Robotics: Basics and Selected Advanced Concepts
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Lecture – 12
Direct Kinematics of Parallel Manipulators

Welcome to this NPTEL course on Robotics: Basics and Advanced Concepts. In the last lecture, we had looked at how we had introduced parallel robots and we had discussed how to derive loop closure constraint equations in a parallel robot. In this lecture, we look at Direct Kinematics of Parallel robots.

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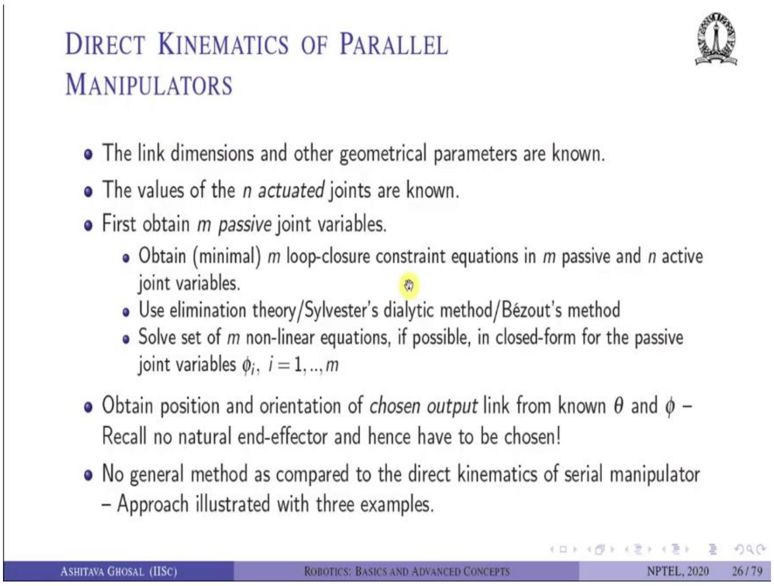

So, in the direct kinematics of parallel manipulators, the link dimensions and other geometrical parameters are known. The values of the n actuated joints are known. So, remember in the parallel robot as discussed in the last lecture, there will be some n actuated joints and m passive joints.

So, the first task in the direct kinematics of parallel robot is to obtain the m passive joint variables. So, basically how do I solve? For the m passive joint variables given the n actuated joint variables. So, what do we need to do? We need to first obtain minimal number of m loop closure constraint equations in m passive variables and n active joint variables ok. So, m should be as small as possible so, as to make our life simpler and easier.

Then, we use elimination theory for example, the Sylvester's dialytic method or the Bezout's method to eliminate $(m-1)$ passive joint variables to obtain a single equation in one of the joint, passive joint variables and we would like to solve these m non-linear equations in closed-form for the passive joint variables.

Once we find the single eliminant, we can solve that equation and then by back substitution, find all the m passive joint variables. So, once the actuated joint variables and the passive joint variables are now known, we obtain the position and orientation of a chosen output link from the known θ's and ϕ's ok. So, recall in a parallel robot, there is no natural end-effector not like in the serial robot where the free end is typically the end effector.

Hence, we have to say which one is the out natural output link ok. Most of the time, it is sort of obvious, but nevertheless we have to say that this is the output link of the parallel robot. So, there are no known general method as compared to direct kinematics of serial robot, why? Because of this problem of choosing this m loop closure constraint equations and also to find the minimal polynomial in one passive joint variable is not always very clear and obvious.

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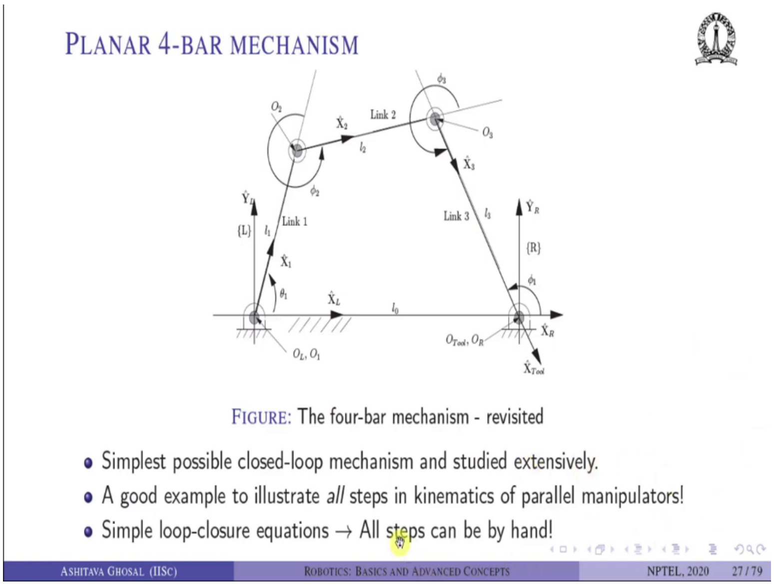

Let us look at three examples of direct kinematics and as usual, we will start with the simplest possible parallel robot which is a planar 4-bar mechanism. So, as discussed earlier, these are the three-moving links; link 1, link 2, link 3 and there is a fixed link.

And we have this one fixed coordinate system O_L, the first joint axis and the first coordinate system is at the same place, then we have the second coordinate system on the second joint axis, the third coordinate system on the third joint axis. So, these are shown as O_1, O_2, O_3 and so on.

Only the X axis for the coordinate system is shown, the Y-axis will be normal to the X -axis with the Z-axis pointing out of the paper. So, we have it will be very by now, we should be able to assign the coordinate system and the origins at each link ok.

So, this is the reason why everybody works on this because this is the simplest possible closed loop mechanism and has been studied extensively, there are ways to ensure that the results that you are developing or getting using some formal techniques like Sylvester's method and so on match with whatever is known in literature.

It is a good example to illustrate all steps in kinematics of parallel robots as we will see later. The loop closure equations are very simple, and all steps can be done by hand ok. You can do it on paper by paper and pencil.

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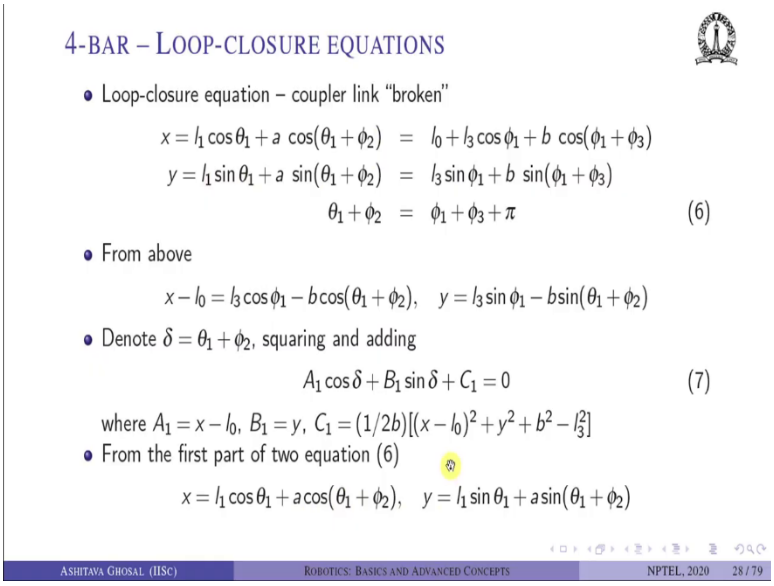

So, as I had shown earlier that the loop closure equation with the coupler link broken can be written in the following form. So, if the point where the coupler link is broken is at a distance a from one end and b from the other end so, the x and y components of that point can be written as $(l_1 \cos\theta_1 + a\cos(\theta_1 + \phi_2))$.

And from the other direction, we can go l_0 along the fixed link and $(l_3 \cos\phi_1 + b\cos(\phi_1 + \phi_3))$ ok. So, we can write the x component likewise we can write the y components which will have l_1, l_2, l_3, a, b and now $\sin\theta$ and sin of the angles and finally, we have the matching of the orientation of the coordinate system where we have broken the link as $(\theta_1 + \phi_2 = \phi_1 + \phi_3 + \pi)$ ok.

So, from the above, we can easily see that this $(x - l_0 = l_3\cos\phi_1 - b\cos(\theta_1 + \phi_2))$. Where did I get this $-b\cos(\theta_1 + \phi_2)$? Because $(\phi_1 + \phi_3 + \pi = \theta_1 + \phi_2)$ so, we can substitute $(\phi_1 + \phi_3)$ we will get θ_1, ϕ_2, and π and then, that will give you minus cos.

Likewise for y, we can write $y = l_3\sin\phi_1 + b\sin(\phi_1 + \phi_3)$ can be again represented as $(\theta_1 + \phi_2 - \pi)$ which will give you $-b\sin(\theta_1 + \phi_2)$ ok. So, we can derive these two equations from this loop closure constraint equations.

We denote $\delta = \theta_1 + \phi_2$. So, we see that there is a term always occurring which is $(\theta_1 + \phi_2)$. So, let us call it δ and we can square and add these two equations and we will get $(A_1\cos\delta + B_1\sin\delta + C_1 = 0)$ ok. So, basically, what we have is we have one δ and if you see if you square and add, you will see that this $\cos\phi_1$ angle will vanish ok. So, we have $(l_3\cos\phi_1 - b\cos\delta)$ and likewise and when you square and add, one of these angles will vanish.

And we will be left with one single equation in cos and sin δ where the A, B, C related to x, l_0, l_1 and so on and A and B, there is no angle in A, B and C. From the first part of again the loop closure equations which is this that $(x = l_1\cos\theta_1 + a\cos(\theta_1 + \phi_2))$ and $(y = l_1\sin\theta_1 + a\sin(\theta_1 + \phi_2))$ ok, we can take a look at these two equations.

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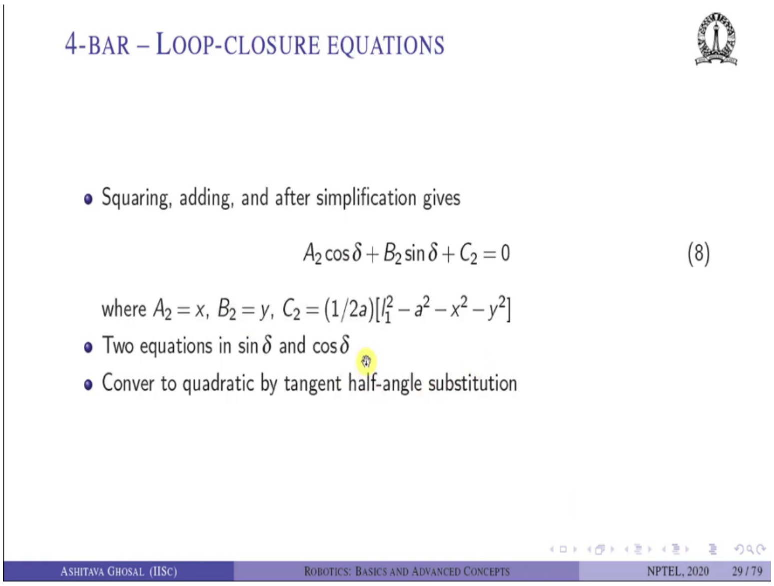

And again, square and add these two equations, we will get another equation in cos and sin δ ok. So, it will be $(A_2 \cos\delta + B_2 \sin\delta + C_2 = 0)$ and again, A_2, B_2, C_2 are quantities where x, y, l_1, a and so on other quantities appear. There are no angles again in A_2, B_2 and C_2. So, we have two equations in $\sin\delta$ and $\cos\delta$.

We can convert both these two equations to quadratics by tangent half-angle substitution. Remember I had shown you this in the inverse kinematics of serial robot. So, we can say something like some $x = \tan(\delta/2)$ and hence, you will get $\sin\delta = \frac{2x}{1+x^2}$ and so on and $\cos\delta = \frac{1-x^2}{1+x^2}$. So, we can substitute all this back in this two equations and we will get two quadratic equations in $\tan(\delta/2)$.

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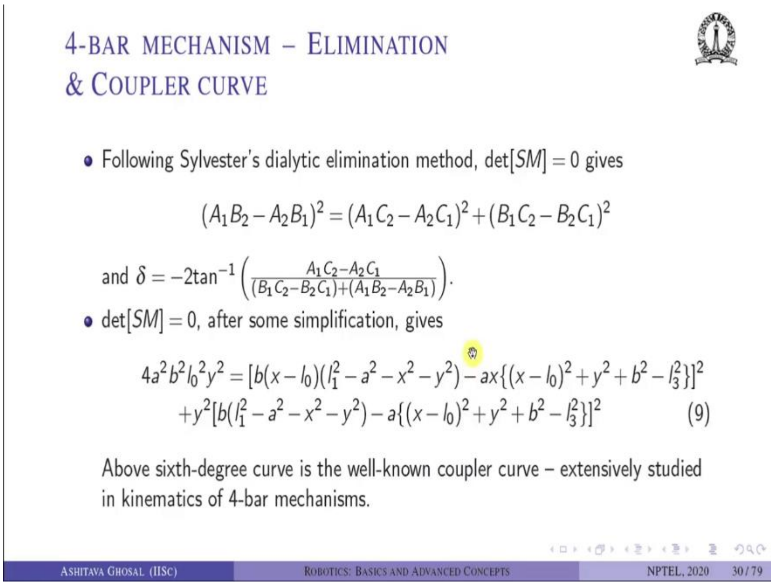

And we can use the Sylvester dialytic method, which is, obtain the Sylvester's matrix, this in this case, it will be 4 by 4 Sylvester's matrix and determinant of SM equal to 0 gives you the eliminate ok. So, we have managed to now eliminate delta ok and hence, we can and we can also solve for delta which is δ is given by

$$\delta = -2\tan^{-1}\left(\frac{A_1C_2 - A_2C_1}{(B_1C_2 - B_2C_1) + (A_1B_2 - A_2B_1)}\right)$$

So, this can be obtained. And the Sylvester's eliminate the determinant of SM equals to 0 gives you an expression in terms of A_1, B_1, C_1 and A_2, B_2, C_2 specifically of this form and what does it contain? It contains all the link lengths; it also contains A and B where the link was broken, and it also contains x and y. There are no known, there are no angles in the 4-bar which appear like θ_1, ϕ_1, ϕ_2, ϕ_3 in this elimination procedure.

So, after some simplification, you can see that this determinant of SM equals to 0 gives you an equation which contains terms like $4a^2b^2l_0^2y^2$ or even terms like $(l_1^2 - a^2 - x^2 - y^2)^2$. So, if you look at it a little carefully, you will see that this is a sixth-degree curve in (x, y) ok. So, there are powers which contains sixth-degree terms in this equation ok.

So, can we see one sixth-degree term? Yes. So, for example, if you look at this term so, this has y^2, then the whole thing is squared here so that is to the power 4 and then, it is multiplied by again by y^2 outside. So, there will be a term which is y^6 ok. So, it is a sixth-degree equation, and this is the very well-known sixth-degree curve or equation or a 'coupler curve' ok. This has been extensively studied in the kinematics of 4-bar mechanism ok.

So, what have we done, what we have achieved is the following. We have taken the loop closure equation by breaking the coupler link at one point ok, then we have derived an expression of that point as the 4-bar mechanism moves and what is the relationship between x and y of that point ok? So, it is a one-degree of freedom mechanism 4-bar. So, there must be a relationship between x and y because there can be only one independent variable and that equation is this coupler equation.

So, given any x ok, you choose an x, and we can find out the y from this equation and we can plot that point as the 4-bar mechanism moves ok. So, what so, we have not used the traditional geometric or algebraic approach, which is used by kinematicians, we have used this Sylvester's dialytic method before that loop closure equations and we have obtained the coupler curve ok.

So, this is the key step in kinematics of parallel robots. We can find the passive joints and completely describe the mechanism called the parallel robot.

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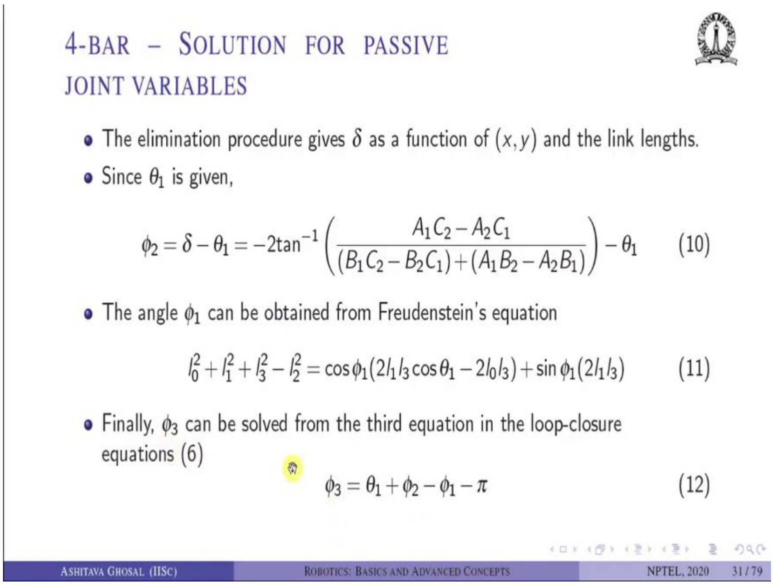

We can also continue, and we showed you that the elimination procedure gives δ as a function of x, y and the link lengths. We can also find out the joint angles. So, given since θ_1 is given, we can find out ϕ_2 which is $(\delta - \theta_1)$ and

$$\phi_2 = -2\tan^{-1}\left(\frac{A_1C_2 - A_2C_1}{(B_1C_2 - B_2C_1) + (A_1B_2 - A_2B_1)}\right) - \theta_1$$

So, choose x, find y, then from that x and y, find δ and then from the δ, find θ_1.

The angle ϕ_1 can be obtained from the Freudenstein equation ok. This is a very nice equation because it directly relates θ and ϕ ok. Remember we had broken it up at the two ends of the coupler and insisted that the position vector of the one end and the position vector of the other, the distance is l_2^2 ok. So, we can find from the Freudenstein equation directly ϕ_1.

Finally, we need to solve for ϕ_3 which is from the loop closure equation ok, we can solve for $\phi_3 = \theta_1 + \phi_2 - \phi_1 - \pi$ ok. So, this is one way to solve the direct kinematics of a 4-bar mechanism.

So, we have started with the choice of one independent coordinates in this case x, solve for y, then from that we solve for ϕ_2, θ_1 is given and so on. We can also start from the Freudenstein equation. We could have solved for ϕ_1 given θ_1 and then after some algebra, you can show that you can solve for the other angles also.

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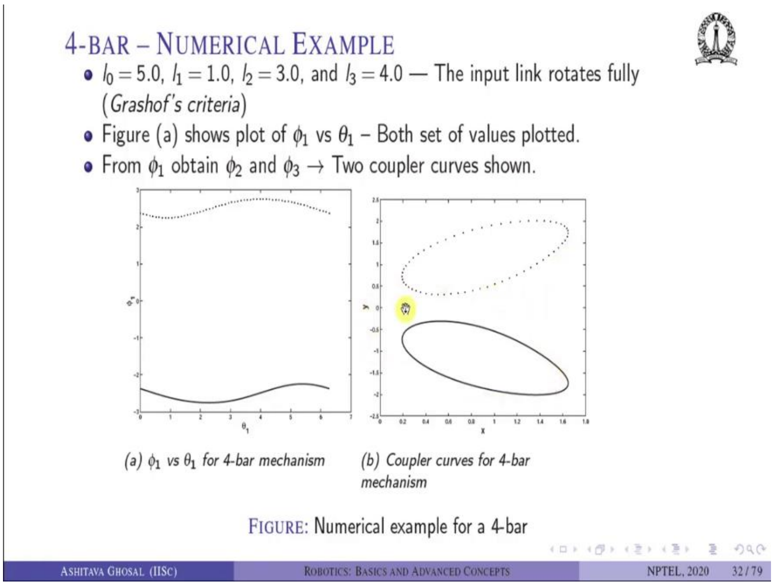

Let us look at an example. So, we have $l_0 = 5.0$ so, the base link is chosen as 5, $l_1 = 1.0$, $l_2 = 3.0$ and $l_3 = 4.0$ arbitrarily chosen but chosen with the goal that the input link will rotate fully. So, this is something called as a Grashof's criteria, it satisfies the Grashof's criteria ok. We will derive this Grashof's criteria next class, next lecture.

But in kinematics of 4-bar mechanism, there is this Grashof's criteria which tells you that for certain sums of certain lengths if it is less than or equal to sum of some other two lengths, then the crank or the input link will rotate fully ok. So, this Figure (a) shows a plot of ϕ_1 versus θ_1 ok, this is from the Freudenstein equation and there are two possible solutions, and both of these solutions are plotted. So, one is this dotted and one is this ok.

And then, from ϕ_1, we obtain ϕ_2 and ϕ_3 by looking at those kinematic equations and we can plot x and y so, this is the other way of doing it. We have chosen started with θ_1, solved ϕ_1 and then this.

In the analysis which I showed you, we first obtained the coupler curve and then went to the joint angles, but we could have done it the other way around also, we could have started with the actuated joint variables, solved for the passive joint variables and then, obtain the x and y coupler curves and in this case, the coupler curve looks like this ok. So, there are two of these, one solid line which corresponds to this solution and one dotted line which corresponds to this solution.

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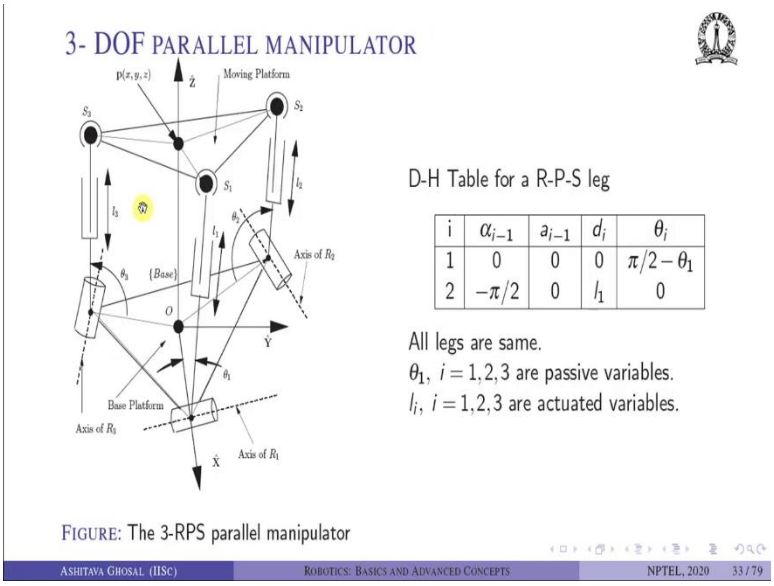

Let us take another example. So, we looked at of planar case, now we look at a spatial case. So, the first spatial case we will consider is a 3-degree of freedom parallel robot. So, this is a 3-RPS parallel robot. So, again the 3-RPS means that there is a fixed base, there is a moving platform and the corners of the base lie on a triangle and then, there is an R, P and an S chain ok. So, this is the third chain which is R, P and S. So, the actuated joint variable in this case is l_3 in this chain.

Similarly, we can have an R, P, S. So, in this actuated joint variable is l_1 ok. So, there are three of them. So, basically that is why there is a 3-RPS so, this like some sort of a convention to denote that there are three serial chains connecting a base and a moving platform. So, we can find the D-H parameter for one of these RPS legs ok.

So, first joint, will be 0, 0, 0 and this angle θ_1, but actually the D-H parameter angle is ϕ_1 which is in this case not shown here, but that is same as $(\pi/2 - \theta_1)$. So, you can go back and see the D-H parameters we had derived in week 2 for some parallel robots and this is one of them.

The second link which is second row of this D-H table, $\alpha = -\pi/2$, $a = 0$ and this is l_1, l_1 is the translation along this prismatic joint and we stop at the origin of the spherical joint. So, the spherical joint angles do not appear in this D-H table. All these three legs are same in all of them θ's are the passive joint variables and l's are the actuated joint variables ok.

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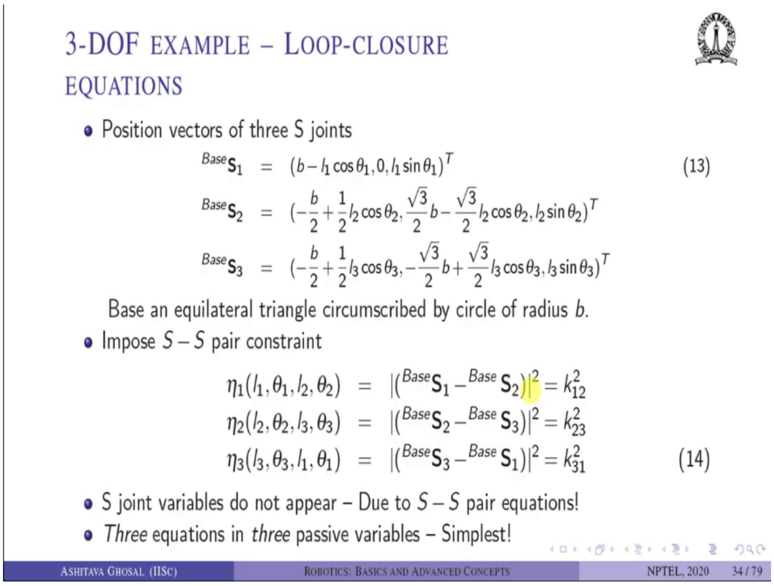

So, we want to analyze the kinematics of this 3-RPS robot. So, first we obtain the loop closure equations ok. How do we obtain the loop closure equations? We find the position vector of three S joints from a fixed base, from this origin. So, what is the position vector of this point?

We go by some distance along X and then, we go along this link to this point ok. How do I find from here to here? We use the D-H table ok. So, how about second link? We have to go in this direction by some distance along the base, along the fixed base and then, along the leg ok.

So, we have chosen an X, Y and Z coordinate system in the fixed base with the origin O. So, the first one is along this X direction itself. So, I have taken this distance as b ok. So, then this will be some b along the X component and then, the Y and Z likewise for the other one.

The other one b is not along X and Y, but at some angle. So, I have chosen these three points to lie on an equilateral triangle of this distance from the centroid as b and the angles are 120 degrees ok. But it need not be. So, we could have chosen some other way of choosing the fixed base.

We can also choose the top moving platform also in this case as an equilateral triangle of some sides ok, equal sides and the chosen point of interest is the centroid of this equilateral triangle. So, let us get back. So, the position vector of the three S joints can be obtained as some $(b - l_1 \cos\theta_1 , 0, l_1 \sin\theta_1)^T$.

The 2nd spherical joint is $-(b/2)$ because I said they are equilateral triangle so, the angles between the two directions is 120 degrees. So, $\left(-\frac{b}{2} + \frac{1}{2} l_2 \cos\theta_2 , \frac{\sqrt{3}}{2} b - \frac{\sqrt{3}}{2} l_2 \cos\theta_2 , l_2 \sin\theta_2\right)^T$ and likewise, the third spherical joint from the origin of the fixed coordinates is given by $\left(-\frac{b}{2} + \frac{1}{2} l_3 \cos\theta_3 , -\frac{\sqrt{3}}{2} b + \frac{\sqrt{3}}{2} l_3 \cos\theta_3 , l_3 \sin\theta_3\right)^T$.

So, I can find the position vector in terms of the actuated variables which is l_1, l_2, l_3 and the passive variables which is θ_1, θ_2, θ_3 which are the rotations of the rotary joints at the base.

So, now, we impose this S-S pair constraint. Remember in one of the previous lectures, we had said that if there are two S joints in a chain . So, the constraint with these two S-S joints or S-S pair impose is the distance between the centers of the two S joints are constant. So, exactly with the same thing. So, the distance between S_1 and S_2 is given by this vectors difference ($^{Base}S_1 - {}^{Base}S_2$) and the square of this distance is assumed to be a constant k_{12}^2 ok.

So, now, let us carefully look at this constraint equation. So, what does this constraint equation contain? So, we have to look at this vector and this vector. So, it must contain θ_1, it must contain l_1 ok, it must contain θ_2 and it must contain l_2 and that is all ok, the rest are all constant b and $\sqrt{3}$ and all these things are constant.

So, the first constraint equation contains l_1, θ_1, l_2 and θ_2. Likewise, the distance between spherical joint 2 and spherical joint 3 is again a constant and this will contain l_2, θ_2, l_3 and θ_3 and the third constraint equation which is the distance between spherical joint 3 and spherical joint 1 is again constant. So, we are assuming the constants are k_{12}^2, k_{23}^2 and k_{31}^2 and the third constraint equation should contain l_3, θ_3, l_1 and θ_1.

So, notice the first equation does not contain l_3 and θ_3, the second constraint equation does not contain θ_1 and l_1 and the third constraint equation does not contain l_2 and θ_2 ok, this is the important piece of observation ok. S joint variables also do not appear because of the S-S pair equation, this we have seen earlier. So, these three equations are in three passive joint variables ok.

What are the passive joints variables? θ_1, θ_2 and θ_3. So, l_1, l_2 and l_3 are given to you, they are the actuated joint variables. So, in the direct kinematics problem, the actuated joint variables and the geometry is given. So, this is the simplest possible loop closure constraint equations for this mechanism. So, we have the minimal set of equations which is three and there are three passive joint variables.

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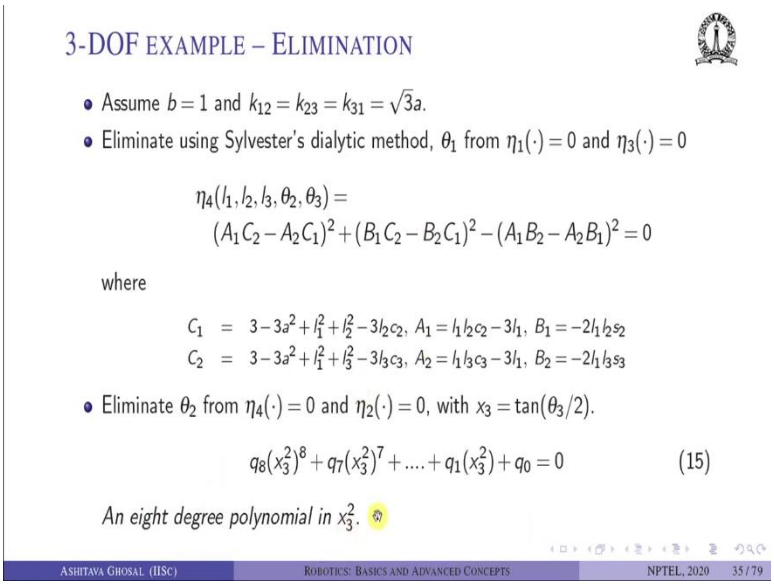

So, this is sort of hard to do an analytical elimination, we have to assume some numbers. So, we will assume that $b = 1$, it is not a very serious assumption. So, everything is scaled by the size of the base platform and $k_{12} = k_{23} = k_{31} = \sqrt{3}a$. So, basically, we are assuming that the top platform is also equilateral triangle of side a.

So, we eliminate using Sylvester's dialytic method, θ_1 from first and the third constraint equation ok. So, you can see. So, the first constraint equation contains l_1, θ_1, l_2, θ_2 and the third constraint equation contains l_3, θ_3, l_1 and θ_1. So, we can eliminate θ_1 which is the passive joint variables from the first and third constraint equation. What will be left with? We will be left with l_1, l_2, θ_2, l_3 and θ_3 ok. So, that is what is mentioned here.

So, we use Sylvester's dialytic method to eliminate θ_1 from the first and the third constraint equation and we will be generating a constraint equation or an equation which contains l_1, l_2, l_3, θ_2 and θ_3, this will be of this form ok, I am not going to go into the details of how each of them are obtained, they are obtained using a symbolic manipulation software called MAPLE and we will have $(A_1C_2 - A_2C_1)^2$ and so on this is equal to 0 where C_1, C_2, A_1, A_2, B_1, B_2 are now functions of only l_1, l_2, l_3 and θ_2 and θ_3 ok, θ_1 is no longer there.

Next, we eliminate θ_2 from this 4th equation and the second equation. We call the second equation contained θ_2, did not contain θ_1, second equation contain θ_2 and θ_3 ok.

So, we can eliminate θ_2 from this 4th equation which we have derived and the second constraint equation and then, substitute $x_3 = \tan(\theta_3/2)$ and then, do a lot of simplification and you will be ending up with a sixteenth-degree polynomial, but it is an eighth-degree polynomial in $(x_3)^2$ ok. So, recall $x_3 = \tan(\theta_3/2)$. So, we will have $q_8(x_3^2)^8 + q_7(x_3^2)^7 + \cdots$ and so on ok.

So, what is this equation? This equation contains only l_1, l_2, l_3 and the geometry a and b and so on ok, it does not contain any other passive joint variables. So, this is the equation which we are looking for and we have obtained this using this Sylvester's dialytic method in two steps. So, first eliminated θ_1, then we eliminated θ_2 and we obtain a single equation in θ_3 or $\tan(\theta_3/2)$ and this is an eighth-degree polynomial in $(x_3)^2$.

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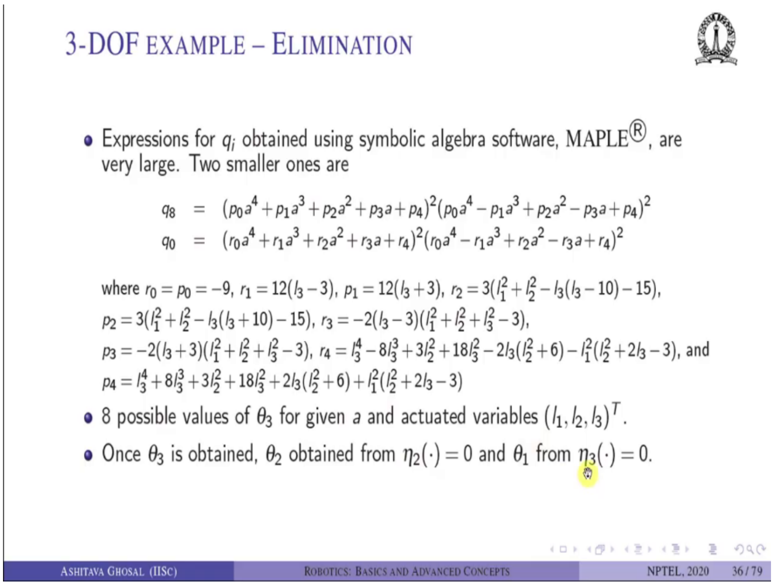

So, this expressions for this coefficients are very very big ok. If this was obtained using this symbolic algebra software called MAPLE as I told you. So, two small once so, which is this q_8 which is the leading term which is $q_8(x_3^2)^8$ is given by this long expression, q_0 is also given by this horribly long expressions where r_0, p_0 and etcetera are again given by complicated expressions.

But the only important thing to realize is we can find all the coefficients q_0, q_1 all the way to q_8 and all these coefficients are only functions of constants or l_1, l_2, l_3 ok. So, for example, you can see that this r_0 here is constant whereas, r_1 here which is r_1a^3 is $12(l_3 - 3)$ ok. So, we can find these expressions, and this is done mechanically using this symbolic algebra software called MAPLE.

So, what is the end result? So, given values of l_1, l_2, l_3, we can find 8 possible values of θ_3 and once θ_3 is obtained, θ_2 can be obtained from the second constraint equation and θ_1 can be obtained from the third constraint equation ok. So, this is the direct kinematics of the 3-RPS. So, given the actuator joint variables l_1, l_2, l_3, we first obtain θ_3, then we obtain θ_1 and then we obtain θ_2 and then we obtain θ_1 ok, the three passive joint variables.

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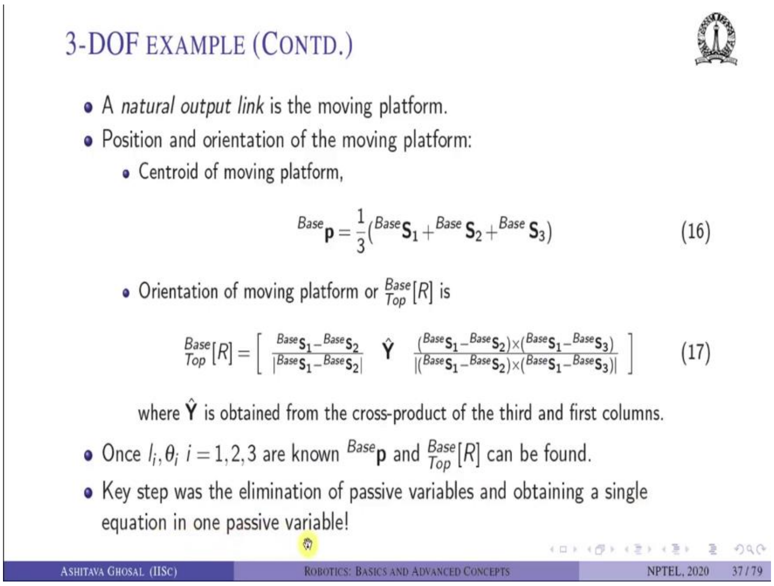

The problem is not yet finished. We need to find the position and orientation of the output link or the end effector. So, in this case, the natural output link is the moving platform, the top moving platform. The position and orientation of the top moving platform, we can assume that the position is the centroid of the top moving platform, which is nothing, but the sum of the vectors connecting the three points on the top platform.

So, basically, if I know the location of the three spherical joints, this centroid of the top platform is given by $(1/3)(^{Base}S_1 + ^{Base}S_2 + ^{Base}S_3)$ ok. If you do not choose the centroid, then we do not have one-third, we will have some ratios of the three vectors and the orientation of the top moving platform can also be obtained.

So, what is the orientation? So, we say that the X vector, the X axis of the rotation matrix $\{Top\}$ to the $\{Base\}$ is along S_1 to S_2 is the vector unit vector from S_1 point to S_2 point, from the first spherical joint to the second spherical joint so, this is the unit vector.

Between S_1 and S_3 is not necessarily perpendicular to between S_1 and S_2, but we can find the cross product of this vector S_1 to S_2 and S_1 to S_3 and find the unit vector along this and the cross product will be normal to that moving platform so, this is the Z axis, and the Y-axis = Z × X again right-handed coordinate system.

So, we can define the orientation of the top platform by X vector between first and second spherical joint, the Z axis is normal to that plane and the Y axis which is normal to both Z and X ok. Think about it, it is quite natural.

So, we have chosen the output link, which is the top platform and in particular, we have said that I am interested in the motion of the centroid and the orientation of the top platform ok. So, once l's and θ's the actuated and the passive joint variables are known, we can substitute all those things in these expressions here and we can find this vector $\boldsymbol{p}$ which is the location of the centroid of the top platform and the rotation matrix of the top platform with respect to the fixed base.

So, the key step was the elimination of the passive variables and obtaining a single equation in one passive variable in this case θ_3 and this is where the Sylvester's method or the general theory of elimination has been used.

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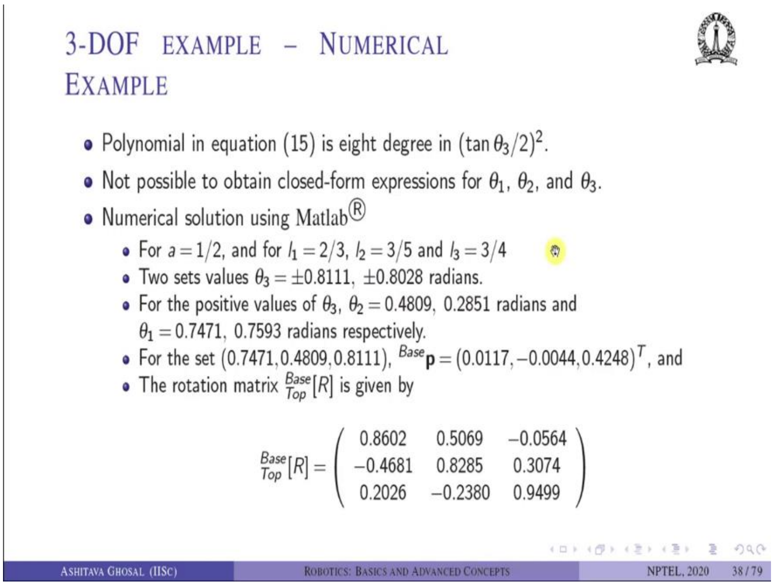

So, let us look at a numerical example. The polynomial in equation (15) is eighth degree in $(\tan(\theta_3/2))^2$ ok. So, we cannot solve eights degree polynomials in closed form. So, we go to MATLAB and we use some program ok, standard MATLAB tools like `fsolve` to solve for that numerically and to solve for that numerically, we have to assume some values of a and l_1, l_2, l_3.

So, we have chosen $a = 1/2$, $l_1 = 2/3$, $l_2 = 3/5$, $l_3 = 3/4$, we could choose any other values of that a and that l_1, l_2, l_3 ok. Remember $b = 1$, the fixed base is scaled. So, all the dimensions are in the b is related to 1, fixed base. So, we get two sets of values of θ_3. So, we will get $\pm$0.8111 radians, $\pm$0.8028 radians and for the positive values of θ_3, θ_2 is this and θ_1 is this ok.

So, for the set $\theta_1 = 0.7471$, $\theta_2 = 0.4809$ and $\theta_3 = 0.811$, the origin of the centroid of the top platform is located X coordinate is 0.0117, Y coordinate is -0.0044 and the Z coordinate is 0.4248 and the rotation matrix can be also obtained by solving for the X axis, then the normal to the plane and the Y axis ok.

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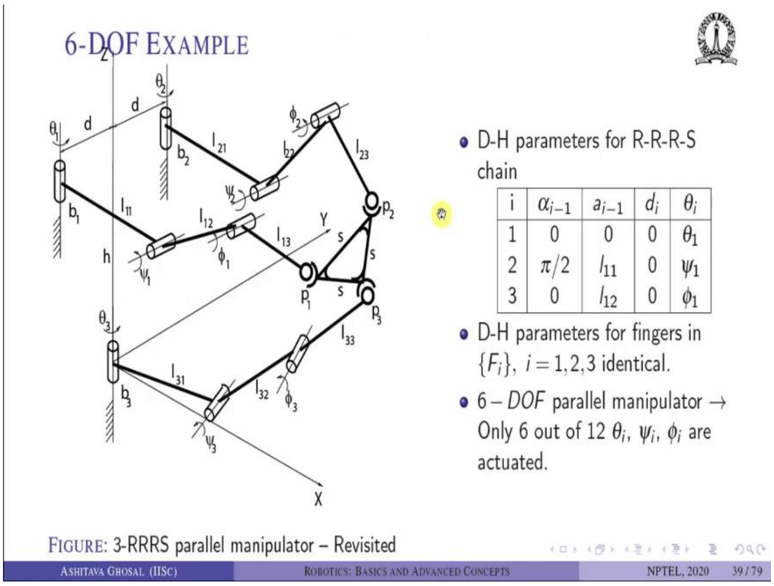

So, let us continue. Hence, look at another example which is the 6-degree of freedom example. So, this as I have mentioned earlier, it models a three fingered hand gripping an object. So, these are the three fingers b_1 which is fixed base b_1, then l_{11}, l_{12}, l_{13} these are the finger elements or the link lengths in the finger and likewise, we have another finger, and we have another finger third finger ok.

The distance between these two fingers is $2d$ and this distance is h so, these are constant and it is gripping an object with point contact with friction. So, that can be modeled as a spherical joint ok. So, how do we start? We first obtain the D-H parameters for each one of these fingers ok. So, finger is R, R, R and S chain. So, previously, we had R, P, S in the 3 RPS example here each chain is R, R, R and S ok.

So, we can find the D-H table which is first link 0, 0, 0, θ_1. second link is $\pi/2$ because the second rotation axis is perpendicular to the first rotation axis so, $\pi/2$, this is l_{11}, the distance is $d_i = 0$ and this angle is ψ_1 and the third link is parallel to the second so, third joint is parallel to the second joint.

So, we have $\alpha_{i-1} = 0$, $a_{i-1} = l_{12}$, $d_i = 0$ and $\theta_i = \phi_1$. So, as usual with all D-H convention, l_{13} does not appear, but when we find this position vector, when we assign a tool coordinate system, l_{13} will appear.

So, what is the basic idea? We will fix one coordinate system F_i at the base of each of these fingers. So, this is F_1, this is F_2, and this is F_3 ok. We know this is a 6 degree of freedom robot ok. So, when you grip the object, you can manipulate the gripped object, you can change the X, Y, Z and you can also orient the object.

So, you can think of you are holding a small ball and we will look at these things later in more detail and you can see that you can change the orientation of the ball and you can also move the center of the ball in some sense.

So, out of these three, θ_1, ψ_1, ϕ_1 in this first finger, then θ_2, ψ_2, ϕ_2 in the second finger and likewise, in the third finger. So, if it is 6 degree of freedom, 6 out of these 9 joint angles are actuated ok. It is a 6 degree of freedom system. So, there can be only 6 actuated joints ok.

So, in this example we will assume that the first two joints are actuated. So, θ_1 and ψ_1 is actuated and ϕ_1 is passive likewise, θ_2 and ψ_2 are actuated and ϕ_2 is passive and so on ok. So, what is the task? We need to find loop closure constraint equations to solve for the three passive joint variables ok.

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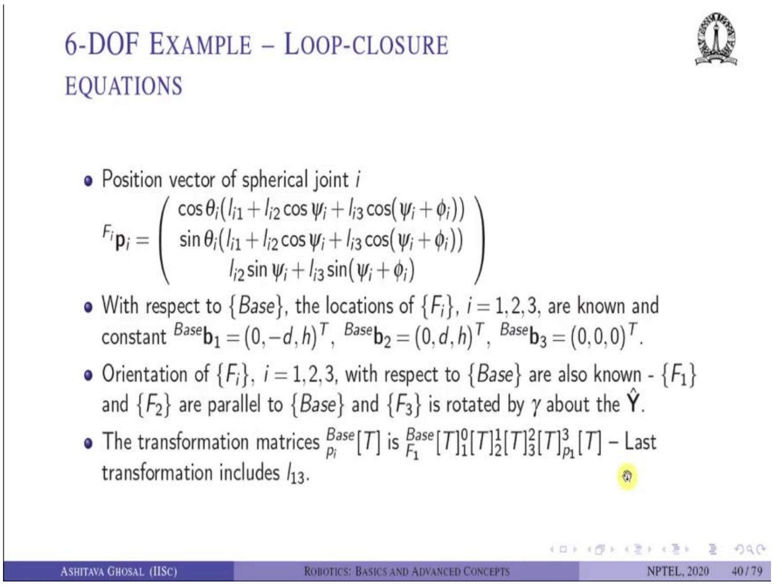

So, how do we go about? We can find that the position vector of the spherical joint p_1 with respect to the fixed coordinate system at the base of the finger can be written in terms of this l_{i1} - l_{11}, l_{12} and θ_1, ψ_1, ϕ_1 ok. So, this is obtained by finding the D-H table, finding the transformation matrices, you multiply three transformation matrices and then pick the last column which is the position vector with respect to a coordinate system which is fixed at the base of the finger.

So, with respect to another $\{Base\}$ which is somewhere in between you know like the hand or the palm which is fixed to the palm, let us denote that coordinate system by $\{Base\}$. We can show that this first joint is located at $(0, -d, h)^T$, second joint is $(0, d, h)^T$ and the third joint is $(0, 0, 0)^T$ ok.

Orientation of F_1 with respect to this $\{Base\}$ coordinate system is also known ok. So, we have chosen the $\{Base\}$ coordinate system which is sort of the midpoint or some point connecting the three finger base points. And it turns out to actually model a three fingered hand the F_3 coordinate system must be rotated by an angle γ about the Y axis ok, you can look at your three-finger ok, the thumb, the index and the middle finger. The thumb initially starts off at a different angle.

So, we can find the transformation matrices $\{Base\}$ to the spherical joints 4×4 homogeneous transformation matrix by going from $\{Base\}$ to $\{F_1\}$, then $\{0\}$ to $\{1\}$, $\{1\}$

to $\{2\}$, $\{2\}$ to $\{3\}$ and then, $\{3\}$ to $\{p_1\}$. So, $\{3\}$ to $\{p_1\}$ will contain the last l_{13} or l_{23} and l_{33} ok. The last transformation includes the last link length l_{13}.

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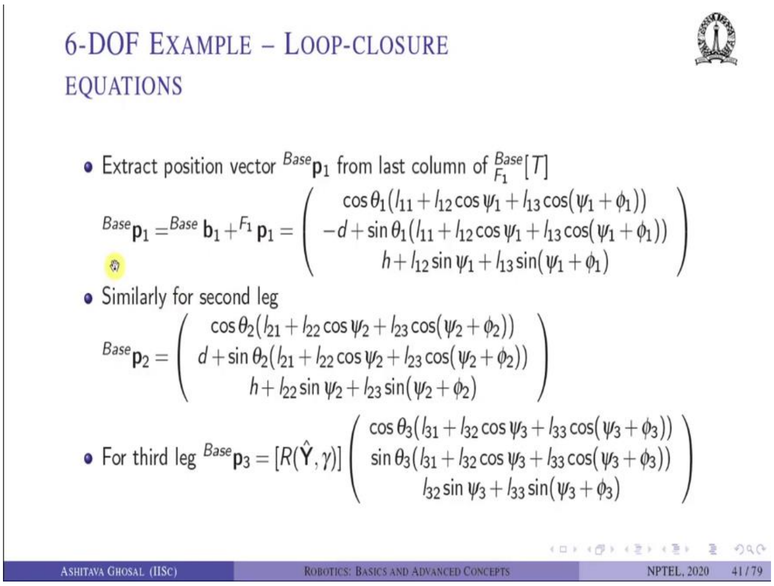

So, we multiply all this four this how many are there? So one, two, three, four and five transformation matrices and then, we extract the position vector of the last link last transformation matrix, the resultant transformation matrix. So, this will give the position vector of the spherical joint 1 with respect to the $\{Base\}$.

So, this is given by ${}^{Base}\boldsymbol{p}_1$ can be written in more detail ${}^{Base}\boldsymbol{b}_1$ which is the position vector of the base of the finger and then ${}^{F_1}\boldsymbol{p}_1$ and it turns out that these are not very complicated, but reasonably complicated expressions containing θ_1, ψ_1 and ϕ_1 and the link lengths l_{11}, l_{12}, l_{13} and so on and this distance d and h.

Similarly, for the second leg, we can find ${}^{Base}\boldsymbol{p}_2$. This will contain θ_2, ψ_2, ϕ_2 and this again this d and h and the link lengths in the second finger and, but third one as I said, we need to pre multiply by a rotation matrix gamma because the base thumb finger starts at a different angle ok. So, the basic idea is we have now the position vectors of the three contact points modeled as spherical joints.

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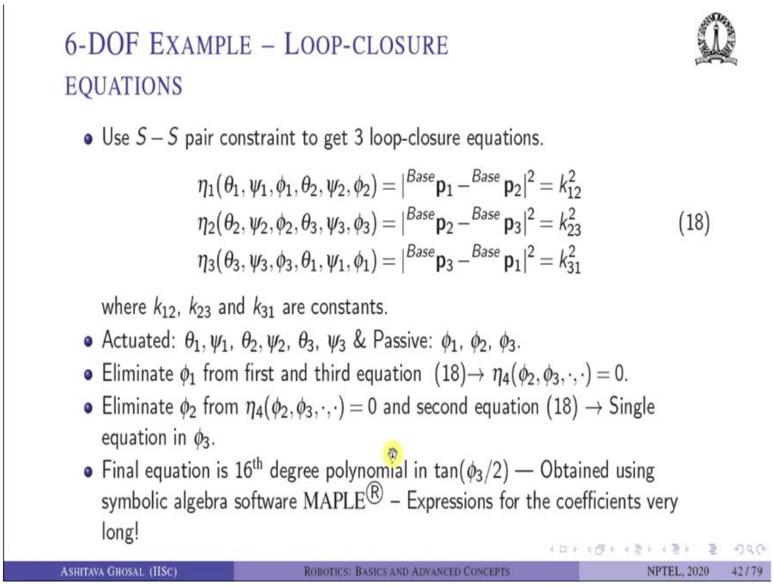

Again we use the S-S pair constraint. So, between the first contact point and the second contact point, these are modeled as an S-S pair. So, the distance between that must be constant. So, between $\boldsymbol{p}_1$ and $\boldsymbol{p}_2$ ok, the distance is k_1^2 very similar to what we did for the 3 RPS case. So, this equation will contain θ_1, ψ_1, ϕ_1, θ_2, ψ_2 and ϕ_2.

The second constraint equation is between point 2, spherical joint 2 and the spherical joint 3 and that is given by k_{23}^2 and the third one is between 3 and 1 and this is k_{31}^2 so, where these distances k_{12}, k_{23}, k_{31} are constants. So, we have three equations in three passive joint variables ok.

So, as I have said, we will assume that the first two joints are actuated and the third one is passive ok. So, the passive joints are ϕ_1, ϕ_2, ϕ_3 and the actuated joints are this θ_1, ψ_1, θ_2, ψ_2, θ_3, ψ_3 ok. The first two joints in each finger.

So, again we can use this Sylvester's dialytic method to eliminate ϕ_1 from the first and third equation and we get an equation which contains ϕ_2 and ϕ_3 and all the actuated joint variables. Then, we can eliminate ϕ_2 from this equation and the second equation and we get a single equation in ϕ_3 ok. So, this is again a two-step process. We eliminate one from two equations and then, we eliminate the other one from the resultant equation and one of the original equation ok.

So, this turns out to be a 16th degree polynomial in $\tan(\phi_3/2)$ ok. Again, we have obtained this equation using symbolic algebra software MAPLE, coefficients are very very long as you can see.

So, the main difference between this example and the previous 3-RPS example was there were three actuated joints, and three passive joints which we needed to find out. In this case, there are six actuated joints and three passive joints, and we obtained a sixteenth-degree polynomial. In the previous case, it was actually eighth-degree polynomial in x_3^2.

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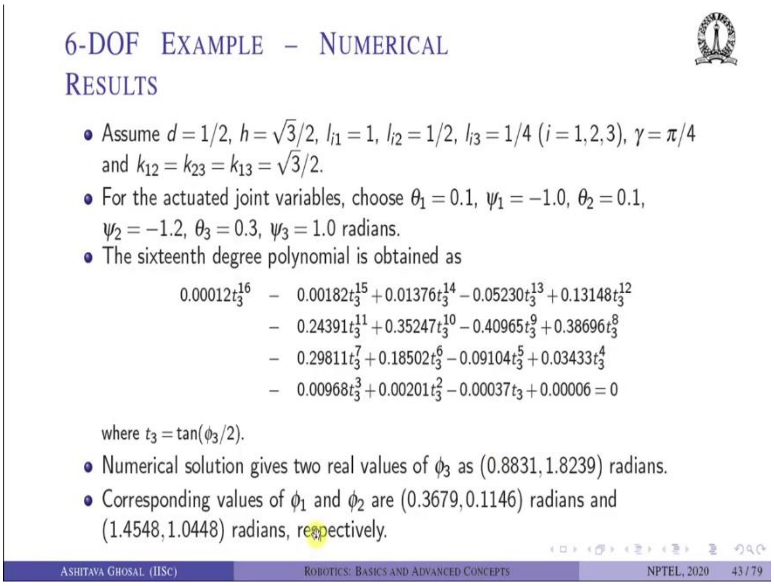

So, let us do a numerical example. We assume ;$d = 1/2$, $h = \sqrt{3}/2$ and the actuated joint variables are arbitrarily chosen as 1, 1/2, 1/4 and so on and then, this distance between spherical joints are assumed to be equal to $\sqrt{3}/2$. So, for the chosen actuated joint variables θ_1, ψ_1, θ_2 and ψ_2 all in radians ok.

We can find that the sixteenth-degree polynomial is obtained in this horribly complicated form ok. Nevertheless, we can find it ok, it takes a while, it takes a lot of simplification and computation, but we can show that there is a t_3^{16}, $t_3 = \tan(\phi_3/2)$, the coefficient is 0.000012. So, for example, t_3^6 is 0.18502 only the first five places of decimal are shown here.

We can solve numerically this equation in MATLAB and we will get two real values of ϕ_3 so, 0.8831 radians and 1.8239 radians ok. So, there is a root solver in MATLAB which given a polynomial, it will give you all the roots and it turns out in this case, there are two real roots only.

And once you have found out ϕ_3, we can find out ϕ_1 and ϕ_2 from the previous two equations from the generated equation and one of the original equation and ϕ_1 and ϕ_2 are given by 0.3679, 0.1146 and likewise, ϕ_2 is 1.45 and so on ok. So, this is a numerical basically you just give the equations to MATLAB, give the known values and it will tell you all the unknown values, it will solve those equations.

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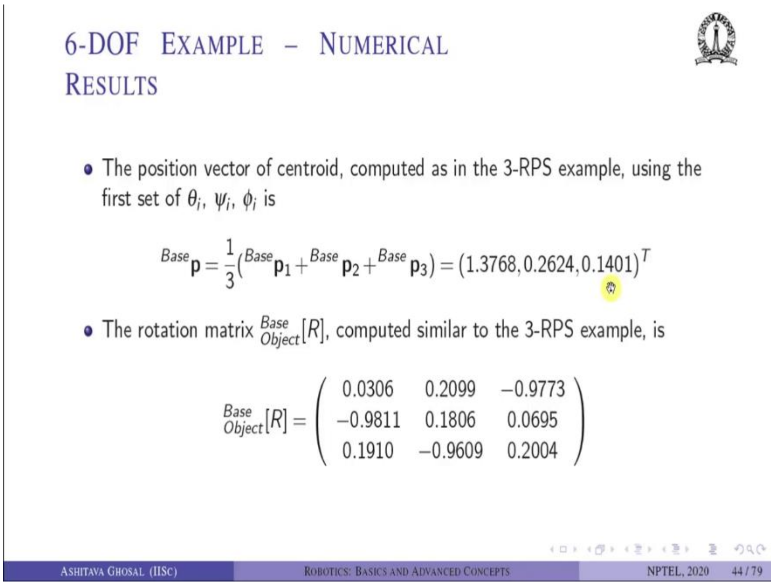

So, as we see in the direct kinematics, we now need to find the position and orientation of the moving platform which is the chosen output link. So, again we will assume that the $^{\text{Base}}\boldsymbol{p}$ which is the centroid of the top platform is given $(1/3)(^{Base}\boldsymbol{p}_1 + {}^{Base}\boldsymbol{p}_2 + {}^{Base}\boldsymbol{p}_3)$ and we will get an X, Y, Z coordinate of this form. So, X = 1.37 so on, Y = 0.261 and Z = 0.1401.

We can also find the rotation matrix of the gripped object and this is $^{\text{Base}}_{\text{Object}}[\text{R}]$ and again, we choose the X axis between first and second joint, Z axis is normal to that plane, formed by those three points and Y axis is perpendicular to Z and X and we can find this rotation matrix.

So, in this lecture, we have discussed how to obtain the direct kinematics of parallel robots ok. So, the key ideas where that we need to derive the loop closure constraint equations from the loop closure constraint equation, we find the passive joint variables and then, knowing the active and passive joint variables, we obtain the position and orientation of the chosen output link. In the next lecture, we will look at mobility of parallel manipulators.