Robotics: Basics and Selected Advanced Concepts
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Lecture – 11
Introduction, Loop-closure Equations

Welcome to the NPTEL course on Robotics Basics and Advanced concepts. This week, we will look at Kinematics of Parallel Robots. Last week, we had looked at Kinematics of Serial Robots ok. So, the contents of these lectures, in this week are first, we will have some Introduction to parallel robots; then, we look at the Loop-closure Constraint Equations in this first lecture.

In the second lecture, we look at Direct Kinematics of Parallel Manipulators. Third lecture, an important concept in Parallel Manipulators called Mobility. Fourth lecture, we will look at Inverse Kinematics of Parallel Robots and last, we will look at this very well-known problem of Direct Kinematics of Stewart Platform Manipulators.

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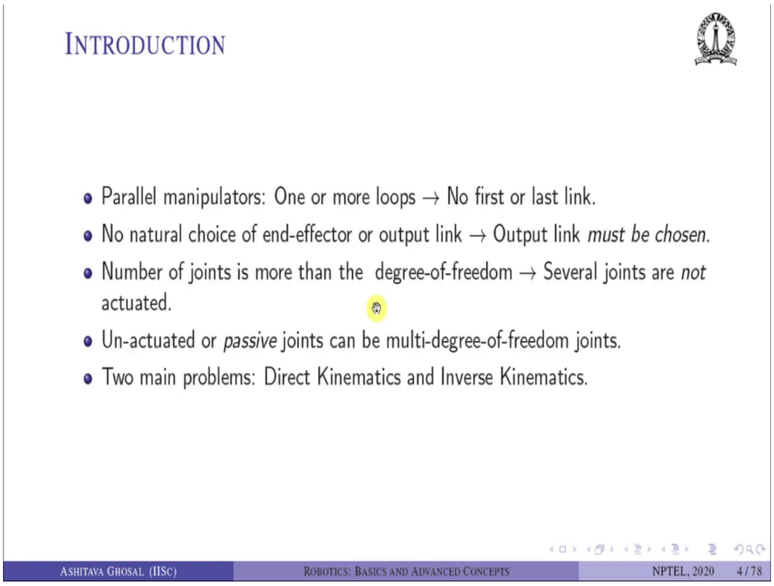

So, as I had mentioned these, there will be 5 lectures in this part of kinematics of parallel robots. So, let us start introduction. So, what is a parallel robot? What is the main difference between parallel and serial robots which we looked at last week? In a parallel robot or a parallel manipulator, there are one or more loops ok. So, there is no real first or last link.

In the serial robot, remember we had a fixed link and then a link and a joint and a link and a joint all the way to a free end which was the end effector. In a parallel manipulators such an arrangement is not there. So, as a result, there is no natural choice of end effector or output link ok. So, we need to choose or say this is my output link.

Another consequence of one or more loops is that the number of joints is more than the degree of freedom and as a consequence, several joints are not actuated. Remember for serial robots or for in general, Grubler Kutzbach criteria, we found that we could find the degree of freedom of any mechanism and the degree of freedom was same as the number of actuated joints which are possible.

So, we have several un-actuated or passive joints and in a multi-degree-of-freedom parallel manipulator ok. So, hence, we can also use multi-degree-of-freedom joints. So, if the joint is passive, we do not need to put in actuators at that place and hence, we can use multi-degree-of-freedom joints.

So, you can think of a spherical joint which has three-degrees-of-freedom, but we do not need to put 3 motors to implement that three-degrees-of-freedom. So, hence, such as spherical joint was not possible in a serial robot. But in a parallel robot, it is possible because it need not be actuated. Similar to serial robots, there are two main problems; one is direct kinematics and one is inverse kinematics.

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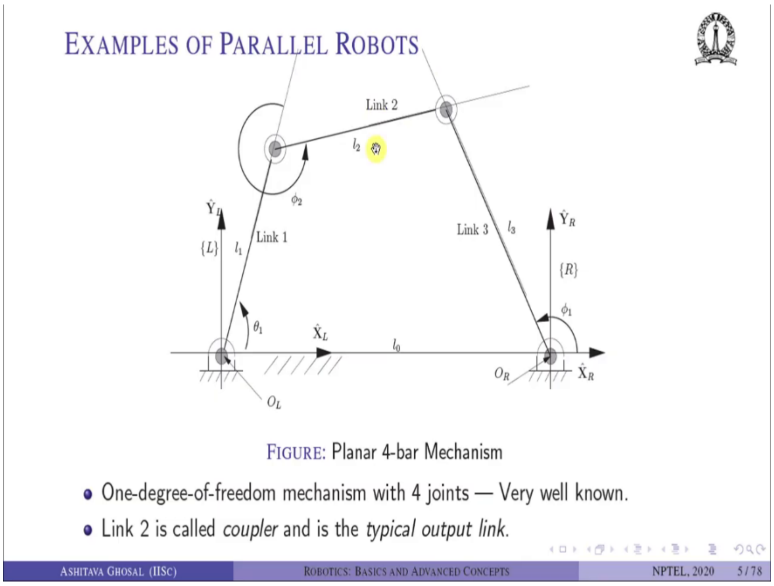

Before we go into these problems and look at it in more detail, let us look at some simple parallel robots; examples of parallel robot. So, the first is this 4-bar mechanism. This is a planar 4-bar mechanism. It is one-degree-of-freedom and it is very well-known ok. So, many people will not say that this is a parallel robot, but let us just extend our definition of parallel robot to include one-degree-of-freedom closed loop mechanisms also ok.

We will be using this 4-bar mechanism extensively in this week because it is very simple to analyze. We can do all the calculations and derivation of equations by hand and also, we can very easily see what we are getting whether they make sense or not. So, in a 4-bar mechanism, we have a left fixed joint here which is denoted by O_l. Then, we have another joint, rotary joint which is here which is O_2. or second joint. Then, we have a third joint and a fourth joint.

So, this forms a loop. So, we start from the fixed end, go to the second joint, go to the third joint and come back to the fixed base again ok. So, this is a one single loop and in this loop, we have we can define a coordinate system L which is the lefts X and Y axis. The origin is at O_l.

We can also likewise define a coordinate system R, which is on the right fixed point and this is X R and Y R with the origin O R and then, we have Link 1, Link 2, Link 3 ok; these are the moving links ok so with link-lengths $l_1, l_2\ and\ l_3$. There is also a fixed link which is of length l_0. So, that is the distance between the left fixed point and the right fixed point ok.

So, this is a very well-known example. This has been studied by many many researchers in both mechanisms and robotics community and this Link 2 is also called the coupler. And typically in a 4-bar mechanism, this Link 2 is the output, chosen output link; not always, sometimes Link 3 could also be the chosen output link.

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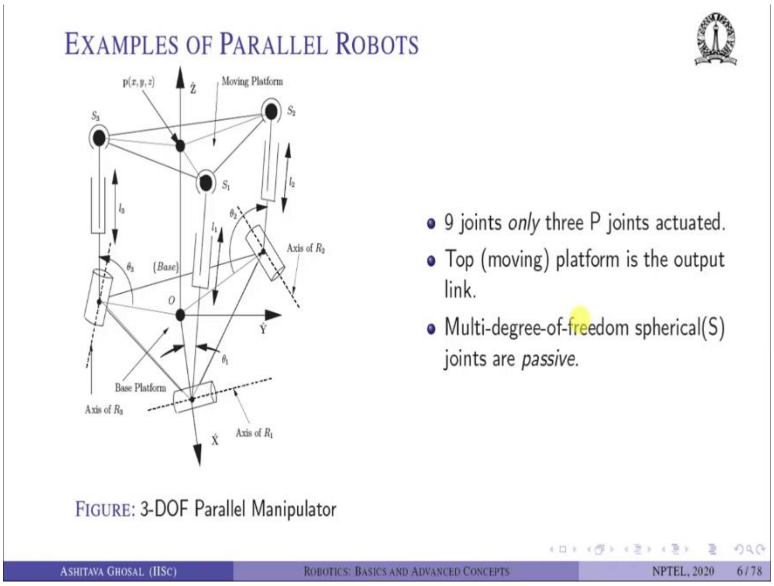

Another example this is a three-degree-of-freedom parallel robot. It is a spatial parallel robot. It consists of a fixed base. This is called as the base platform and a moving platform and this base to moving platform is connected by a rotary joint, a prismatic joint and a spherical joint ok.

So, in this chain R, P and S, the P joint is the one which is actuated. So, we can again define an axis of this rotary joint and some rotation angle θ_3; similarly axis of the rotary joint 2 and an axis θ_2 and θ_1 likewise and we can also define a fixed coordinate system which is O X, O Y and O Z ok, at the same maybe the center of this triangle.

And similarly, we can define a point P on the moving platform which is p (x, y, z) ok. So, this has 3 spherical joints on the top. So, these spherical joints are not actuated ok. So, hence, we can use multi-degree-of-freedom joints. In this example, the moving platform typically is the output link ok. So, in this example, the moving platform is the output link and as I said, there are multi-degree-of-freedom spherical joints which are passive.

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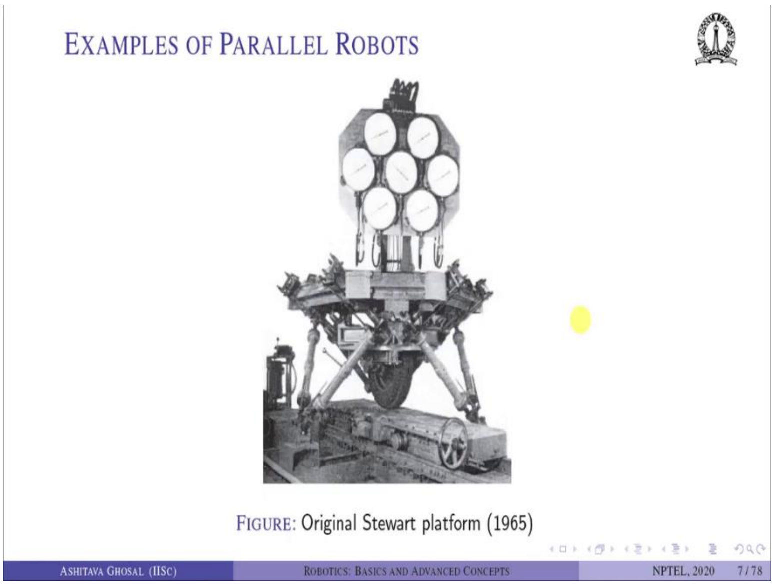

The Original Stewart platform was a six-degree-of-freedom parallel robot and it was invented in 1965 by this person called Stewart and it consist of a prismatic joint or a sliding joint in each one of these legs. At the two ends, there is a hook joint and a spherical joint.

So, this is a UPS arrangement in each leg. There are 6 of these legs and by sliding the prismatic joint, we can make this top platform go up and down have x and y, z motion and also, 3 rotary angles ok; roll, pitch, yaw for example. This was used to test tires. So, basically you mount a tyre at the bottom of this top platform and then, you can tilt the tyre and you can make the tyre go in all the 3 motions x, y and z and also, the 3 rotations. So, it was the tyre testing machine.

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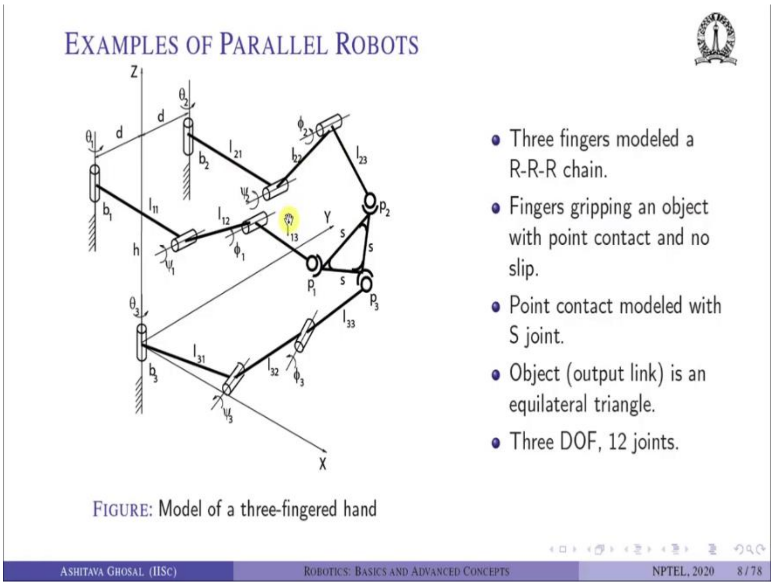

Third example is that of a robot which is a Model of a three-fingered hand ok. So, we had discussed this once in like in week 2. So, I have a three-fingered hand; each finger consists of a rotary joint along the z axis theta 1, one more joint along the finger. So, these two joint axes are perpendicular to each other and a third which is again parallel to the second joint ok.

So, if you look at your finger the first one is at the base, the second one is the middle and the third one is not considered ok. So, the first joint can both rotate about the z axis and also rotate about the perpendicular axis and the second joint is parallel to the second rotation. So, the small finger, small portion in any finger is not considered in this model.

So, we have these three fingers ok. So, and the link lengths are l_{31}, l_{32} and l_{33} so for the third finger l_{11}, l_{12}, $l_{13}$for the second finger and l_{21} sorry for the first finger and l_{21}, l_{22}, l_{23} for the third finger ok. These two fingers index and the middle finger are separated by a distance 2 d.

And similarly, the from the center of that point of intersection to the thumb is a distance of h ok. So, you can think of these three fingers grasping up grasping an object and one of the models of grasping an object is this point contact with friction ok, point contact with no slip.

So, if you have these three fingers grasping an object with point contact and no slip, we can show that it can be modeled using a spherical joint. So, there is a spherical joint. So, this is the object this triangular piece is the object and $p_1, p_2\ and\ p_3$ are 3 spherical joints ok. So, in this case, the output link is clearly this triangular piece ok. So, this has three-degrees-of-freedom.

You can use the Grubler Kutzbach criteria and show that it is three-degrees-of-freedom. However, there are 12 joints. So, there are several joints which are passive, which are not actuated. So, for example, these 3 spherical joints are not actuated, and we will see later that only 2 joints in a finger are actuated.

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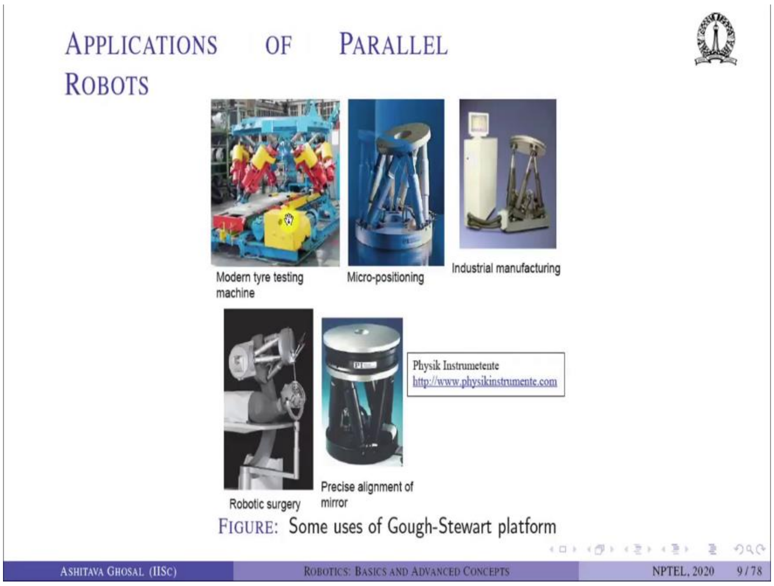

There are many actuated, there are many applications of parallel robots. So, a modern tyre testing machine looks like this. So, for example, these are the U P and S and this is the top platform. The tyre is mounted from the top platform and this is the place which will apply the you know rubbing or pressure to the tyre.

We can also have Stewart platforms which are used for micro positioning ok. We can also use Stewart platform or 6 parallel robots for robotic surgery and for precise alignment ok.

So, there are several advantages in using parallel robots; one of the main advantage is that the load which the parallel robot can carry is much more, it is basically shared by these 6 legs in this case of a Stewart platform. Another big advantage is that the error in a parallel robot is only the maximum error in each leg, the errors do not add up ok. We had discussed this in the first week also.

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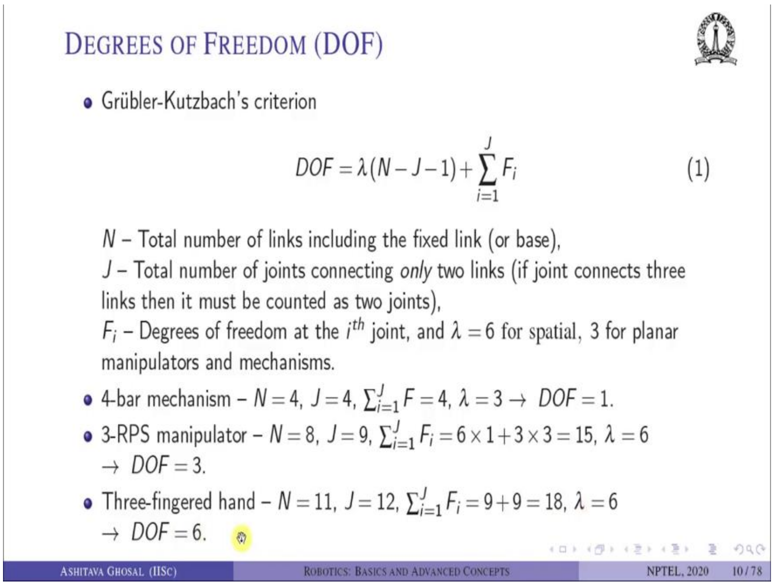

So, let us continue. So, the degree of freedom for parallel robot can be obtained using the Grubler Kutzbach criteria. So, DOF is $\lambda(N - J - 1)$ plus sum of all the degrees of freedom at the J joints. So, this is similar to the same formula used for serial robots.

So, N is the total number of links including the base; J is the total number of joints connecting only 2 links ok; F_i is the degree of freedom at the ith joint and λ is equal to 6 for spatial motion and 3 for planar manipulators and mechanisms. So, for the 4-bar mechanism, we can see that N is 4, J is 4 the sum of the degrees-of-freedom in each joint is 4, there is 4 rotary joints, λ is 3.

So, if you add all these things to this equation, you will get degree of freedom is 1. For the 3-RPS robot, N is 8, J is 9, the sum of the degrees of freedom in each leg is 6 into 1 and then, 3 into 3, 6 because there is an R and a P ok. So, you can think about it, we will get 6 into 1 plus 3 into 3 and then, λ is 6.

So, the degree of freedom is 3. For the three-degree-of-freedom hand, N is 11, J is 12, the degrees of freedom in each of the joints ok, so there are 3 spherical joints which is 9 and then, there are 3 rotary joints in each finger. So, that is again 9. So, λ is 6. So, we get degree of freedom as 6.

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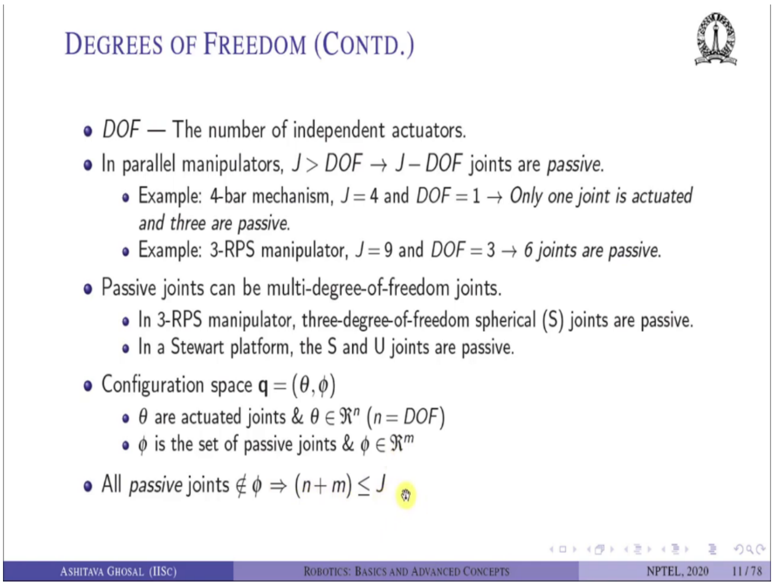

As I said in the case of the serial robot, the degree of freedom is basically the number of independent actuators that we can have in this robot or in this mechanism. So, in a parallel manipulator, if J is greater than DOF, J minus DOF joints are passive. So, example in the 4-bar mechanism, J was 4, but DOF is 1.

So, only one joint is actuated and three are passive. In the 3-RPS manipulator, J was 9, degree of freedom was 3. So, 6 joints are passive ok. So, this passive joints can be multi-degree-of-freedom joints as I have mentioned because we do not need to put actuators to implement a multi-degree-of-freedom joint.

So, in the 3-RPS manipulator, the three-degree-of-freedom spherical joints are passive. In a Stewart platform, the U and the S joints are passive ok. So, let us continue. So, we divide the configuration space basically the set of all joint variables which completely describe the mechanism or this or the parallel robot into θ and ϕ.

So, θ are the actuated joints, let us say if it is n degree of freedom, there are θ from an n dimensional space; ϕ is the set of passive joints and ϕ could be m of them ok. So, all passive joints ϕ and n plus m is less than or equal to J ok. So, sometimes, we will see later that all the joints need not be passive, they do need not appear in this m dimensional space of passive joints ok. So, hence, n plus m is less than or equal to J ok.

So, the next topic in this lecture is loop-closure constraint equation. This is a very important concept and again, we will use the 4-bar example mainly to show the concept of a loop closure constraint equation.

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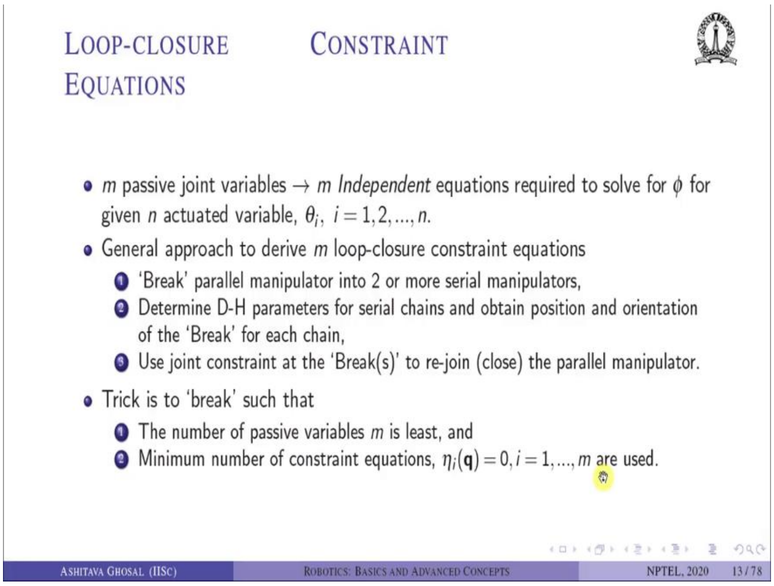

So, what is the motivation? We have m passive joint variables. So, m independent equations are required to solve for the m ϕ ok. So, we can at most give or specify the n actuated variables θ_i 1 to n; but we need somehow to evaluate what those values of the m passive joint variables are ok.

So, the general approach is to derive m loop-closure constraint equations and what is the way to derive this m loop-closure constraint equation. The standard way is to break the parallel robot into 2 or more serial manipulators ok. We determine the D-H parameters for the serial chain and obtain position and orientation of the Break for each chain.

So, suppose if I break it at some point, I can put a coordinate system at that place, where you are broken and we can find the position and orientation of that coordinate system. Then, we use the join constraints at the Breaks to re-join or close the parallel manipulator ok.

So, if you want to analyze what happens due to the break, you have to rejoin it somehow in the during the analysis process. So, the main trick is to break such that the number of passive variables m is least and I will show you some example how we can get different m's depending on how we break the mechanism or parallel robot into serial robots.

The minimum number of constraint equation $n_i(q)$ is equal to 0, i equals 1 through m are to be used ok. So, this is a standard thing. I do not want to break in a way such that I get many many passive variables ok, if I can get with a lesser number of passive variables ok.

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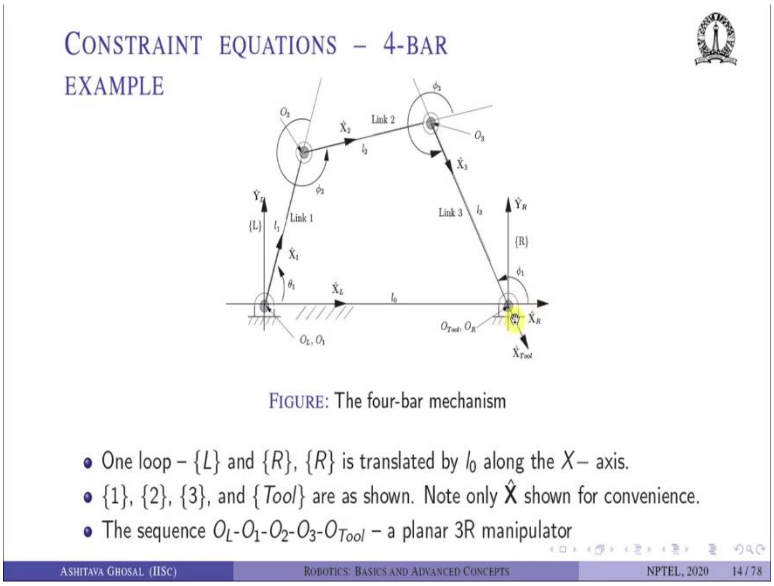

As I said, we will look at this 4-bar mechanism many times. So, let us see what are the possible constraint equations for the 4-bar example. So, 4-bar was Link 1, Link 2, Link 3 and then, this sixth link ok. So, we have one loop which is {L}, then {R}, it goes to through this loop and we can go from {L} to {R} using this along this translation along the X axis ok.

So, we fix coordinate systems {1}, {2}, {3} and {Tool} as shown. So, we fix the coordinate system 1 at O_L; 2 at the second joint; 3 at the third joint, very similar to a serial robot. 3R planar robot and then, this {tool} coordinate system is at the end is so the X axis is shown for convenience only. So, the sequence O_L, O_1 at the same place, then O_2, O_3 and O_{Tool} basically is a 3R parallel robot. So, we are assuming in some sense conceptually that we have broken it at this place ok, at this fourth rotary joint.

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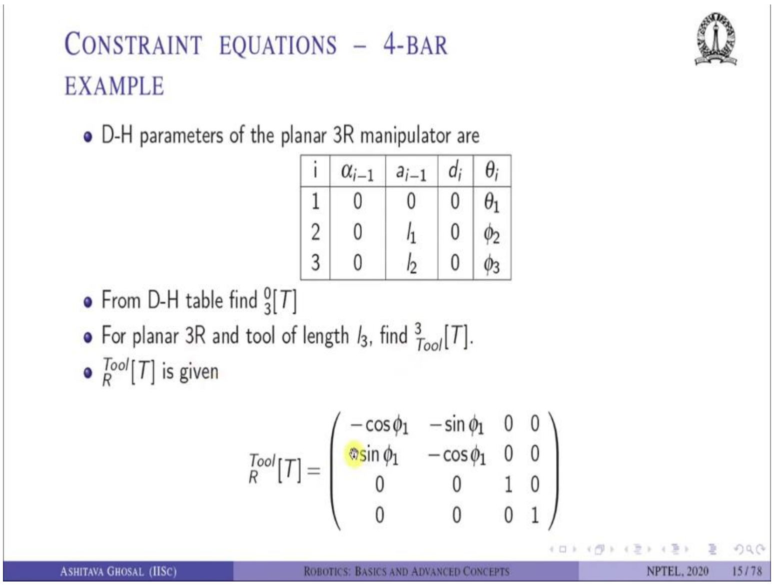

So, the D-H parameters for the planar 3R robot are well known, we have looked at in the serial robot case except that we are do not have θ_1, θ_2, θ_3; we have θ_1, ϕ_2, ϕ_3 ok. We are using different variable names. So, we can see this, what is ϕ_2, ϕ_2 is the angle between the X_1 axis and the X_2 axis, exactly same as what we did for the D-H parameters for any robot serial robot, ϕ_3 is the angle between the X_2 and the X_3 axis.

This is ϕ_3 that is the way these angles are drawn in this form ok. So, we can obtain the D-H table. So, from this D-H table, we can find ${}^0_3[T]$. So, first row will give ${}^0_1[T]$, second row will give ${}^1_2[T]$and the third row will give ${}^2_3T$and then, we multiply those three transformation matrices and we obtain ${}^0_3[T]$ ok.

So, for the planar 3R, the tool of length l_3 is given. So, we can also find ${}^3_{Tool}[T]$ and then, we can also find Tool with respect to the right handed coordinate system and if you look at it little bit carefully, the tool coordinate system and the right coordinate system; so, the tool X axis of the tool coordinate system is like this.

The X axis of the right handed reference coordinate system is like this and there is an angle of $\pi - \phi_1$ ok. So, we can find the transformation matrix between Tool and the right coordinate system, R coordinate system and this is given by minus $-\cos\phi_1$, $-\sin\phi_1$ $\sin\phi_1$ $\cos\phi_1$,again because of the angle which is $\pi - \phi_1$ ok.

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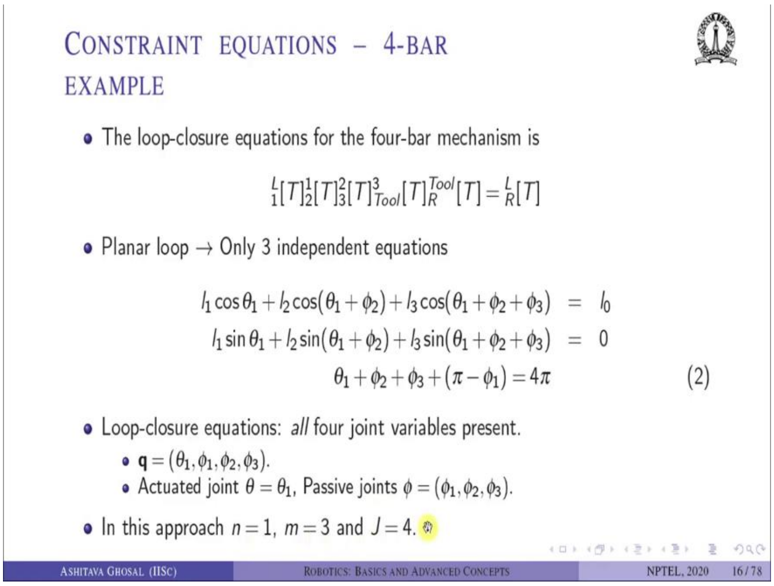

So, what is the loop-closure equation for the four-bar? Very straight forward. We go from l to 1, 1 to 2, 2 to 3, 3 to Tool and then, Tool to R that should be equal to l to R ok. So, let us go back and see this drawing once more. What am I talking about? So, we have a coordinate system l, a coordinate system 1.

So, l to 1, then 1 to 2, we have a coordinate system here 2; then 2 to 3, then 3 to Tool and then, Tool to R. So, this loop multiplication of all this transformation matrix must be equal to the transformation matrix between l to R so directly going from this along the fixed base ok. So, this is a planar equation. This corresponds to a planar system planar loop.

So, although this is a 4 by 4 homogeneous transformation matrix, there are only 3 independent equations in this matrix equation. So, and these three independent equations, once you do all this can be shown to be equal to this $l_1 \cos\theta_1 + l_2 \cos(\theta_1 + \phi_2) + l_3 \cos(\theta_1 + \phi_2 + \phi_3) = l_0$

Similarly, the sin component $l_1 \sin\theta_1 + l_2 \sin(\theta_1 + \phi_2) + l_3 \sin(\theta_1 + \phi_2 + \phi_3) = 0$ and $\theta_1 + \phi_2 + \phi_3 + (\pi - \phi_1) = 4\pi$ ok. So, let us go back and see the figure once more and see whether it makes sense or not. So, I am taking one vector from here to here, from here to here, from here to here.

So, the X component of the vector will have $l_1 \cos\theta_1$, then $l_2 \cos(\theta_1 + \phi_2)$ and $l_3 \cos(\theta_1 + \phi_2 + \phi_3)$ to reach the X component and the X component should be equal to l_0 which is this distance along the X axis. The Y component I go from here to here, here to here, here to here, the Y component will become 0 ok.

The last equation tells you that the angle ok, the sum of the angles interior angles of a quadrilateral is given by this formula. So, this is a slight difference between the planar 3R and in this case, the plane 3R broken up broken at the right hand right joint ok. So, we will get $\theta_1 + \phi_2 + \phi_3 + (\pi - \phi_1)$ because the sum of the interior angles should be equal to 4 π.

So, in this case, what you can see is we have this loop-closure equation contains all four joint variables $\theta_1, \phi_2\ \phi_3, \phi_1$ ok, all the four joint variables are shown here. The actuated joint variable is θ_1. Most of the time in a 4-bar, the crank which is θ_1 is the one which is actuated. So, we have 1 actuated joint variable and 3 passive joint variables ok.

So, in this approach, if I break it at the joint at the R coordinate system at the other end of the 4-bar, I have n equals 1 which is n is the number of actuated joint variables, m is equal to 3 and the total number of joints is 4. So, in this case n plus m is equal to 4 ok.

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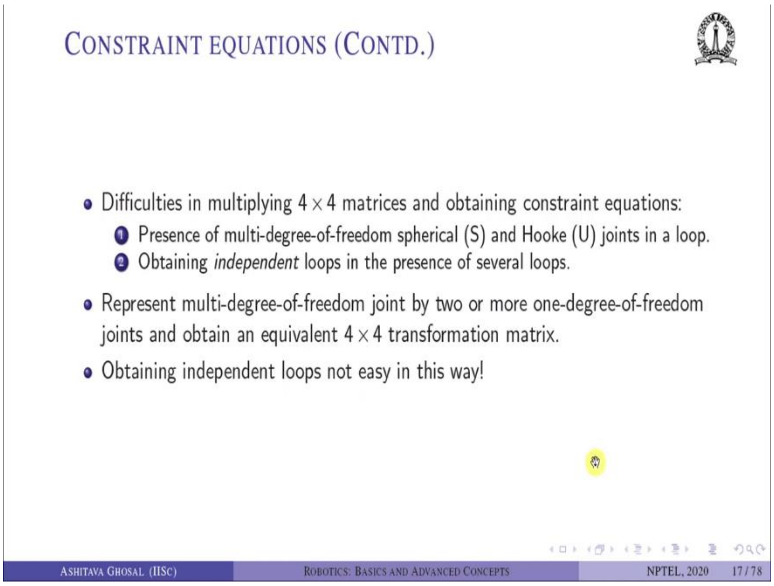

So, let us see what have we done? We have done the D-H parameters for one loop. We have obtained the 4 by 4 transformation matrices and now we have obtained the constraint equation based on the geometry. In a general or complicated parallel robot, we can also have multi-degree-of-freedom spherical and Hooke joint, U joint ok.

So, and more importantly, it is sort of hard to obtain which are the independent loops in the presence of several loops. In the case of a 4-bar there is exactly 1 loop. So, we can break it there and we show that this is the loop closure equation.

So, to represent multi-degree-of-freedom joints, but two or more one-degree-of-freedom joint and obtain an equivalent transformation matrix is one way of solving this problem of S or U joint. However, to obtain the independent loops is not easy ok. There are no standard ways or not very easy to check which is the independent loop ok.

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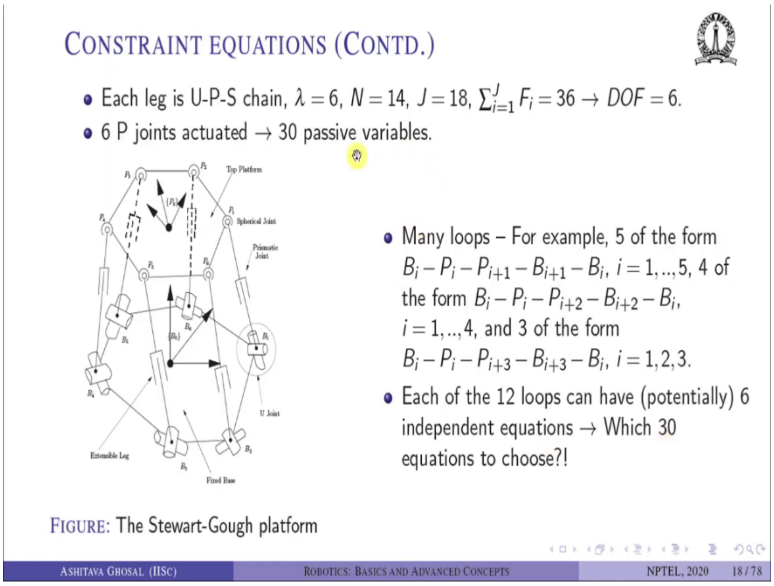

So, let us consider what happens in a 3 six-degree-of-freedoms Stewart Gough platform. So, as I said, there is a fixed base. There is a moving platform and each of these points; so, P_4 which is connected to B_4 by means of a spherical joint and a Hooke joint and a prismatic joint ok.

So, we have a six sided top platform and a six sided bottom fixed base and corner points are attached by U, P and S chain ok. So, in this case, λ is 6; N is 14 we can count. So, 1, 2, 3 and then so on and you can count that the number of links are 14; the number of joints is 18, Why? There are 6 here, there are 6 here, 12 and then there are 6 prismatic joints; so, 18.

The sum of degrees of freedom at each one of this joint is 36. So, you can see this is 6 into 3, 18; 2 into 6, 12 plus 6 into 1. So, this is 36. So, if we substitute in the Grubler's formula, we will get degree of freedom of 6 ok. So, if you have six-degrees-of-freedom, we can actuate 6 joints and in a Stewart platform the 6 prismatic joints are actuated.

So, which means out of this 36 degrees of freedom in the joints, 30 are passive variables and only 6 are actuated. More importantly, we have many loops. So, for example, 5 loops are of the form B_i; let us say B_1 P_1, P_2, B_2 and then back to B_1 ok. So, we can start from B_1 P_1, P_2, B_2 and then back to B_1 and there are 5 similar ones just by changing i ok.

Then, we can have 4 loops of the form B_1 P_1, P_3, B_3 and then, back to here, ok. So, we can skip the middle one and then similarly, we can have 3 where you skip 2 middle ones and go to the third one after the first and then, come back. So, we have several possible loops and there are 12 of these if you think a little bit about it.

So, each of the 12 loops can have potentially 6 independent equations because this is in 3D space. So, the transformation matrices when we multiply, we will get a matrix equation and each transformation matrix has 6 independent equation ok. So, we can have 12 such loops and there are potentially 6 independent equations in each loop. So, which 30 should we use for the 30 passive variables? So, it is not clear ok. It is quite complicated.

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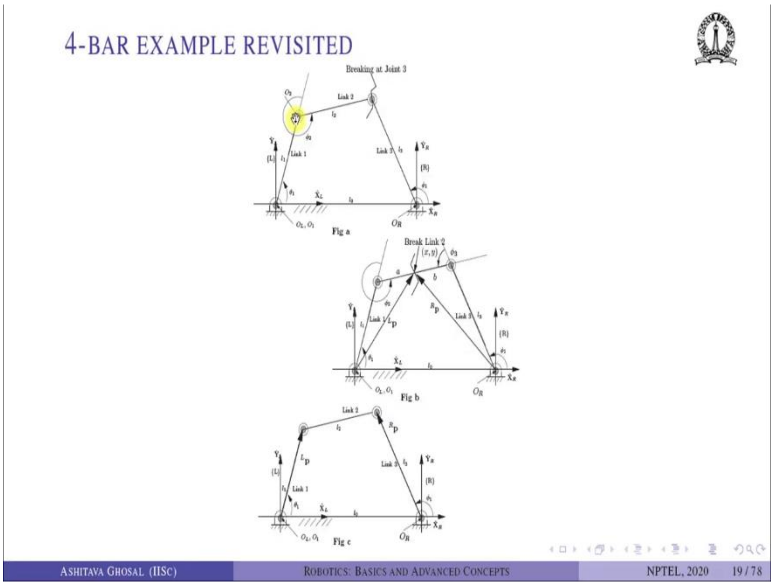

Let us go back and look at the 4-bar example again. So, the first time, I showed you we broke at the O_R the last link. We could also break at the joint 3. So, if I break at joint 3, then I have a planar two-degree-of-freedom and a planar single 1 R robot so 2 R and 1 R.

I could have also broken it in the coupler link. So, at a distance a and b, somewhere here I can break it at Link 3, Link 2 sorry.

So, then what do we have we have 2 planar 2R robots. So, 1R, 1R and some end effector point similarly 1R, 1R and then, another point here. We can also break at 2 places; so, we can break at the second joint and third joint. So, these are possible ways of breaking this 4-bar example and let us try to find out the loop closure equations, the relevant equations for each of these 3 cases.

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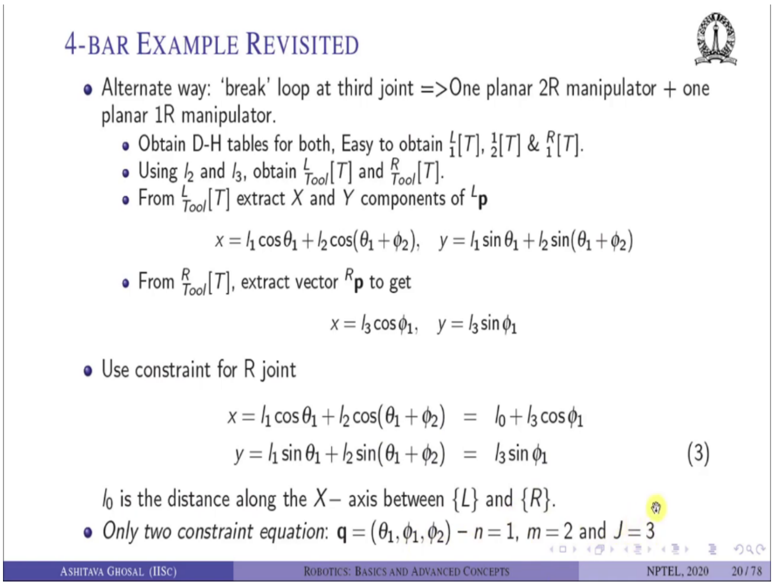

So, first is we break the loop at the third joint which is case one. So, I have a planar 2R manipulator and one planar 1R manipulator. So, we can again obtain the D-H table for both is very easy to obtain l to 1, 1 to 2 and R to 1. Using l 2 and l 3 we obtain L Tool transformation matrix and R Tool transformation matrix so from L Tool extract X and Y components of this point ok.

So, we can extract the X and Y component of this point here ok. So, what will it be? It will be similar to $l_1 \cos\theta_1 + l_2 \cos(\theta_1 + \phi_2)$. So, the x component of this point will be equal to $l_0 + l_3 cos(\phi_1)$ that is what I am trying to get at ok. So, that is what is there, the x component from the left side is this, y component from the left side is this and for the R coordinate system the same point is $l_3 cos(\phi_1)$ $l_3 sin(\phi_1)$.

So, now, we can write the loop-closure equation which is $l_1 \cos\theta_1 + l_2\cos(\theta_1 + \phi_2)$ is equal to $l_0 + l_1\cos\theta_1 + l_2\cos(\theta_1 + \phi_2)$because you have to move along the x axis by l_3 and then, if you equate the y ok, we have $l_1 \sin\theta_1 + l_2 \sin(\theta_1 + \phi_2)$ is equal to $l_3 sin(\phi_1)$ ok.

So, what have we used here, so we have basically used the constraint for an R joint. So, recall in week 2, we had looked at what are the constraints imposed by a R joint. So, we are at the point ok, where we have broken. So, at this point basically the XY position coming from this side should be equal to the XY position coming from this side ok. We do not have to worry about the orientation.

Remember, the rotary joint imposed 3 position constraints; in this case since it is plane, 2 position constraint and we also had orientation, but that is not useful here ok. So, we have 2 in this case; θ_1, ϕ_1 ϕ_2. So, you can see the equations do not contain the rotation at the joint, where we have broken ok. So, θ_1 is actuated. So, n is 1 and there are 2 passive joints ϕ_1 and ϕ_2. So, n is here n plus m and J is 3 ok.

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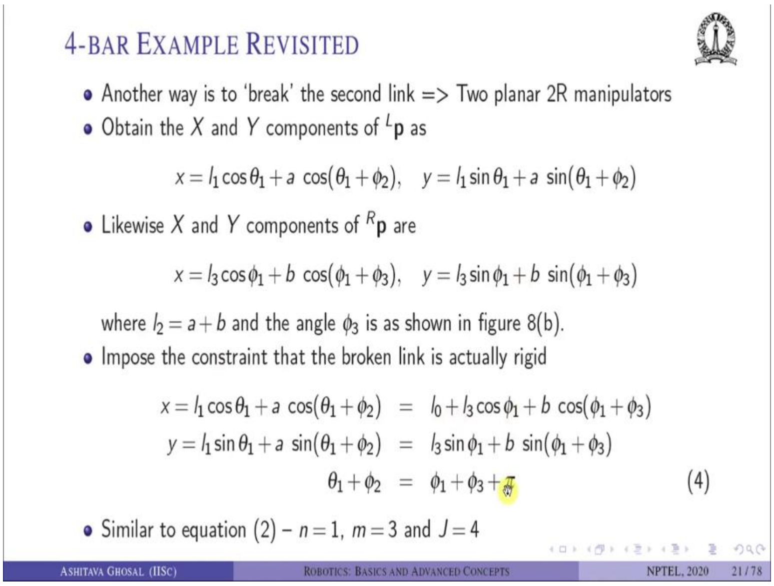

So, as I said we can also break the second link the coupler link and we get two planar 2R robot. Again, we can obtain the X and Y components of the point, where we are breaking from the left side and also from the right side ok. So, the from the left side it is $l_1 \cos\theta_1 + a\cos(\theta_1 + \phi_2)$ and y is equal to $l_1 \sin\theta_1 + a \sin(\theta_1 + \phi_2)$ ok.

Likewise, from the right side, we can write the x and y components in terms of angle ϕ_1 and b and ϕ_3 ok. Here a comes because you have broken distance a and b and we can impose the constraint that the broken link is actually rigid. So, what is the constraint when you have a rigid link? The positions are same and the orientations are the same from both sides ok.

So, x from left side will be equal to right side y; from left side will be equal to y from the right side and the orientation of the coordinate system, where we have broken is given by $\theta_1 + \phi_2$ and now from the other side, it is $\phi_1 + \phi_3 + \pi$ ok.

This π comes because the X axis is pointed opposite in the two coordinate systems. So, in this case, we have similar to the case when we broke it at the third joint. So, we have n is 1, m is 3 and J is 4, why? Because we have $\theta_1, \phi_2, \phi_1, \phi_3$, so all the joint angles appear in these 3 constraint equations.

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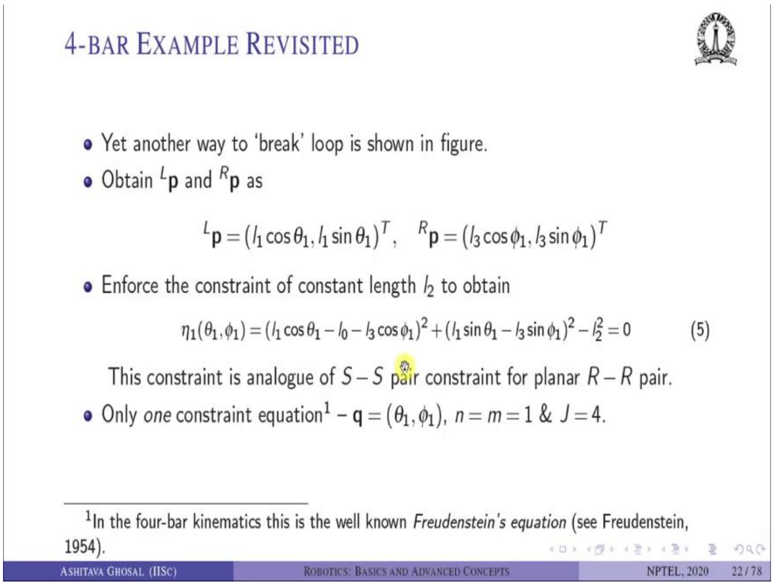

Finally, we can break it at the sec end of the first link and end of the last link ok. So, we can obtain the points l from the left side which is nothing but $l_1 \cos\theta_1$ and this is $l_1 \sin\theta_1$ and from the right side, we have $l_3 cos(\phi_1)$ and $l_3 sin(\phi_1)$ ok. So, what is the constraint we want to enforce? We want to enforce that the link length 2 is always constant ok.

So, the link 2 has link length 2. So, basically, we the constraint is x from this x minus this x square plus this y minus this y square is equal to l_2^2 which is what is written here.

So, $(l_1 + \cos(\theta_1) - l_0 - l_3\,cos(\phi_3))^2 + \left(l_1 \sin\,(\theta_1 - l_3 sin(\phi_1)\right)^2 = l_2^2$ or minus l_2^2 equal to 0 ok.

So, this is very very similar to the constraint S-S pair introduces in a loop. So, in a plane the S-S joint, S-S pair is similar to a R-R pair. So, what do we have here? Here, we have only one constraint equation with 1 θ_1 variable and 1 ϕ_1 variable. So, q contains θ_1 and ϕ_1. So, n and m equal to 1 and J is 4 here, we still have 4 joints in the mechanism.

So, this is a very well-known equation. This is called as the Freudenstein equation ok. So, if you look at 4-bar kinematics and if you go and search for Freudenstein equation, so in 1954, he derived this equation for a 4-bar mechanism, more importantly, he went on to use this equation to design 4-bar mechanisms. So, lots of possibilities of using this equation to design a 4-bar for 1 position, 2 position and 3 or 4 specified positions are possible ok.

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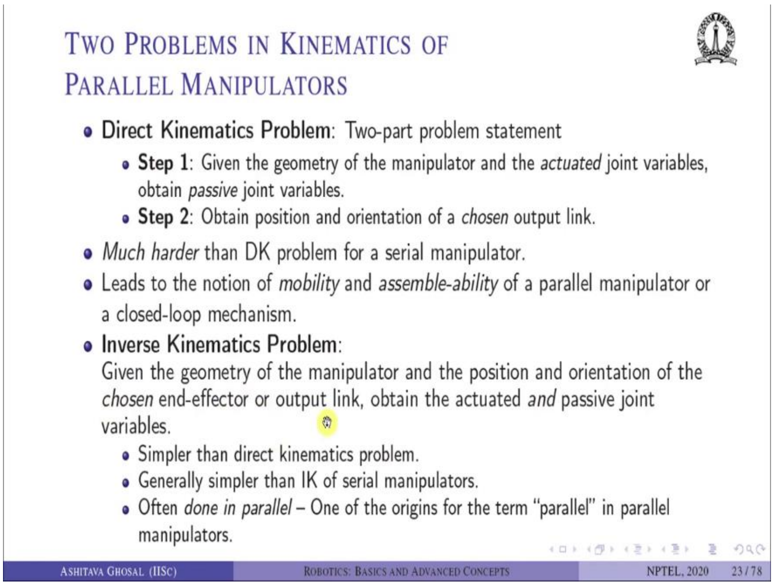

So, as I said, we have two problems in kinematics of parallel robots; one is the direct kinematics problem it is a two-part problem in this case as opposed to serial robot. First, Step 1: Given the geometry of the manipulator and the actuated joint variables obtain the passive joint variable ok.

So, for example, in this equation, if you are given θ_1, we can use this equation θ_1 comma ϕ_1 equal to 0 to obtain ϕ_1 somehow ok; one equation, one unknown. So, now, but we need to do some more work to obtain the other two passive joint variables, nevertheless we can obtain all the passive joint variables.

Second step is once you obtain the position and orientation of the chosen link, you need to obtain that. So, once you obtain the passive joint variables, we can go back and substitute in the general equations of a 4-bar mechanism without breaking. So, every variable is there and we can find the position and orientation of a chosen output link ok. So, this is a much harder than the direct kinematics for the serial robot because why? It is two steps.

In the direct kinematics of a serial robot, just the actuated joint variables are given and we find the position and orientation of the end effector that is not enough. First you have to find the passive joint variables, using the loop closure constraint equation and then, obtain the position and orientation of the chosen output link.

The solution of the direct kinematics problems specifically the Step 1 gives rise to something called the notion of mobility and assemble ability of a parallel robot or a closed-loop mechanism. So, we will see later that this when you try to obtain the passive joint variables, we will obtain conditions that some of the passive joint variables do not have real values ok. So, that leads to this notion of mobility and whether you can assemble the parallel robot.

The inverse kinematics problem is different again. So, what are we given? The given the geometry of the manipulator and the position of orientation of the chosen end effector or the output link ok. So, depending on which is the chosen output link, the inverse kinematics problem will be slightly different ok.

So, if I choose the fourth link in a 4-bar mechanism as the output link, the inverse kinematics is different than if I choose the coupler link. So, first we have to obtain the actuated and passive joint variables and that is the problem, once you choose the output link.

This is simpler than the direct kinematics problem, we will see later ok. It is also normally simpler than the inverse kinematics of serial robots and this could be often done in parallel ok. So, depending and I we will discuss this notion of doing it in parallel and this is one of the origin of the term parallel in parallel robots ok. Why? Because we can solve the inverse kinematics problem using parallel computation or in parallel.

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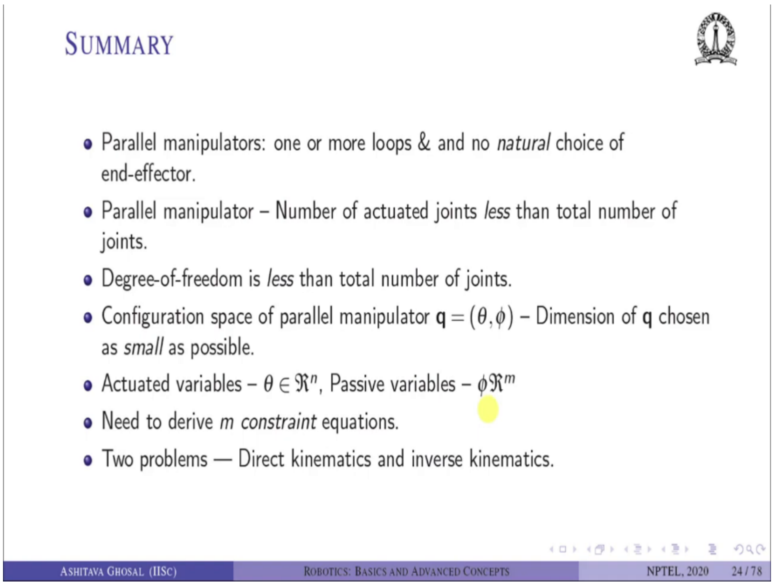

So, in summary, parallel manipulators have one or more loops and no natural choice of end effector ok. You have to say this is my end effector and you have to say which are the loops, we are you know there are more than one loop. Parallel manipulators the number of actuated joints is less than the total number of joints.

Hence, there are some joints which are actuated and some joints which are passive ok. The degree of freedom is less than the total number of joint, this is also very important. In a serial robot with one-degree-of-freedom joint in a PUMA, we had 6 rotary joints and the degree of freedom was 6.

In a planar 3R robot, we had 1 degree of rotary joints, 1 degree of freedom rotary joints and the degree of freedom of the planar 3R was 3; whereas, in a parallel robot, the number of joints is much more than the degree of freedom. So, this leads us to this notion of a configuration space of a parallel manipulator; basically, we call this configuration space q, it is divided into two parts; one is theta and one is phi ok.

So, thetas are the actuated joints and phis are the passive joints. So, and we would like to choose the dimension of q as small as possible. So, thetas are fixed you know if you have 3 degrees-of-freedom parallel robot, the dimension of theta will be 3, but we could arrive at different phis as I showed you for the 4-bar mechanism ok.

So, we could have just 1 phi for the 4-bar and 1 actuated or we could have 1 actuate and 2 passive or you can have 1 actuated and 3 passive joints. So, we would like to choose the dimension of q as small as possible ok. So, there are typically n degrees of freedom in a parallel robot, then theta will be n dimensional and if you have passive variables, they form a m dimensional space.

So, we need to derive m constraint equation. So, here phi should be element of R m ok. So, in order to solve these passive variables, we must have m constraint equations and as I said there are two problems; one is direct kinematics and one is inverse kinematics of parallel robots ok.

So, with this, we will stop and in the next lecture, we will look at Direct Kinematics of Parallel Manipulators.