with this basically we have understood how a simple controller can be designed using the knowledge of the vibration. Thank you.